where \( N(t) \) denotes the number of crimes committed in year \( t \), with \( t = 0 \) corresponding to the beginning of 2001. Find where the function \( N \) is increasing and where it is decreasing.

**Solution** The derivative \( N' \) of the function \( N \) is

\[
N'(t) = -0.3t^2 + 3t = -0.3(t - 10)
\]

Since \( N'(t) > 0 \) for \( t \) in the interval \((0, 7)\), the function \( N \) is increasing throughout that interval (Figure 25).

### 4.1 Self-Check Exercises

1. Find the intervals where the function \( f(x) = \frac{3}{4}x^3 - x^2 - 12x + 3 \) is increasing and the intervals where it is decreasing.

2. Find the relative extrema of \( f(x) = \frac{x^2}{1 - x^2} \).

*Solutions to Self-Check Exercises 4.1 can be found on page 261.*

### 4.1 Concept Questions

1. Explain each of the following:
   a. \( f \) is increasing on an interval \( I \).
   b. \( f \) is decreasing on an interval \( I \).

2. Describe a procedure for determining where a function is increasing and where it is decreasing.

3. Explain each term: (a) relative maximum and (b) relative minimum.

4. a. What is a critical number of a function \( f \)?
   b. Explain the role of a critical number in determining the relative extrema of a function.

5. Describe the first derivative test and describe a procedure for finding the relative extrema of a function.

### 4.1 Exercises

In Exercises 1–8, you are given the graph of a function \( f \). Determine the intervals where \( f \) is increasing, constant, or decreasing.

1. 

2. 

3. 

4. 

5. The graph of the function $f$ shown in the accompanying figure gives the elevation of that part of the Boston Marathon course that includes the notorious Heartbreak Hill. Determine the intervals (stretches of the course) where the function $f$ is increasing (the runner is laboring), where it is constant (the runner is taking a breather), and where it is decreasing (the runner is coasting).

10. Aircraft Structural Integrity Among the important factors in determining the structural integrity of an aircraft is its age. Advancing age makes planes more likely to crack. The graph of the function $f$, shown in the accompanying figure, is referred to as a “bathtub curve” in the airline industry. It gives the fleet damage rate (damage due to corrosion, accident, and metal fatigue) of a typical fleet of commercial aircraft as a function of the number of years of service.

9. The Boston Marathon The graph of the function $f$ shown in the accompanying figure gives the elevation of that part of the Boston Marathon course that includes the notorious Heartbreak Hill. Determine the intervals (stretches of the course) where the function $f$ is increasing (the runner is laboring), where it is constant (the runner is taking a breather), and where it is decreasing (the runner is coasting).

11. Refer to the following figure:

What is the sign of the following?

a. $f''(2)$

b. $f'(x)$ in the interval (1, 3)

c. $f''(4)$

d. $f'(x)$ in the interval (3, 6)

e. $f''(7)$

f. $f'(x)$ in the interval (6, 9)

g. $f''(x)$ in the interval (9, 12)

12. Refer to the following figure:

a. What are the critical numbers of $f$. Give reasons for your answers.

b. Draw the sign diagram for $f''$.

c. Find the relative extrema of $f$.

In Exercises 13–36, find the interval(s) where the function is increasing and the interval(s) where it is decreasing.

13. $f(x) = 3x + 5$

14. $f(x) = 4 - 5x$

15. $f(x) = x^2 - 3x$

16. $f(x) = 2x^2 + x + 1$

17. $g(x) = x - x^3$

18. $f(x) = x^3 - 3x^2$

19. $g(x) = x^3 + 3x^2 + 1$

20. $f(x) = x^3 - 3x + 4$

21. $f(x) = \frac{1}{3}x^3 - 3x^2 + 9x + 20$
24. \( g(x) = x^4 - 2x^2 + 4 \)

26. \( h(x) = \frac{1}{2x + 3} \)

28. \( g(r) = \frac{2r}{r^2 + 1} \)

30. \( f(x) = x^{2/3} + 5 \)

32. \( f(x) = (x - 5)^{2/3} \)

34. \( g(x) = x\sqrt{x + 1} \)

36. \( h(x) = \frac{x^2}{x - 1} \)

In Exercises 45–48, match the graph of the function with the graph of its derivative in (a)–(d).

45.

46.

47.

48.

(a) \( y' \)

(b) \( y' \)

(c) \( y' \)

(d) \( y' \)
4.2 Self-Check Exercises

1. Determine where the function \( f(x) = 4x^3 - 3x^2 + 6 \) is concave upward and where it is concave downward.

2. Using the second derivative test, if applicable, find the relative extrema of the function \( f(x) = 2x^3 - 3x^2 - 12x - 10 \).

3. A certain country's gross domestic product (GDP) (in millions of dollars) in year \( t \) is described by the function

\[ G(t) = -2t^3 + 45t^2 + 20t + 6000 \quad (0 \leq t \leq 15) \]

where \( t = 0 \) corresponds to the beginning of 1995. Find the inflection point of the function \( G \) and discuss its significance.

