6.1 Self-Check Exercises

1. Evaluate \[ \int \left( \frac{1}{\sqrt{x}} - \frac{2}{x} + 3e^x \right) \, dx. \]

2. Find the rule for the function \( f \) given that (1) the slope of the tangent line to the graph of \( f \) at any point \( P(x, f(x)) \) is given by the expression \( 3x^2 - 6x + 3 \) and (2) the graph of \( f \) passes through the point \( (2, 9) \).

3. Suppose United Motors’ share of the new cars sold in a certain country is changing at the rate of \( f(t) = -0.01875t^2 + 0.15t - 1.2 \) \( (0 \leq t \leq 12) \) percent/year at year \( t \) (\( t = 0 \) corresponds to the beginning of 1996). The company’s market share at the beginning of 1996 was 48.4%. What was United Motors’ market share at the beginning of 2008?

Solutions to Self-Check Exercises 6.1 can be found on page 411.

6.1 Concept Questions

1. What is an antiderivative? Give an example.

2. If \( f'(x) = g'(x) \) for all \( x \) in an interval \( I \), what is the relationship between \( f \) and \( g \)?

3. What is the difference between an antiderivative of \( f \) and the indefinite integral of \( f \)?

4. Can the power rule be used to integrate \( \frac{1}{x} \, dx \)? Explain your answer.

6.1 Exercises

In Exercises 1–4, verify directly that \( F \) is an antiderivative of \( f \).

1. \( F(x) = \frac{1}{3} x^3 + 2x^2 - x + 2; f(x) = x^2 + 4x - 1 \)

2. \( F(x) = xe^x + \pi; f(x) = e^x(1 + x) \)

3. \( F(x) = \sqrt{2x^2 - 1}; f(x) = \frac{2x}{\sqrt{2x^2 - 1}} \)

4. \( F(x) = x \ln x - x; f(x) = \ln x \)

In Exercises 5–8, (a) verify that \( G \) is an antiderivative of \( f \), (b) find all antiderivatives of \( f \), and (c) sketch the graphs of a few of the family of antiderivatives found in part (b).

5. \( G(x) = 2x; f(x) = 2 \)

6. \( G(x) = 2x^2; f(x) = 4x \)

7. \( G(x) = \frac{1}{3} x^3; f(x) = x^2 \)

8. \( G(x) = e^x; f(x) = e^x \)

In Exercises 9–50, find the indefinite integral.

9. \( \int 6 \, dx \)

10. \( \int \sqrt{x} \, dx \)

11. \( \int 3x^3 \, dx \)

12. \( \int 2x^2 \, dx \)

13. \( \int x^{-4} \, dx \)

14. \( \int 3t^{-7} \, dt \)

15. \( \int x^{3/2} \, dx \)

16. \( \int 2u^{2/3} \, du \)

17. \( \int x^{-5/4} \, dx \)

18. \( \int 3x^{-10} \, dx \)

19. \( \int \frac{2}{x^2} \, dx \)

20. \( \int \frac{1}{3x^3} \, dx \)

21. \( \int \pi \sqrt{t} \, dt \)

22. \( \int \frac{3}{\sqrt{t}} \, dt \)

23. \( \int (3 - 2x) \, dx \)

24. \( \int (1 + u + u^2) \, du \)

25. \( \int (x^2 + x + x^{-3}) \, dx \)

26. \( \int (0.3t^2 + 0.02t + 2) \, dt \)

27. \( \int 4e^x \, dx \)

28. \( \int (1 + e^x) \, dx \)

29. \( \int (1 + x + e^x) \, dx \)

30. \( \int (2 + x + 2x^2 + e^x) \, dx \)

31. \( \int \left( 4x^3 - \frac{2}{x^2} - 1 \right) \, dx \)

32. \( \int \left( 6x^3 + \frac{3}{x} - x \right) \, dx \)

33. \( \int (x^{3/2} + 2x^{1/2} - x) \, dx \)

34. \( \int (t^{1/2} + 2t^{1/2} - 4t^{-1/2}) \, dt \)

35. \( \int \left( \sqrt{x} + \frac{3}{\sqrt{x}} \right) \, dx \)

36. \( \int \left( \sqrt{x^2 - \frac{1}{x^2}} \right) \, dx \)

37. \( \int \left( \frac{u^3 + 2u^2 - u}{3u} \right) \, du \)

38. \( \int \left( \frac{x^4 - 1}{x^3} \right) \, dx \)

39. \( \int (2t + 1)(t - 2) \, dt \)

40. \( \int u^{-2}(1 - u^2 + u^4) \, du \)
41. \[ \int \frac{1}{x^2} (x^2 - 2x + 1) \, dx \]
42. \[ \int \sqrt{t^2 + t - 1} \, dt \]
43. \[ \int \frac{ds}{(s + 1)^2} \]
44. \[ \int \left( \frac{\sqrt{x + \frac{3}{x}}}{x} - 2e^x \right) \, dx \]
45. \[ \int (e^t + t^4) \, dt \]
46. \[ \int \left( \frac{1}{x^2} - \frac{1}{\sqrt{x^2}} + \frac{1}{\sqrt{x}} \right) \, dx \]
47. \[ \int \left( \frac{x^3 + x^2 - x + 1}{x^2} \right) \, dx \]
Hint: Simplify the integrand first.
48. \[ \int \frac{\sqrt{t}}{t^2} \, dt \]
Hint: Simplify the integrand first.
49. \[ \int \frac{(\sqrt{x} - 1)^2}{x^2} \, dx \]
Hint: Simplify the integrand first.
50. \[ \int (x + 1)^2 \left( 1 - \frac{1}{x} \right) \, dx \]
Hint: Simplify the integrand first.

