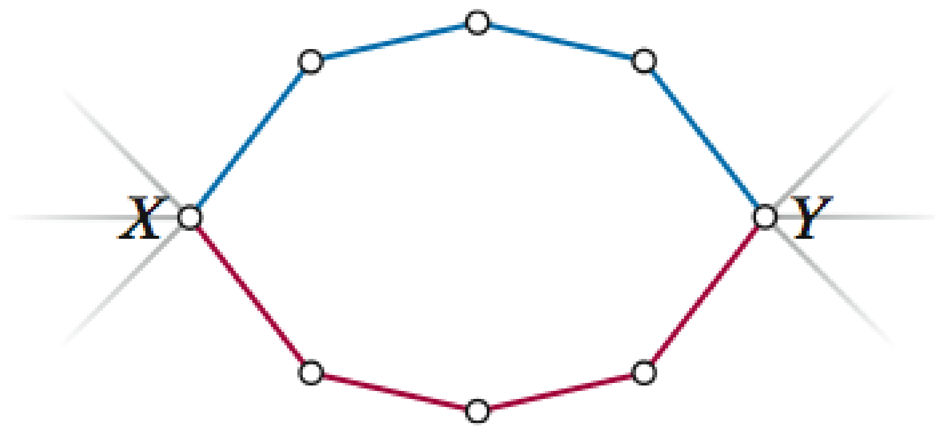
Hour-Exam on Friday, June 21

- Covers material in Chapters 1, 2, 3
- Bring #2 pencil and picture ID.
- You may use a calculator.
- You may NOT use cell phones or other wireless devices.
- You may NOT use books or notes.
- There will be room on the exam paper for calculations.

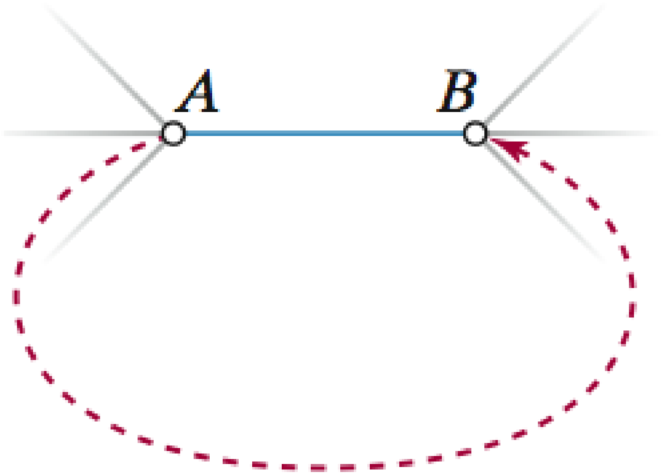
Property 1 of Trees

For any two vertices *X* and *Y* of a tree, there is one and *only one* path joining *X* to *Y*. (If there were *two different* paths joining *X* and *Y*, then these two paths would form a circuit, as shown.)



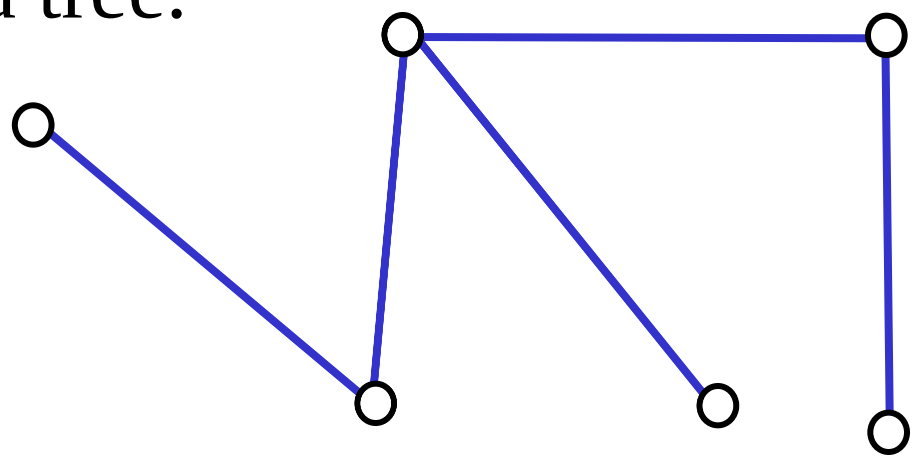
Property 2 of Trees

Every edge of a tree is a *bridge*, i.e., if the edge is removed, then the graph becomes disconnected. (If the graph is still connected without the edge *AB*, then there must be an alternative path from *A* to *B*. This would imply that the edge *AB* is part of a circuit as illustrated.)