Solutions to Self-Check Exercises 4.2 can be found on page 281.

4.2 Concept Questions

1. Explain what it means for a function \( f \) to be (a) concave upward and (b) concave downward on an open interval \( I \). Given that \( f \) has a second derivative on \( I \) (except at isolated numbers), how do you determine where the graph of \( f \) is concave upward and where it is concave downward?

2. What is an inflection point of the graph of a function \( f \)? How do you find the inflection point(s) of the graph of a function \( f \) whose rule is given?

3. State the second derivative test. What are the pros and cons of using the first derivative test and the second derivative test?

4.2 Exercises

In Exercises 1–8, you are given the graph of a function \( f \). Determine the intervals where \( f \) is concave upward and where it is concave downward. Also, find all inflection points of \( f \), if any.

1.

2.

3.

4.

5.

6.

7.

8.
Spread of a Rumor: Initially, a handful of students heard a rumor on campus. The rumor spread, and after \( t \) hr, the number had grown to \( N(t) \). The graph of the function \( N \) is shown in the following figure:

\[
y = N(t)
\]

Describe the spread of the rumor in terms of the speed it was spread. In particular, explain the significance of the inflection point \( P \) of the graph of \( N \).

Exercises 19–22, show that the function is concave upward wherever it is defined.
1. \( f(x) = 4x^2 - 12x + 7 \)
2. \( g(x) = x^4 + \frac{1}{2}x^2 + 6x + 10 \)
3. \( f(x) = \frac{1}{x^2} \)
4. \( g(x) = -\sqrt{4 - x^2} \)

Exercises 23–42, determine where the function is concave upward and where it is concave downward.
5. \( f(x) = 2x^2 - 3x + 4 \)
6. \( g(x) = -x^2 + 3x + 4 \)
7. \( f(x) = x^3 - 1 \)
8. \( g(x) = x^3 - x \)
9. \( f(x) = x^4 - 6x^2 + 2x + 8 \)
10. \( g(x) = 3x^4 - 6x^2 + x - 8 \)
11. \( f(x) = x^{17} \)
12. \( g(x) = \sqrt{x} \)
13. \( f(x) = \sqrt{4 - x} \)
14. \( g(x) = \sqrt{x - 2} \)
15. \( f(x) = \frac{1}{x - 2} \)
16. \( g(x) = \frac{x}{x + 1} \)
17. \( f(x) = \frac{1}{2 + x^2} \)
18. \( g(x) = \frac{1}{1 + x} \)
19. \( k(t) = \frac{t^2}{t - 1} \)
20. \( f(x) = \frac{x + 1}{x - 1} \)
21. \( g(x) = \frac{x + 1}{x^2} \)
22. \( h(r) = \frac{1}{(r - 2)^2} \)
23. \( g(t) = (2t - 4)^{1/3} \)
24. \( f(x) = (x - 2)^{2/3} \)

Exercises 43–54, find the inflection point(s), if any, of each function.
25. \( f(x) = x^3 - 2 \)
26. \( g(x) = x^2 - 6x \)
27. \( f(x) = 6x^2 - 18x^4 + 12x - 15 \)
28. \( g(x) = 2x^2 - 3x^3 + 18x - 8 \)
29. \( f(x) = 3x^4 - 4x^2 + 1 \)
30. \( f(x) = x^4 - 2x^3 + 6 \)
31. \( g(t) = \sqrt{t} \)
32. \( f(x) = \sqrt{x} \)
33. \( f(x) = (x - 1)^3 + 2 \)
34. \( f(x) = (x - 2)^{1/3} \)
35. \( f(x) = \frac{2}{1 + x^2} \)
36. \( f(x) = \frac{2}{x} \)
37. \( f(x) = \frac{2}{x^2 + 1} \)
38. \( f(x) = \frac{16}{x} \)
39. \( g(x) = x^2 + \frac{2}{x} \)
40. \( g(x) = \frac{1}{1 + x^2} \)
41. \( f(x) = \frac{x^2}{x - 1} \)
42. \( f(x) = \frac{x^2}{x^2 + 1} \)

In Exercises 71–76, sketch the graph of a function having the given properties.
71. \( f(2) = 4, f'(2) = 0, f''(x) < 0 \) on \((-\infty, \infty)\)
72. \( f(2) = 2, f'(2) = 0, f''(x) > 0 \) on \((-\infty, 2), f''(x) < 0 \) on \((2, \infty), f''(x) > 0 \) on \((-\infty, \infty)\)
73. \( f(-2) = 4, f(3) = -2, f'(-2) = 0, f'(3) = 0, f''(x) > 0 \) on \((-\infty, -2) \cup (3, \infty), f''(x) < 0 \) on \((-2, 3), \) inflection point at \((1, 1)\)
74. \( f(0) = 0, f''(0) \) does not exist, \( f''(x) < 0 \) if \( x \neq 0 \)
75. \( f(0) = 1, f'(0) = 0, f(x) > 0 \) on \((-\infty, \infty), f''(x) < 0 \) on \((-\sqrt{2}/2, \sqrt{2}/2), f''(x) > 0 \) on \((-\infty, -\sqrt{2}/2) \cup (\sqrt{2}/2, \infty)\)
76. \( f \) has domain \([-1, 1], f(-1) = -1, f(-1/2) = -2, f'(1) = 0, f'(x) > 0 \) on \((-1, 1)\)

