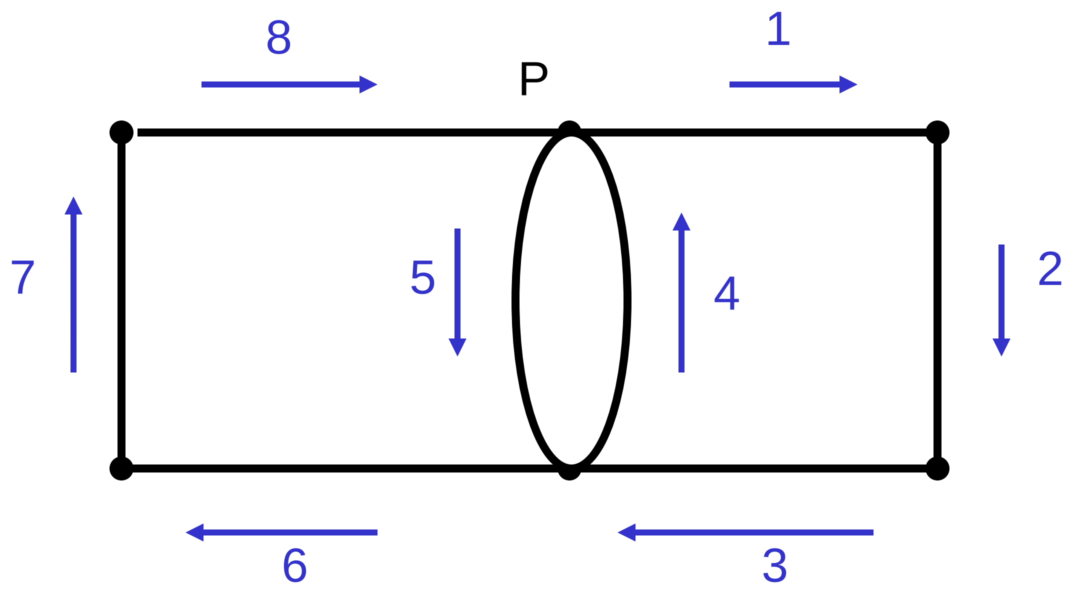
Review

An Euler Circuit



A circuit that covers each edge exactly once is called an Euler circuit.

Can we tell when an Euler circuit is possible?

EULER'S CIRCUIT THEOREM

- If a graph is *connected* and *every vertex is even*, then it has an Euler circuit (at least one, usually more).
- If a graph has *any odd vertices*, then it does not have an Euler circuit.

Illustration using the Theorem

This graph is connected but it has odd vertices (e.g. C). This graph has no Euler circuits.

