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An existence theory for nonlinear elasticity that allows for cavitation. (English. English summary)

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FEATURED REVIEW.

This paper contains interesting generalizations of the existence theory for nonlinear elasticity due to J. M. Ball [Arch. Rational Mech. Anal. 63 (1976/77), no. 4, 337–403; MR 57#14788], and also interesting results regarding regularity properties of certain classes of Sobolev mappings arising naturally in the generalized existence theory. The mathematical model is the following. Assume that an elastic body, when subject to no forces, occupies a region $\Omega \subset \mathbf{R}^n$. The deformations of the body are described by mappings $u: \Omega \to \mathbf{R}^n$. (The position of the particle which was at $x \in \Omega$ before the deformation is $u(x)$ after the deformation.) It is assumed that the behavior of the material is governed by an energy functional of the form $E(u) = \int_\Omega W(x, \nabla u(x))\, dx +$ surface energy, where, for almost every $x \in \Omega$, the stored energy function $F \to W(x, F)$, defined on $n \times n$ matrices, is polyconvex (a notion introduced in the above-mentioned paper of Ball) and satisfies certain coercivity assumptions. (Recall that a function defined on matrices is said to be polyconvex if it can be expressed as a convex function of subdeterminants.) The surface-energy term is basically the area of the boundary of the deformed configuration $u(\Omega)$. Important properties of W, which one does not want to sacrifice for obtaining rigorous results, are frame indifference (the condition that $W(x, F) = W(x, RF)$ for each rotation R) and the condition that $W(x, F) \to +\infty$ as $\det F \to 0+$. In the above-mentioned paper, Ball was the first to establish general lower-semicontinuity theorems for functionals with polyconvex stored energy functions satisfying these conditions. Using these theorems, one can prove, for many different types of boundary conditions, the existence of admissible deformations which minimize the energy functional. The existence theory of Ball requires certain coercivity of W at infinity. These coercivity conditions imply that the deformations which have finite energy belong to the classes $\mathcal{A}^+_{p,q}(\Omega) = \{u \in W^{1,p}(\Omega, \mathbf{R}^n),\ \mathrm{Cof}\, \nabla u \in L^q(\Omega),$ and $\det \nabla u > 0$ a.e. in $\Omega\}$, for some $p \geq n-1$ and $q \geq p/(p-1)$. These classes are interesting from the point of view of the theory of Sobolev spaces. Roughly speaking, the functions belonging to $\mathcal{A}^+_{p,q}(\Omega)$ are more regular than is suggested by the classical imbedding theorems. One of the first results in this direction, due to S. K. Vodopyanov and V. M. Goldshtein [Sibirsk. Mat. Zh. 17 (1976), no. 3, 515–531, 715; MR

54#2961], is that every function $u \in W^{1,n}(\Omega, \mathbf{R}^n)$ satisfying $\det \nabla u > 0$ a.e. in Ω is continuous. Further results can be found for example in the paper by the reviewer [Arch. Rational Mech. Anal. 100 (1988), no. 2, 105–127; MR 89g:73013]. All this has close connections with the theory of quasi-regular mappings. See, for example, the papers of J. J. Manfredi and E. Villamor [Bull. Amer. Math. Soc. (N.S.) 32 (1995), no. 2, 235–240; MR 95m:30033] and J. Heinonen and P. Koskela [Arch. Rational Mech. Anal. 125 (1993), no. 1, 81–97; MR 94i:30020].

The principal achievement of the paper under review is that the authors can obtain a good existence theory under weaker coercivity assumptions than those of Ball and, at the same time, as is further proved in the paper, the classes of functions in which the existence theory is established still have many of the nice properties of the classes $\mathcal{A}^+_{p,q}(\Omega)$. The authors use many new ideas to prove these results.

One of the interesting features of this work is that the weaker coercivity assumptions allow, in certain circumstances, the creation of cavities in the deformed body. This is not possible if we work with the classes $\mathcal{A}^+_{p,q}(\Omega)$.

The paper also contains a thorough discussion of the connection between mathematical properties of various classes of mappings and the deformations observed in the physical world.

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