In Exercises 51–58, find \( f(x) \) by solving the initial value problem.
51. \( f'(x) = 2x + 1; f(1) = 3 \)
52. \( f'(x) = 3x^2 - 6x; f(2) = 4 \)
53. \( f'(x) = 3x^2 + 4x - 1; f(2) = 9 \)
54. \( f'(x) = \frac{1}{\sqrt{x}}; f(4) = 2 \)
55. \( f'(x) = 1 + \frac{1}{x^2}; f(1) = 2 \)
56. \( f'(x) = e^x + 2x; f(0) = 2 \)
57. \( f'(x) = \frac{x + 1}{x}; f(1) = 1 \)
58. \( f'(x) = 1 + e^x + \frac{1}{x}; f(1) = 3 + e \)

In Exercises 59–62, find the function \( f \) given that the slope of the tangent line to the graph of \( f \) at any point \( (x, f(x)) \) is \( f'(x) \) and that the graph of \( f \) passes through the given point.
59. \( f'(x) = \frac{1}{2} x^{4/3}; (2, \sqrt{2}) \)
60. \( f'(i) = i^2 - 2i + 3; (1, 2) \)
61. \( f'(x) = e^x + x; (0, 3) \)
62. \( f'(x) = \frac{2}{x} + 1; (1, 2) \)

63. **Bank Deposits** Madison Finance opened two branches on September 1 \( (t = 0) \). Branch A is located in an established industrial park, and branch B is located in a fast-growing new development. The net rate at which money was deposited into branch A and branch B in the first 180 business days is given by the graphs of \( f \) and \( g \), respectively (see the figure). Which branch has a larger amount on deposit at the end of 180 business days? Justify your answer.

[Graph showing rate of deposit, thousands of dollars per day over time]

64. **Velocity of a Car** Two cars, side by side, start from rest and travel along a straight road. The velocity of car A is given by \( v = f(t) \), and the velocity of car B is given by \( v = g(t) \). The graphs of \( f \) and \( g \) are shown in the figure below. Are the cars still side by side after \( T \) sec? If not, which car is ahead of the other? Justify your answer.

[Graph showing velocity over time]

65. **Velocity of a Car** The velocity of a car (in feet/second) \( t \) sec after starting from rest is given by the function
\[ f(t) = 2\sqrt{t} \quad (0 \leq t \leq 30) \]
Find the car’s position, \( s(t) \), at any time \( t \). Assume \( s(0) = 0 \).

66. **Velocity of a Maglev** The velocity (in feet/second) of a maglev is
\[ v(t) = 0.2t + 3 \quad (0 \leq t \leq 120) \]
At \( t = 0 \), it is at the station. Find the function giving the position of the maglev at time \( t \), assuming that the motion takes place along a straight stretch of track.

67. **Cost of Producing Clocks** Lorimar Watch Company manufactures travel clocks. The daily marginal cost function associated with producing these clocks is
\[ C'(x) = 0.00000009x^2 - 0.009x + 8 \]
where \( C'(x) \) is measured in dollars/unit and \( x \) denotes the number of units produced. Management has determined that the daily fixed cost incurred in producing these clocks is $120. Find the total cost incurred by Lorimar in producing the first 500 travel clocks/day.
68. **Revenue Functions** The management of Lorimar Watch Company has determined that the daily marginal revenue function associated with producing and selling their travel clocks is given by

\[ R'(x) = -0.009x + 12 \]

where \( x \) denotes the number of units produced and sold and \( R'(x) \) is measured in dollars/unit.

**a.** Determine the revenue function \( R(x) \) associated with producing and selling these clocks.

**b.** What is the demand equation that relates the wholesale unit price with the quantity of travel clocks demanded?

69. **Profit Functions** Cannon Precision Instruments makes an automatic electronic flash with Thyrister circuitry. The estimated marginal profit associated with producing and selling these electronic flashes is

\[ P'(x) = -0.004x + 20 \]

dollars/unit/month when the production level is \( x \) units per month. Cannon's fixed cost for producing and selling these electronic flashes is $16,000/month. At what level of production does Cannon realize a maximum profit? What is the maximum monthly profit?

70. **Cost of Producing Guitars** Carlota Music Company estimates that the marginal cost of manufacturing its Professional Series guitars is

\[ C'(x) = 0.002x + 100 \]

dollars/month when the level of production is \( x \) guitars/month. The fixed costs incurred by Carlota are $4000/month. Find the total monthly cost incurred by Carlota in manufacturing \( x \) guitars/month.

71. **Health Costs** The national health expenditures are projected to grow at the rate of

\[ r(t) = 0.0058t + 0.159 \quad (0 \leq t \leq 13) \]

trillion dollars/year from 2002 through 2015. Here, \( t = 0 \) corresponds to 2002. The expenditure in 2002 was $1.60 trillion.

**a.** Find the function \( f \) giving the projected national health expenditures in year \( t \).

**b.** What does your model project the national health expenditure to be in 2015?

**Source:** National Health Expenditures

72. **Quality Control** As part of a quality-control program, the chess sets manufactured by Jones Brothers are subjected to a final inspection before packing. The rate of increase in the number of sets checked per hour by an inspector \( t \) hr into the 8 a.m. to 12 noon morning shift is approximately

\[ N'(t) = -3t^2 + 12t + 45 \quad (0 \leq t \leq 4) \]

**a.** Find an expression \( N(t) \) that approximates the number of sets inspected at the end of \( t \) hours.

**Hint:** \( N(0) = 0 \).

73. **Satellite Radio Subscriptions** Based on data obtained by polling automobile buyers, the number of subscribers of satellite radios is expected to grow at the rate of

\[ r(t) = -0.375t^2 + 2.1t + 2.45 \quad (0 \leq t \leq 5) \]

million subscribers/year between 2003 \((t = 0)\) and 2008 \((t = 5)\). The number of satellite radio subscribers at the beginning of 2003 was 1.5 million.

