S. J. Spector* (`sspector@math.siu.edu`), Department of Mathematics, Southern Illinois University, Carbondale, IL 62901, and **J. Sivaloganathan** (`js@maths.bath.ac.uk`), Department of Mathematical Sciences, University of Bath, BA2 7AY Bath, England. *Necessary Conditions for a Minimum at a Radial Cavitating Singularity in Nonlinear Elasticity.*

It is still not known if the radial cavitating minimizers obtained by Ball [Phil. Trans. R. Soc. Lond. A 306 (1982), 557–611] (and subsequently by many others) are global minimizers of any physically reasonable energy

$$E(\mathbf{u}) = \int_B W(\nabla \mathbf{u}(\mathbf{x})) \, d\mathbf{x}$$

($B \subset \mathbb{R}^3$ is the unit ball). We thus consider the related problem of obtaining necessary conditions for radial solutions to be minimizers with respect to non-radial perturbations. A standard blowup argument applied to either an inner or an outer variation yields a new inequality that has yet to be verified for most W. However, in the special case when $W(\mathbf{F}) = \frac{\mu}{2}|\mathbf{F}|^2 + h(\det \mathbf{F})$ we show that the inequality produced by an inner variation is a well-known inequality, first proven by Brezis, Coron, and Lieb [Comm. Math. Phys. 107 (1986), 649–705] and which arises in the theory of nematic liquid crystals:

$$\int_B |\nabla \mathbf{n}(\mathbf{x})|^2 \, d\mathbf{x} \geq \int_B \left| \nabla \left(\frac{\mathbf{x}}{|\mathbf{x}|} \right) \right|^2 d\mathbf{x} = 8\pi$$

for all $\mathbf{n} \in W^{1,2}(B, \partial B)$ that satisfy $\mathbf{n}(\mathbf{x}) = \mathbf{x}/|\mathbf{x}|$ on ∂B. (Received July 29, 2005)

1