Need more time in My Labs Plus?

Sunday	Monday	Tuesday	Wednesday	Thursday
Recitation Sessions in Faner:				
	Faner 1032 11:00-12:00 Sec 201		Faner 1032 11:00-12:00 Sec 203	Faner 1024 1:20-2:20 Sec 206
Open Labs (in Morris Library Room 174):				
6 – 9 pm	1:30 – 4 pm	2 – 5 pm	1 – 4 pm	2 – 5 pm

Or access your homework from almost any computer connected to the internet.
(Perform a browser check in My Labs Plus first to make sure that you have everything you need.)

Problems Using Graphs

1. For a network of city streets, visit each service address to read the water meter.
2. Use the network of streets in a city to check each traffic light following a power outage.

Solutions:

1. Construct a graph where each edge represents a street with water meters. Find an Euler circuit (or Eulerize the graph and find an Euler circuit).
2. Construct a graph where each vertex is a corner with a traffic light and the edges represent routes between vertices. Find a circuit which visits each vertex exactly once.

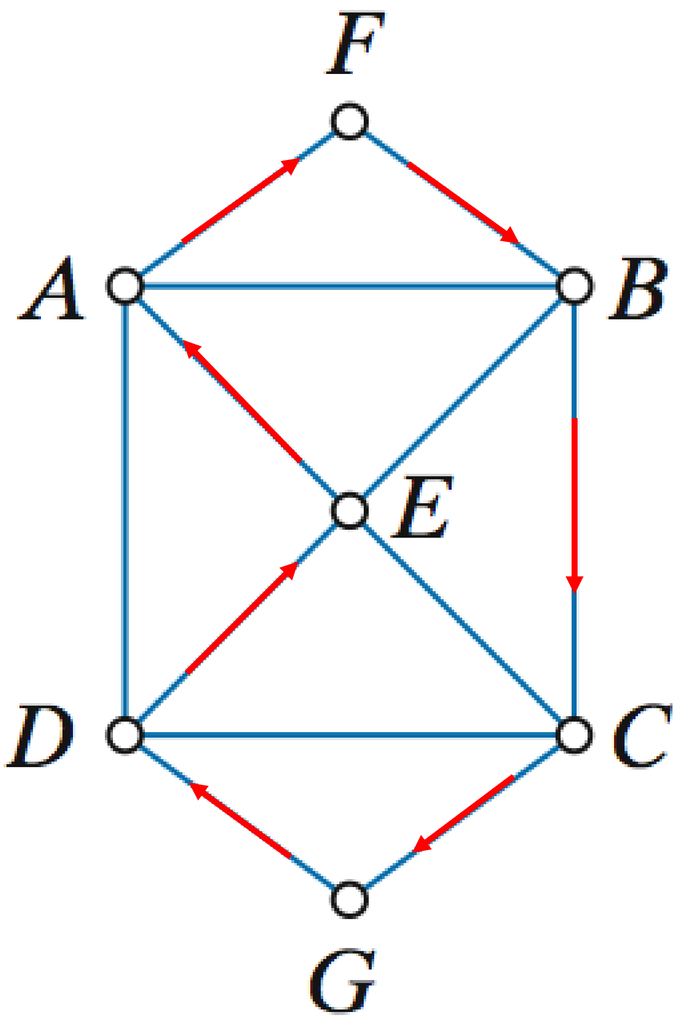
HAMILTONIAN CIRCUITS

A Hamiltonian circuit is a circuit that includes each vertex of the graph once and only once. (At the end, of course, the circuit must return to the starting vertex.)

Example 2.1 Hamilton versus Euler

The figure shows a graph that (1) has Euler circuits (the vertices are all even) and (2) has Hamiltonian circuits.

One such Hamiltonian circuit is *A*, *F*, *B*, *C*, *G*, *D*, *E*, *A* – there are plenty more.

