



ELSEVIER

Electrical Power and Energy Systems 25 (2003) 443–461

---

---

**ELECTRICAL POWER  
&  
ENERGY SYSTEMS**

---

---

[www.elsevier.com/locate/ijepes](http://www.elsevier.com/locate/ijepes)

# Bifurcations, chaos, and system collapse in a three node power system

Zhujun Jing<sup>a,\*</sup>, Dashun Xu<sup>b</sup>, Yu Chang<sup>c</sup>, Luonan Chen<sup>d</sup>

<sup>a</sup>*Institute of Mathematics, Chinese Academy of Sciences, Beijing 100080, People's Republic of China*

<sup>b</sup>*Department of Mathematics and Statistics, Memorial University of Newfoundland, NF, A1C5S7, Canada*

<sup>c</sup>*Institute of Applied Physics and Computational Mathematics, Beijing 100088, People's Republic of China*

<sup>d</sup>*Department of Electrical Engineering and Electronics, Osaka Sangyo University, 3-1-1 Nakagaito, Daito, Osaka 574-8530, Japan*

---

## Abstract

A model of the three node power system proposed by Rajesh and Padiyar [Electr. Power Energy Syst. 21 (1999) 375] is studied. As the bifurcation parameter  $P_m$  (input power to the generator) is changed, the system including the effects of the non-linearity exhibits complex dynamics emerging from static and dynamic bifurcations which link with the system collapse. The analyses for the model exhibit dynamical bifurcations, including three Hopf bifurcations, cyclic fold bifurcations, torus bifurcations and period-doubling bifurcations, and complex dynamical behaviors, including periodic orbits, period-doubling orbits, quasi-periodic orbits, phase-locked phenomena and two chaotic regions between two Hopf bifurcations, i.e. in the 'Hopf window' and intermittency chaos. Moreover, one of the two chaotic regions results from period-doubling bifurcations, and another results from quasi-periodic orbits emerging from a torus bifurcation. Simulations are given to illustrate the various types of dynamic behaviors associated with the power system collapse for the model. In particular, we first shown that the oscillatory transient may play a role in the collapse, and there are different critical points for different dominated state variables. Besides, the hard-limits and increases of the damping factor widen the feasible operating region of the power system, and prevent the torus bifurcation to occur so that some complex dynamical phenomena can be inhibited.

© 2003 Elsevier Science Ltd. All rights reserved.

*Keywords:* Power systems; Chaos; Bifurcations; Hard-limits

---

## 1. Introduction

Rajesh and Padiyar investigated dynamical phenomena of the 'two-axis model', the 'one-axis model' and the classical model for power systems in Refs. [16,17]. In the choice of bifurcation parameters, they indicated that the reactive power demand at the load bus as a bifurcation parameter is unrealistic and cannot characterize a wide range of operating conditions. Hence, they took input power to the generator ( $P_m$ ), the constant real and reactive powers of the motor ( $P_{ld}$ ,  $Q_{ld}$ ) and the reference voltage to the AVR ( $V_{ref}$ ) as the bifurcation parameters, and found out the conditions leading to chaotic behaviors. In particular, they gave a comprehensive study of bifurcations for two-axis model, and also found a torus bifurcation resulting in the emergence of quasi-periodic solutions after a torus bifurcation (TR) induced by a Hopf bifurcation (HB) (corresponding to TR<sup>5</sup> and HB<sup>4</sup> here) for the one-axis model. However, there are still no detail analyses on the one-axis

model. In particular, some important behaviors involving phase-locked phenomena and chaos are not given in Ref. [17]. Therefore, Section 4 in this paper aims to investigate the dynamical behaviors emerging from dynamic bifurcations for one-axis model in details. Specifically we first present sustained oscillations and the two chaotic regions via period-doubling and quasi-periodic routes, and show phase-locked phenomena in which the quasi-periodic orbit becomes a complex periodic orbit. Moreover this paper indicates that global dynamic phenomena are drastically affected by the bifurcation parameter  $P_m$ , and the state variables dominating system behavior are different at different critical points. In Section 5 of this paper, the effects of the damping factor on the system are considered. When the damping factor is greater than some value, the Hopf bifurcation and the torus bifurcation will be inhibited so that the complex behaviors resulted from the bifurcations can be prevented. However, this role of the damping factor is not as drastic as that of machine damping reported in Ref. [19].

To tackle the problems of voltage stability, the power industry utilizes an emerging control to set the control gains of the excitation voltage profiles and faster voltage

---

\* Corresponding author. Address: Department of Mathematics, Hunan Normal University, Hunan Changsha 410081, People's Republic of China.

*E-mail address:* jingzj@math03.math.ac.cn (Z. Jing).