**a.** Find an expression giving the number of satellite radio subscribers in year \( t \) \((0 \leq t \leq 5)\).

**b.** Based on this model, what was the number of satellite radio subscribers in 2005?

**Source:** Cable Group

74. **Risk of Down Syndrome** The risk at which the risk of Down syndrome is changing is approximated by the function

\[ r(x) = 0.00464x^2 - 0.3012x + 4.9 \quad (20 \leq x \leq 45) \]

where \( r(x) \) is measured in percentage of all births/year and \( x \) is the maternal age at delivery.

**a.** Find a function \( f \) giving the risk as a percentage of all births when the maternal age at delivery is \( x \) years, given that the risk of down syndrome at age 30 is 0.14% of all births.

**b.** Based on this model, what is the risk of Down syndrome when the maternal age at delivery is 40 years? 45 years?

**Source:** New England Journal of Medicine

75. **Credit Card Debt** The average credit card debt per U.S. household between 1990 \((t = 0)\) and 2003 \((t = 13)\) was growing at the rate of approximately

\[ D(t) = -4.479t^2 + 69.8t + 279.5 \quad (0 \leq t \leq 13) \]

dollars/year. The average credit card debt per U.S. household stood at $2917 in 1990.

**a.** Find an expression giving the approximate average credit card debt per U.S. household in year \( t \) \((0 \leq t \leq 13)\).

**b.** Use the result of part (a) to estimate the average credit card debt per U.S. household in 2003.

**Source:** Enron Capital Group

76. **Genetically Modified Crops** The total number of acres of genetically modified crops grown worldwide from 1997 through 2003 was changing at the rate of

\[ R(t) = 2.718t^2 - 19.86t + 30.18 \quad (0 \leq t \leq 6) \]

million acres/year. The total number of acres of such crops grown in 1997 \((t = 0)\) was 27.2 million acres. How many acres of genetically modified crops were grown worldwide in 2003?

**Source:** International Services for the Acquisition of Agri-biotech Applications
The number of computers that Universal can expect to sell in the first year is given by
\[ N(12) = 2000(12) + 30,000(e^{-0.05(12)} - 1) = 10,464 \]

### 6.2 Self-Check Exercises

1. Evaluate \( \int \sqrt{2x + 5} \, dx \).

2. Evaluate \( \int \frac{x^2}{(2x^3 + 1)^{3/2}} \, dx \).

3. Evaluate \( \int xe^{2x-1} \, dx \).

4. According to a joint study conducted by Oxnard’s Environmental Management Department and a state government agency, the concentration of carbon monoxide (CO) in the air due to automobile exhaust is increasing at the rate given by

\[ f(t) = \frac{8(0.1t + 1)}{300(0.2t^2 + 4t + 64)^{1/2}} \]

parts per million (ppm) per year \( t \). Currently, the CO concentration due to automobile exhaust is 0.16 ppm. Find an expression giving the CO concentration \( t \) years from now.

**Solutions to Self-Check Exercises 6.2 can be found on page 420.**

### 6.2 Concept Questions

1. Explain how the method of substitution works by showing the steps used to find \( \int f(g(x))g'(x) \, dx \).

2. Explain why the method of substitution works for the integral \( \int xe^{-x^2} \, dx \), but not for the integral \( \int e^{-x^2} \, dx \).

### 6.2 Exercises

**In Exercises 1–50, find the indefinite integral.**

1. \( \int 4(4x + 3)^4 \, dx \)
2. \( \int 4x(2x^2 + 1)^7 \, dx \)
3. \( \int (x^3 - 2x)^2(3x^2 - 2) \, dx \)
4. \( \int (3x^2 - 2x + 1)(x^3 - x^2 + x)^4 \, dx \)
5. \( \int \frac{4x}{(2x^2 + 2)^3} \, dx \)
6. \( \int \frac{3x^2 + 2}{(x^3 + 2x)^2} \, dx \)
7. \( \int 3t^2 \sqrt{t^2 + 2} \, dt \)
8. \( \int 3r^3(r^2 + 2)^{3/2} \, dt \)
9. \( \int (x^2 - 1)^6 x \, dx \)
10. \( \int x^3(2x^3 + 3)^4 \, dx \)
11. \( \int \frac{x^4}{1 - x^7} \, dx \)
12. \( \int \frac{x^2}{\sqrt{x^3 - 1}} \, dx \)
13. \( \int \frac{2}{x - 2} \, dx \)
14. \( \int \frac{x^2}{x^3 - 3} \, dx \)
15. \( \int \frac{0.3x - 0.2}{0.3x^2 - 0.4x + 2} \, dx \)
16. \( \int \frac{2x^3 + 1}{0.2x^4 + 0.3x} \, dx \)
17. \( \int \frac{x}{3x^2 - 1} \, dx \)
18. \( \int \frac{x - 1}{x^3 - 3x + 1} \, dx \)
19. \( \int e^{2x} \, dx \)
20. \( \int e^{-0.05x} \, dx \)
21. \( \int e^{-x} \, dx \)
22. \( \int e^{(x + 1)} \, dx \)
23. \( \int xe^{-x^2} \, dx \)
24. \( \int x^2e^{-x^2} \, dx \)
25. \( \int (e^t - e^{-t}) \, dt \)
26. \( \int (e^{2t} + e^{-3t}) \, dt \)
27. \( \int \frac{e^t}{1 + e^t} \, dt \)
28. \( \int \frac{e^{2x}}{1 + e^{2x}} \, dx \)
29. \( \int \frac{e^{3t}}{\sqrt{x}} \, dx \)
30. \( \int \frac{e^{-10x}}{x^2} \, dx \)
31. \( \int \frac{e^{x} + x^2}{e^{3x} + x^3} \, dx \)
32. \( \int \frac{e^{x} - e^{-x}}{(e^{3x} + e^{-x})^{3/2}} \, dx \)
33. \( \int e^{(e^x + 1)^3} \, dx \)
34. \( \int e^{x^2}(1 + e^{-x}) \, dx \)
35. \( \int \frac{\ln 5x}{x} \, dx \)
36. \( \int \frac{(\ln u)^3}{u} \, du \)
37. \( \int \frac{1}{x} \ln x \, dx \)
38. \( \int \frac{1}{x(\ln x)^2} \, dx \)
39. \( \int \frac{\sqrt{\ln x}}{x} \, dx \)
40. \( \int \frac{(\ln x)^{7/2}}{x} \, dx \)
41. \( \int \left( xe^{x^2} - \frac{x}{x^2 + 2} \right) \, dx \)
42. \( \int \left( xe^{-x^2} + \frac{e^x}{e^x + 2} \right) \, dx \)
43. \[\int \frac{x + 1}{\sqrt{x} - 1} \, dx\]  
Hint: Let \( u = \sqrt{x} - 1 \).