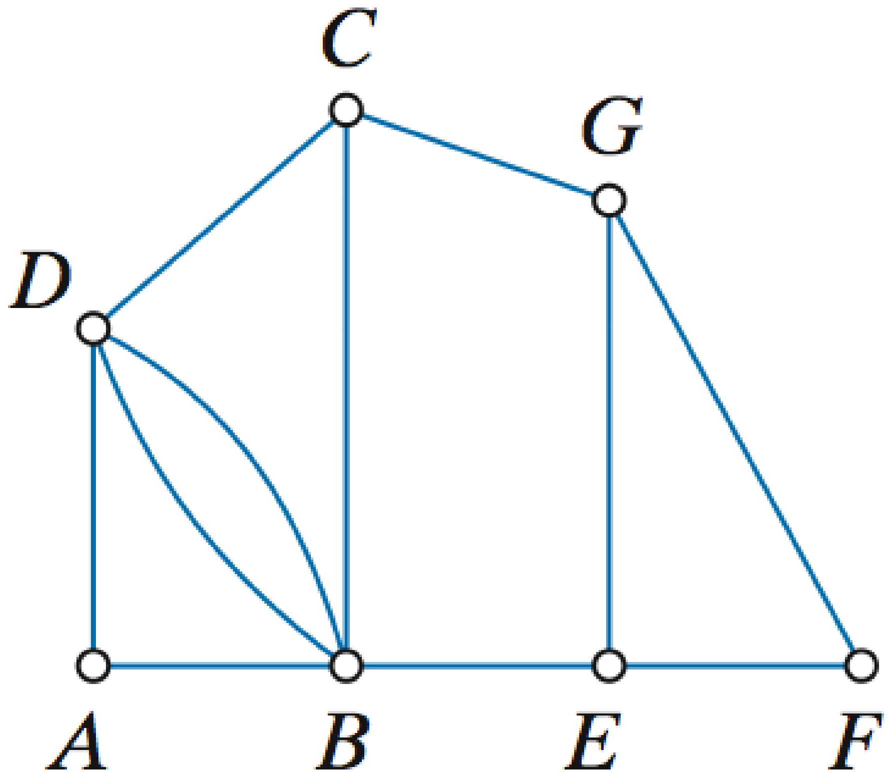


Figure 1-15(b) in text.

Illustration using the Theorem

This graph is connected and all of the vertices are even. This graph does have Euler circuits.

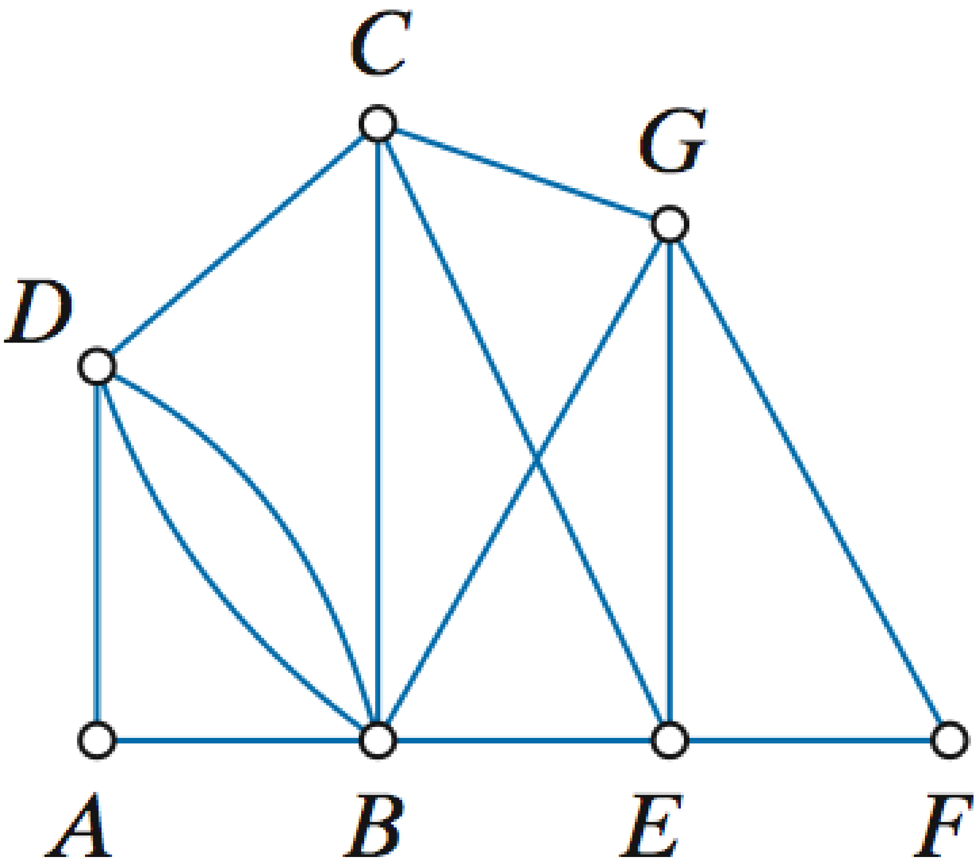
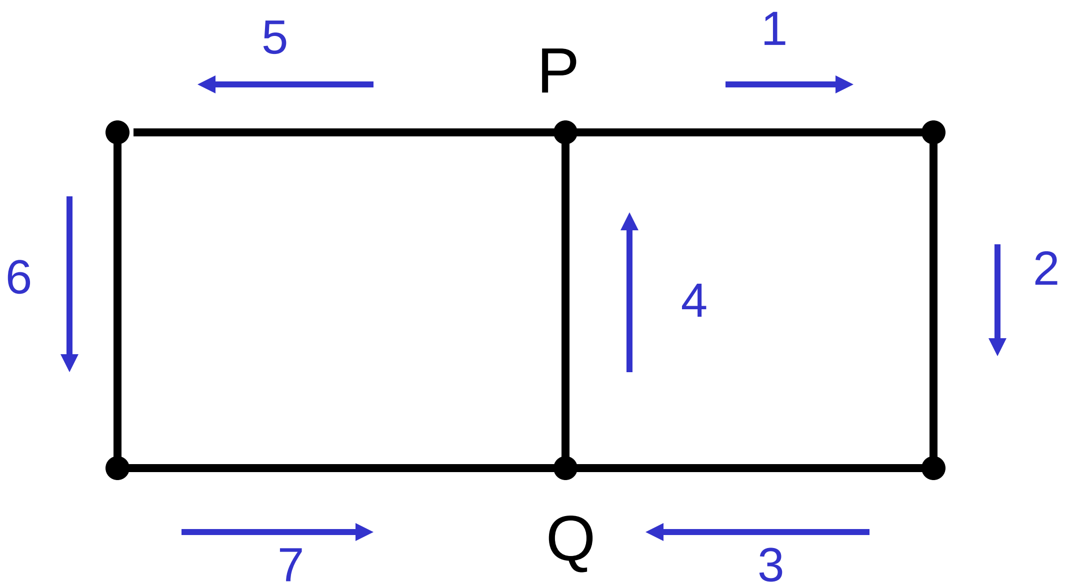


