MATH 101: Introduction to Contemporary Mathematics

Sections 201 - 206

Instructor: **H. R. Hughes**

Course web page: http://www.math.siu.edu/hughes/math101.htm

Summer 2013

Lecture sessions meet:

MTWF 12:10 - 1:10 p.m., Lawson 151

Recitation sessions meet:

Faner (access inside computer lab)

Section	Time	Place	Instructor
201	M 11 - 12	Faner 1032	Matthew Walker
203	W 11 - 12	Faner 1032	Matthew Walker
206	R 1:20-2:20	Faner 1024	Oday Hazaimah

Open labs in Morris Library Room 174

(see web site)

Use i>clicker2 to register participation in lectures.

Homework will be assigned in MyLabsPlus.

Quizzes will be given in lecture with i>clicker2.

There will be three hour-exams.

Grading Policy:

Lab (HW)	15%,
Quiz and Lecture	15%
Hour-Exams	45%,
Final Exam	25%,

Quizzes cannot be made up.

The lowest 20% of quiz, HW, and participation grades will be dropped.

Text Book

Introduction to Contemporary Mathematics

(Custom Edition, Pearson 2012)

ISBN-13: 978-125-673-9838

(comes with MyLabsPlus access code)

Chapter 1: The Mathematics of Getting Around

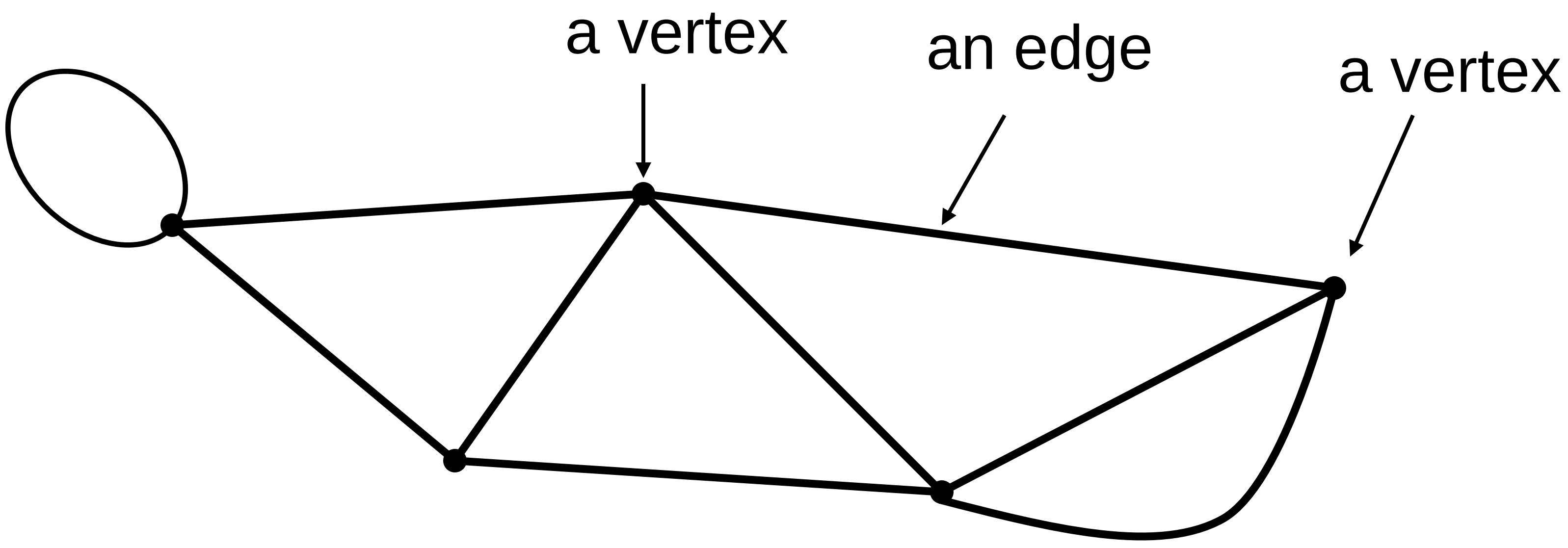
General goal: Find the most efficient way to traverse a street network while checking parking meters, delivering mail, etc.

Our goal: Find the most efficient route through a network of streets that satisfies the following two conditions:

1) Each street is traversed exactly once.
2) You begin and end at the same place.

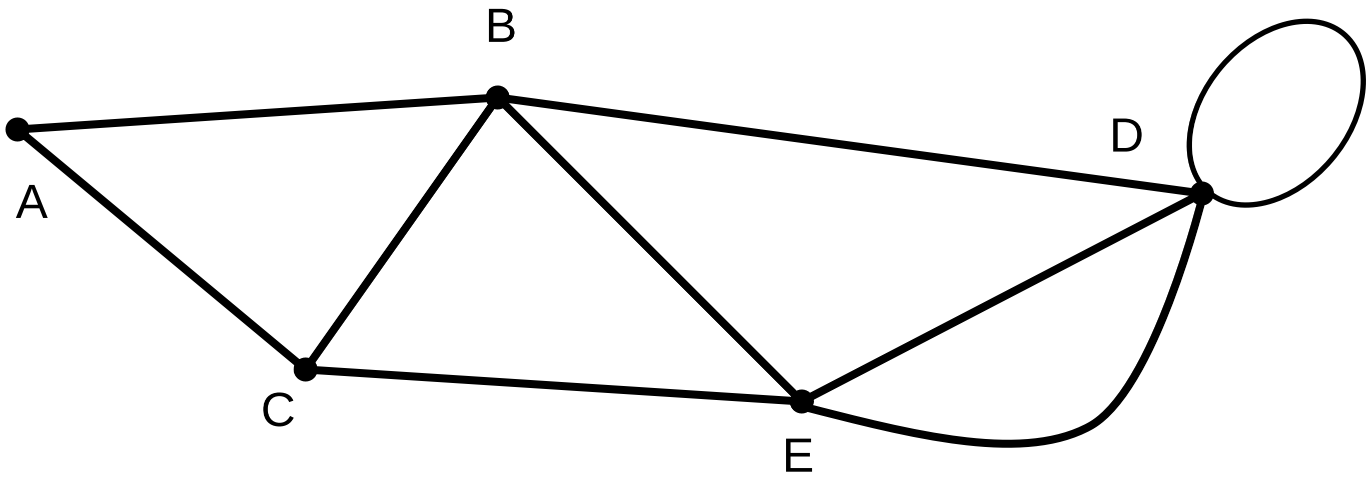
Mathematical modeling: Finding a mathematical object or equation that captures the essential characteristics of a problem, but eliminates all unnecessary characteristics.