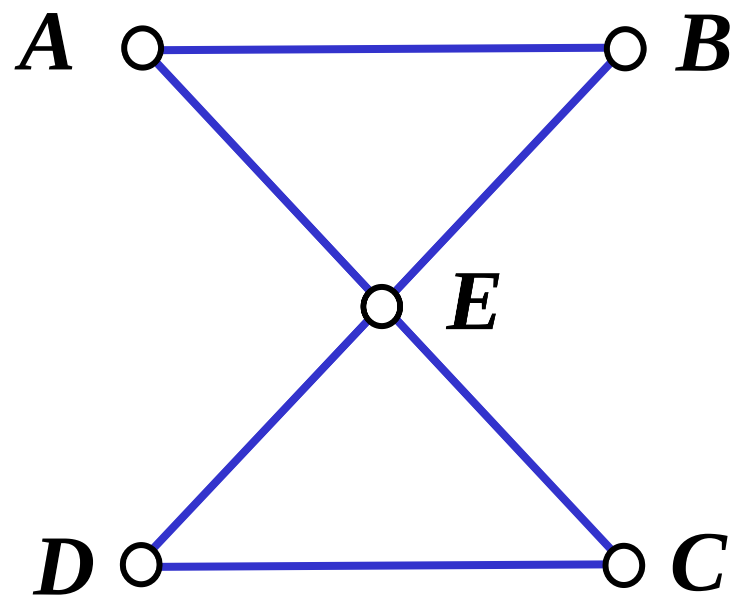


Example 2.1 Hamilton versus Euler

This graph:

(1) has Euler circuits (the vertices are all even) and (2) has no Hamiltonian circuits.

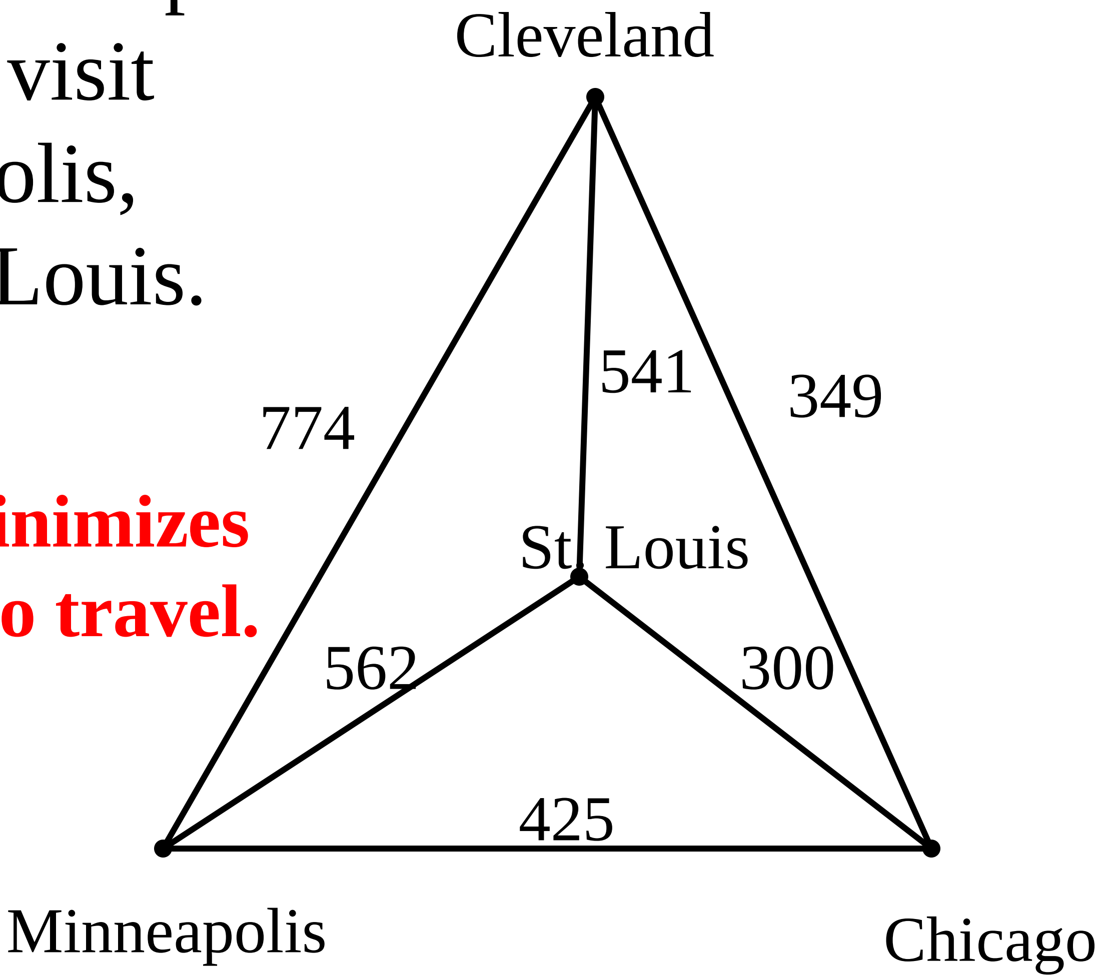
For example, the path *A*, *B*, *E*, *D*, *C* visits each vertex, but you have to go through *E* again to get back to *A* and complete the circuit.