44. \[\int \frac{e^{-x} - 1}{e^{-x} + u} \, du\]  
Hint: Let \( v = e^{-x} + u \).

45. \[\int x(x - 1)^3 \, dx\]  
Hint: \( u = x - 1 \) implies \( x = u + 1 \).

46. \[\int \frac{t}{t + 1} \, dt\]  
Hint: \( \frac{t}{t + 1} = 1 - \frac{1}{t + 1} \).

47. \[\int \frac{1 - \sqrt{x}}{1 + \sqrt{x}} \, dx\]  
Hint: Let \( u = 1 + \sqrt{x} \).

48. \[\int \frac{1 + \sqrt{x}}{1 - \sqrt{x}} \, dx\]  
Hint: Let \( u = 1 - \sqrt{x} \).

49. \[\int \frac{7(1 - \sqrt{x})}{6} \, dx\]  
Hint: Let \( u = 1 - \sqrt{x} \).

50. \[\int x^3(x^2 + 1)^{\frac{3}{2}} \, dx\]  
Hint: Let \( u = x^2 + 1 \).

In Exercises 51–54, find the function \( f \) given that the slope of the tangent line to the graph of \( f \) at any point \((x, f(x))\) is \( f'(x) \) and that the graph of \( f \) passes through the given point.

51. \( f'(x) = 5(2x - 1)^3 \); \((1, 3)\)

52. \( f'(x) = \frac{3x^2}{2\sqrt{x^3 - 1}} \); \((1, 1)\)

53. \( f'(x) = -2xe^{-x^2+1} \); \((1, 0)\)

54. \( f'(x) = 1 - \frac{2x}{x^2 + 1} \); \((0, 2)\)

55. Cable Telephone Subscribers The number of cable telephone subscribers stood at 3.2 million at the beginning of 2004 \((t = 0)\). For the next 5 yr, the number was projected to grow at the rate of

\[ R(t) = 3.36(t + 1)^{0.05} \quad (0 \leq t \leq 5) \]

million subscribers/year. If the projection held true, how many cable telephone subscribers were there at the beginning of 2008 \((t = 4)\)?

Source: Sanford Bernstein

56. TV Viewers: News Magazine Shows The number of viewers of a weekly TV newsmagazine show, introduced in the 2003 season, has been increasing at the rate of

\[ s\left(2 + \frac{1}{2}\right)^{-t^{13}} \quad (1 \leq t \leq 6) \]

million viewers/year in its \( t \)th year on the air. The number of viewers of the program during its first year on the air is given by \( 9(5/2)^{13} \) million. Find how many viewers were expected in the 2008 season.

57. Student Enrollment The registrar of Kellogg University estimates that the total student enrollment in the Continuing Education division will grow at the rate of

\[ N'(t) = 2000(1 + 0.2t)^{-3/5} \]

students/year, \( t \)yrs from now. If the current student enrollment is 1000, find an expression giving the total student enrollment \( t \) yrs from now. What will be the student enrollment \( 5 \)yrs from now?

58. TV on Mobile Phones The number of people watching TV on mobile phones is expected to grow at the rate of

\[ N'(t) = \frac{5.4145}{\sqrt{1 + 0.9t}} \quad (0 \leq t \leq 4) \]

million/year. The number of people watching TV on mobile phones at the beginning of 2007 \((t = 0)\) was 11.9 million.

a. Find an expression giving the number of people watching TV on mobile phones in year \( t \).

b. According to this projection, how many people will be watching TV on mobile phones at the beginning of 2011?

Source: International Data Corporation, U.S. forecast

59. Demand: Women’s Boots The rate of change of the unit price \( p \) (in dollars) of Apex women’s boots is given by

\[ p'(x) = \frac{-250}{(10 + x^2)^{3/2}} \]

where \( x \) is the quantity demanded daily in units of a hundred. Find the demand function for these boots if the quantity demanded daily is 300 pairs \((x = 3)\) when the unit price is $50/pair.

60. Population Growth The population of a certain city is projected to grow at the rate of

\[ r(t) = 400\left(1 + \frac{2t}{24 + t^2}\right) \quad (0 \leq t \leq 5) \]

people/year, \( t \) years from now. The current population is 60,000. What will be the population 5 yr from now?

61. Oil Spill In calm waters, the oil spilling from the ruptured hull of a grounded tanker forms an oil slick that is circular in shape. If the radius \( r \) of the circle is increasing at the rate of

\[ r'(t) = \frac{30}{\sqrt{2t + 4}} \]

feet/minute \( t \) min after the rupture occurs, find an expression for the radius at any time \( t \). How large is the polluted area 16 min after the rupture occurred?

Hint: \( r(0) = 0 \).

