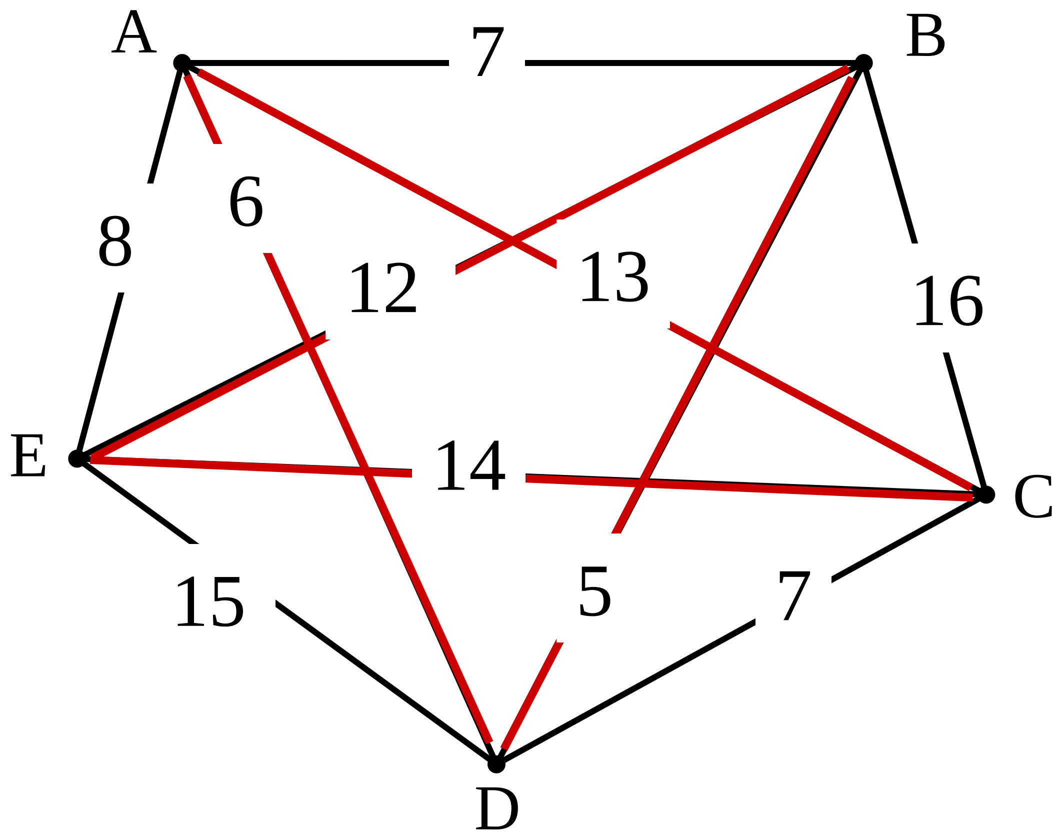
Hour-Exam on Friday, June 21

- Covers material in Chapters 1, 2, 3
- Bring #2 pencil and picture ID.
- You may use a calculator.
- You may NOT use cell phones or other wireless devices.
- You may NOT use books or notes.
- There will be room on the exam paper for calculations.

Last Friday's Question 2

What is the total weight for the Hamilton circuit generated by the Nearest-Neighbor Algorithm starting at A?