77. Demand for RNs: The following graph gives the total number of help-wanted ads for RNs (registered nurses) in 22 cities over the past 12 mo as a function of time \( t \) (measured in months).

a. Explain why \( N''(t) \) is positive on the interval \((0, 12)\).

b. Determine the signs of \( N''(t) \) on the interval \((0, 6)\) and the interval \((6, 12)\).
4.3 Self-Check Exercises

1. Find the horizontal and vertical asymptotes of the graph of the function
   \[ f(x) = \frac{2x^2}{x^2 - 1} \]

2. Sketch the graph of the function
   \[ f(x) = \frac{2}{3}x^3 - 2x^2 - 6x + 4 \]

Solutions to Self-Check Exercises 4.3 can be found on page 294.

4.3 Concept Questions

1. Explain the following terms in your own words:
   a. Vertical asymptote  
   b. Horizontal asymptote

2. a. How many vertical asymptotes can the graph of a function \( f \) have? Explain using graphs.
   b. How many horizontal asymptotes can the graph of a function \( f \) have? Explain using graphs.

3. How do you find the vertical asymptotes of a rational function?

4. Give a procedure for sketching the graph of a function.

4.3 Exercises

In Exercises 1–10, find the horizontal and vertical asymptotes of the graph.

1. \[ y = \frac{2}{1 + 0.5(x)} \]

2. \[ y = \frac{1}{(x + 1)^2} \]

3. \[ y = \frac{1}{x^3} \]

4. \[ y = \frac{1}{x^2 + 1} \]

5. \[ y = \frac{x}{x^2 - 1} \]

6. \[ y = \frac{x}{x^2 + 1} \]
In Exercises 11–28, find the horizontal and vertical asymptotes of the graph of the function. (You need not sketch the graph.)

11. \( f(x) = \frac{1}{x} \)
12. \( f(x) = \frac{1}{x + 2} \)
13. \( f(x) = -\frac{2}{x^2} \)
14. \( g(x) = \frac{1}{1 + 2x^2} \)
15. \( f(x) = \frac{x - 1}{x + 1} \)
16. \( g(t) = \frac{t + 1}{2t - 1} \)
17. \( h(x) = x^3 - 3x^2 + x + 1 \)
18. \( g(x) = 2x^3 + x^2 + 1 \)

19. \( f(t) = \frac{t^2}{t^2 - 9} \)
20. \( g(x) = \frac{x^3}{x^2 - 4} \)
21. \( f(x) = \frac{3x}{x^2 - x - 6} \)
22. \( g(x) = \frac{2x}{x^2 + x - 2} \)
23. \( g(t) = 2 + \frac{5}{(t - 2)^2} \)
24. \( f(x) = 1 + \frac{2}{x - 3} \)
25. \( f(x) = \frac{x^2 - 2}{x^2 - 4} \)
26. \( h(x) = \frac{2 - x^2}{x^2 + 1} \)
27. \( g(x) = \frac{x^3 - x}{x(x + 1)} \)
28. \( f(x) = \frac{x^4 - x^2}{x(x - 1)(x + 2)} \)

In Exercises 29 and 30, you are given the graphs of two functions \( f \) and \( g \). One function is the derivative function of the other. Identify each of them.

29.

30.

31. **Terminal Velocity** A skydiver leaps from the gondola of a hot-air balloon. As she free-falls, air resistance, which is proportional to her velocity, builds up to a point where it balances the force due to gravity. The resulting motion may be described in terms of her velocity as follows: Starting at rest (zero velocity), her velocity increases and approaches a constant velocity, called the terminal velocity. Sketch a graph of her velocity \( v \) versus time \( t \).

32. **Spread of a Flu Epidemic** Initially, 10 students at a junior high school contracted influenza. The flu spread over time, and the total number of students who eventually contracted the flu approached but never exceeded 200. Let \( P(t) \) denote the number of students who had contracted the flu after \( t \) days, where \( P \) is an appropriate function.
   
a. Make a sketch of the graph of \( P \). (Your answer will not be unique.)
b. Where is the function increasing?
c. Does P have a horizontal asymptote? If so, what is it?
d. Discuss the concavity of P. Explain its significance.
e. Is there an inflection point on the graph of P? If so, explain its significance.

