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BIFURCATIONS OF PERIODIC SOLUTIONS AND CHAOS IN JOSEPHSON SYSTEM

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Abstract. The Josephson equation is investigated in detail: the existence and bifurcations for harmonic and subharmonic solutions under small perturbations are obtained by using second-order averaging method and subharmonic Melnikov function, and the criterion of existence for chaos is proved by Melnikov analysis; the bifurcation curves about n-subharmonic and heteroclinic orbits and the driving frequency ω effects to the forms of chaotic behaviors are given by numerical simulations.

1. Introduction. In this paper we consider the Josephson system

$$\begin{cases} \dot{x} = y \\ \dot{y} = -sinx - ksin2x + \beta - \alpha(cosx + 2kcos2x)y + fsin\omega t. \end{cases} \tag{A}$$

Where $x(t)$ is the phase-error process (i.e., an angular variable); $sinx+ksin2x$ is the Hybrid loop which represents the phase-detector characteristics; ω and f are angular frequency and amplitude of the driving current (force) respectively; $sin\omega t$ represents a sinus plus noise; $fsin\omega t$ is a small sinusoidal force; $-\alpha(cosx + 2kcos2x)y + \beta$ is a characteristic of transfer functions of the ideal filter.

The domain of definition of the Josephson system is the tangent bundle of the circle $\mathbf{TS^1} = \mathbf{R^2}\times\mathbf{S}^1$, i.e., the cylinder. The detailed descriptive surrey of Josephson System (A) may be found in [1, 3, 4, 11, 16]. The Josephson junction was first proposed by Josephson (see[18]), then the system has investigated by many authors, for example, see[1, 2, 4-15, 19-22, 24-29] and references there.

As is well known, the synchronous electric motor models of a single machine infinite bus [23], single point Josephson function [11], superconducting derive [8], forced pendulum [20, 25] and many other applications, can readily be described by the model, or analogous ones.

The study of Josephson system is of fundamental and even practical interest. On one the hand, the eminent characteristics of the Josephson system have a rich content of nonlinear properties which are suitable for a detailed investigating various dynamical states. On the other hand, an understanding of the dynamical behavior will be directly useful in the Josephson devices. We therefore think it is worthwhile to undertake a detailed discussion for the System (A) in order to point out which range of parameter corresponds to a certain behavior.

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