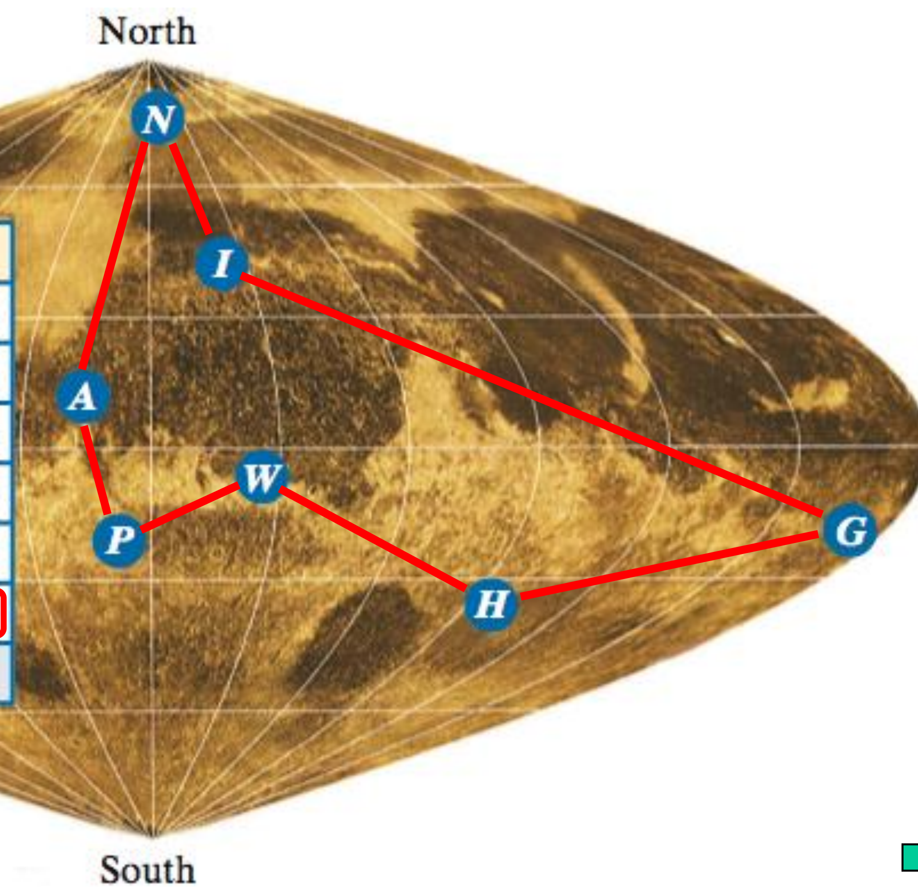
6 + 5 + 12 + 14 + 13 = 50

Example 2.12 Roving the Red Planet

What is the tour if we use the Nearest-Neighbor Algorithm starting at A?

Total length: 20,100 miles

	A	G	H	I	N	P	W
A	*	7500	5000	2800	3500	1500	2200
G	7500	*	3000	6000	8000	6500	5000
H	5000	3000	*	4000	4800	3500	2800
I	2800	6000	4000	*	2000	3000	2900
N	3500	8000	4800	2000	*	4000	3200
P	1500	6500	3500	3000	4000	*	1300
W	2200	5000	2800	2900	3200	1300	*



