E.g. ① Solve $\frac{dy}{dx} = \frac{4x - x^3}{4 + y^3}$

Rewrite as $(4x - x^3)dx = (4 + y^3)dy$.

Integrate it: $\int (4x - x^3)dx = \int (4 + y^3)dy$

$$2x^2 - \frac{1}{4}x^4 + c = 4y + \frac{1}{4}y^4.$$

or.

$$y^4 + 16y = 8x^2 - x^4 + c.$$

which gives the general solution implicitly. Here c is arbitrary.

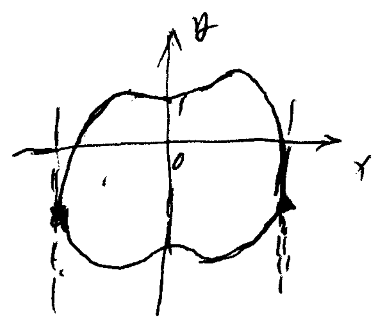
② Find the solution passing through the point $(0,1)$, i.e. $y(0) = 1$.

From the general solution:

$$1^4 + 16 \cdot 1 = 8 \cdot 0^2 - 0^4 + c.$$

$\therefore\ c = 17.$

i.e. the solution is implicitly given by $y^4 + 16y = 8x^2 - x^4 + 17$.



③ Find the interval where the solution exists

Observe: the graph has two vertical tangent lines; implying the derivatives of y do not exist.

Thus: $y^3 + 4 = 0 \Rightarrow y = -4^{\frac{1}{3}}$.

substituting into the solution:

$$-4^{\frac{4}{3}} - 16 \cdot 4^{\frac{1}{3}} = 8x^2 - x^4 + 7.$$

Thus $x \approx \pm 3.3$.

That is, the solution exists on $(-3.3, 3.3)$.

HW: 22/10, 15 (a), (c).