A Graph



A **graph** is a finite set of "dots" and connecting "lines." The mathematical term for dot is **vertex** and for line is **edge**.

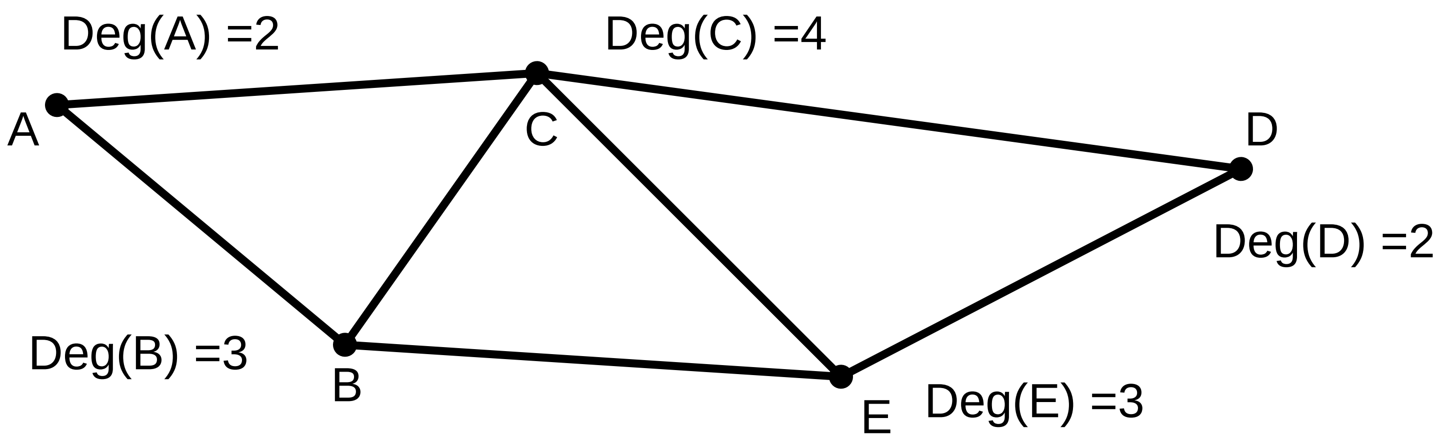
More about Graphs



Vertices are labeled with letters for reference. Edges can then be labeled with the pair of vertices at the ends. The edges for the graph above are AB, AC, BC, CE, BE, BD, ED, ED, DD

Notice: Graphs can have multiple edges (ED, ED) and "loops" that begin and end at the same vertex (DD).

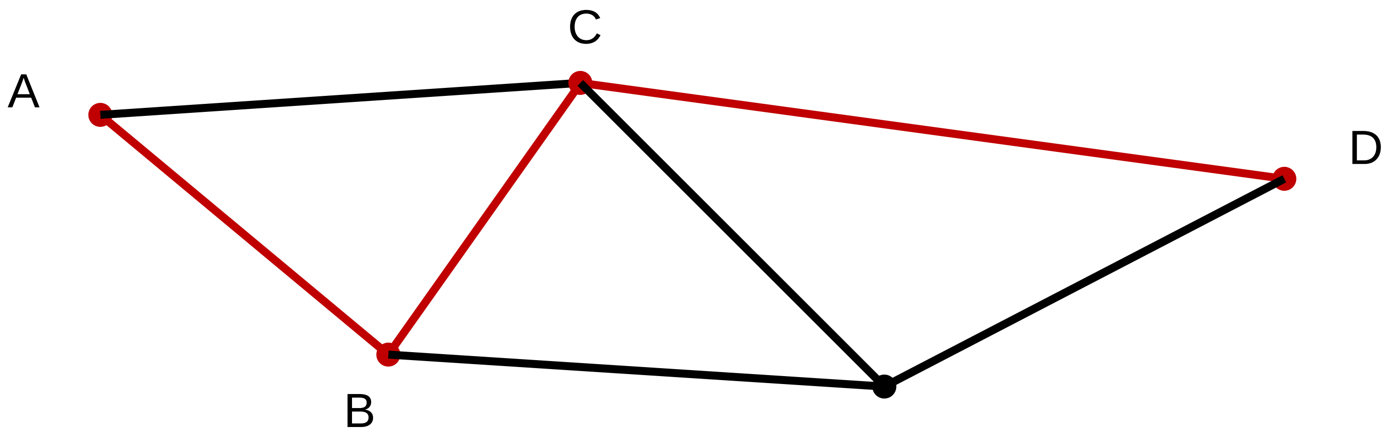
Graph Concepts and Terminology



1) Two vertices are **adjacent** if there is an edge connecting them. Vertices A and B are adjacent, but A and E are not.