**Exercises 33–36, use the information summarized in the table to sketch the graph of f.**

### Exercise 33

$f(x) = x^3 - 3x^2 + 1$

**Domain:** $(-\infty, \infty)$

**Intercepts:** $y$-intercept: 1

**Asymptotes:** None

**Intervals where $f$ is $\uparrow$ and $y$:**

$\uparrow$ on $(-\infty, 0) \cup (2, \infty)$; $\Downarrow$ on (0, 2)

**Relative extrema:** Rel. max. at (0, 1); rel. min. at (2, -3)

**Concavity:** Downward on $(-\infty, 1)$; upward on $(1, \infty)$

**Point of inflection:** (1, -1)

### Exercise 34

$f(x) = \frac{1}{9}(x^2 - 4x^3)$

**Domain:** $(-\infty, \infty)$

**Intercepts:** $x$-intercepts: 0, 4; $y$-intercept: 0

**Asymptotes:** None

**Intervals where $f$ is $\uparrow$ and $y$:**

$\uparrow$ on $(3, \infty)$; $\Downarrow$ on $(-\infty, 0) \cup (0, 3)$

**Relative extrema:** Rel. min. at $(3, -3)$

**Concavity:** Downward on $(0, 2)$; upward on $(-\infty, 0) \cup (2, \infty)$

**Points of inflection:** $(0, 0)$ and $(2, -\frac{15}{2})$

### Exercise 35

$f(x) = -\frac{4x - 4}{x^2}$

**Domain:** $(-\infty, 0) \cup (0, \infty)$

**Intercept:** $x$-intercept: 1

**Asymptotes:** $x$-axis and $y$-axis

**Intervals where $f$ is $\uparrow$ and $y$:**

$\uparrow$ on $(0, 2)$; $\Downarrow$ on $(-\infty, 0) \cup (2, \infty)$

**Relative extrema:** Rel. max. at $(2, 1)$

**Concavity:** Downward on $(-\infty, 0) \cup (0, 3)$; upward on $(3, \infty)$

**Point of inflection:** $(3, \frac{1}{2})$

### Exercise 36

$f(x) = x - 3x^{\frac{1}{3}}$

**Domain:** $(-\infty, \infty)$

**Intercepts:** $x$-intercepts: $\pm 3\sqrt{3}, 0$

**Asymptotes:** None

**Intervals where $f$ is $\uparrow$ and $y$:**

$\uparrow$ on $(-\infty, -1) \cup (1, \infty)$; $\Downarrow$ on $(-1, 1)$

**Relative extrema:** Rel. max. at $(-1, 2)$; rel. min. at $(1, -2)$

**Concavity:** Downward on $(-\infty, 0)$; upward on $(0, \infty)$

**Point of inflection:** $(0, 0)$

In Exercises 37–60, sketch the graph of the function, using the curve-sketching guide of this section.

### Exercise 37

$g(x) = 4 - 3x - 2x^3$

### Exercise 38

$f(x) = x^2 - 2x + 3$

### Exercise 39

$h(x) = x^3 - 3x + 1$

### Exercise 40

$f(x) = 2x^3 + 1$

### Exercise 41

$f(x) = -2x^3 + 3x^2 + 12x + 2$

### Exercise 42

$f(t) = 2t^3 - 15t^2 + 36t - 20$

### Exercise 43

$h(x) = \frac{3}{2}x^4 - 2x^3 - 6x^2 + 8$

### Exercise 44

$f(t) = 3t^4 + 4t^3$

### Exercise 45

$f(x) = \sqrt{x^2 - 4}$

### Exercise 46

$f(x) = \sqrt{x^2 + 5}$

### Exercise 47

$g(x) = \frac{1}{2}x - \sqrt{x}$

### Exercise 48

$g(x) = \sqrt{x^2}$

### Exercise 49

$g(x) = \frac{2}{x - 1}$

### Exercise 50

$g(x) = \frac{1}{x + 1}$

### Exercise 51

$h(x) = \frac{x + 2}{x - 2}$

### Exercise 52

$g(x) = \frac{x}{x - 1}$

### Exercise 53

$f(t) = \frac{t^2}{1 + t^2}$

### Exercise 54

$g(x) = \frac{x}{x^2 - 4}$

### Exercise 55

$g(t) = \frac{t^2 - 2}{t - 1}$

### Exercise 56

$g(x) = \frac{x^2 - 9}{x^2 - 4}$

### Exercise 57

$g(t) = \frac{t^2}{t^2 - 1}$

### Exercise 58

$h(x) = \frac{1}{x^3 - x - 2}$

### Exercise 59

$h(x) = (x - 1)^2 + 1$

### Exercise 60

$g(x) = (x + 2)^2 + 1$

61. **Cost of Removing Toxic Pollutants** A city’s main well was recently found to be contaminated with trichloroethylene (a cancer-causing chemical) as a result of an abandoned chemical dump that leached chemicals into the water. A proposal submitted to the city council indicated that the cost, measured in millions of dollars, of removing $x$% of the toxic pollutants is given by

$$C(x) = \frac{0.5x}{100 - x}$$

a. Find the vertical asymptote of $C(x)$.

b. Is it possible to remove 100% of the toxic pollutant from the water?