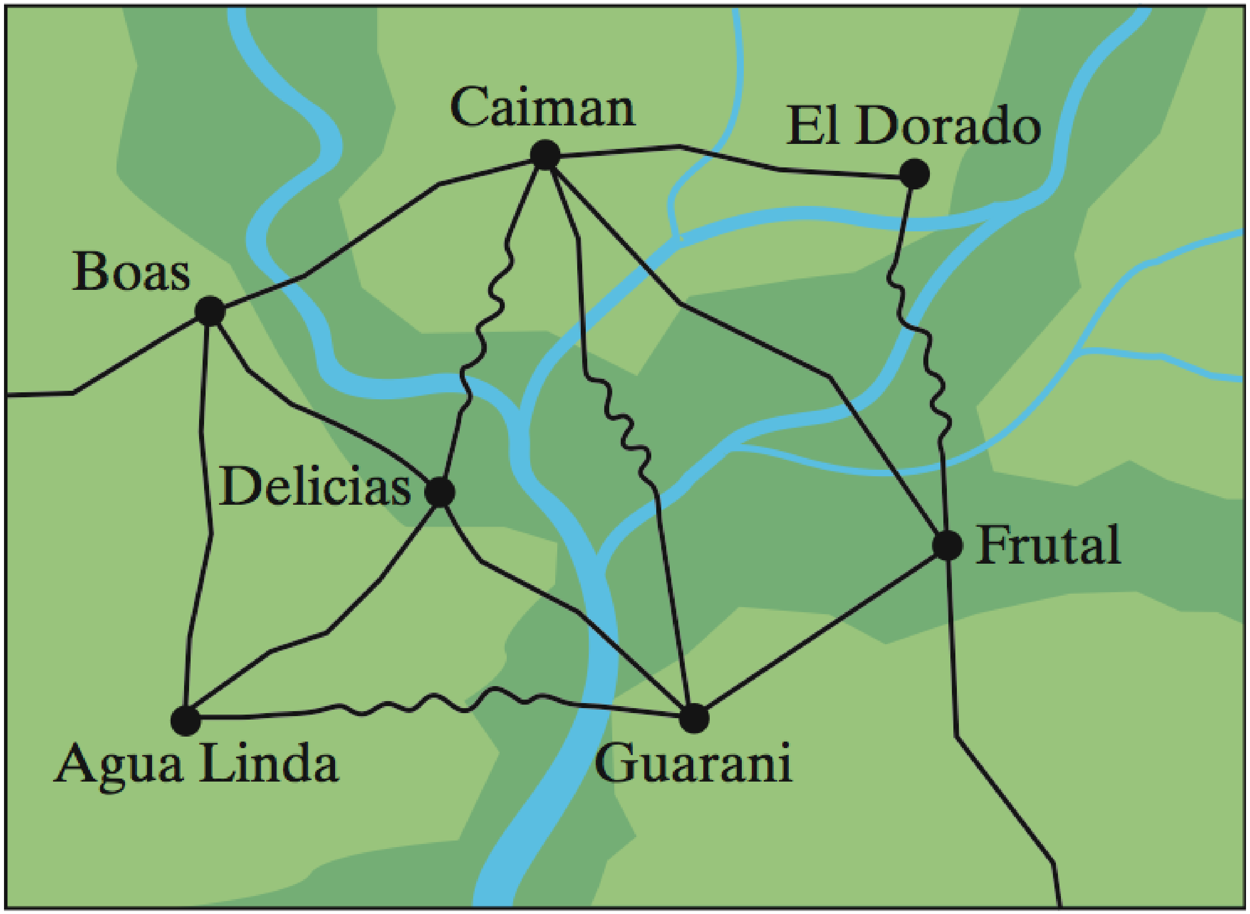
Network

A network is a graph that is connected.

Typically, the vertices of a network (sometimes called nodes or terminals) are “objects” – transmitting stations, computer servers, places, cell phones, people, and so on. The edges of a network (which in this context are often called links) indicate connections among the objects – wires, cables, roads, Internet connections, social connections, and so on.