Planning a trip

We live in Chicago and plan to take a car trip to visit friends in Minneapolis, Cleveland, and St. Louis.

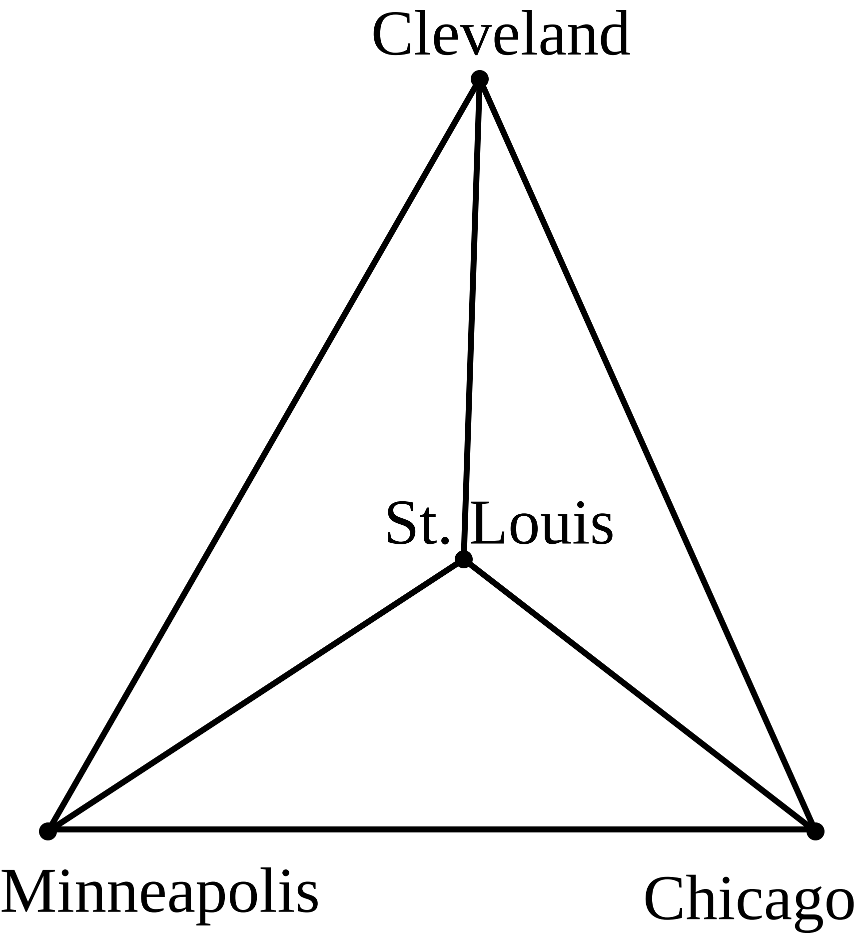
Design a route that minimizes the distance we have to travel.