62. **Average Cost of Producing DVDs** The average cost per disc (in dollars) incurred by Herald Media Corporation in pressing $x$ DVDs is given by the average cost function

$$C(x) = 2.2 + \frac{2500}{x}$$

a. Find the horizontal asymptote of $C(x)$.

b. What is the limiting value of the average cost?

63. **Concentration of a Drug in the bloodstream** The concentration (in milligrams/cubic centimeter) of a certain drug in a patient’s bloodstream $t$ hr after injection is given by

$$C(t) = \frac{0.2t}{t + 1}$$

a. Find the horizontal asymptote of $C(t)$.

b. Interpret your result.
b. To find the maximum velocity attained by the rocket, find the largest value of the function that describes the rocket’s velocity at any time \( t \) — namely,

\[ v = f'(t) = -3t^2 + 192t + 195 \quad (t \geq 0) \]

We find the critical number of \( v \) by setting \( v' = 0 \). But

\[ v' = -6t + 192 \]

and the critical number of \( v \) is 32. Since

\[ v'' = -6 < 0 \]

the second derivative test implies that a relative maximum of \( v \) occurs at \( t = 32 \). Our computation has in fact clarified the property of the “velocity curve.” Since \( v'' < 0 \) everywhere, the velocity curve is concave downward everywhere. With this observation, we assert that the relative maximum must in fact be the absolute maximum of \( v \). The maximum velocity of the rocket is given by evaluating \( v \) at \( t = 32 \),

\[ f'(32) = -3(32)^2 + 192(32) + 195 \]

or 3267 feet per second. The graph of the velocity function \( v \) is sketched in Figure 67.

### 4.4 Self-Check Exercises

1. Let \( f(x) = x - 2\sqrt{x} \)
   
   a. Find the absolute extrema of \( f \) on the interval \([0, 9]\).
   
   b. Find the absolute extrema of \( f \).

2. Find the absolute extrema of \( f(x) = 3x^4 + 4x^3 + 1 \) on \([-2, 1]\).

3. The operating rate (expressed as a percent) of factories, mines, and utilities in a certain region of the country on the \( t \)th day of 2006 is given by the function

\[ f(t) = 80 + \frac{1200t}{t^2 + 40000} \quad (0 \leq t \leq 250) \]

On which of the first 250 days of 2006 was the manufacturing capacity operating rate highest?

Solutions to Self-Check Exercises 4.4 can be found on page 310.

### 4.4 Concept Questions

1. Explain the following terms: (a) absolute maximum and (b) absolute minimum.

2. Describe the procedure for finding the absolute extrema of a continuous function on a closed interval.

### 4.4 Exercises

In Exercises 1–8, you are given the graph of a function \( f \) defined on the indicated interval. Find the absolute maximum and the absolute minimum of \( f \), if they exist.

1. 

2. 

[Graphs of functions are shown for Exercises 1 and 2.]
In Exercises 9–38, find the absolute maximum value and the absolute minimum value, if any, of each function.

9. \( f(x) = 2x^2 + 3x - 4 \)  
10. \( g(x) = -x^2 + 4x - 7 \)

11. \( h(x) = x^{3/5} \)  
12. \( f(x) = x^{2/3} \)

13. \( f(x) = \frac{1}{1 + x^2} \)  
14. \( f(x) = \frac{x}{1 + x^2} \)

15. \( f(x) = x^2 - 2x - 3 \) on \([-2, 3]\)  
16. \( g(x) = x^2 - 2x - 3 \) on \([0, 4]\)

17. \( f(x) = -x^2 + 4x + 6 \) on \([0, 5]\)  
18. \( f(x) = -x^2 + 4x + 6 \) on \([3, 6]\)  
19. \( f(x) = x^3 + 3x^2 - 1 \) on \([-3, 2]\)

20. \( g(x) = x^3 + 3x^2 - 1 \) on \([-3, 1]\)  
21. \( g(x) = 3x^4 + 4x^3 \) on \([-2, 1]\)

22. \( f(x) = \frac{1}{2}x^4 - \frac{2}{3}x^3 - 2x^2 + 3 \) on \([-2, 3]\)

23. \( f(x) = \frac{x + 1}{x - 1} \) on \([2, 4]\)  
24. \( g(t) = \frac{t}{t - 1} \) on \([2, 4]\)

25. \( f(x) = 4x + \frac{1}{x} \) on \([1, 3]\)  
26. \( f(x) = 9x - \frac{1}{x} \) on \([1, 3]\)

27. \( f(x) = \frac{1}{2}x^2 - 2\sqrt{x} \) on \([0, 3]\)  
28. \( g(x) = \frac{1}{8}x^2 - 4\sqrt{x} \) on \([0, 9]\)

29. \( f(x) = \frac{1}{x} \) on \((0, \infty)\)  
30. \( g(x) = \frac{1}{x + 1} \) on \((0, \infty)\)

31. \( f(x) = 3x^{2/3} - 2x \) on \([0, 3]\)