Figure 1-15(c) in text.

An Euler Path



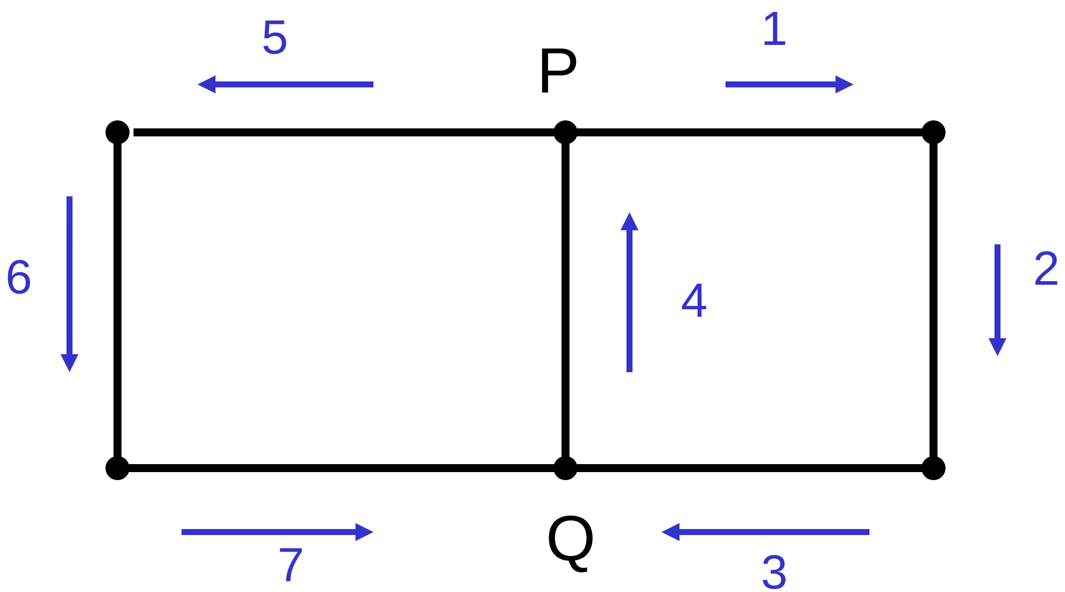
A path that covers each edge exactly once is called an Euler path.

Can we tell when an Euler path is possible?