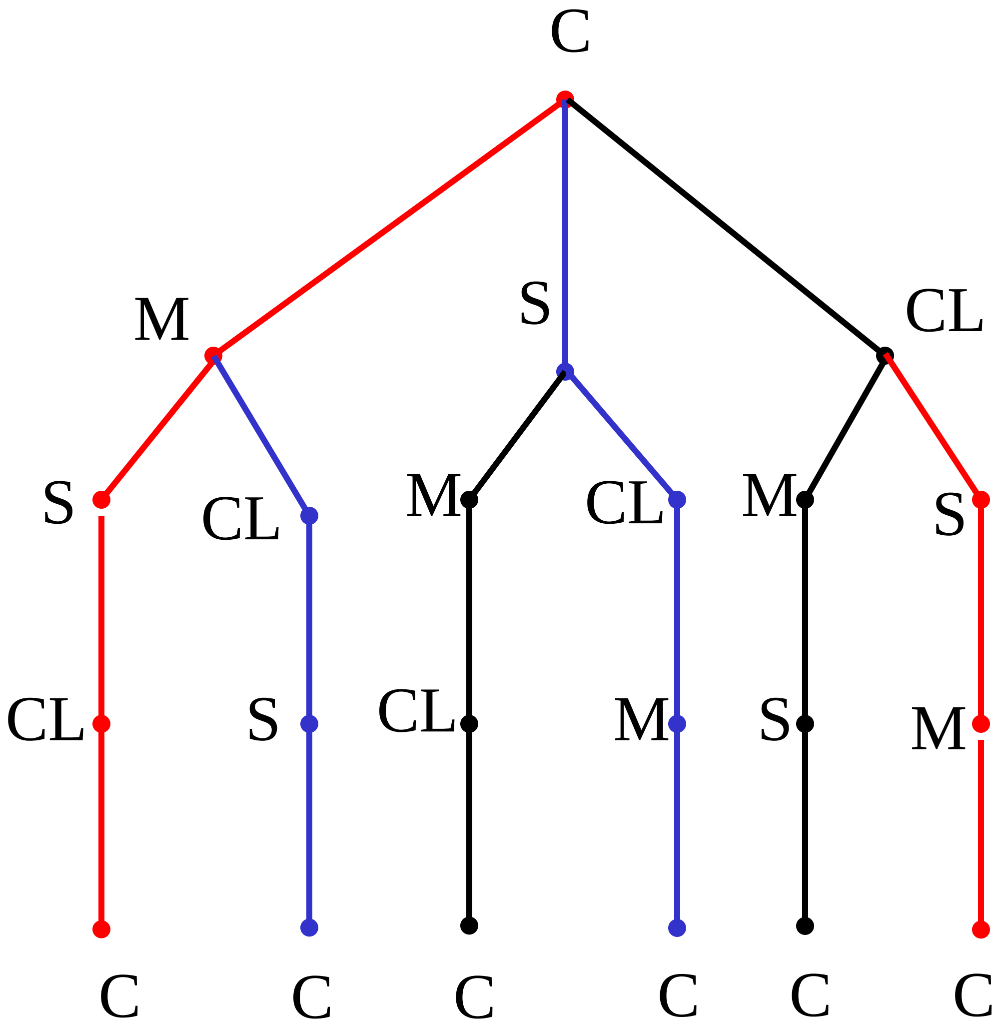


Algorithm

1. Generate all possible Hamiltonian tours (starting and ending in Chicago)
2. Add up the distances (costs) on the edges of each tour.
3. Choose the tour of minimum distance (cost).

Method of Trees





Solutions

1. **Chicago** $\xrightarrow{425}$ **Minneapolis** $\xrightarrow{562}$ **St. Louis** $\xrightarrow{541}$ **Cleveland** $\xrightarrow{349}$ **Chicago**

Total distance for this circuit = 1877. ⟵ **Optimal solution**

2. **Chicago** $\xrightarrow{300}$ **St. Louis** $\xrightarrow{562}$ **Minneapolis** $\xrightarrow{774}$ **Cleveland** $\xrightarrow{349}$ **Chicago**