Property 3 of Trees

- Among all networks with N vertices, a tree is the one with the *fewest* number of edges.
- If a tree has N vertices, it has exactly $N - 1$ edges.
- If a network has N vertices and $N - 1$ edges, then it is a tree.



Redundancy of a network

If a network has N vertices and M edges, then $M \geq N - 1$.

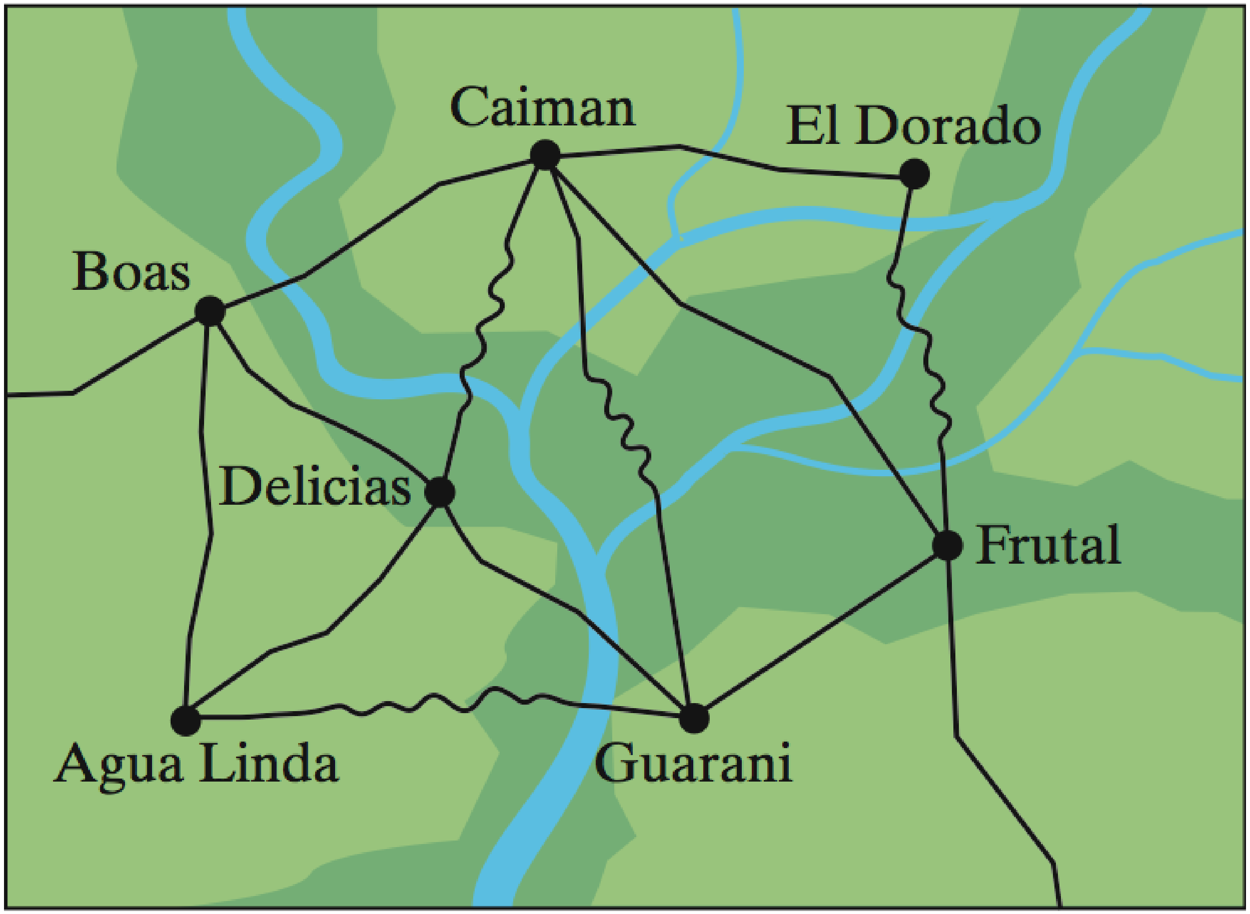
The difference $R = M - (N - 1)$ is called the **redundancy** of the network.

If $M = N - 1$, then $R = 0$ and the network is a tree.

If $M > N - 1$, then $R > 0$ and the network has circuits and is not a tree.