2) The **degree** of a vertex is the number of edge connections at the vertex. A vertex is **even** if its degree is an even number and **odd** if the degree is an odd number. In the above graph, vertices A, C, and D are even and B and E are odd.

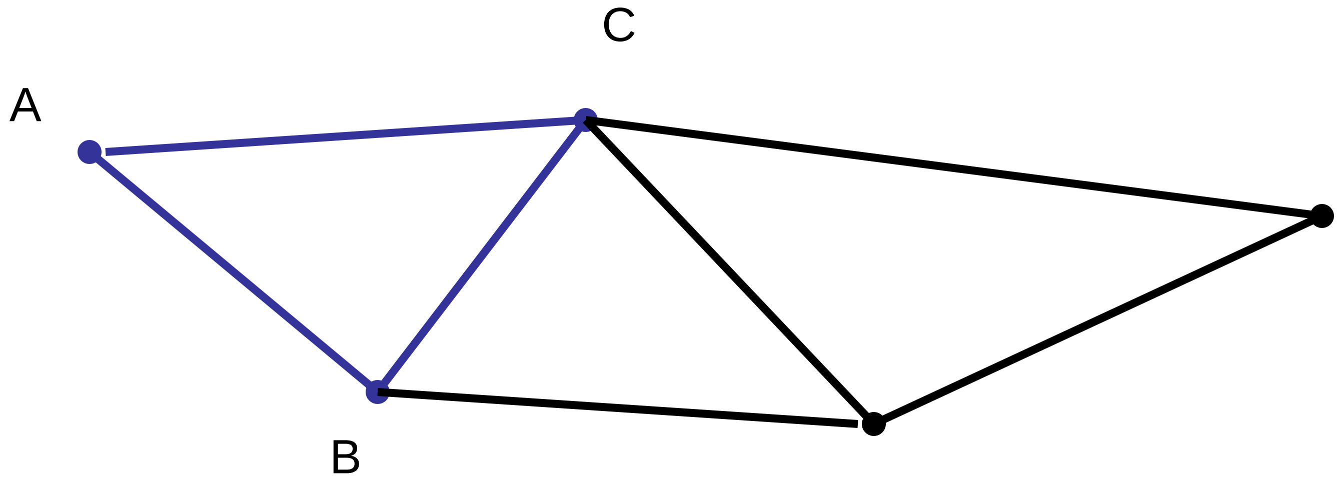
A Path



A->B->C->D is called a path

A path is a connected sequence of distinct edges in a graph that starts at one vertex and ends at a different vertex.