62. Life Expectancy of a Female Suppose in a certain country the life expectancy at birth of a female is changing at the rate of

\[ g'(t) = \frac{5.45218}{(1 + 1.09t)^{0.05}} \]

years/year. Here, \( t \) is measured in years, with \( t = 0 \) corresponding to the beginning of 1900. Find an expression \( g(t) \) giving the life expectancy at birth (in years) of a female in that country if the life expectancy at the beginning of 1900 is 50.02 yr. What is the life expectancy at birth of a female born in 2000 in that country?
6.4 Concept Questions

1. State the fundamental theorem of calculus.
2. State the net change formula and use it to answer the following questions:
   a. If a company generates income at the rate of \( R(t) \) dollars/day, explain what \( \int_a^b R(t) \, dt \) measures, where \( a \) and \( b \) are measured in days with \( a < b \).
   b. If a private jet airplane consumes fuel at the rate of \( R \) gal/min, write an integral giving the net fuel consumption by the airplane between times \( t = a \) and \( t = b \) (\( a < b \)), where \( t \) is measured in minutes.

6.4 Exercises

In Exercises 1–4, find the area of the region under the graph of the function \( f \) on the interval \([a, b]\), using the fundamental theorem of calculus. Then verify your result using geometry.

1. \( f(x) = 2; [1, 4] \)
2. \( f(x) = 4; [-1, 2] \)
3. \( f(x) = 2x; [1, 3] \)
4. \( f(x) = \frac{1}{4}x + 1; [1, 4] \)

In Exercises 5–16, find the area of the region under the graph of the function \( f \) on the interval \([a, b]\).

5. \( f(x) = 2x + 3; [-1, 2] \)
6. \( f(x) = 4x - 1; [2, 4] \)
7. \( f(x) = -x^2 + 4; [-1, 2] \)
8. \( f(x) = 4x - x^2; [0, 4] \)
9. \( f(x) = \frac{1}{x^2}; [1, 2] \)
10. \( f(x) = \frac{1}{x^2}; [2, 4] \)
11. \( f(x) = \sqrt{x}; [1, 9] \)
12. \( f(x) = x^3; [1, 3] \)
13. \( f(x) = 1 - \sqrt{x}; [-8, -1] \)
14. \( f(x) = \frac{1}{\sqrt{x}}; [1, 9] \)
15. \( f(x) = e^x; [0, 2] \)
16. \( f(x) = e^x - e; [1, 2] \)

In Exercises 17–40, evaluate the definite integral.

17. \( \int_2^4 3x \, dx \)
18. \( \int_{-2}^2 -2x \, dx \)
19. \( \int_1^3 (2x + 3) \, dx \)
20. \( \int_1^2 (4 - x) \, dx \)
21. \( \int_{-1}^3 2x^2 \, dx \)
22. \( \int_0^2 8x^3 \, dx \)
23. \( \int_{-2}^2 (x^2 - 1) \, dx \)
24. \( \int_0^4 \sqrt{u} \, du \)
25. \( \int_1^4 4x^{1/3} \, dx \)
26. \( \int_1^4 2x^{-3/2} \, dx \)
27. \( \int_0^1 (x^3 - 2x^2 + 1) \, dx \)
28. \( \int_1^2 (r^5 - r^3 + 1) \, dt \)
29. \( \int_0^3 \frac{1}{x} \, dx \)
30. \( \int_1^3 \frac{2}{x} \, dx \)

31. \( \int_0^4 x(x^2 - 1) \, dx \)
32. \( \int_0^2 (x - 4)(x - 1) \, dx \)
33. \( \int_1^3 (r^2 - r)^2 \, dr \)
34. \( \int_1^4 (x^2 - 1)^2 \, dx \)
35. \( \int_0^1 1 \, dx \)
36. \( \int_0^2 \frac{2}{x^3} \, dx \)
37. \( \int_4^\infty \left( \frac{\sqrt{x} - \frac{1}{\sqrt{x}}}{} \right) \, dx \)
38. \( \int_9^\infty \sqrt{2x} \, dx \)
39. \( \int_3^\infty \frac{3x^3 - 2x^2 + 4}{x^2} \, dx \)
40. \( \int_1^\infty \left( \frac{1 + x + 1}{u^3} \right) \, du \)

41. MARGINAL COST A division of Ditton Industries manufactures a deluxe toaster oven. Management has determined that the daily marginal cost function associated with producing these toaster ovens is given by

\[ C'(x) = 0.0003x^2 - 0.12x + 20 \]

where \( C'(x) \) is measured in dollars/unit and \( x \) denotes the number of units produced. Management has also determined that the daily fixed cost incurred in the production is $800.

a. Find the total cost incurred by Ditton in producing the first 300 units of these toaster ovens per day.
b. What is the total cost incurred by Ditton in producing the 201st through 300th units/day?

42. MARGINAL REVENUE The management of Ditton Industries has determined that the daily marginal revenue function associated with selling \( x \) units of their deluxe toaster ovens is given by

\[ R'(x) = -0.1x + 40 \]

where \( R'(x) \) is measured in dollars/unit.

a. Find the daily total revenue realized from the sale of 200 units of the toaster oven.
b. Find the additional revenue realized when the production (and sales) level is increased from 200 to 300 units.

43. MARGINAL PROFIT Refer to Exercise 41. The daily marginal profit function associated with the production and sales of the deluxe toaster ovens is known to be

\[ P'(x) = -0.0003x^2 + 0.02x + 20 \]
where \( x \) denotes the number of units manufactured and sold daily and \( P'(x) \) is measured in dollars/unit.

a. Find the total profit realizable from the manufacture and sale of 200 units of the toaster ovens per day. Hint: \( P(200) = P(0) - \int_0^{200} P'(x) \, dx \), \( P(0) = -800 \).

b. What is the additional daily profit realizable if the production and sale of the toaster ovens are increased from 200 to 220 units/day?