Example 3.1 The Amazonian Cable Network

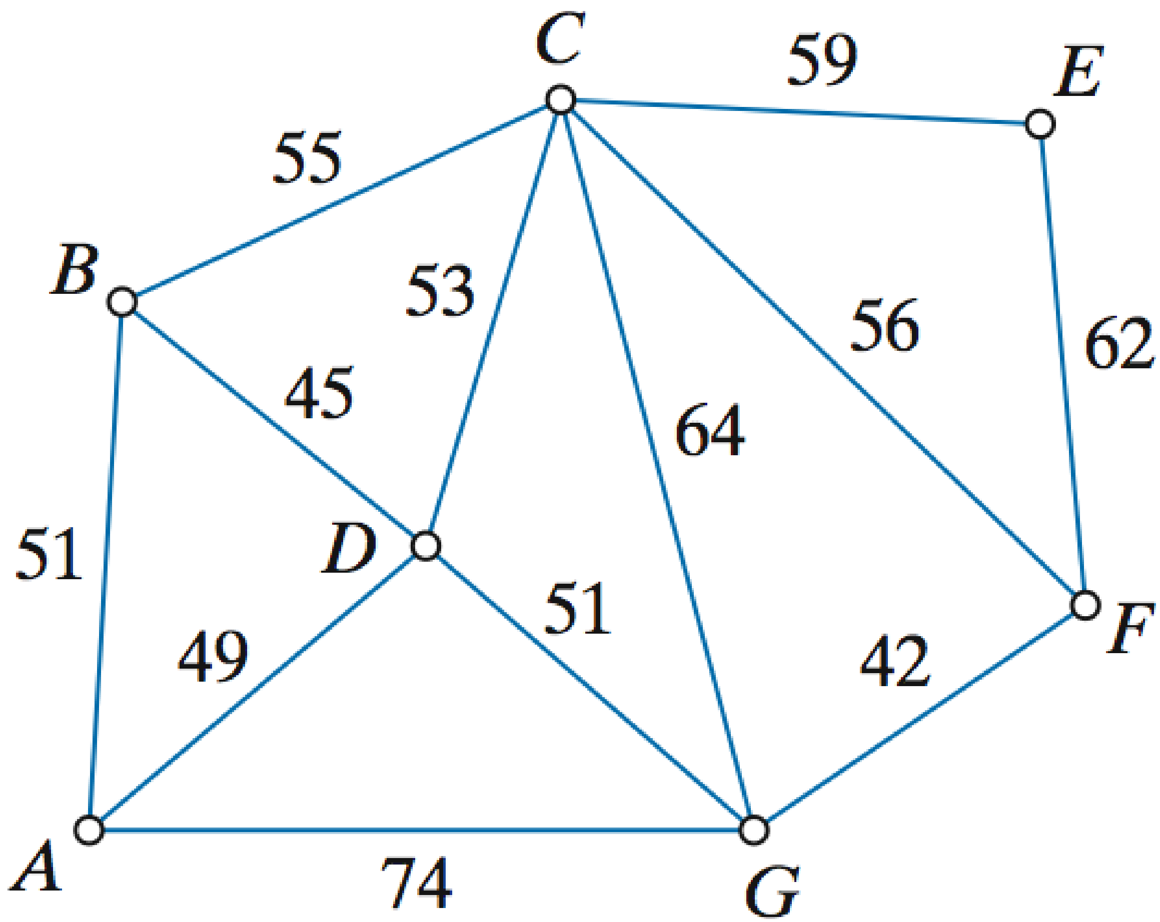
The Amazonia Telephone Company is contracted to provide telephone, cable, and Internet service to the seven small mining towns shown.



Example 7.1 The Amazonian Cable Network

Weighted graph model:

The vertices represent the towns, the edges represent the existing roads, and the weights represent the cost of building a link along that edge.



Our goals

1. Use predetermined pathways for the links.
   Choose a subgraph of the original graph, i.e. choose a graph that only contains edges from the original graph.
2. Provide service between any pair of cities.
   The subgraph must be connected and span all vertices (include all vertices).
3. Minimize the total cost of building the links.
   Find the subgraph with the smallest total weight.