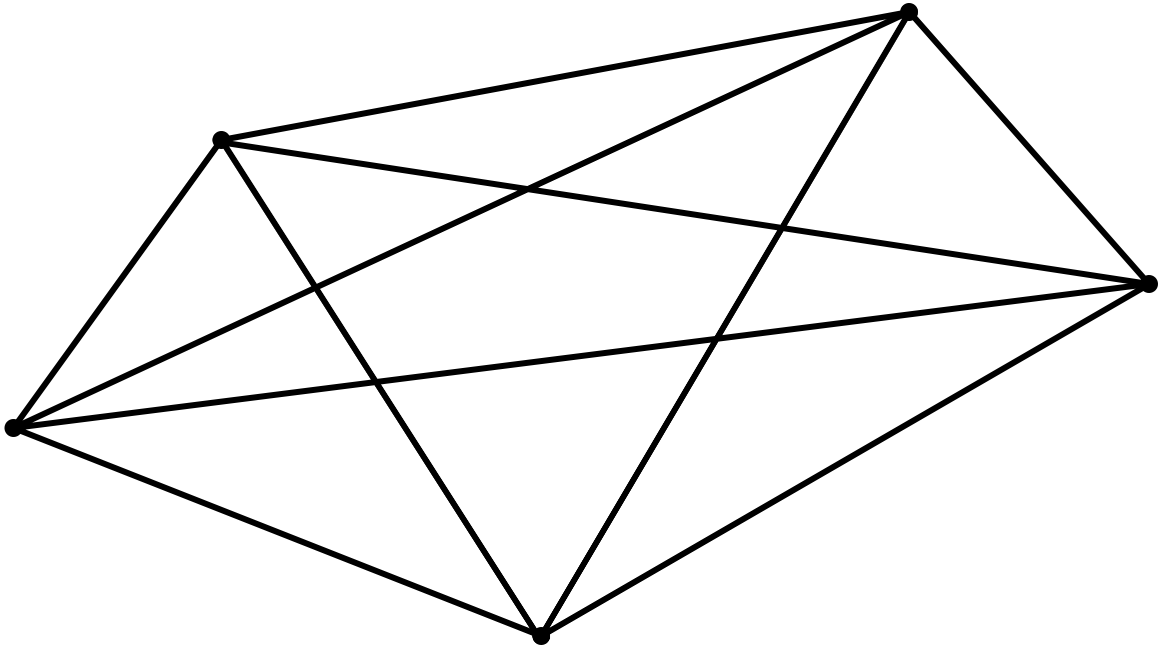
Total distance = 1985

3. **Chicago** $\xrightarrow{300}$ **St. Louis** $\xrightarrow{541}$ **Cleveland** $\xrightarrow{774}$ **Minneapolis** $\xrightarrow{425}$ **Chicago**

Total distance = 2040

Complete Graph

A graph in which every pair of vertices is joined by an edge. The complete graph with *N* vertices is denoted K_N .



K_5

Traveling Salesman Problem

Problem: determine the trip of minimum cost that a salesperson can make visiting all cities in a sales territory, starting and ending the trip in the same city.

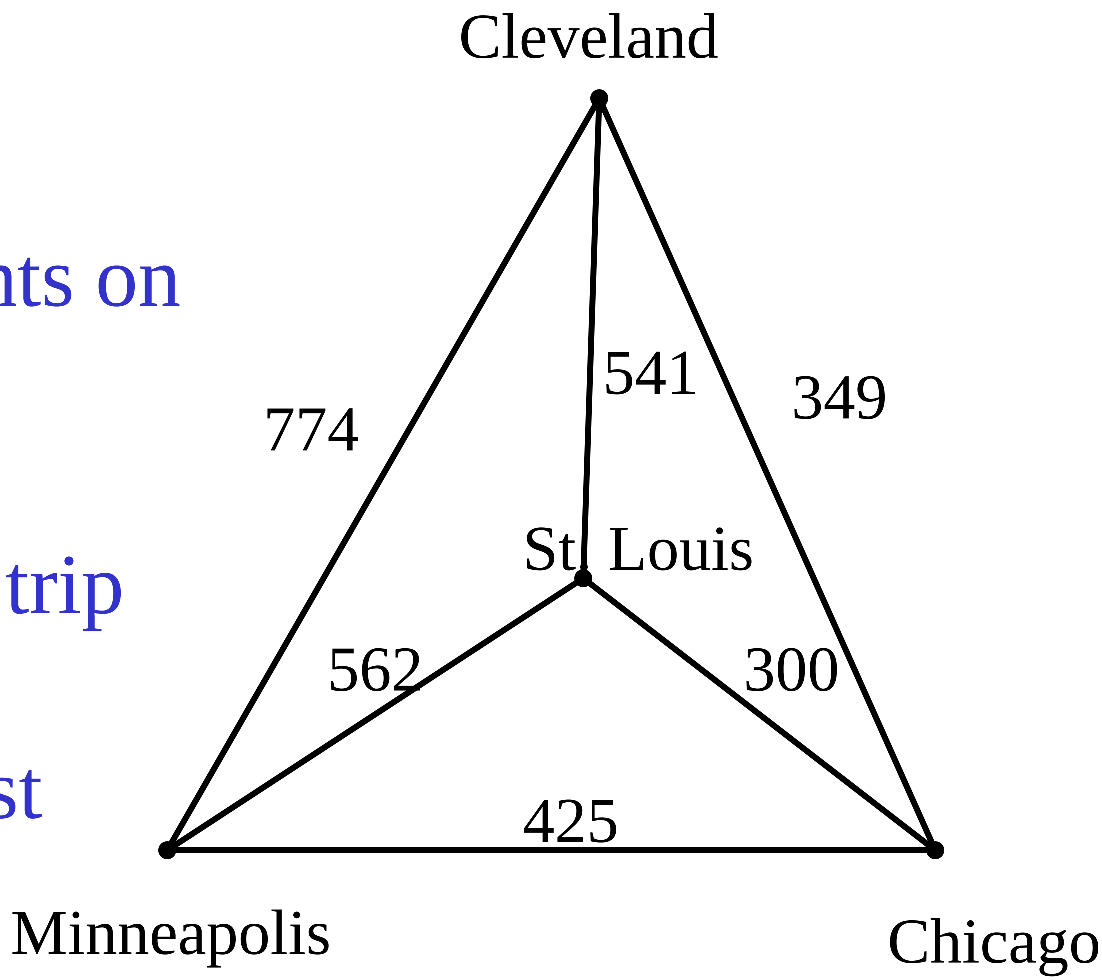
Finding a minimum-cost Hamiltonian circuit in a complete graph with *N* vertices is usually called the traveling salesman problem (TSP).