EULER'S PATH THEOREM

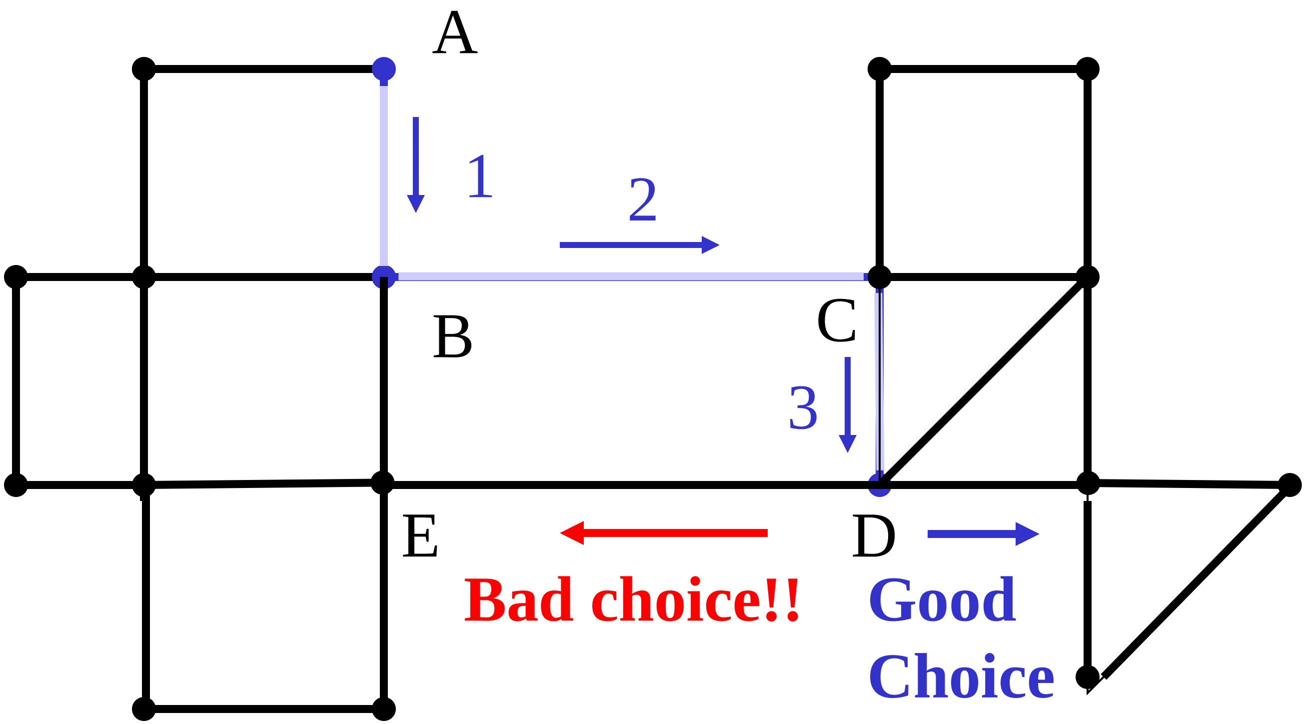
- If a graph is *connected* and has *exactly two odd vertices*, then it has an Euler path (at least one, usually more). Any such path must start at one of the odd vertices and end at the other one.
- If a graph has *more than two* odd vertices, then it cannot have an Euler path.

