The convergence of regularised minimisers for cavitation problems in nonlinear elasticity

Jeyabal Sivaloganathan
Department of Mathematical Sciences
University of Bath
Bath BA2 7AY,
U.K.,

Scott J. Spector
Department of Mathematics
Southern Illinois University
Carbondale, IL 62901-4408,
U.S.A.

and

V. Tilakraj
Abingdon Technology Centre
Schlumberger Geoquest
Abingdon OX14 1DZ,
UK

Abstract. Consider a nonlinearly elastic body which occupies the region $\Omega \subset \mathbb{R}^m$ in its reference state and which is held in tension under prescribed boundary displacements on $\partial\Omega$. Let $\mathbf{x}_0 \in \Omega$ be any fixed point in the body. It is known from variational arguments that, for sufficiently large boundary displacements, there may exist discontinuous weak solutions of the equilibrium equations corresponding to a hole forming at $\mathbf{x}_0$ in the deformed body (this is the phenomenon of cavitation). For each $\epsilon > 0$, define the regularised domains $\Omega_\epsilon = \Omega \backslash \overline{B_\epsilon(\mathbf{x}_0)}$ which contain a pre-existing hole of radius $\epsilon > 0$ centred on $\mathbf{x}_0$. Now consider the corresponding mixed displacement/traction problem on Ω_ϵ in which the boundary $\partial\Omega$ is subject to the same boundary displacements and the deformed cavity surface (i.e. the image of ∂B_ϵ) is required to be stress free. It follows from variational arguments that there exists a weak solution $\mathbf{u}_\epsilon$ of this problem for each $\epsilon > 0$. In this paper we prove convergence of these regularised minimisers $\mathbf{u}_\epsilon$ in the limit as $\epsilon \to 0$. In particular, we show that if $\epsilon_n \searrow 0$ then, passing to a subsequence, $\mathbf{u}_{\epsilon_n} \to \mathbf{u}$ where $\mathbf{u}$ is a minimiser for the original pure displacement problem on Ω. Finally, we study the effect on cavitation of regularising the variational problem by introducing a surface energy term which penalises the formation and growth of cavities.

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