GRAPH MODEL OF A TRAVELING SALESMAN PROBLEM

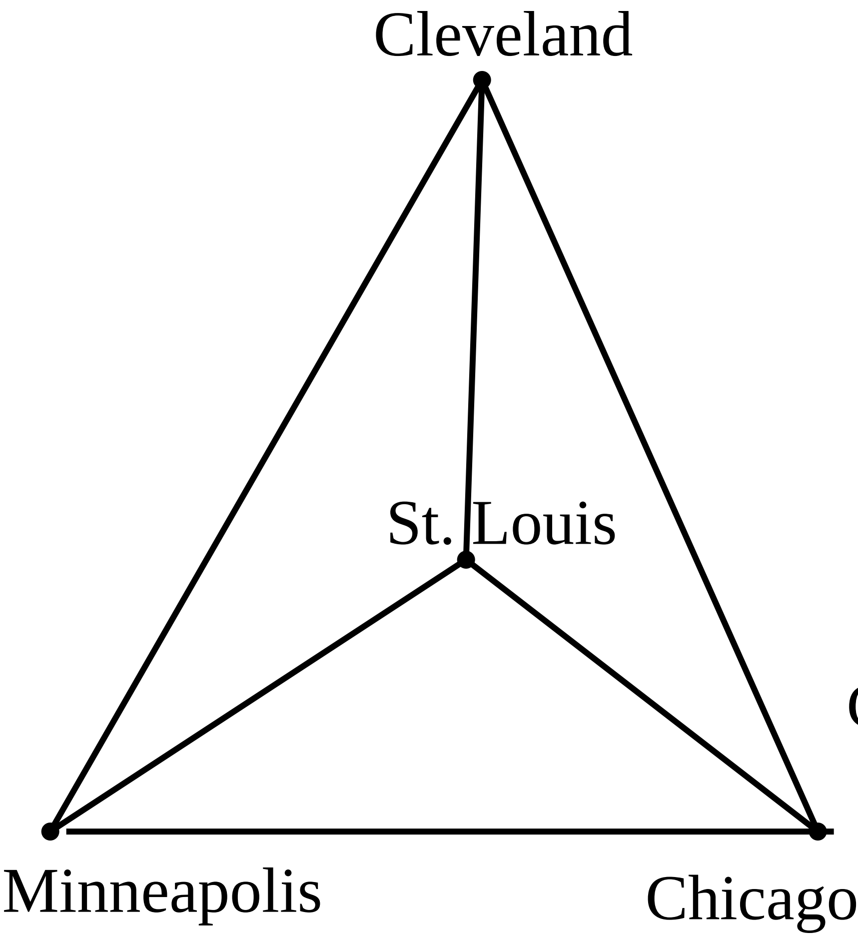
- Sites → vertices of the graph.
- Costs → weights of the edges.
- Tour → Hamilton circuit.
- Optimal tour → Hamilton circuit of least total weight