44. INTERNET ADVERTISING U.S. Internet advertising revenue grew at the rate of

\[
R(t) = 0.82t + 1.14 \quad (0 \leq t \leq 4)
\]

billion dollars/year between 2002 \( (t = 0) \) and 2006 \( (t = 4) \). The advertising revenue in 2002 was $5.9 billion.

a. Find an expression giving the advertising revenue in year \( t \).

b. If the trend continued, what was the Internet advertising revenue in 2007?

Source: Interactive Advertising Bureau

45. MOBILE-PHONE AD SPENDING Mobile-phone ad spending is expected to grow at the rate of

\[
R(t) = 0.8256t^0.04 \quad (1 \leq t \leq 5)
\]

billion dollars/year between 2007 \( (t = 1) \) and 2011 \( (t = 5) \). The mobile-phone ad spending in 2007 was $0.9 billion.

a. Find an expression giving the mobile-phone ad spending in year \( t \).

b. If the trend continued, what will be the mobile-phone ad spending in 2012?

Source: Interactive Advertising Bureau

46. EFFICIENCY STUDIES Terapeo Electronics, a division of Tempeo Toys, manufactures an electronic football game. An efficiency study showed that the rate at which the games are assembled by the average worker \( t \) hr after starting work at 8 a.m. is

\[
-\frac{3}{2} t^2 + 6t + 20 \quad (0 \leq t \leq 4)
\]

units/hour.

a. Find the total number of games the average worker can be expected to assemble in the 4-hr morning shift.

b. How many units can the average worker be expected to assemble in the first hour of the morning shift? In the second hour of the morning shift?

47. SPEEDBOAT RACING In a recent pretrial run for the world water speed record, the velocity of the Sea Falcon II \( t \) sec after firing the booster rocket was given by

\[
v(t) = -t^2 + 20t + 440 \quad (0 \leq t \leq 20)
\]

feet/second. Find the distance covered by the boat over the 20-sec period after the booster rocket was activated.

Hint: The distance is given by \( \int_0^{20} v(t) \, dt \).

48. POCKET COMPUTERS Annual sales (in millions of units) of pocket computers are expected to grow in accordance with the function

\[
f(t) = 0.18t^2 + 0.16t + 2.64 \quad (0 \leq t \leq 6)
\]

where \( t \) is measured in years, with \( t = 0 \) corresponding to 1997. How many pocket computers were sold over the 6-yr period between the beginning of 1997 and the end of 2002?

Source: Dataplace, Inc.

49. SINGLE FEMALE-HEADED HOUSEHOLDS WITH CHILDREN The percentage of families with children that are headed by single females grew at the rate of

\[
R(t) = 0.8499r^2 - 3.872r + 5 \quad (0 \leq t \leq 3)
\]

households/decade between 1970 \( (t = 0) \) and 2000 \( (t = 3) \). The number of such households stood at 5.6% of all families in 1970.

a. Find an expression giving the percentage of these households in the \( t \)th decade.

b. If the trend continued, estimate the percentage of these households in 2010.

c. What was the net increase in the percentage of these households from 1970 to 2000?

Source: U.S. Census Bureau

50. AIR PURIFICATION To test air purifiers, engineers ran a purifier in a smoke-filled 10-ft \( \times \) 20-ft room. While conducting a test for a certain brand of air purifier, it was determined that the amount of smoke in the room was decreasing at the rate of

\[
R(t) = -0.00032t^4 - 0.01872t^3 + 0.3948t^2
\]

\[ \quad - 3.83t + 17.63 \quad (0 \leq t \leq 20) \]

percent of the (original) amount of smoke per minute, \( t \) min after the start of the test. How much smoke was left in the room 5 min after the start of the test? Ten min after the start of the test?

Source: Consumer Reports

51. TV SET-TOP BOXES The number of television set-top boxes shipped worldwide from the beginning of 2003 until the beginning of 2009 is projected to be

\[
f(t) = -0.05586t^3 + 0.262t^2 + 17.46t + 63.4
\]

million units/year, where \( t \) is measured in years, with \( t = 0 \) corresponding to 2003. If the projection held true, how many set-top boxes were expected to be shipped from the beginning of 2003 until the beginning of 2009?

Source: InStat

52. CANADIAN OIL-SANDS PRODUCTION The production of oil (in millions of barrels per day) extracted from oil sands in Canada is projected to grow according to the function

\[
P(t) = \frac{4.76}{1 + 4.11e^{-22t}} \quad (0 \leq t \leq 15)
\]

where \( t \) is measured in years, with \( t = 0 \) corresponding to 2005. What is the total production of oil from oil sands over the years from 2005 until 2020 \( (t = 15) \)?

Hint: Multiply the integrand by \( \frac{e^{22t}}{e^{22t}} \).

Source: Canadian Association of Petroleum Producers.
6.5 Concept Questions

1. Describe two approaches used to evaluate a definite integral using the method of substitution. Illustrate with the integral \( \int_0^1 x^2(x^3 + 1)^2 \, dx \).

2. Define the average value of a function \( f \) over an interval \([a, b]\). Give a geometric interpretation.

6.5 Exercises

In Exercises 1–28, evaluate the definite integral.

