

On Bifurcation in Finite Elasticity: Buckling of a Rectangular Rod

by

Henry C. Simpson
Department of Mathematics
University of Tennessee
Knoxville, TN 37996–1300, USA

and

Scott J. Spector
Department of Mathematics
Southern Illinois University
Carbondale, IL 62901, USA

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ABSTRACT. Although there is an extensive literature on the linearization instability of the nonlinear system of partial differential equations that governs an elastic material, there are very few results that prove that a second branch of solutions actually bifurcates from a known solution branch when the known branch becomes unstable. In this paper the implicit function theorem in a Banach space setting is used to prove that the quasistatic compression of a rectangular elastic rod between rigid frictionless plates leads to the buckling of the rod as is observed in experiment and as first predicted by Euler.

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