Example 3.1 The Amazonian Cable Network

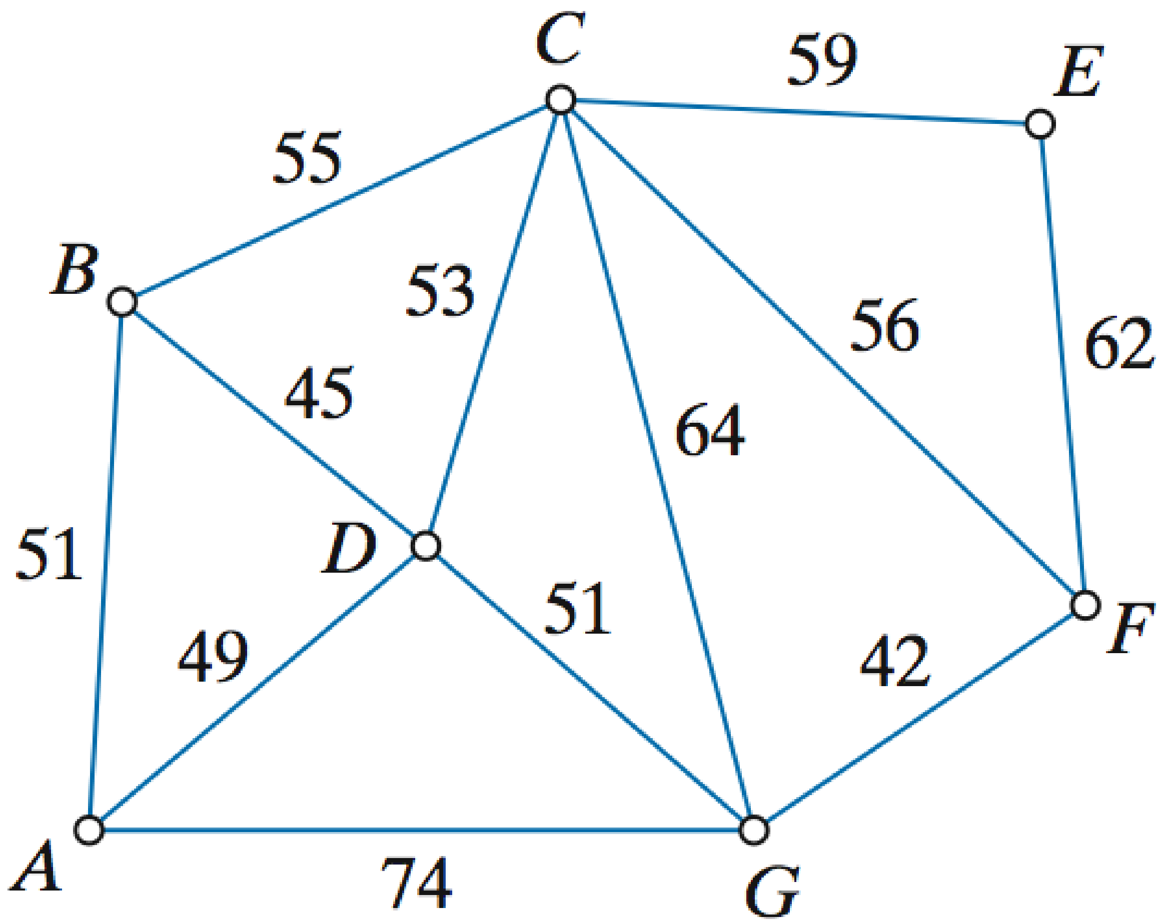
The Amazonia Telephone Company is contracted to provide telephone, cable, and Internet service to the seven small mining towns shown.



Example 7.1 The Amazonian Cable Network

Weighted graph model:

The vertices represent the towns, the edges represent the existing roads, and the weights represent the cost of building a link along that edge.