An Euler Path



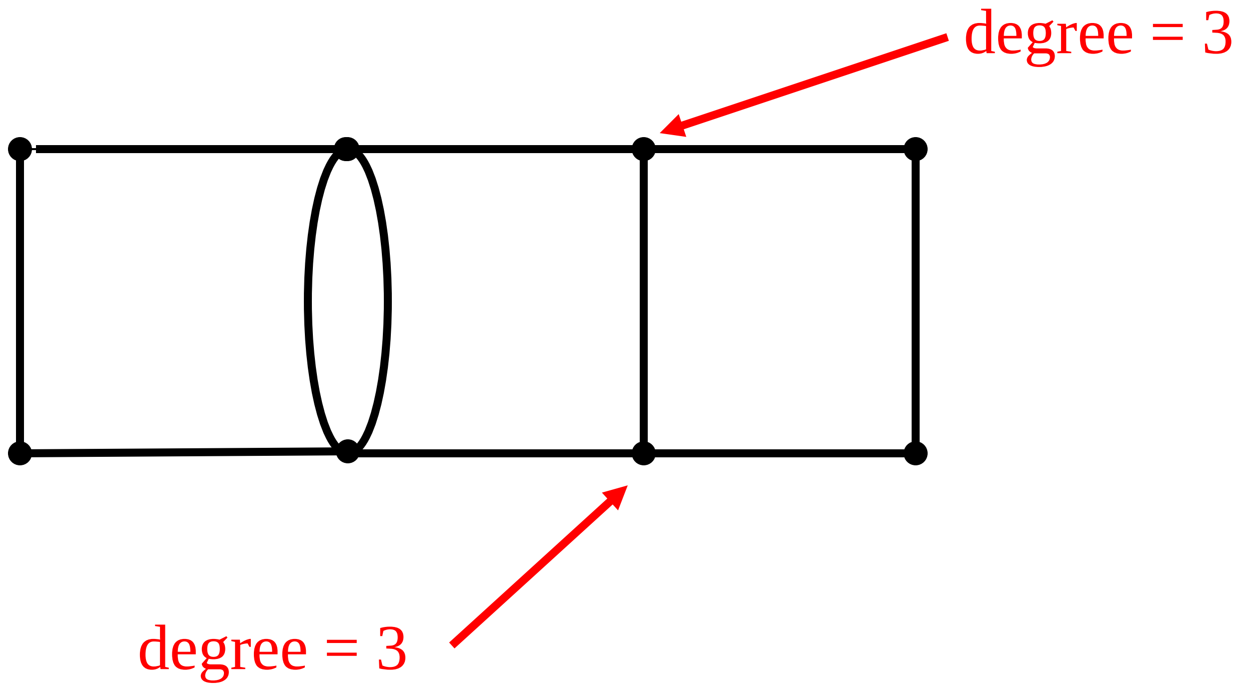
There are two odd vertices: P and Q
This Euler path begins at P and ends at Q.

Guidance for Finding Euler Circuits



Never use an edge that is the only link between two parts of the graph that still need to be covered.

Does this graph have an Euler Circuit?



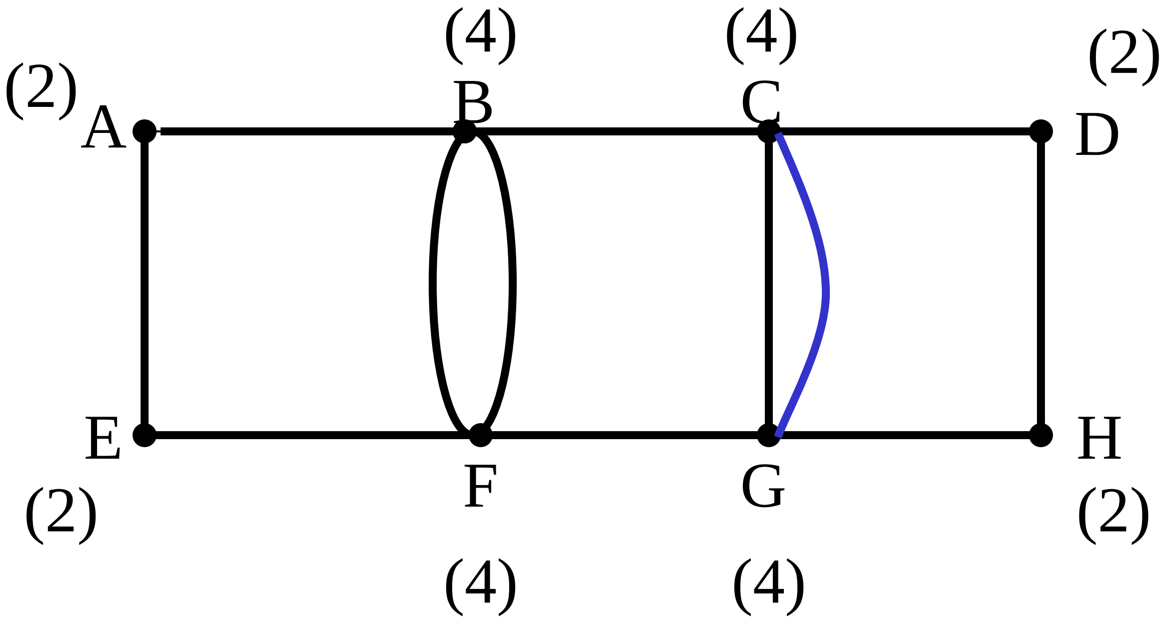
No, according to **Euler's Theorem**

In order to make a circuit that covers all edges in this graph, some edges must be reused!

