On cavitation, configurational forces and implications for fracture in a nonlinearly elastic material

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Abstract. Experiments on polymers indicate that large tensile stress can induce cavitation, that is, the appearance of voids that were not previously evident in the material. This phenomenon can be viewed as either the growth of pre-existing infinitesimal holes in the material or, alternatively, as the spontaneous creation of new holes in an initially perfect body. In this paper our approach is to adopt both views concurrently within the framework of the variational theory of nonlinear elasticity. We model an elastomer on a macroscale as a void-free material and, on a microscale, as a material containing certain defects that are the only points at which hole formation can occur. Mathematically, this is accomplished by the use of deformations whose point singularities are constrained. One consequence of this viewpoint is that cavitation may then take place at a point that is not energetically optimal. We show that this disparity will generate configurational forces, a type of force identified previously in dislocations in crystals, in phase transitions in solids, in solidification, and in fracture mechanics.

As an application of this approach we study the energetically optimal point for a solitary hole to form in a homogeneous and isotropic elastic ball subject to radial boundary displacements. We show, in particular, that the center of the ball is the unique optimal point. Finally, we speculate that the configurational force generated by cavitation at a non-optimal material point may be sufficient to result in the onset of fracture. The analysis utilizes the energy-momentum tensor, the asymptotics of an equilibrium solution with an isolated singularity, and the linear theory of elasticity at the stressed configuration that the body occupies immediately prior to cavitation.

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