1. \( \int_0^1 x^2(x^2 - 1)^3 \, dx \)

2. \( \int_0^1 x^2(2x^3 - 1)^3 \, dx \)

3. \( \int_0^1 x \sqrt{5x^2 + 4} \, dx \)

4. \( \int_0^1 x \sqrt{3x^2 - 2} \, dx \)

5. \( \int_0^1 x^2(x^3 + 1)^2 \, dx \)

6. \( \int_0^1 (2x - 1)^5/2 \, dx \)

7. \( \int_0^1 \frac{1}{\sqrt{2x + 1}} \, dx \)

8. \( \int_0^2 \frac{x}{\sqrt{x^2 + 5}} \, dx \)

9. \( \int_1^2 (2x - 1)^4 \, dx \)

10. \( \int_1^2 (2x + 4)(x^2 + 4x - 8)^3 \, dx \)

11. \( \int_{-1}^1 x^2(x^3 + 1)^2 \, dx \)

12. \( \int_{-1}^1 \left( x^3 + \frac{3}{4} \right)(x^5 + 3x)^{-2} \, dx \)

13. \( \int_{-1}^1 x \sqrt{x - 1} \, dx \)

14. \( \int_{-1}^1 x \sqrt{x + 1} \, dx \)

Hint: Let \( u = x + 1 \).

15. \( \int_0^1 xe^2 \, dx \)

16. \( \int_0^1 e^{-x} \, dx \)

17. \( \int_0^1 (e^{2x} + x^2) \, dx \)

18. \( \int_0^1 \left( e^{t} - e^{-t} \right) \, dt \)

19. \( \int_{-1}^1 xe^{x^2 + 1} \, dx \)

20. \( \int_{-1}^1 \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx \)

21. \( \int_{-1}^1 \frac{2}{x^2} \, dx \)

22. \( \int_{-1}^1 \frac{x}{1 + 2x^2} \, dx \)

23. \( \int_{-1}^1 \frac{x^2}{x^2 + 3x^2 - 1} \, dx \)

24. \( \int_{-1}^1 e^{2x} \, dx \)

25. \( \int_{-1}^1 \left( 4e^{2u} - \frac{1}{u} \right) \, du \)

26. \( \int_{-1}^1 \left( 1 + \frac{1}{x} + e^x \right) \, dx \)

27. \( \int_{-1}^1 \left( 2e^{-u} - \frac{1}{x} \right) \, dx \)

28. \( \int_{-1}^1 \frac{\ln x}{x} \, dx \)

In Exercises 29–34, find the area of the region under the graph of \( f \) on \([a, b]\).

29. \( f(x) = x^2 - 2x + 2; [-1, 2] \)

30. \( f(x) = x^3 + x; [0, 1] \)

31. \( f(x) = \frac{1}{x^2}; [1, 2] \)

32. \( f(x) = 2 + \sqrt{x + 1}; [0, 3] \)

33. \( f(x) = e^{-x^2}; [-1, 2] \)

34. \( f(x) = \frac{\ln x}{4x}; [1, 2] \)

In Exercises 35–44, find the average value of the function \( f \) over the indicated interval \([a, b]\).

35. \( f(x) = 2x^2 + 3; [0, 2] \)

36. \( f(x) = 8 - x; [1, 4] \)

37. \( f(x) = 2x^2 - 3; [1, 3] \)

38. \( f(x) = 4 - x^2; [-2, 3] \)

39. \( f(x) = x^2 + 2x - 3; [-1, 2] \)

40. \( f(x) = x^5; [-1, 1] \)

41. \( f(x) = \sqrt{2x + 1}; [0, 4] \)

42. \( f(x) = e^{-x}; [0, 4] \)

43. \( f(x) = xe^{-x}; [0, 2] \)

44. \( f(x) = \frac{1}{x + 1}; [0, 2] \)

45. **World Production of Coal.** A study proposed in 1980 by researchers from the major producers and consumers of the world's coal concluded that coal could and must play an important role in fueling global economic growth over the next 20 yr. The world production of coal in 1980 was 3.5 billion metric tons. If output increased at the rate of 3.5\% annual billion metric tons/year in year \( t \) (\( t = 0 \) corresponding to 1980), determine how much coal was produced worldwide between 1980 and the end of the 20th century.

46. **Newton's Law of Cooling.** A bottle of white wine at room temperature (68°F) is placed in a refrigerator at 4 p.m. Its temperature after \( t \) hr is changing at the rate of

\[ -18e^{-0.6t} \text{°F/hour} \]

How many degrees will the temperature of the wine have dropped by 7 p.m.? What will the temperature of the wine be at 7 p.m.?

47. **Net Investment Flow.** The net investment flow (rate of capital formation) of the giant conglomerate LTF incorporated is projected to be

\[ f \sqrt{\frac{1}{\frac{1}{2} t^2 + 1}} \]
6.6 Self-Check Exercises

1. Find the area of the region bounded by the graphs of \( f(x) = x^2 + 2 \) and \( g(x) = 1 - x \) and the vertical lines \( x = 0 \) and \( x = 1 \).

2. Find the area of the region completely enclosed by the graphs of \( f(x) = -x^2 + 6x + 5 \) and \( g(x) = x^2 + 5 \).

3. The management of Kane Corporation, which operates a chain of hotels, expects its profits to grow at the rate of \( 1 + t^{0.5} \) million dollars/year \( t \) years from now. However, with renovations and improvements of existing hotels and proposed acquisitions of new hotels, Kane’s profits are expected to grow at the rate of \( t - 2\sqrt{t} + 4 \) million dollars/year in the next decade. What additional profits are expected over the next 10 yr if the group implements the proposed plans?

Solutions to Self-Check Exercises 6.6 can be found on page 463.

6.6 Concept Questions

1. Suppose \( f \) and \( g \) are continuous functions such that \( f(x) \geq g(x) \) on the interval \([a, b]\). Write an integral giving the area of the region bounded above by the graph of \( f \), below by the graph of \( g \), and on the left and right by the lines \( x = a \) and \( x = b \).

2. Write an expression in terms of definite integrals giving the area of the shaded region in the following figure:

6.6 Exercises

In Exercises 1–8, find the area of the shaded region.

