Addition Rule of Probability (for *A* or *B*)

If A and B are **mutually exclusive** events, then

$$P(A \text{ or } B) = P(A) + P(B).$$

If A and B are **any** two events, then

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$

Example: Probability Involving "Or"

When a single card is drawn from a standard 52-card deck, what is the probability that it will be a king or a diamond?

Solution

$$P(\text{king or diamond}) = P(\text{K}) + P(\text{D}) - P(\text{K and D})$$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$

$$= \frac{16}{52} = \frac{4}{13}.$$

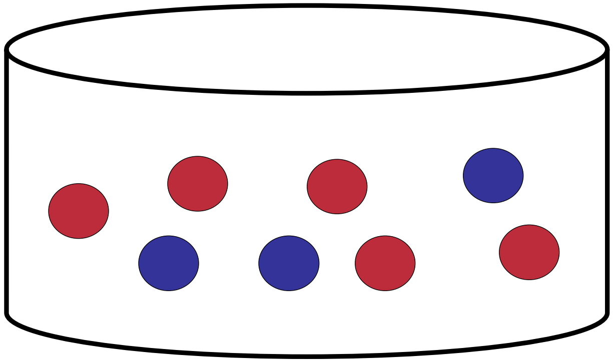
Conditional Probability

Sometimes the probability of an event must be computed using the knowledge that some other event has happened.

The probability of event *B*, computed on the assumption that event *A* has happened, is called the **conditional probability of *B*, given *A*,** and is denoted ***P*(*B* | *A*).**

Example: Selecting From a Jar of Balls

Jeff draws two balls from the jar *without* replacement. Given that he draws a red ball first, find the probability that he draws a blue ball second.



5 red and 3 blue

R_1 = [first ball is red]

B_2 = [second ball is blue]

If Jeff draws a red ball first, there are now 4 red and 3 blue balls.

$$P(B_2 \mid R_1) = \frac{3}{7}$$

Multiplication Rule of Probability

If A and B are any two events, then

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A).$$

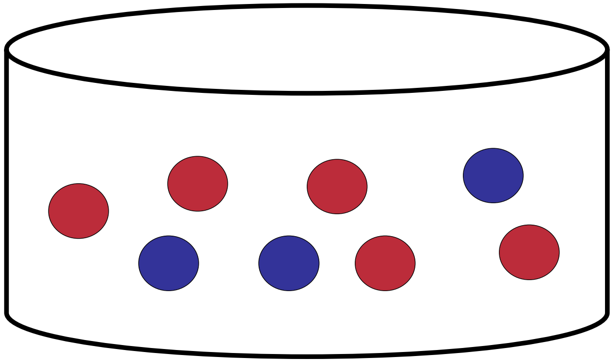
Two events A and B are called **independent events** if the occurrence of one of them has no effect on the probability of the other one. If A and B are independent, then

$$P(A \text{ and } B) = P(A) \cdot P(B).$$

Example: Selecting From a Jar of Balls

Jeff draws two balls from the jar *without* replacement. Find the probability that he draws a red ball and then a blue ball, in that order.

5 red and 3 blue



Using the multiplication rule:

$$P(R_1 \text{ and } B_2) = P(R_1) \cdot P(B_2 \mid R_1) = \frac{5}{8} \cdot \frac{3}{7} = \frac{15}{56}$$

Example: Selecting From a Jar of Balls

Jeff draws two balls from the jar *without* replacement. We summarize all of the cases with a tree:

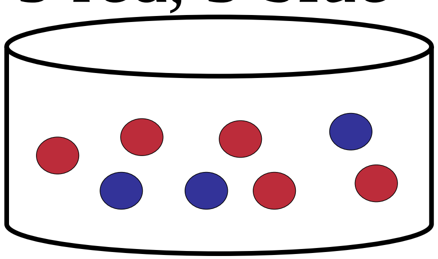
5/8 → R_1

- 4/7 → R_2 $\quad P(R_1 \text{ and } R_2) = \frac{5}{8} \cdot \frac{4}{7} = \frac{20}{56}$
- 3/7 → B_2 $\quad P(R_1 \text{ and } B_2) = \frac{5}{8} \cdot \frac{3}{7} = \frac{15}{56}$

