On the Global Stability of Incompressible Elastic Bars in Uniaxial Extension

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Abstract

When a rectangular bar is subjected to uniaxial tension, the bar usually deforms (approximately) homogeneously and isoaxially until a critical load is reached. A bifurcation, such as the formation of shear bands or a neck, may then be observed. In this paper such an experiment is modelled as the in-plane extension of a two-dimensional, homogeneous, isotropic, incompressible, hyperelastic material in which the length of the bar is prescribed, the ends of the bar are assumed to be free of shear, and the sides are left completely free. It is shown that standard, additional constitutive hypotheses on the stored-energy function imply that no such bifurcation is possible in this model due to the fact that the homogeneous isoaxial deformation is the unique absolute minimizer of the elastic energy. Thus, in order for a bifurcation to occur either the material must cease to be elastic or the stored-energy function must violate the additional hypotheses. The fact that no local bifurcations can occur under the assumptions used herein was known previously, since these assumptions prohibit the load on the bar from reaching a maximum value. However, the fact that the homogeneous deformation is the absolute minimizer of the energy appears to be a new result.

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