Percent, Sales Tax, & Discounts

Many applications involving percent are based on the following formula:

A is *P* percent of B.

$$A = P \cdot B.$$

Note that "of" implies multiplication.

Suppose that the local sales tax rate is 7.5% and you purchase a bicycle for \$894.

Sales tax amount = tax rate × item's cost

7.5% × \$894 = 0.075 × \$894 = \$67.05

The tax paid is \$67.05.

Percent and Sales Price

- Businesses reduce prices, or ***discount***, to attract customers and to reduce inventory.

A computer with an original price of \$1460 is on sale at 15% off.

a. Discount amount = discount rate × original price

$$= 15\% \times \$1460 = 0.15 \times \$1460 = \$219$$

15% of the original price, or 15% of \$1460

The discount amount is \$219.

Sale price = \$1460 – \$219 = \$1241.

Percent and Change

If a quantity changes, its ***percent increase*** or its ***percent decrease*** can be found as follows:

1. Find the fraction for the percent increase or decrease:

$$\frac{\text{amount of increase}}{\text{original amount}} \quad \text{or} \quad \frac{\text{amount of decrease}}{\text{original amount}}.$$

2. Find the percent increase or decrease by expressing the fraction in step 1 as a percent.

Example: Percent Change

- A store owner buys a calculator at a price of $90.00 and offers it for sale at $112.50. What is the percentage increase from wholesale to retail price?

$$\frac{112.5 - 90}{90} = \frac{22.5}{90} = 0.25 = 25\%$$

- The calculator hasn't sold so the store owner discounts the price back to the wholesale price of $90.00. What is the percent discount from the retail price?

$$\frac{112.5 - 90}{112.5} = \frac{22.5}{112.5} = 0.20 = 20\%$$

Formulas: percentage rates

- To increase an amount by a given percentage, convert the percentage to a decimal, *i*, and multiply by **$1 + i$**. For example, when \$100 increases by 8%, the resulting total is

$$\$100 \times (1 + 0.08) = \$100 \times 1.08 = \$108.$$

- To decrease an amount by a given percentage, use a negative value of *i*. For example, to discount or depreciate \$60 by 20%, multiply

$$\$60 \times (1 - 0.20) = \$60 \times 0.80 = \$48.$$

Interest

- **Interest** is a payment made for the use of a saver's or investor's money.
- The **rate** of interest describes what proportion of the original amount or **principal** is paid.
- With **simple interest**, interest is paid only on the original balance.
- With **compound interest**, interest is paid on the original balance, plus any accumulated interest.

Example: simple vs. compound interest

Suppose $100 is deposited into a savings account with an interest rate of 6% per year. How much has accumulated after 3 years?

	Simple Interest		Compound Interest	
	Interest	Balance	Interest	Balance
Start		$ 100.00		$ 100.00
after 1 year	$ 6.00	106.00	$ 6.00	106.00
after 2 years	6.00	112.00	6.36	112.36
after 3 years	6.00	118.00	6.74	119.10
⋮				
after 20 years	6.00	220.00	18.15	320.71

Simple Interest (Arithmetic Growth)

A = accumulated (future) value
P = principal amount (present value)
r = annual rate of interest (as decimal)
t = number of years

$$A = P + Prt = P(1 + rt)$$

The total amount of interest is $I = Prt$.

Sample Question

If you deposit \$2000 at 7% simple interest, what is the balance after 2 years?

A) \$2280.00

B) \$2289.80

C) \$2295.05

D) \$2300.52

$$A = P + Prt$$

$$\$2000 + \$2000 \times 0.07 \times 2 = \$2280.00$$

$$A = P(1 + rt)$$

$$\$2000(1 + 0.07 \times 2) = \$2000 \times 1.14$$

$$= \$2280.00$$

Sample Question

If you borrow \$3000 and pay back \$3225 in 3 years, what rate of simple interest will you pay?

$$\frac{I}{P} = \frac{3225 - 3000}{3000} = \frac{225}{3000} = 0.075 = 7.5\%$$

$$r = \frac{7.5\%}{3} = 2.5\% \text{ per year}$$

Discounted Loan

The principal minus interest is loaned to the borrower. The borrower must repay the entire principal.

P = principal

r = annual rate of interest

t = number of years

$P - Prt$ = the proceeds (amount borrower receives)

Example: Suppose you borrow $9,000 on a 6% discounted loan for 18 months. What is the net amount you receive?

$$P - Prt = 9000 - 9000 \times 0.06 \times 1.5 = 9000 - 810 = 8190$$

Compound Interest

A **nominal rate, *r*,** is the stated rate of interest for a specified length of time, not taking into account whether or how often interest is compounded.

Suppose interest at a nominal annual rate ***r*** is compounded ***n*** times per year. Then the rate per compounding period is $\boldsymbol{i = r / n}$.

For example, if a savings account pays 6%:

- compounded quarterly, then $i = 0.06/4 = 0.015$;
- compounded monthly, then $i = 0.06/12 = 0.005$.

Compound Interest (Exponential Growth)

A = accumulated (total) amount
P = principal (beginning) amount
i = r/n = interest rate per compounding period
nt = number of compounding periods

	Interest	Balance
Start		P
after 1st period	$P \times i$	$P + Pi = P(1 + i)$
after 2nd period	$P(1 + i) \times i$	$P(1 + i)\,(1 + i) = P(1 + i)^2$
after 3rd period	$P(1 + i)^2 \times i$	$P(1 + i)^2\,(1 + i) = P(1 + i)^3$
. . .	. . .	. . .
after nt periods	$P(1 + i)^{nt-1} \times i$	$\boldsymbol{A = P(1 + i)^{nt}}$

Compound Interest (Exponential Growth)

A = accumulated (total) amount

P = principal (beginning) amount

r = nominal annual interest rate

i = r/n = interest rate per compounding period

nt = number of compounding periods

$$A = P(1+i)^{nt} = P\left(1+\frac{r}{n}\right)^{nt}$$

Sample Question

If you deposit $2000 at 4% compounded quarterly, what is the balance after 2 years (to the nearest dollar)?

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A. $2160

B. $2163

C. $2166

D. $2167

$$\$2000\left(1 + \frac{0.04}{4}\right)^{4\times 2} = \$2000 \times 1.01^{8}$$

$$= \$2000 \times 1.082857$$

$$= \$2165.71$$

Note: $2160 is the amount accrued using 4% simple interest, $A = P(1 + rt)$: $2000(1 + 2 × 0.04) = $2160