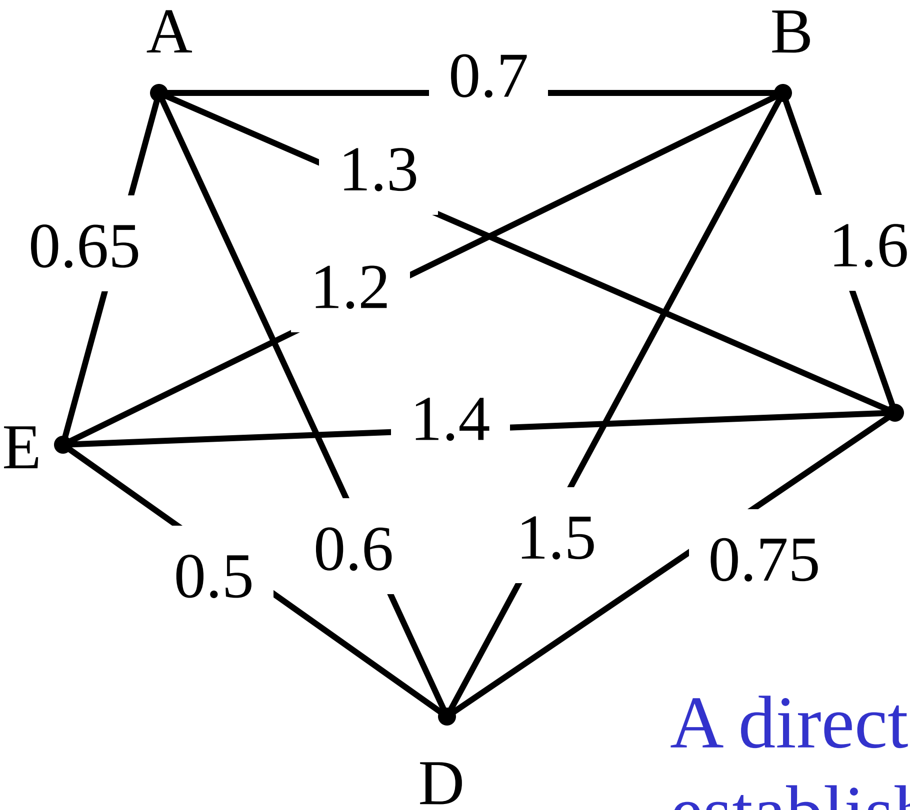


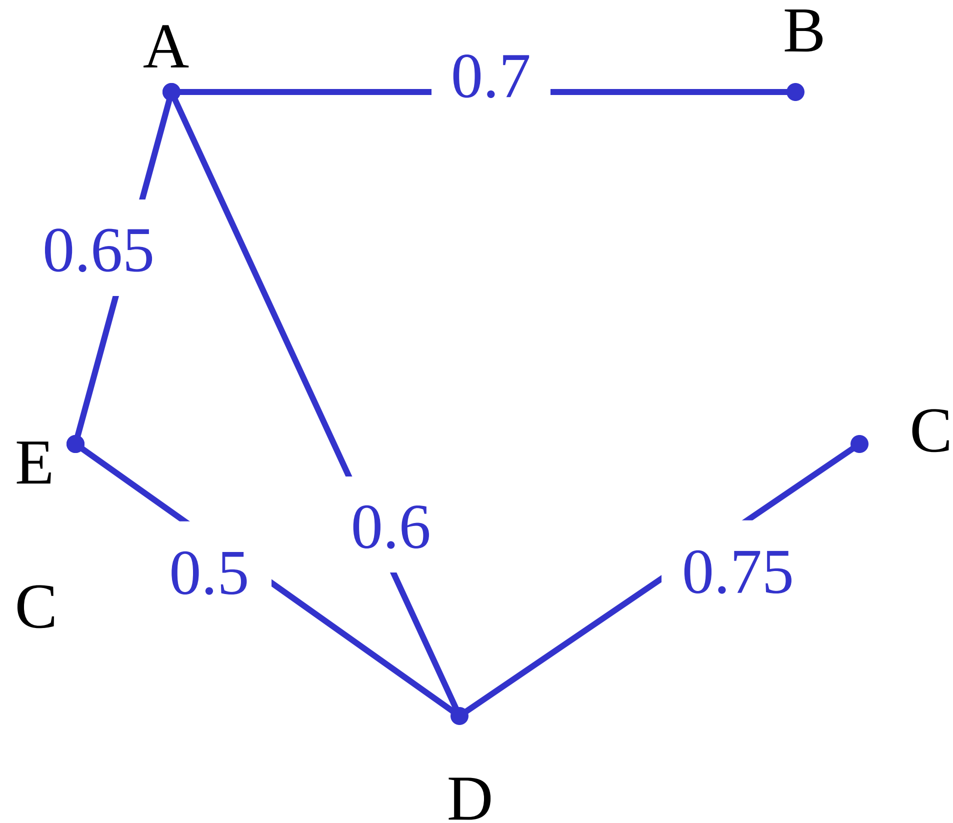
Communication Network

1. A **direct** communication link is not necessary for communicating between two cities.
2. The cost of building the links is our primary concern.
3. The cost of relaying a message can be neglected.

Connection Subgraph

Complete weighted graph:





A direct connection is not needed to establish a connection between every pair of cities.

Our goals

1. Use predetermined pathways for the links.
   Choose a subgraph of the original graph, i.e. choose a graph that only contains edges from the original graph.
2. Provide service between any pair of cities.
   The subgraph must be connected and span all vertices (include all vertices).
3. Minimize the total cost of building the links.
   Find the subgraph with the smallest total weight.

Minimal Network - No Circuits

A minimal network *cannot have any circuits*. Why not? A circuit containing the edge *XY* gives two paths to connect *X* and *Y*. The edge *XY* is redundant – it is not required to make all connections in the network.