Eulerizing a Graph

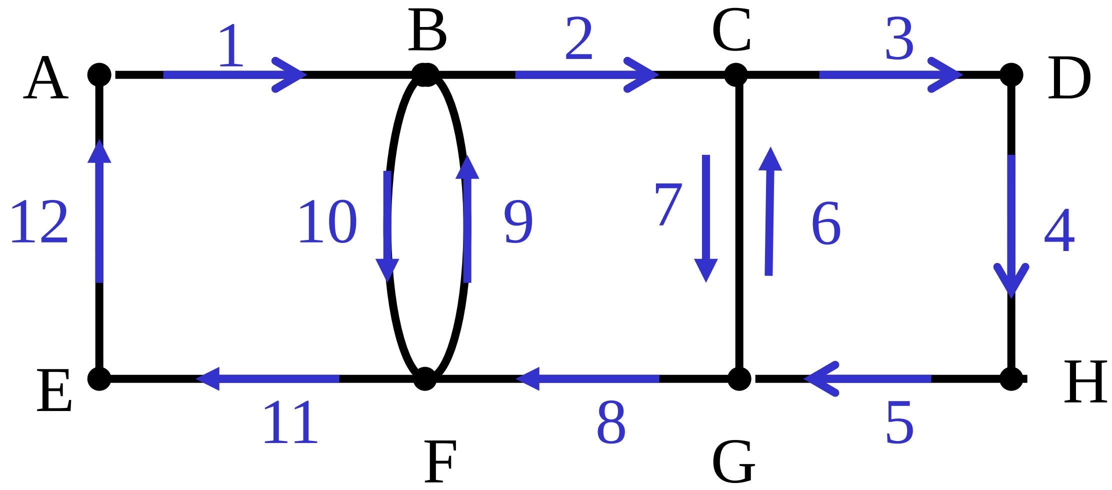
- Duplicating an existing edge on a graph can be interpreted as reusing an existing edge.
- Duplicating edges on a graph in order to make all vertices **even** is called **eulerizing** the graph.
- In this course, eulerizing a graph is limited to duplicating existing edges.

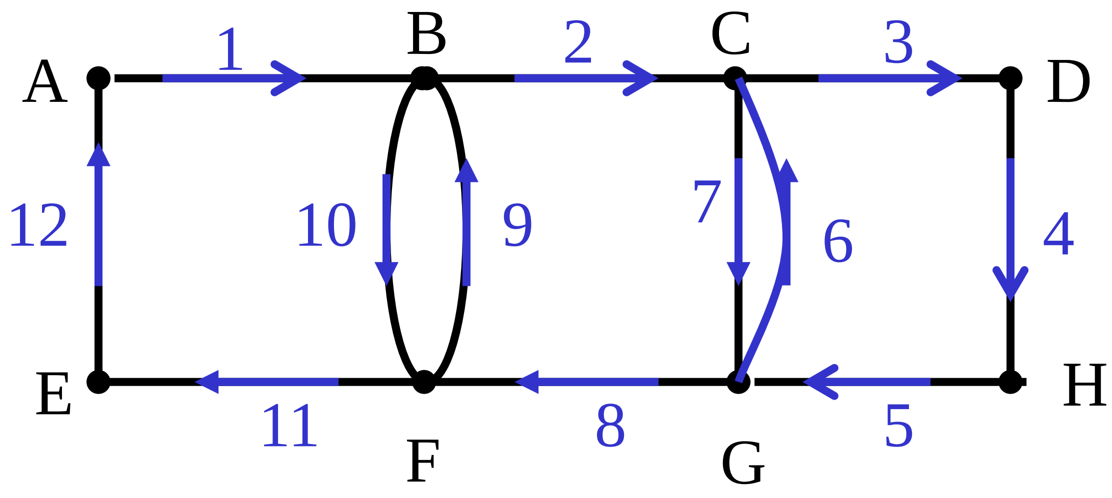
Can it become "Eulerized?"