To find the minimum spanning tree (MST):

KRUSKAL'S ALGORITHM

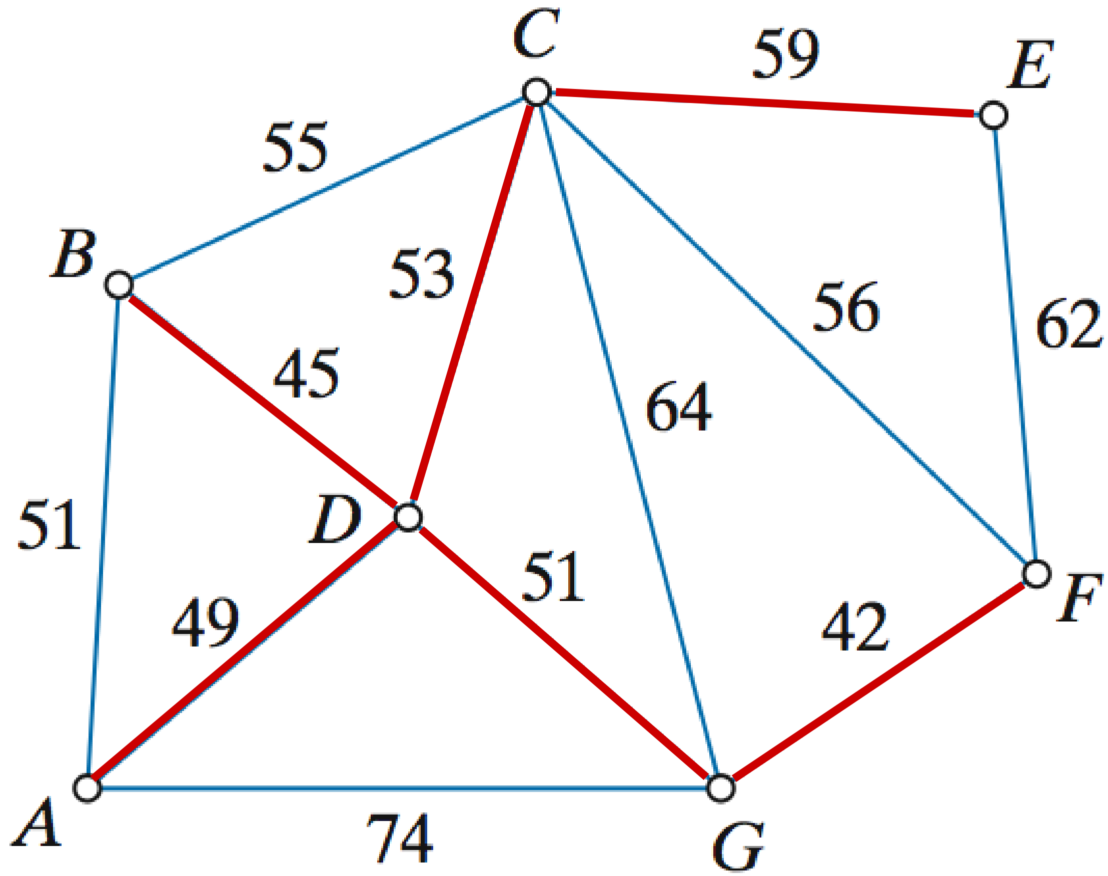
Steps At each stage, pick the cheapest link available. (In case of a tie, pick one at random.)

Rule Do not pick a link if it creates a circuit.

End Stop when all vertices have been spanned.

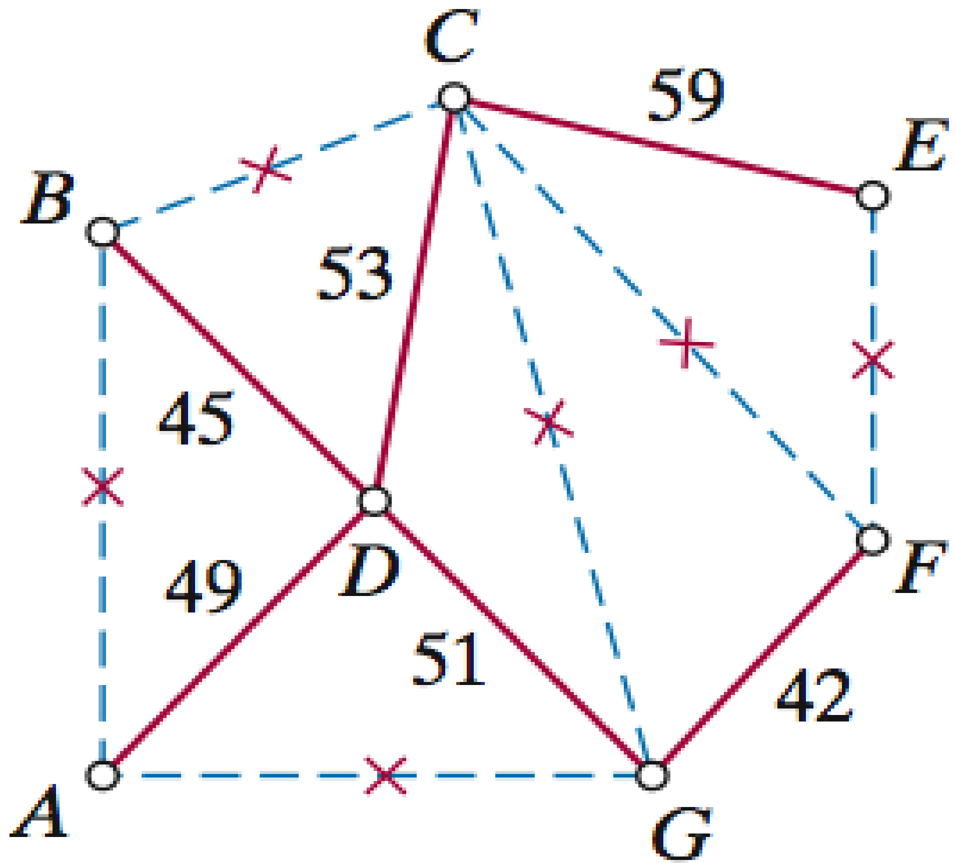
Example 3.7 The Amazonian Cable Network: Part 2

