Decoding received words

The generator matrix for a (3, 6) block code is given. Suppose that a 3 bit message is encoded and transmitted. The received word is 101010. We find the check matrix and decode the received word if it is a codeword or differs from a codeword in a single digit.

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$\begin{matrix} \mathbf{1} \\ \mathbf{0} \\ \mathbf{1} \\ \mathbf{0} \\ \mathbf{1} \\ \mathbf{0} \end{matrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{matrix} 1 & 0 & 0 \end{matrix}$$

The corrected codeword is 101110 and the message is 101.

Chapter 8: Voting Methods

Assume that each voter has a rank ordering of candidates, i.e., casts a **preference list ballot**.

Ballot

1st *Buchanan*

2nd *Coolidge*

3rd *Adams*

Rank	Vote
First Choice	*B*
Second Choice	*C*
Third Choice	*A*

Possible Vote Outcome

Number of votes	4	3	2
First choice	*A*	*B*	*C*
Second choice	*B*	*C*	*B*
Third choice	*C*	*A*	*A*

4 voters ranked the candidates in order A, B, C
3 voters ranked the candidates in order B, C, A
2 voters ranked the candidates in order C, B, A

Which candidate is preferred by the group?

Majority Rule Method

Only first-place votes are considered, and the candidate receiving **more than half of the total first-place votes** is declared the winner.

Majority Rule with Three Candidates

Consider the following preference list ballots:

Number of Votes	4	3	2
First Choice	*A*	*B*	*C*
Second Choice	*B*	*C*	*B*
Third Choice	*C*	*A*	*A*

To win by majority rule, a candidate must have at least five (more than half of 9) first-place votes. Majority rule produces no winner in this case.

Plurality Method

Only first-place votes are considered. The candidate with the **most first-place votes** wins.

Example: Suppose that three candidates, *A*, *B*, and *C* are ranked as follows:

Number of Votes	4	3	2
First Choice	*A*	*B*	*C*
Second Choice	*B*	*C*	*B*
Third Choice	*C*	*A*	*A*

Using plurality voting, *A* is the winner.
However, the majority of voters are unhappy – they ranked *A* last!
If C were to drop out, B would win over A with a majority vote.

Borda Count Method

- Each voter gives
  - the voter's last-place candidate 1 point;
  - the next-to-last place candidate 2 points;
  - the third-to-last place candidate 3 points;
  - and so on.
- Total the points from all voters for each candidate.
- The candidate with the most points wins.
- The point total for each candidate is known as the candidate's **Borda score**.

Example: Borda Count

Number of Votes	4	3	2
First Choice	*A* (3)	*B* (3)	*C* (3)
Second Choice	*B* (2)	*C* (2)	*B* (2)
Third Choice	*C* (1)	*A* (1)	*A* (1)

A: $(4 \times 3) + (3 \times 1) + (2 \times 1) = 17$

B: $(4 \times 2) + (3 \times 3) + (2 \times 2) = 21$ ← Borda Winner

C: $(4 \times 1) + (3 \times 2) + (2 \times 3) = 16$

Check: $17 + 21 + 16 = 54 = 6 \times 9$

Plurality-with-Elimination

Candidates are eliminated in an order based on **the number of first-place votes**.

Example: Using the preference schedule in the following figure, which candidate will win by this method?

Votes	7	5	4	1
1st	*A*	*C*	*B*	
2nd		*A*	*C*	*A*
3rd	*B*	*B*		*B*
4th	*C*		*A*	*C*

Step 1. No candidate has a majority. *D* has the fewest 1st place votes so *D* is eliminated.
Remove *D* from the chart and move others up.

Votes	7	5	4	1
1st	*A*	*C*		*A*
2nd		*A*	*C*	
3rd	*C*		*A*	*C*

Step 2. ***B*** **now has the fewest 1st place votes so** ***B*** **is eliminated. Remove** ***B*** **from the chart and move others up.**

Votes	7	5	4	1
1st	*A*	*C*	*C*	*A*
2nd	*C*	*A*	*A*	*C*

Step 3. ***C*** **now has a majority of the votes.**

C **wins!**

Head-to-Head Winner

A candidate is a **Head-to-Head winner** if he or she beats all other candidates by majority rule when they meet head-to-head (one-on-one).

To decide if a Head-to-Head winner exists:

Every candidate is matched on a one-on-one basis with every other candidate.

Drawback:* *there may not exist a Head-to-Head winner.

Example – Head-to-Head Winner

Example: Suppose that three candidates, *A*, *B*, and *C* are ranked as follows:

Number of Votes	4	3	2
First Choice	*A*	*B*	*C*
Second Choice	*B*	*C*	*B*
Third Choice	*C*	*A*	*A*

***A* vs. *B*:** *B* wins 5 to 4
***A* vs. *C*:** *C* wins 5 to 4
***B* vs. *C*:** *B* wins 7 to 2

B is the Head-to-Head winner.

Note: if *C* were to drop out, the result is unchanged;
the Plurality winner is not the Head-to-Head winner.

Monotonicity

When a candidate wins an election and, in a reelection, the only changes are changes that favor that candidate, then that same candidate should win the reelection.

Number of votes	~~5~~ 6	~~4~~ 3
First choice	*A*	*B*
Second choice	*B*	*A*

Majority rule is monotone and is the only method for two-candidate elections that is monotone, treats voters equally, and treats both candidates equally.

Plurality-with-Elimination is Not Monotone

Monotonicity: When a candidate wins an election and, in a reelection, the only changes are changes that favor that candidate, then that same candidate should win the reelection.

Number of Votes	7	6	5	3
First choice	A	B	C	D
Second choice	B	A	B	C
Third choice	C	C	A	B
Fourth choice	D	D	D	A

D is eliminated.

B is eliminated.

A is the Winner.

Number of votes	7	6	5	3
First choice	A	B	C	D
Second choice	B	A	B	C
Third choice	C	C	A	B
Fourth choice	D	D	D	A

A is the winner, so now suppose the voters in the last column raise A to first place.

Number of Votes	7	6	5	3
First choice	A	B	~~C~~	A
Second choice	B	A	B	~~D~~
Third choice	~~C~~	~~C~~	A	~~C~~
Fourth choice	~~D~~	~~D~~	~~D~~	B

Eliminate D.

Eliminate C.

B wins!