Answer: Yes, if we duplicate (reuse) an edge!

Circuits with Reused Edges



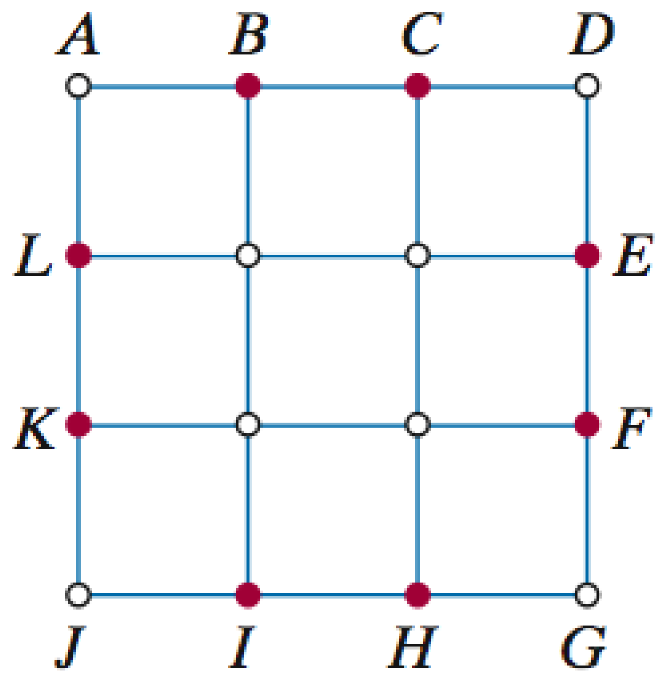


Example 1.22 Covering a 3 by 3 Street Grid

The graph represents a 3 block by 3 block street grid.

How can we find an optimal route that covers all the edges of the graph and ends back at the starting vertex?

Our first step is to identify the odd vertices. (Shown in red.)

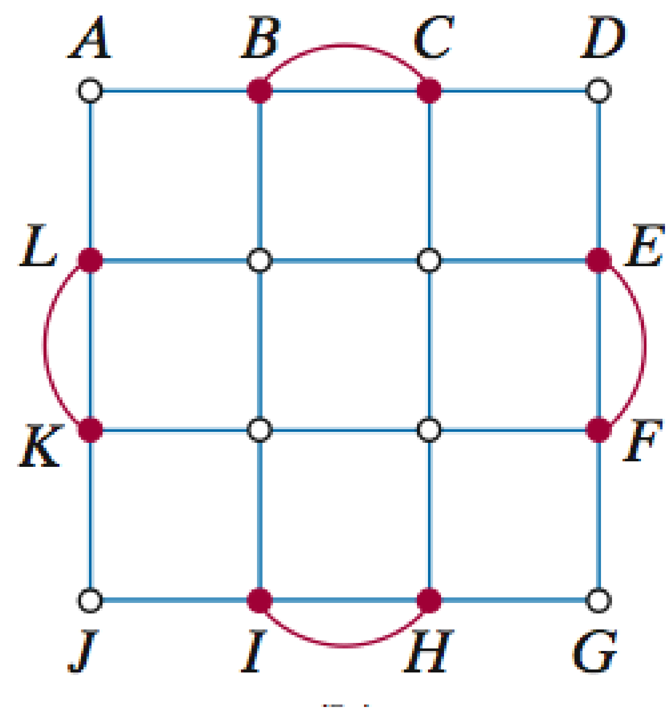


Example 1.22 Covering a 3 by 3 Street Grid

When we duplicate edges *BC*, *EF*, *HI*, and *KL*, we get this graph.

This is a *eulerized* version of the original graph–its vertices are all even, so it has an Euler circuit.

Could we have done this with fewer than four duplicate edges?



Eulerizing a Street Network

Algorithm:

- "Walk" around the outside edges of the network.
- Duplicate an edge whenever you encounter an odd degree by linking to the next vertex.
- If the degree of this vertex becomes even, skip to the next vertex.