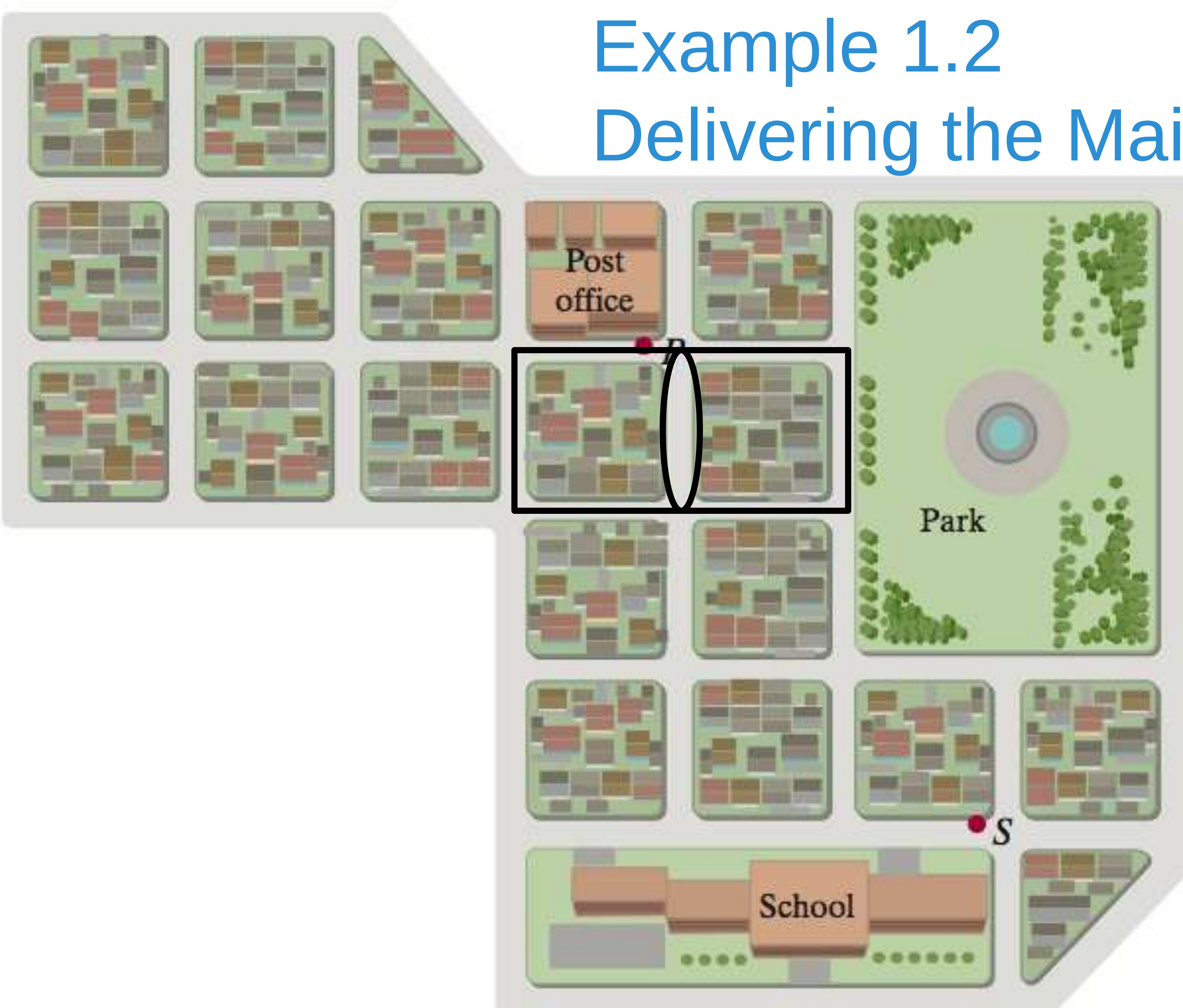
A Circuit



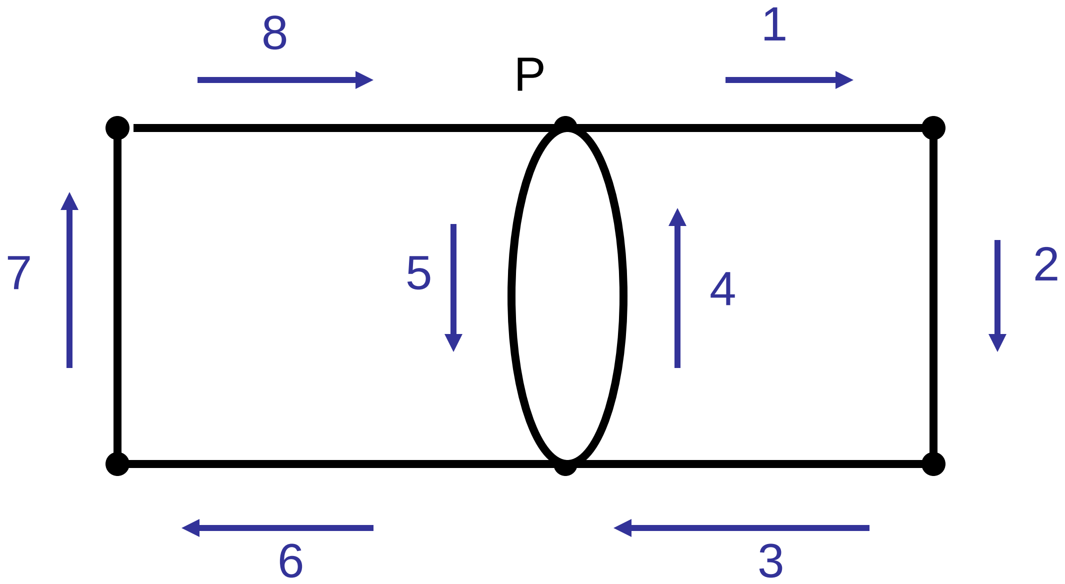
The trip A->B->C->A is called a circuit.

A trip that starts and ends at the same vertex is called a circuit.

Example 1.2
Delivering the Mail



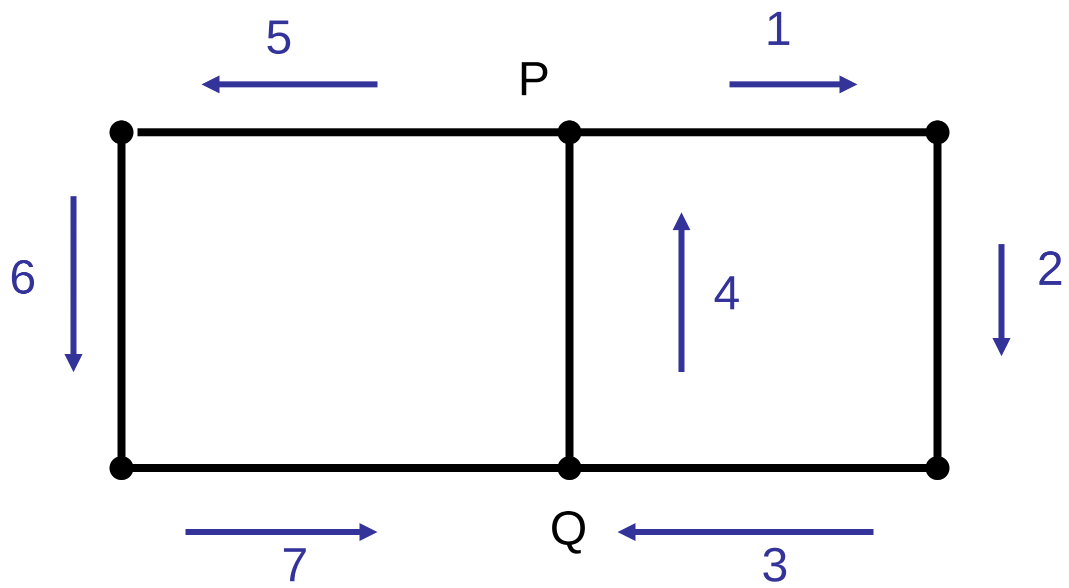
An Euler Circuit



A circuit that covers each edge exactly once is called an Euler circuit.

