Math 405 Sample Final

Sections 1

Instructions:

1. Write your name on this answer booklet.
2. Read each question carefully.
3. Please write legibly.
4. TO ENSURE FULL CREDIT, EXPLAIN YOUR WORK FULLY.
5. This exam has 8 pages.
6. The total number of points on this exam is 100.
7. Books and notes are not allowed in this exam.
8. Independent work is expected.

Name: _____

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

Total:

1. Find the general solution of

$$\mathbf{x}' = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{pmatrix} \mathbf{x}$$

2. Find the general solution of the given system. Sketch a phase portait of the system. Find the solution satisfying the initial condition: $\mathbf{x}(0) = (1, 1)^T$, and describe the behavior of the solution as $t \to \infty$. (**please review more related homework problems such as about saddle, nodes, etc**)

$$\mathbf{x}' = \begin{pmatrix} 4 & -3 \\ 3 & 4 \end{pmatrix} \mathbf{x}.$$

3. Find the general solution of the given system. Sketch a phase portait of the system. Find the solution satisfying the initial condition: $\mathbf{x}(0) = (2, 3)^T$, and describe the behavior of the solution as $t \to \infty$.

$$\mathbf{x}' = \begin{pmatrix} 1 & -3 \\ 3 & 7 \end{pmatrix} \mathbf{x}, \qquad .$$

4. Find the general solution of the given system (using variation of parameters, undertermined coefficients): (See more practice problems for Sect. 7.9)

$$\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} e^t \\ t \end{pmatrix}.$$

5. Find the critical points for the system

$$\frac{dx}{dt} = x + x^2 + y^2, \qquad \frac{dy}{dt} = y - xy,$$

and discuss the type and stability of these critical points. (You should review all similar problems of the homeworks in Sect. 9.3)