32. \( g(x) = x^2 + 2x^{2/3} \) on \([-2, 2]\)

33. \( f(x) = x^{2/3}(x^2 - 4) \) on \([-1, 2]\)

34. \( f(x) = x^{2/3}(x^2 - 4) \) on \([-1, 3]\)

35. \( f(x) = \frac{x}{x^2 + 2} \) on \([-1, 2]\)

36. \( f(x) = \frac{1}{x^2 + 2x + 5} \) on \([-2, 1]\)
37. \( f(x) = \frac{x}{\sqrt{x^2 + 1}} \) on \([-1, 1]\)

38. \( g(x) = x \sqrt{4 - x^2} \) on \([0, 2]\)

39. A stone is thrown straight up from the roof of an 80-ft building. The height (in feet) of the stone at any time \( t \) (in seconds), measured from the ground, is given by

\[ h(t) = -16t^2 + 64t + 80 \]

What is the maximum height the stone reaches?

40. **Maximizing Profits** Lynbrook West, an apartment complex, has 100 two-bedroom units. The monthly profit (in dollars) realized from renting out \( x \) apartments is given by

\[ P(x) = -10x^2 + 1760x - 50,000 \]

To maximize the monthly rental profit, how many units should be rented out? What is the maximum monthly profit realizable?

41. **Seniors in the Workforce** The percentage of men, age 65 yr and older, in the workforce from 1950 \((t = 0)\) through 2000 \((t = 50)\) is approximately

\[ P(t) = 0.0135t^2 - 1.126t + 41.2 \quad (0 \leq t \leq 50) \]

Show that the percentage of men, age 65 yr and older, in the workforce in the period of time under consideration was smallest around mid-September 1991. What is that percent?

*Source: U.S. Census Bureau*

42. **Flight of a Rocket** The altitude (in feet) attained by a model rocket \( t \) sec into flight is given by the function

\[ h(t) = \frac{1}{3}t^3 + 4t^2 + 20t + 2 \quad (t \geq 0) \]

Find the maximum altitude attained by the rocket.

43. **Female Self-Employed Workforce** Data show that the number of nonfarm, full-time, self-employed women can be approximated by

\[ N(t) = 0.81t - 1.1414t + 1.53 \quad (0 \leq t \leq 6) \]

where \( N(t) \) is measured in millions and \( t \) is measured in 5-yr intervals, with \( t = 0 \) corresponding to the beginning of 1963. Determine the absolute extrema of the function \( N \) on the interval \([0, 6]\). Interpret your results.

*Source: U.S. Department of Labor*

44. **Average Speed of a Vehicle** The average speed of a vehicle on a stretch of Route 134 between 6 a.m. and 10 a.m. on a typical weekday is approximated by the function

\[ f(t) = 20t - 40 \sqrt{t} + 50 \quad (0 \leq t \leq 4) \]

where \( f(t) \) is measured in miles per hour and \( t \) is measured in hours, with \( t = 0 \) corresponding to 6 a.m. At what time of the morning commute is the traffic moving at the slowest rate? What is the average speed of a vehicle at that time?

45. **Maximizing Profits** The management of Trappee and Sons, producers of the famous TexaPep hot sauce, estimate that their profit (in dollars) from the daily production and sale of \( x \) cases (each case consisting of 24 bottles) of the hot sauce is given by

\[ P(x) = -0.0000002x^3 + 6x - 400 \]

What is the largest possible profit Trappee can make in 1 day?

46. **Maximizing Profits** The quantity demanded each month of the Walter Serkin recording of Beethoven’s Moonlight Sonata, manufactured by Phonola Record Industries, is related to the price/compact disc. The equation

\[ p = -0.000042x + 6 \quad (0 \leq x \leq 12,000) \]

where \( p \) denotes the unit price in dollars and \( x \) is the number of discs demanded, relates the demand to the price. The total monthly cost (in dollars) for pressing and packaging \( x \) copies of this classical recording is given by

\[ C(x) = 600 + 2x - 0.000002x^2 \quad (0 \leq x \leq 20,000) \]

To maximize its profits, how many copies should Phonola produce each month?

*Hint: The revenue is \( R(x) = px \), and the profit is \( P(x) = R(x) - C(x) \).*

47. **Maximizing Profit** A manufacturer of tennis rackets finds that the total cost \( C(x) \) (in dollars) of manufacturing \( x \) rackets/day is given by \( C(x) = 400 + 4x + 0.0001x^2 \). Each racket can be sold at a price of \( p \) dollars, where \( p \) is related to \( x \) by the demand equation \( p = 10 - 0.00004x \). If all rackets that are manufactured can be sold, find the daily level of production that will yield a maximum profit for the manufacturer.