Can we tell when an Euler circuit is possible?

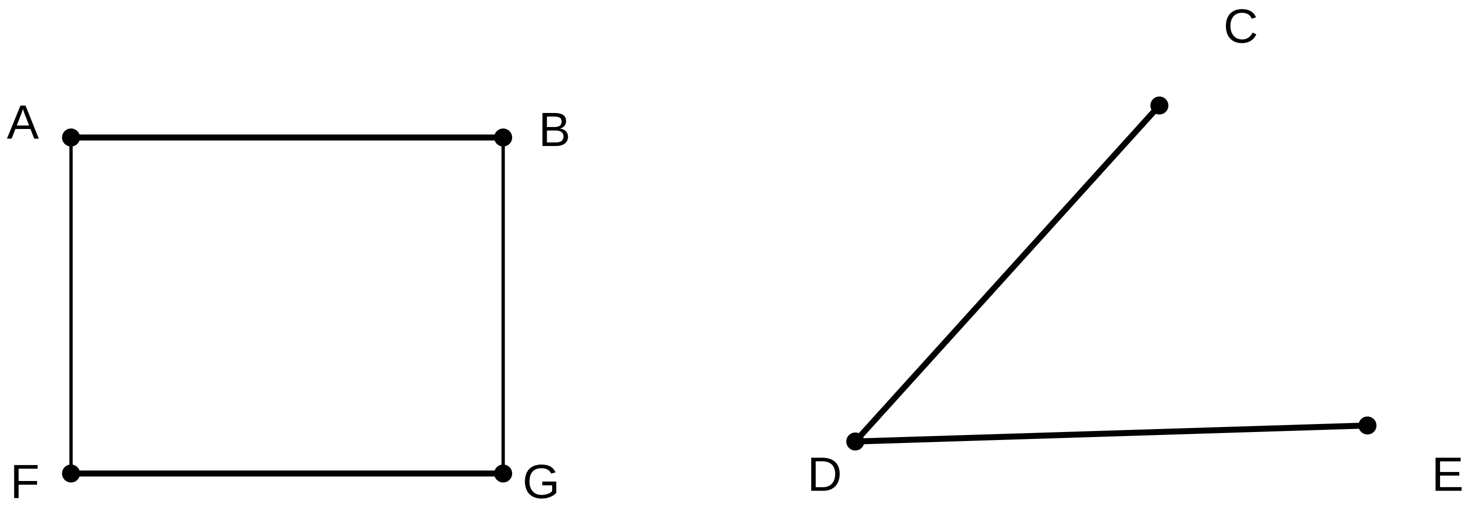
An Euler Path



A path that covers each edge exactly once is called an Euler path.

Can we tell when an Euler path is possible?

A graph is said to be **connected** if for every pair of its vertices there is at least one path connecting the two vertices.



A non-connected graph

Example 1.15 The Seven Bridges of Königsberg

Can a walker take a stroll and cross each of the seven bridges of Königsberg exactly once?

The drawing is not exactly to scale and the angles of some of the bridges are changed. Does it matter?