3/8 → B_1

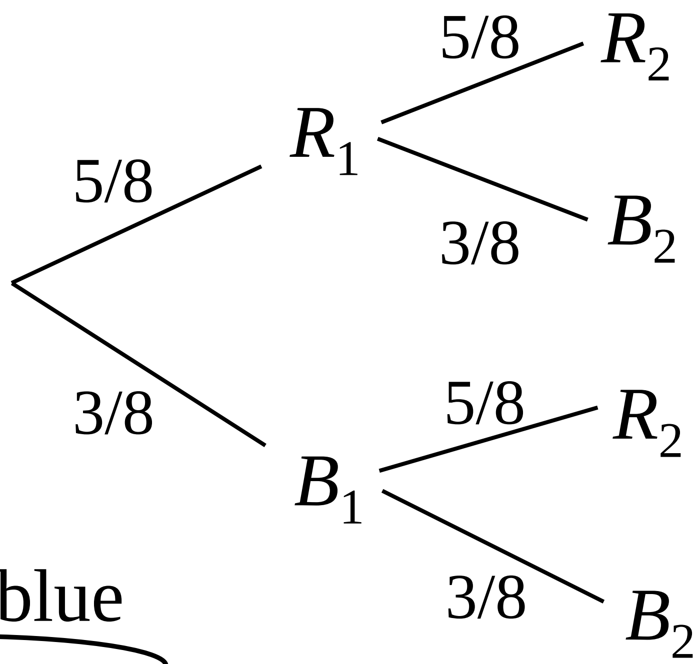
- 5/7 → R_2 $\quad P(B_1 \text{ and } R_2) = \frac{3}{8} \cdot \frac{5}{7} = \frac{15}{56}$
- 2/7 → B_2 $\quad P(B_1 \text{ and } B_2) = \frac{3}{8} \cdot \frac{2}{7} = \frac{6}{56}$

5 red, 3 blue



Example: Selecting From a Jar of Balls

Jeff draws two balls from the jar *with* replacement:



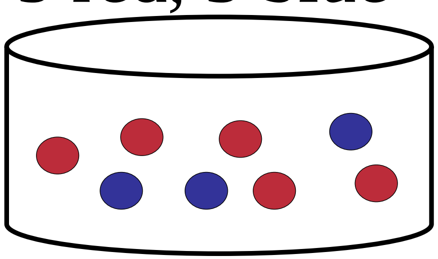
$$P(R_1 \text{ and } R_2) = \frac{5}{8} \cdot \frac{5}{8} = \frac{25}{64}$$

$$P(R_1 \text{ and } B_2) = \frac{5}{8} \cdot \frac{3}{8} = \frac{15}{64}$$

$$P(B_1 \text{ and } R_2) = \frac{3}{8} \cdot \frac{5}{8} = \frac{15}{64}$$

$$P(B_1 \text{ and } B_2) = \frac{3}{8} \cdot \frac{3}{8} = \frac{9}{64}$$

5 red, 3 blue



Note that R_2 is independent of R_1 and B_1.

Coin Tossing Model

Experiment: Toss a coin three times in succession.

S = {hhh, hht, hth, htt, thh, tht, tth, ttt}

Let x be the number of heads. We say x is a **random variable**. The **distribution** of x is given by:

x	0	1	2	3
$P(x)$	1/8	3/8	3/8	1/8

$$\frac{1}{8}+\frac{3}{8}+\frac{3}{8}+\frac{1}{8}=\frac{8}{8}=1$$

For example, the probability of getting fewer than two heads is $P(0) + P(1) = 1/8 + 3/8 = 1/2$.

Expected Value

If a random variable x can have any of the values $x_1, x_2, x_3, \ldots, x_n$, and the corresponding probabilities of these values occurring are $P(x_1), P(x_2), P(x_3), \ldots, P(x_n)$, then the **expected value of *x*** is given by

$$E(x) = x_1 \cdot P(x_1) + x_2 \cdot P(x_2) + \cdots + x_n \cdot P(x_n).$$

Example: Finding Expected Value

Find the expected number of heads in three coin tosses.

S = {hhh, hht, hth, htt, thh, tht, tth, ttt}.

The probability distribution is below.

x	0	1	2	3
$P(x)$	1/8	3/8	3/8	1/8
$x \cdot P(x)$	0	3/8	6/8	3/8

$$E(x) = 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = \frac{12}{8} = 1.5$$

Example: Finding Expected Winnings

A player pays $3 to play the following game: He rolls a die and receives $7 if he tosses a 6 and $1 for anything else. Find the player's expected net winnings for the game.

Example: Finding Expected Winnings

Solution

The information for the game is displayed below.

Die Outcome	Payoff	Net	$P(x)$	$x \cdot P(x)$
1, 2, 3, 4, or 5	\$1	–\$2	5/6	–\$10/6
6	\$7	\$4	1/6	\$4/6

Expected value: $E(x) = -\$6/6 = -\$1.00$