48. **Maximizing Profit** The weekly demand for the Pulsar 25-in. color console television is given by the demand equation

\[ p = -0.05x + 600 \quad (0 \leq x \leq 12,000) \]

where \( p \) denotes the wholesale unit price in dollars and \( x \) denotes the quantity demanded. The weekly total cost function associated with manufacturing these sets is given by

\[ C(x) = 0.000002x^3 - 0.03x^2 + 400x + 80,000 \]

where \( C(x) \) denotes the total cost incurred in producing \( x \) sets. Find the level of production that will yield a maximum profit for the manufacturer.

*Hint: Use the quadratic formula.*

49. **Maximizing Profit** A division of Chapman Corporation manufactures a pager. The weekly fixed cost for the division is \$20,000, and the variable cost for producing \( x \) pagers/week is

\[ V(x) = 0.000001x^3 - 0.01x^2 + 50x \]

dollars. The company realizes a revenue of

\[ R(x) = -0.02x^2 + 150x \quad (0 \leq x \leq 7500) \]

dollars from the sale of \( x \) pagers/week. Find the level of production that will yield a maximum profit for the manufacturer.

*Hint: Use the quadratic formula.*
4.5 Concept Questions

1. If the domain of a function \( f \) is not a closed interval, how would you find the absolute extrema of \( f \), if they exist?

2. Refer to Example 4 (page 316). In the solution given in the example, we solved for \( h \) in terms of \( r \), resulting in a function of \( r \), which we then optimized with respect to \( r \). Write \( S \) in terms of \( h \) and re-solve the problem. Which choice is better?

4.5 Exercises

1. Find the dimensions of a rectangle with a perimeter of 100 ft that has the largest possible area.

2. Find the dimensions of a rectangle of area 144 sq ft that has the smallest possible perimeter.

3. **Enclosing the Largest Area** The owner of the Rancho Los Feliz has 3000 yd of fencing with which to enclose a rectangular piece of grazing land along the straight portion of a river. If fencing is not required along the river, what are the dimensions of the largest area that he can enclose? What is this area?

4. **Enclosing the Largest Area** Refer to Exercise 3. As an alternative plan, the owner of the Rancho Los Feliz might use the 3000 yd of fencing to enclose the rectangular piece of grazing land along the straight portion of the river and then subdivide it by means of a fence running parallel to the sides. Again, no fencing is required along the river. What are the dimensions of the largest area that can be enclosed? What is this area? (See the accompanying figure.)

5. **Minimizing Construction Costs** The management of the UNICO department store has decided to enclose an 800-ft² area outside the building for displaying potted plants and flowers. One side will be formed by the external wall of the store, two sides will be constructed of pine boards, and the fourth side will be made of galvanized steel fencing. If the pine board fencing costs \$0/running foot and the steel fencing costs \$3/running foot, determine the dimensions of the enclosure that can be erected at minimum cost.

6. **Packaging** By cutting away identical squares from each corner of a rectangular piece of cardboard and folding up the resulting flaps, an open box may be made. If the cardboard is 15 in. long and 8 in. wide, find the dimensions of the box that will yield the maximum volume.

7. **Metal Fabrication** If an open box is made from a tin sheet 8 in. square by cutting out identical squares from each corner and bending up the resulting flaps, determine the dimensions of the largest box that can be made.

8. **Minimizing Packaging Costs** If an open box has a square base and a volume of 108 in.

9. **Minimizing Packaging Costs** What are the dimensions of a closed rectangular box that has a square cross section, a capacity of 128 in.

\[ \text{volume: } V = xy^2 \]

\[ \text{area: } A = 2xy + x^2 + y^2 \]

\[ \text{cost: } C = 2xy + x^2 + y^2 \]

\[ \text{Find the dimensions that will minimize the cost.} \]
10. **Minimizing Packaging Costs** A rectangular box is to have a square base and a volume of 20 ft³. If the material for the base costs 30¢/square foot, the material for the sides costs 10¢/square foot, and the material for the top costs 20¢/square foot, determine the dimensions of the box that can be constructed at minimum cost. (Refer to the figure for Exercise 9.)

11. **Parcel Post Regulations** Postal regulations specify that a parcel sent by priority mail may have a combined length and girth of no more than 108 in. Find the dimensions of a rectangular package that has a square cross section and the largest volume that may be sent via priority mail. What is the volume of such a package?
   Hint: The length plus the girth is $4x + h$ (see the accompanying figure).

![Figure](image.png)

12. **Book Design** A book designer has decided that the pages of a book should have 1-in. margins at the top and bottom and ½-in. margins on the sides. She further stipulated that each page should have an area of 50 in² (see the accompanying figure). Determine the page dimensions that will result in the maximum printed area on the page.

![Figure](image.png)

13. **Parcel Post Regulations** Postal regulations specify that a parcel sent by priority mail may have a combined length and girth of no more than 108 in. Find the dimensions of the cylindrical package of greatest volume that may be sent via priority mail. What is the volume of such a package? Compare with Exercise 11.
   Hint: The length plus the girth is $2\pi r + l$.

![Figure](image.png)

14. **Minimizing Costs** For its beef stew, Betty Moore Company uses aluminum containers that have the form of right circular cylinders. Find the radius and height of a container if it has a capacity of 36 in³ and is constructed using the least amount of metal.