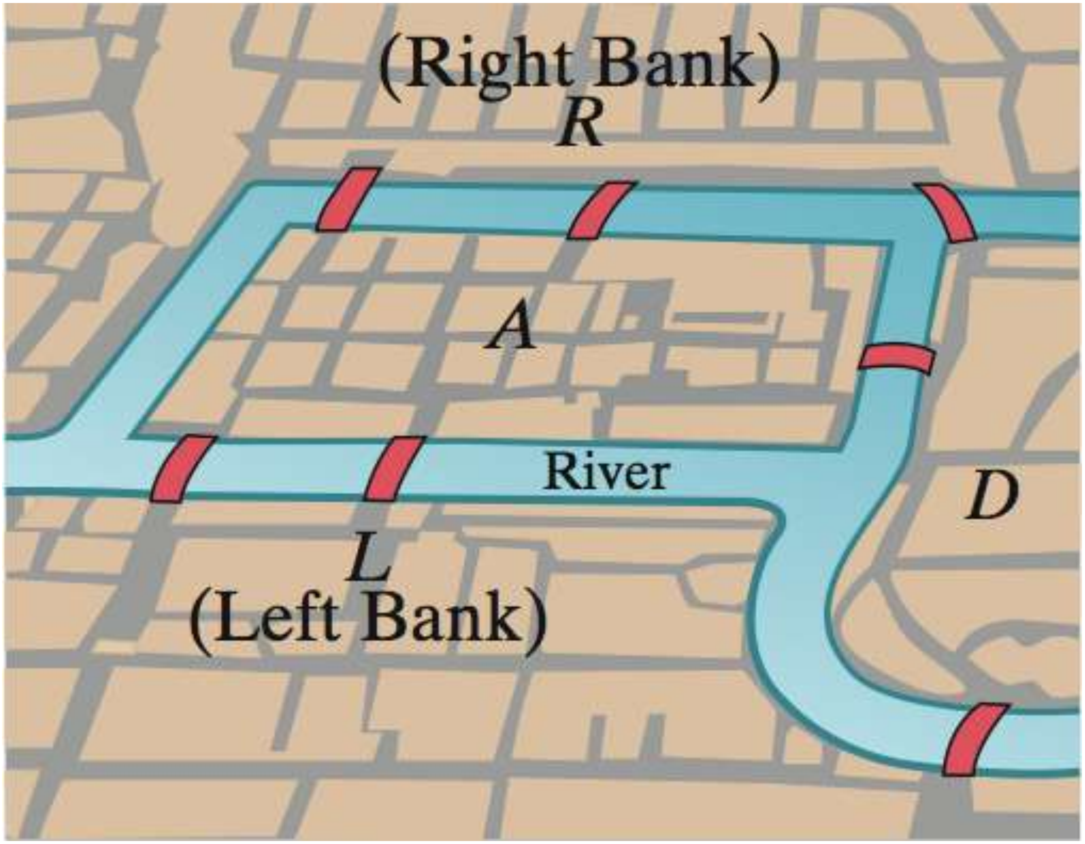


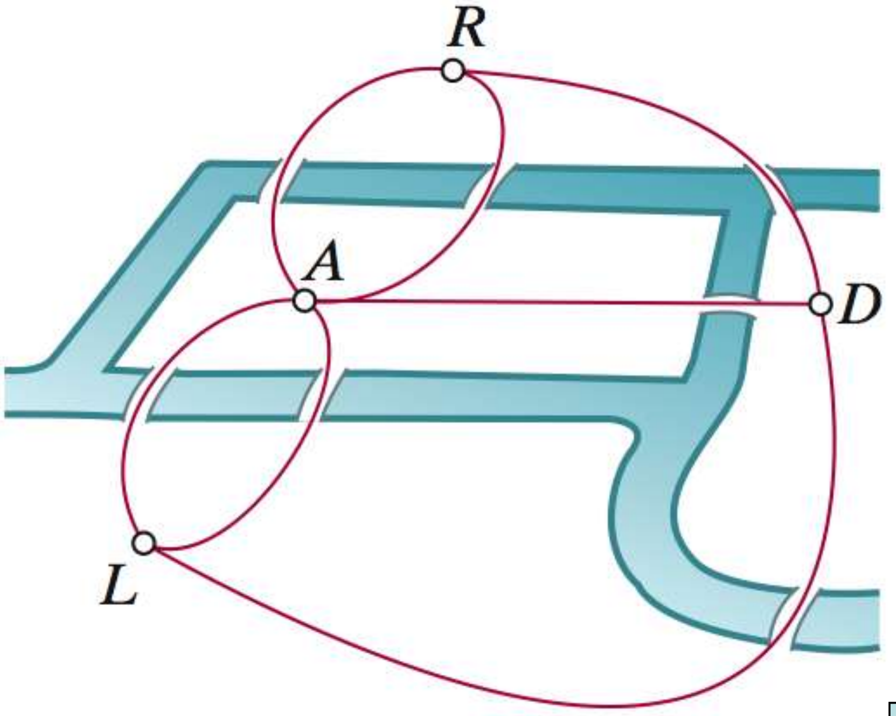
Figure 1-13 from text.

Example 1.15 The Seven Bridges of Königsberg

The only thing that truly matters to the solution of this problem is the relationship between land masses (islands and banks) and bridges.

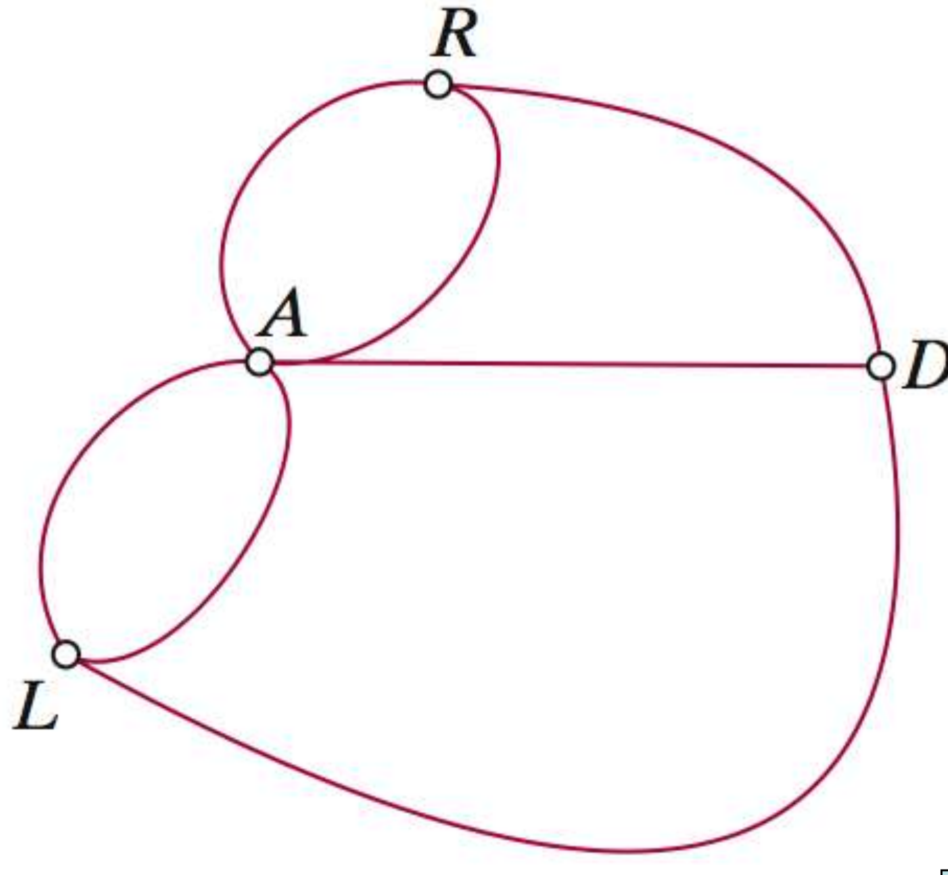
Which land masses are connected to each other and by how many bridges? This information is captured by the graph with the red edges.

