(d) $\lim_{x \to 0}$ — x

7. Compute the following derivatives using the command **diff**:

   (a) $\dfrac{d}{dx}\left(\dfrac{x^3}{x^2+1}\right)$,

   (b) $\dfrac{d}{dx}(\sin(\sin(\sin x)))$,

   (c) $\dfrac{d^3}{dx^3}(\arctan x)$,

   (d) $\dfrac{d}{dx}(\sqrt{1+x^2})$,

   (e) $\dfrac{d}{dx}(e^{x\ln(x)})$.

8. Compute the following integrals using the command **int**:

7. Consider the differential equation

$$y' = \frac{t^2}{1+y^2}$$

(*cf.* Problem 8, Section 2.2 in Boyce & DiPrima).

   (a) Solve it using **dsolve**. Observe that in some sense MATLAB is "too good" in that it finds three rather complicated explicit solutions. Note that two of them are complex-valued. Select the real one. In fact, for this solution it is easier to work with an implicit form; so here are instructions for converting your solution. Substitute $u = t^3$ (or $t = u^{(1/3)}$) to generate an expression in u. Set it equal to y and **solve** for u. Replace u by t^3 to get an implicit solution of the form
   $$f(t, y) = c.$$

   (b) Use **contour** to see what the solution curves look like. For both your t and y

   (c) Plot the solution satisfying the initial condition $y(0.5) = 1$.

   (d) To find a numerical value for the solution $y(t)$ from part (c) at a particular value of t, you can plug the value t into the equation
   $$f(t, y) = f(0.5, 1)$$
   and solve for y. This can be done using **fzero**. (See Section 3.7.) Find $y(-1), y(0), y(1)$. Mark these values on your plot.

8 In this probl…

11. Chapter 6 describes how to plot the direction field for a first order differential equation. For each equation below, plot the direction field on a rectangle large enough (but not too large) to show clearly all of its equilibrium points. Find the equilibria and state whether each is stable or unstable. If you cannot determine the precise value of an equilibrium point from the equation or the direction field, use **fzero** or **solve** as appropriate.

   (a) $y' = -y(y-2)(y-4)/10$,

   (b) $y' = y^2 - 3y + 1$,

   (c) $y' = 0.1y - \sin y$.

12. In this problem, we use the direction field capabilities of MATLAB to study two