

Instructions:

1. Write your name on this answer booklet.
2. Read each question carefully.
3. Please write legibly.
4. TO ENSURE FULL CREDIT, EXPLAIN YOUR WORK FULLY.
5. This exam has 4 pages.
6. The total number of points on this exam is 50.
7. Books and notes are not allowed in this exam.
8. Independent work is expected.

1. (10pts) Show that the general solution of

$$\mathbf{x}' = \mathbf{P}(t)\mathbf{x} + \mathbf{g}(t)$$

is the sum of any particular solution $\mathbf{x}^{(p)}$ of this equation and the general solution $\mathbf{x}^{(c)}$ of the corresponding homogeneous equation.

2. (10pts) Find the general solution of

$$\mathbf{x}' = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \mathbf{x}$$

3. (10pts) Consider the system

$$\mathbf{x}' = \begin{pmatrix} 0 & -5 \\ 1 & -3 \end{pmatrix} \mathbf{x}.$$

Find the general solution of the system, and sketch a phase portrait of the system. Furthermore, find the solution satisfying the initial value condition $\mathbf{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

4. (10pts) Find the general solution of the following system and sketch a phase portrait.

$$\mathbf{x}' = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix} \mathbf{x}.$$

5. (10pts) Find the general solution of the given system:

$$\mathbf{x}' = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} e^t \\ t \end{pmatrix}.$$