(Fig. 1-13(b) in text)



Example 1.15 The Seven Bridges of Königsberg

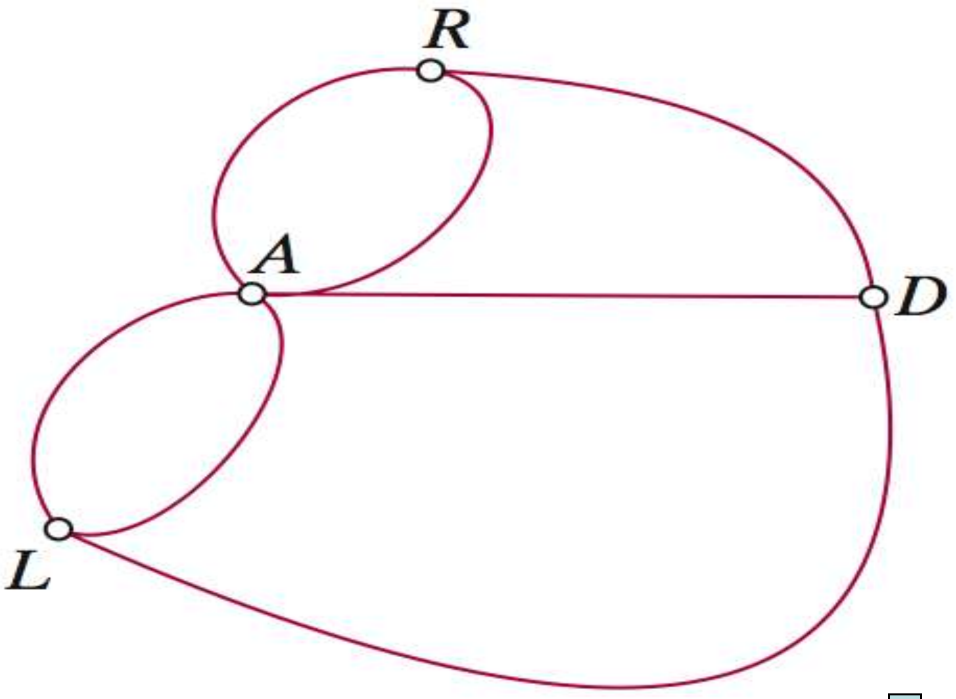
The four vertices of the graph represent each of the four land masses; the edges represent the seven bridges.



Example 1.15 The Seven Bridges of Königsberg: Part 2

An Euler circuit would represent a stroll around the town that crosses each bridge once and ends at the starting point.

An Euler path would represent a stroll that crosses each bridge once but does not return to the starting point.



EULER'S CIRCUIT THEOREM

- If a graph is *connected* and *every vertex is even*, then it has an Euler circuit (at least one, usually more).
- If a graph has *any odd vertices*, then it does not have an Euler circuit.

Illustration using the Theorem

This graph cannot have an Euler circuit for the simple reason that it is disconnected.

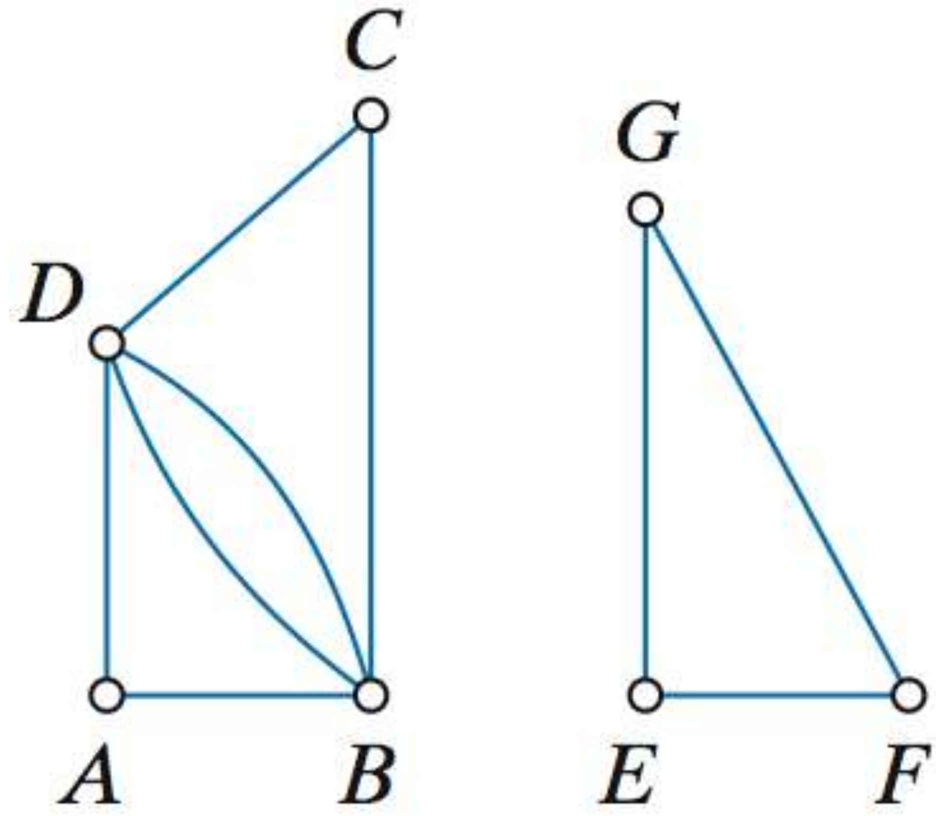


Figure 1-15(a) in text.