1. \( y = x^2 - 6x^2 \)

2. \( y = x^4 - 2x^3 \)

3. \( y = x - 2x^2 \)

4. \( y = \frac{2x}{x^4 + 4} \)
5. \( f(x) = \frac{1}{2} x - \sqrt{x}; a = 0, b = 4 \)
6. \( f(x) = -e^{0.5x}; a = -2, b = 4 \)
7. \( f(x) = -xe^{-x}; a = 0, b = 1 \)

In Exercises 17–26, sketch the graphs of the functions \( f \) and \( g \) and find the area of the region enclosed by these graphs and the vertical lines \( x = a \) and \( x = b \).
8. \( f(x) = x^2 + 3; g(x) = 1; a = 1, b = 3 \)
9. \( f(x) = x + 2; g(x) = x^2 - 4; a = -1, b = 2 \)
10. \( f(x) = -x^2 + 2x + 3; g(x) = -x + 3; a = 0, b = 2 \)
11. \( f(x) = 9 - x^2; g(x) = 2x + 3; a = -1, b = 1 \)
12. \( f(x) = x^2 + 1; g(x) = \frac{1}{3} x^3; a = -1, b = 2 \)
13. \( f(x) = \sqrt{x}; g(x) = \frac{1}{2} x - 1; a = 1, b = 4 \)

In Exercises 27–34, sketch the graph and find the area of the region bounded by the graph of the function \( f \) and the lines \( y = 0, x = a, \) and \( x = b \).
14. \( f(x) = \frac{1}{x}; g(x) = 2x - 1; a = 1, b = 4 \)
15. \( f(x) = x^2; g(x) = \frac{1}{x^2}; a = 1, b = 3 \)
16. \( f(x) = e^x; g(x) = \frac{1}{x}; a = 1, b = 2 \)
17. \( f(x) = x, g(x) = e^{2x}; a = 1, b = 3 \)

In Exercises 35–42, sketch the graph and find the area of the region completely enclosed by the graphs of the given functions \( f \) and \( g \).
18. \( f(x) = x + 2 \) and \( g(x) = x^2 - 4 \)
19. \( f(x) = -x^2 + 4x - 3; a = -1, b = 2 \)
20. \( f(x) = x^3 - x^2; a = -1, b = 1 \)
21. \( f(x) = x^3 - 4x^2 + 3x; a = 0, b = 2 \)
22. \( f(x) = 4x^{1/3} + x^{2/3}; a = -1, b = 8 \)
23. \( f(x) = e^x - 1; a = -1, b = 3 \)
24. \( f(x) = xe^x; a = 0, b = 2 \)
37. \( f(x) = x^2 \) and \( g(x) = x^3 \)
38. \( f(x) = x^3 + 2x^2 - 3x \) and \( g(x) = 0 \)
39. \( f(x) = x^3 - 6x^2 + 9x \) and \( g(x) = x^2 - 3x \)
40. \( f(x) = \sqrt{x} \) and \( g(x) = x^2 \)
41. \( f(x) = x\sqrt{9 - x^2} \) and \( g(x) = 0 \)
42. \( f(x) = 2x \) and \( g(x) = x\sqrt{x + 1} \)

43. **Effect of Advertising on Revenue** In the accompanying figure, the function \( f \) gives the rate of change of Odyssey Travel’s revenue with respect to the amount \( x \) it spends on advertising with their current advertising agency. By engaging the services of a different advertising agency, it is expected that Odyssey’s revenue will grow at the rate given by the function \( g \). Give an interpretation of the area \( A \) of the region \( S \) and find an expression for \( A \) in terms of a definite integral involving \( f \) and \( g \).

44. **Pulse Rate During Exercise** In the accompanying figure, the function \( f \) gives the rate of increase of an individual’s pulse rate when he walked a prescribed course on a treadmill 6 mo ago. The function \( g \) gives the rate of increase of his pulse rate when he recently walked the same prescribed course. Give an interpretation of the area \( A \) of the region \( S \) and find an expression for \( A \) in terms of a definite integral involving \( f \) and \( g \).

45. **Oil Production Shortfall** Energy experts disagree about when global oil production will begin to decline. In the following figure, the function \( f \) gives the annual world oil production in billions of barrels from 1980 to 2050, according to the Department of Energy projection. The function \( g \) gives the world oil production in billions of barrels per year over the same period, according to longtime petroleum geologist Colin Campbell. Find an expression in terms of the definite integrals involving \( f \) and \( g \), giving the shortfall in the total oil production over the period in question heed- ing Campbell’s dire warnings.

Source: U.S. Department of Energy, Colin Campbell

46. **Air Purification** To study the effectiveness of air purifiers in removing smoke, engineers ran each purifier in a smoke-filled 10-ft \( \times \) 20-ft room. In the accompanying figure, the function \( f \) gives the rate of change of the smoke level/minute, \( t \) min after the start of the test, when a brand A purifier is used. The function \( g \) gives the rate of change of the smoke level/minute when a brand B purifier is used.

   a. Give an interpretation of the area of the region \( S \).
   b. Find an expression for the area of \( S \) in terms of a definite integral involving \( f \) and \( g \).

47. Two cars start out side by side and travel along a straight road. The velocity of car 1 is \( f(t) \) ft/sec, the velocity of car 2 is \( g(t) \) ft/sec over the interval \([0, T] \), and \( 0 < T_1 < T \). Furthermore, suppose the graphs of \( f \) and \( g \) are as depicted in the accompanying figure. Let \( A_1 \) and \( A_2 \) denote the areas of the regions (shown shaded).

   a. Write the number
   \[
   \int_{T_1}^{T} [g(t) - f(t)] \, dt - \int_{0}^{T_1} [f(t) - g(t)] \, dt
   \]
   in terms of \( A_1 \) and \( A_2 \).
   b. What does the number obtained in part (a) represent?