15. **Product Design** The cabinet that will enclose the Acrosonic model D loudspeaker system will be rectangular and will have an internal volume of 2.4 ft³. For aesthetic reasons, it has been decided that the height of the cabinet is to be 1.5 times its width. If the top, bottom, and sides of the cabinet are constructed of veneer costing 40¢/square foot and the front (ignore the cutouts in the baffle) and rear are constructed of particle board costing 20¢/square foot, what are the dimensions of the enclosure that can be constructed at a minimum cost?

16. **Designing a Norman Window** A Norman window has the shape of a rectangle surmounted by a semicircle (see the accompanying figure). If a Norman window is to have a perimeter of 28 ft, what should its dimensions be in order to allow the maximum amount of light through the window?

![Figure](image.png)

17. **Optimal Charter-Flight Fare** If exactly 200 people sign up for a charter flight, Leisure World Travel Agency charges $300/person. However, if more than 200 people sign up for the flight (assume this is the case), then each fare is reduced by $1 for each additional person. Determine how many passengers will result in a maximum revenue for the travel agency. What is the maximum revenue? What would be the fare per passenger in this case?
   Hint: Let $x$ denote the number of passengers above 200. Show that the revenue function $R$ is given by $R(x) = (200 + x)(300 - x)$.

18. **Maximizing Yield** An apple orchard has an average yield of 36 bushels of apples/tree if tree density is 22 trees/acre. For each unit increase in tree density, the yield decreases by 2 bushels/tree. How many trees should be planted in order to maximize the yield?

19. **Charter Revenue** The owner of a luxury motor yacht that sails among the 4000 Greek islands charges $600/person/day if exactly 20 people sign up for the cruise. However, if more than 20 people sign up (up to the maximum capacity of 90) for the cruise, then each fare is reduced by $4 for each additional passenger. Assuming at least 20 people sign up for the cruise, determine how many passengers will result in the maximum revenue for the owner of the yacht. What is the maximum revenue? What would be the fare/passenger in this case?

20. **Profit of a Vineyard** Phillip, the proprietor of a vineyard, estimates that the first 10,000 bottles of wine produced this season will fetch a profit of $5/bottle. But if more than
10,000 bottles were produced, then the profit/bottle for the entire lot would drop by $0.0002 for each additional bottle sold. Assuming at least 10,000 bottles of wine are produced and sold, what is the maximum profit?

21. **Optimal Speed of a Truck** A truck gets $600/v mpg when driven at a constant speed of $v$ mph (between 50 and 70 mph). If the price of fuel is $3/gallon and the driver is paid $18/hour, at what speed between 50 and 70 mph is it most economical to drive?

22. **Minimizing Costs** Suppose the cost incurred in operating a cruise ship for one hour is $a + bv^3$ dollars, where $a$ and $b$ are positive constants and $v$ is the ship's speed in miles per hour. At what speed should the ship be operated between two ports, to minimize the cost?

23. **Strength of a Beam** A wooden beam has a rectangular cross section of height $h$ in. and width $w$ in. (see the accompanying figure). The strength $S$ of the beam is directly proportional to its width and the square of its height. What are the dimensions of the cross section of the strongest beam that can be cut from a round log of diameter 24 in.? Hint: $S = kh^2w$, where $k$ is a constant of proportionality.

24. **Designing a Grain Silo** A grain silo has the shape of a right circular cylinder surmounted by a hemisphere (see the accompanying figure). If the silo is to have a capacity of $504\pi$ ft$^3$, find the radius and height of the silo that requires the least amount of material to construct. Hint: The volume of the silo is $\pi r^2h + \frac{2}{3}\pi r^3$, and the surface area (including the floor) is $\pi(3r^2 + 2rh)$.

25. **Minimizing Cost of Laying Cable** In the following diagram, $S$ represents the position of a power relay station located on a straight coast, and $E$ shows the location of a marine biology experimental station on an island. A cable is to be laid connecting the relay station with the experimental station. If the cost of running the cable on land is $1.50/running foot and the cost of running the cable under water is $2.50/running foot, locate the point $P$ that will result in a minimum cost (solve for $x$).

26. **Storing Radioactive Waste** A cylindrical container for storing radioactive waste is to be constructed from lead and have a thickness of 6 in. (see the accompanying figure). If the volume of the outside cylinder is to be $16\pi$ ft$^3$, find the radius and the height of the inside cylinder that will result in a container of maximum storage capacity.

27. **Flights of Birds** During daylight hours, some birds fly more slowly over water than over land because some of their energy is expended in overcoming the downdrafts of air over open bodies of water. Suppose a bird that flies at a constant speed of 4 mph over water and 6 mph over land starts its journey at the point $E$ on an island and ends at its nest $N$ on the shore of the mainland, as shown in the accompanying figure. Find the location of the point $P$ that allows the bird to complete its journey in the minimum time (solve for $x$).