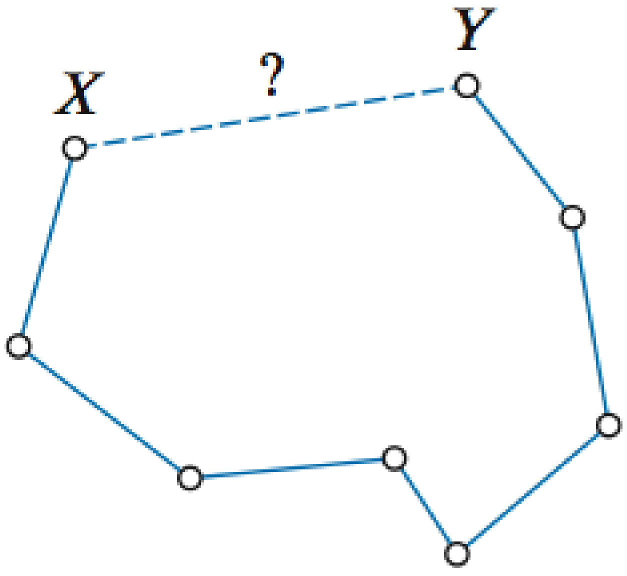
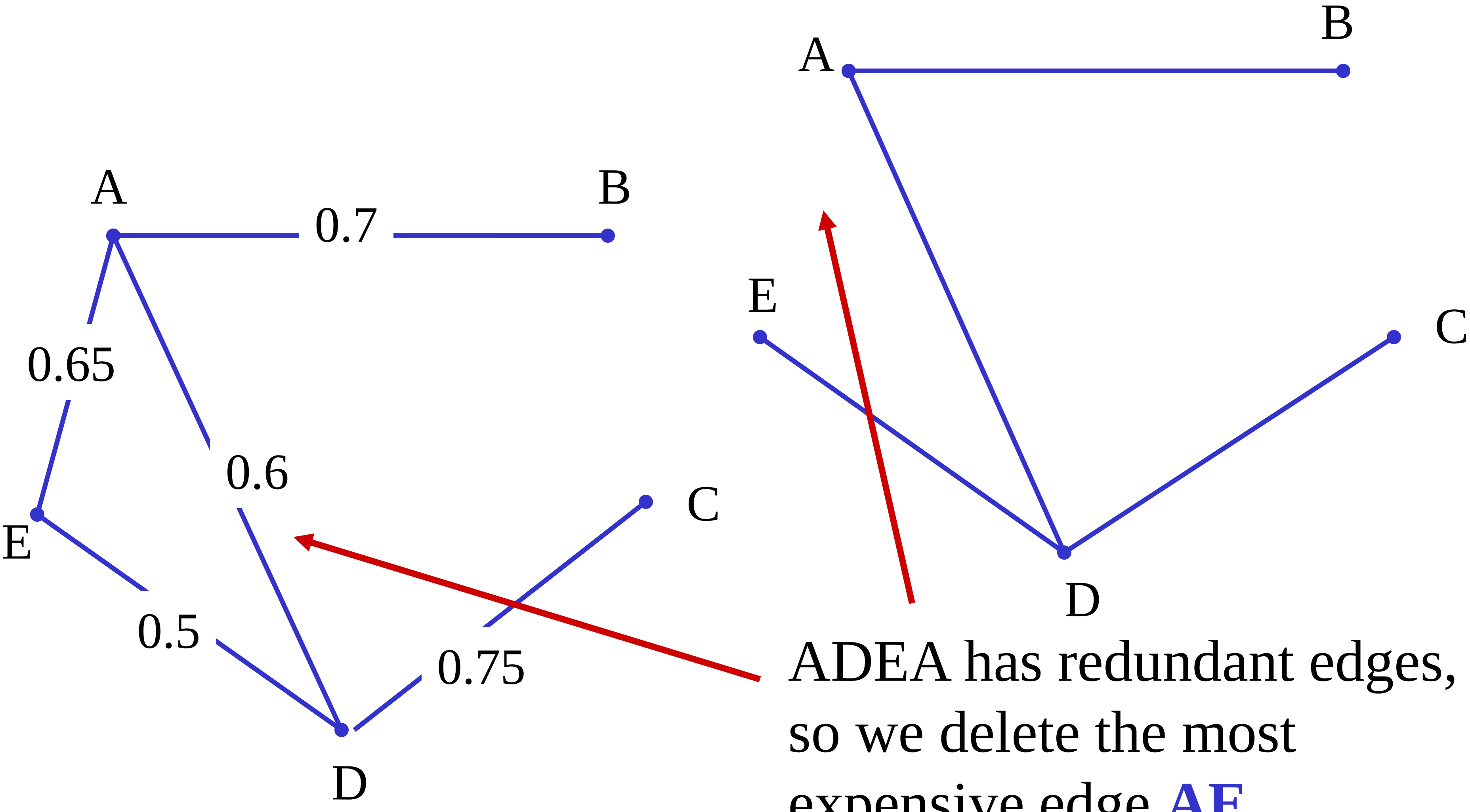