Use Kruskal's algorithm to find the MST. Because there are seven vertices, we use six edges.



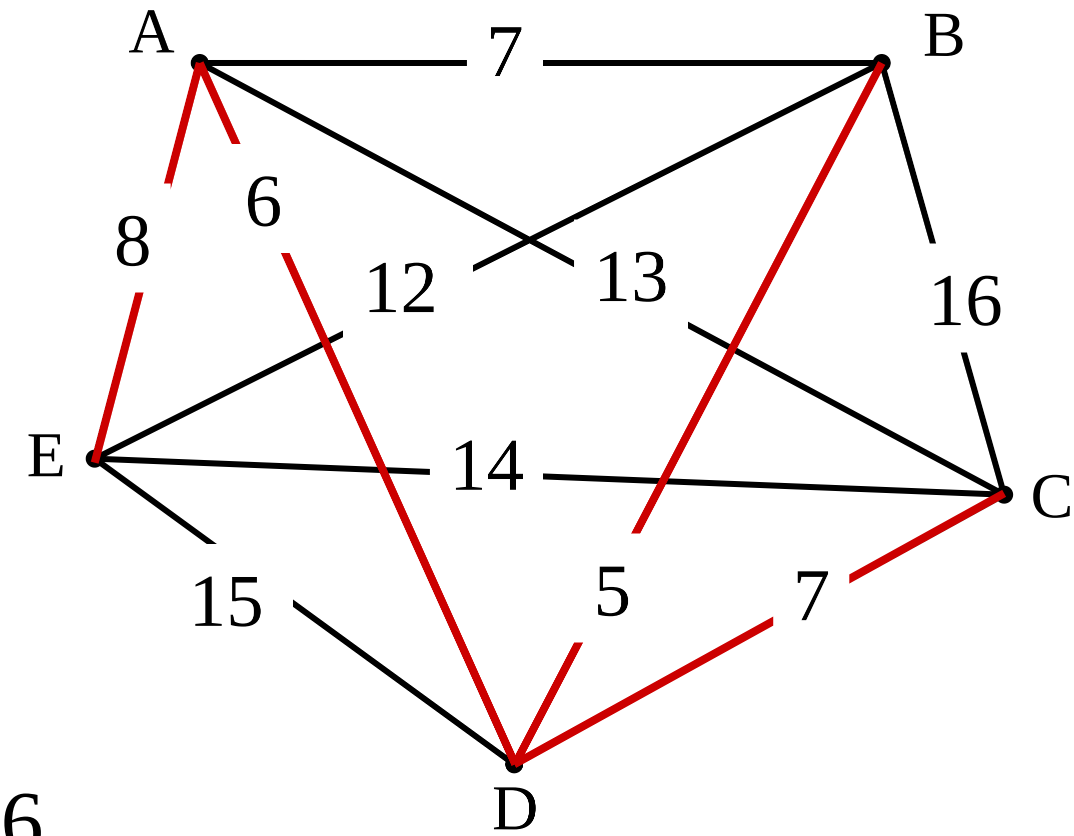
Example 3.7 The Amazonian Cable Network: Part 2

The figure shows the MST in red. The total cost of the network is 299 (million dollars).



Another Example

Use Kruskal's algorithm to find an MST for this graph.



The total cost is 26.