

Complex Dynamics in Swing Equations of a Power System Model *

Yu Chang^[1] Dashun Xu^[2] Zhujun Jing^[3,4]

^[1]*Department of Mathematics and Computer Science, School of Science, Beijing University of Chemical Technology, Beijing 100029, China*

^[2]*Department of Mathematics, Southern Illinois University, Carbondale, IL 62901, USA*

^[3]*Department of Mathematics, Hunan Normal University, Hunan Changsha, 410081, P.R.China*

^[4]*Institute of Mathematics, Academy of Mathematics and Systems Sciences, Chinese Academy of Sciences, Beijing, 100080, P.R.China*

Abstract. *Complex dynamics in the reduced swing equations of a power system are investigated. As the change of bifurcation parameter μ , the system exhibits complex bifurcations, such as saddle-node bifurcation, Hopf bifurcation, cyclic fold bifurcation and torus bifurcations and complex dynamics, such as periodic orbits, quasiperiodic orbits, period-doubling orbits and chaotic attractors which link with the system instability or collapse. The sufficient conditions for saddle-node bifurcation and Hopf bifurcation are derived. Specifically, between the Hopf bifurcations, i.e., in the “Hopf window”, a cascade of period-doubling bifurcation, which leads to chaos, is shown to occur. The periodic orbits, period-doubling orbits, quasiperiodic orbits and chaotic behavior are shown in numerical simulation based on the computations of the orbits, the eigenvalues, the Floquet multipliers and Lyapunov exponents.*

Key words and phrases: *power system, bifurcations, quasiperiodic orbits, chaos, flutter instability*

1 Introduction

From a mathematical perspective, instability or collapse has been viewed as arising from a bifurcation of the power system as a parameter is varied

through some critical value. Thus bifurcations have been widely accepted as important elements of system instability. This paper studies static bifurcations, dynamic bifurcations and, especially, dynamical behaviors in the swing equations. A static bifurcation implies a change in the number of equilibria. A dynamic bifurcation can be a Hopf, cyclic fold, torus and periodic-doubling bifurcation. Static and Hopf bifurcations are local phenomena. The dynamical behaviors emerge from the dynamical bifurcations. The three machines and four buses system presented in [5] is reduced swing equations by Kwatny et.al.in [3],[5],[6],[16]. The swing equations are investigated by several authors. Among others, Kwatny and Yu[5], use the energy analysis to study the system without damping($\gamma = 0$), and explain the unstable neighbors and flutter instability. Kwatny and Piper[3] use the frequency domain method of the Hopf bifurcation to give the flutter instability with positive damping($\gamma \neq 0$). Kwatny et.al.[6,16] analyze the equilibrium solution structure of the swing equations without damping($\gamma = 0$) and show that the parameter space partitions into regions and within each region the number of equilibrium solutions is 0, 2, 4 or 6.

The aim of the paper is to systematically study the dynamical behaviors emerging from the dynamical bifurcations with the variations of the net powers. Specifically, we first give the sufficient conditions of the existence for the saddle-node bifurcation and the Hopf bifurcation by using the bifurcation the-

*This work was supported by the National Science Foundation of China(10671063).