Figure 3-2

Minimizing Cost



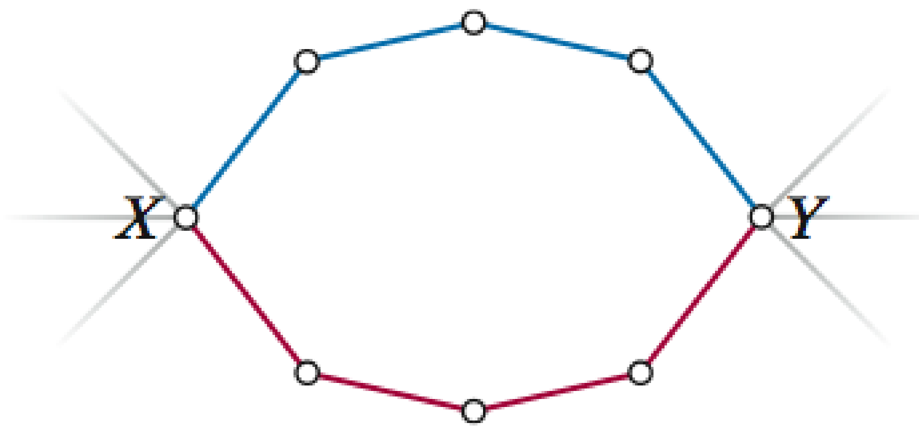
ADEA has redundant edges, so we delete the most expensive edge **AE**.

Definitions

- A **tree** is a network (connected graph) that contains no circuits.
- A subgraph that is a tree and contains all of the vertices of the original graph is called a **spanning tree** of the original graph.
- Among all spanning trees of a weighted graph, one with least total weight is called a **minimum spanning tree** (MST) of the graph.

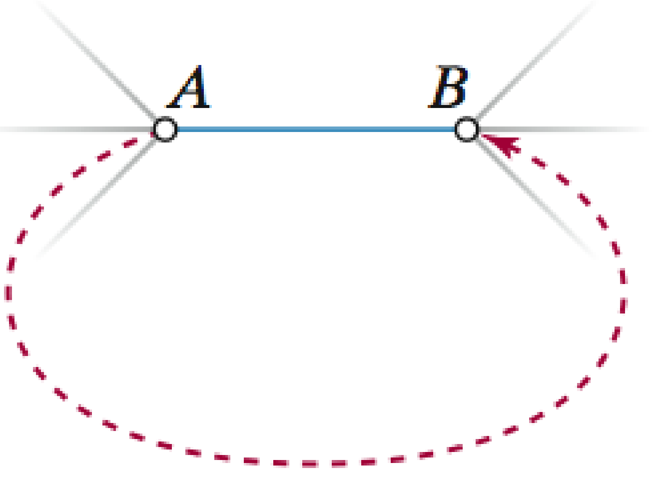
Property 1 of Trees

For any two vertices *X* and *Y* of a tree, there is one and *only one* path joining *X* to *Y*. (If there were *two different* paths joining *X* and *Y*, then these two paths would form a circuit, as shown.)