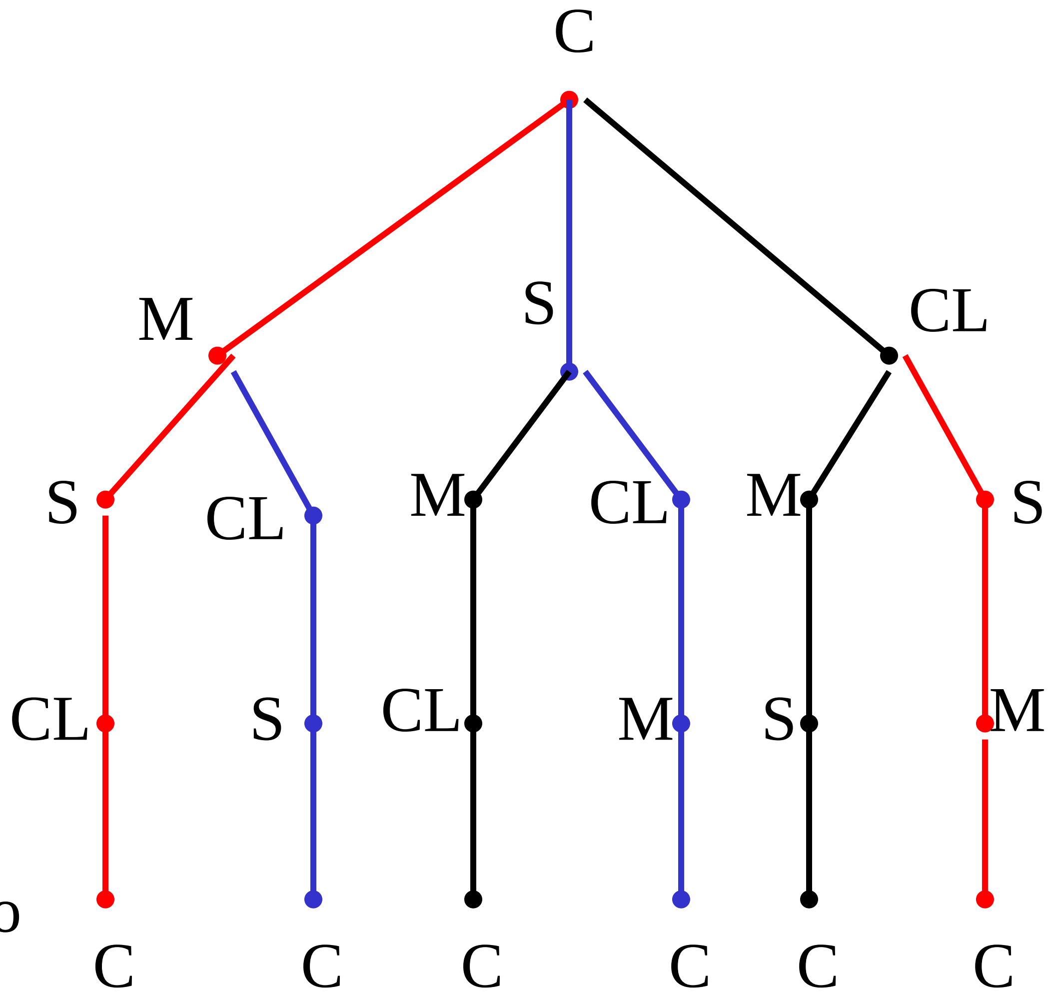
Planning a trip

This problem uses the complete weighted graph with four vertices. The weights on the edges are the distances between cities. The optimal trip is the Hamiltonian circuit with the least total weight.



TSP with four cities





$3 \times 2 \times 1 = 6$ Hamiltonian circuits.

Fundamental Principle of Counting for a Complete Graph

If a complete graph has *N* vertices, then there are (*N* – 1)! Hamiltonian circuits.

$$(N-1)! = (N-1)(N-2)\cdots 3\cdot 2\cdot 1$$

the "factorial of $(N-1)$"