

A Product Property of Sobolev Spaces with Application to Elliptic Estimates

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ABSTRACT. In this paper a Sobolev inequality, which generalizes the Banach algebra property of such spaces, is established; for $p \in [1, \infty)$, $n, m \in \mathbb{Z}^+$, and $m \geq 2$ that satisfy $m > n/p$,

$$\|\phi\psi\|_{m,p,\Omega} \leq K \left[\left(\sup_{\Omega_s} |\phi| \right) \|\psi\|_{m,p,\Omega} + \left(\|\psi\|_{m-1,q,\Omega} + \|\psi\|_{m-1,p,\Omega} \right) \|\phi\|_{m,p,\Omega} \right]$$

for all $\phi, \psi \in W^{m,p}(\Omega)$ that satisfy $\text{spt } \psi \subset \Omega_s \subset \Omega$ and domains $\Omega \subset \mathbb{R}^n$ that are nonempty, open, and satisfy the cone condition. Here $q = p$ if $p > n$, $q \in (n/\Upsilon, pn/(n-p)]$ if $n > p$, $q \in (n/\Upsilon, \infty)$ if $p = n$, $K = K(n, p, m, q, \mathcal{C})$, where \mathcal{C} is the cone from the cone condition, and $\Upsilon := \llbracket n/p \rrbracket$, the largest integer less than or equal to n/p .

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