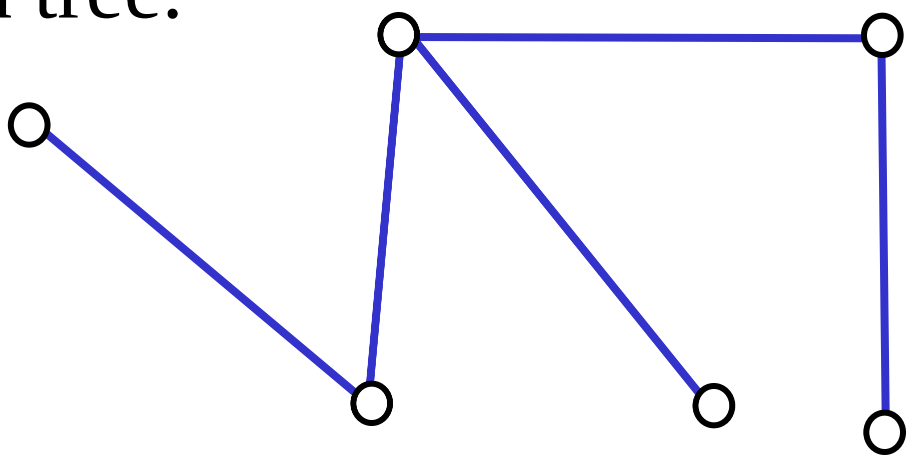
Property 2 of Trees

Every edge of a tree is a *bridge*, i.e., if the edge is removed, then the graph becomes disconnected. (If the graph is still connected without the edge *AB*, then there must be an alternative path from *A* to *B*. This would imply that the edge *AB* is part of a circuit as illustrated.)



Property 3 of Trees

- Among all networks with N vertices, a tree is the one with the *fewest* number of edges.
- If a tree has N vertices, it has exactly $N - 1$ edges.
- If a network has N vertices and $N - 1$ edges, then it is a tree.



Redundancy of a network

If a network has N vertices and M edges, then $M \geq N - 1$.

The difference $R = M - (N - 1)$ is called the **redundancy** of the network.

If $M = N - 1$, then $R = 0$ and the network is a tree.

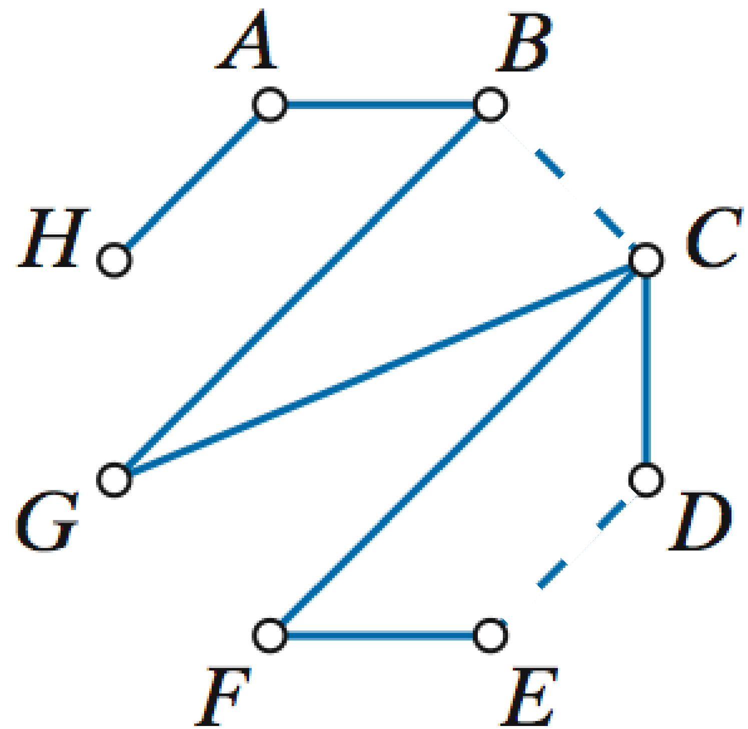
If $M > N - 1$, then $R > 0$ and the network has circuits and is not a tree.

Example 3.4 Spanning Trees

The network in the Figure has $M = 9$ edges and $N = 8$ vertices. The redundancy of the network is

$$R = M - (N - 1) = 2$$

so to find a spanning tree we will have to discard two edges.



For example, BC and DE.