Illustration using the Theorem

This graph is connected but it has odd vertices (e.g. C). This graph has no Euler circuits.

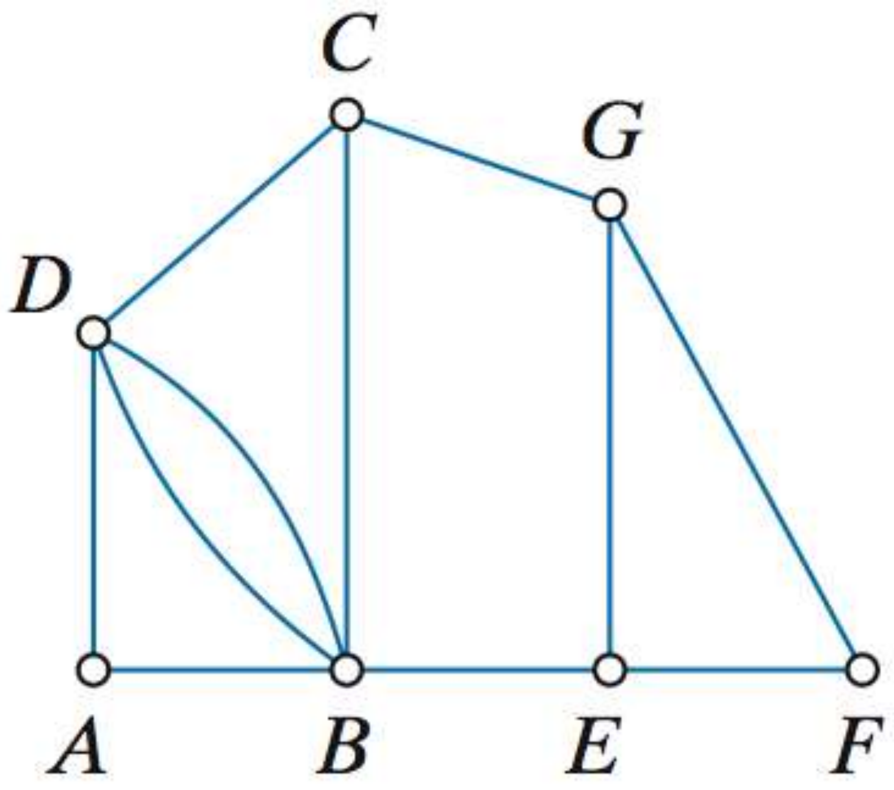


Figure 1-15(b) in text.

Illustration using the Theorem

This graph is connected and all of the vertices are even. This graph does have Euler circuits.

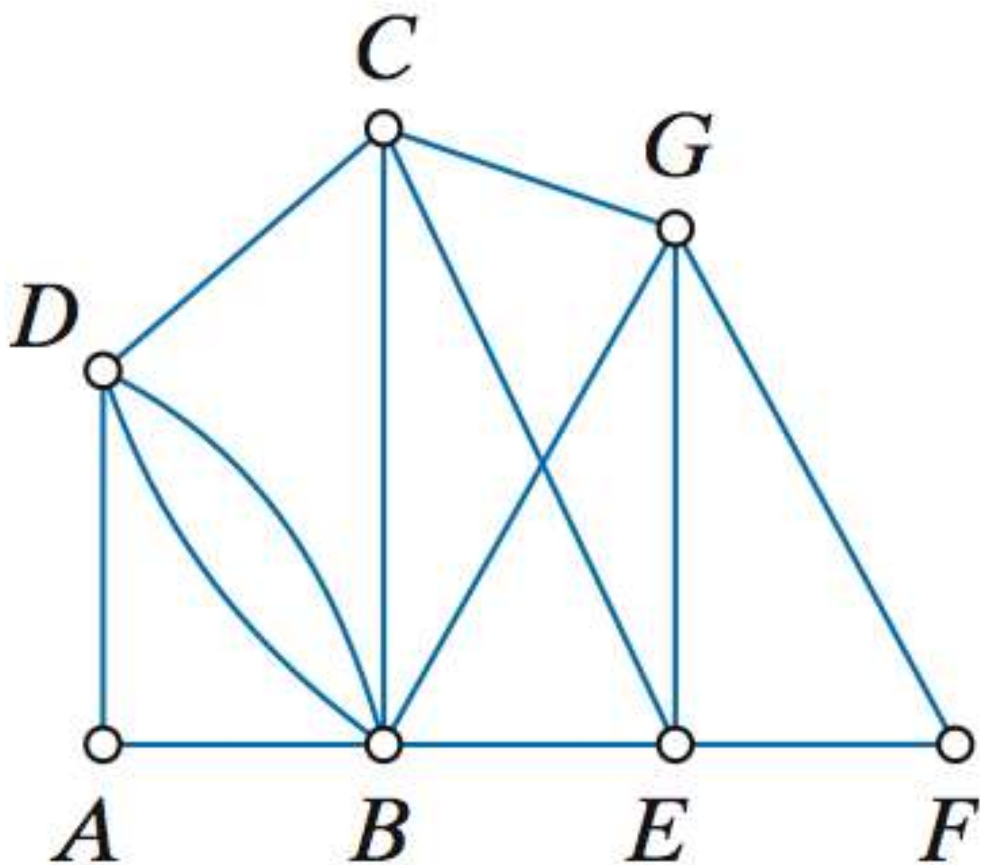


Figure 1-15(c) in text.