On the Stability of Incompressible Elastic Cylinders in Uniaxial Extension

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1st August 2010

Abstract

Consider a cylinder (not necessarily of circular cross-section) that is composed of a hyperelastic material and stretched parallel to its axis of symmetry. Suppose that the elastic material that constitutes the cylinder is homogeneous, transversely isotropic, and incompressible and that the deformed length of the cylinder is prescribed, the ends of the cylinder are free of shear, and the sides are left completely free. In this paper it is shown that mild additional constitutive hypotheses on the stored-energy function imply that the unique absolute minimizer of the elastic energy for this problem is a homogeneous, isoaxial deformation. This extends recent results that show the same result is valid in 2-dimensions. Prior work on this problem had been restricted to a local analysis: in particular, it was previously known that homogeneous deformations are strict (weak) relative minimizers of the elastic energy as long as the underlying linearized equations are strongly elliptic and provided that the load/displacement curve in this class of deformations does not possess a maximum.

Mathematics Subject Classifications (2010): 74B20, 35J50, 49K20, 74G65.

Key words: Incompressible, elastic, uniaxial tension, homogeneous absolute minimizer.