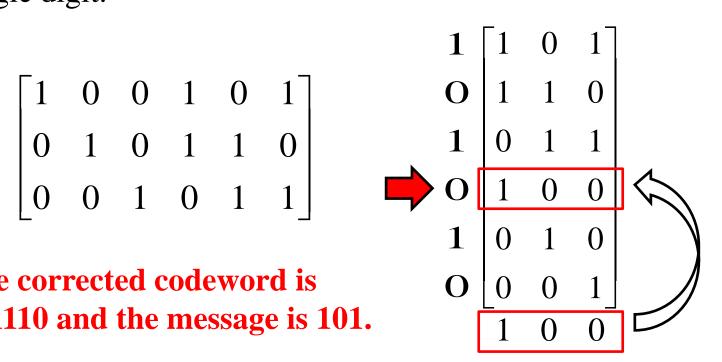
#### Decoding received words

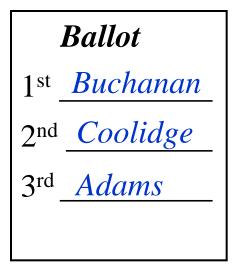
The generator matrix for a (3, 6) block code is given. Suppose that a 3 bit message is encoded and transmitted. The received word is 101010. We find the check matrix and decode the received word if it is a codeword or differs from a codeword in a single digit.

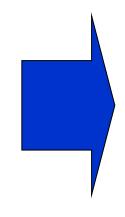
The corrected codeword is **101110 and the message is 101.** 



# Chapter 8: Voting Methods

Assume that each voter has a rank ordering of candidates, i.e., casts a **preference list ballot**.





Rank	Vote
First Choice	В
Second Choice	С
Third Choice	A

### Possible Vote Outcome

Number of votes	4	3	2
First choice	A	В	С
Second choice	В	С	В
Third choice	С	A	A

4 voters ranked the candidates in order A, B, C3 voters ranked the candidates in order B, C, A2 voters ranked the candidates in order C, B, A

Which candidate is preferred by the group?

# **Majority Rule Method**

Only first-place votes are considered, and the candidate receiving **more than half of the total first-place votes** is declared the winner.

#### Majority Rule with Three Candidates

Consider the following preference list ballots:

Number of Votes	4	3	2
First Choice	A	В	С
Second Choice	В	C	В
Third Choice	С	A	A

To win by majority rule, a candidate must have at least five (more than half of 9) first-place votes. Majority rule produces no winner in this case.

#### Plurality Method

Only first-place votes are considered. The candidate with the **most first-place votes** wins.

**Example:** Suppose that three candidates, *A*, *B*, and *C* are ranked as follows:

Number of Votes	4	3	2
First Choice	A	В	С
Second Choice	В	С	В
Third Choice	С	A	A

Using plurality voting, A is the winner.

However, the majority of voters are unhappy – they ranked *A* last! If C were to drop out, B would win over A with a majority vote.

### Borda Count Method

- Each voter gives
  - the voter's last-place candidate 1 point;
  - the next-to-last place candidate 2 points;
  - the third-to-last place candidate 3 points;
  - and so on.
- Total the points from all voters for each candidate.
- The candidate with the most points wins.
- The point total for each candidate is known as the candidate's **Borda score**.

### Example: Borda Count

Number of Votes	4		3		2	
First Choice	A	(3)	B	(3)	С	(3)
Second Choice	В	(2)	С	(2)	В	(2)
Third Choice	С	(1)	A	(1)	A	(1)

$$A: (4 \times 3) + (3 \times 1) + (2 \times 1) = 17$$
  

$$B: (4 \times 2) + (3 \times 3) + (2 \times 2) = 21$$

$$C: (4 \times 1) + (3 \times 2) + (2 \times 3) = 16$$
Borda Winner

Check:  $17 + 21 + 16 = 54 = 6 \times 9$ 

# Plurality-with-Elimination

Candidates are eliminated in an order based on **the number of first-place votes**.

**Example:** Using the preference schedule in the following figure, which candidate will win by this method?

Votes	7	5	4	1
1 st	A	С	B	
$2^{nd}$		A	С	A
3 <sup>rd</sup>	В	B		B
4 <sup>th</sup>	С		A	C

Step 1. No candidate has a majority. D has the fewest 1<sup>st</sup> place votes so D is eliminated. Remove D from the chart and move others up.

Votes	7	5	4	1
1 st	A	С		A
2 <sup>nd</sup>		A	С	
3 <sup>rd</sup>	С		A	C

**Step 2.** *B* now has the fewest 1<sup>st</sup> place votes so *B* is eliminated. Remove *B* from the chart and move others up.

Votes	7	5	4	1
1 st	A	С	С	A
2 <sup>nd</sup>	С	A	A	С

**Step 3.** *C* now has a majority of the votes.

C wins!

### Head-to-Head Winner

A candidate is a **Head-to-Head winner** if he or she beats all other candidates by majority rule when they meet head-to-head (one-on-one).

To decide if a Head-to-Head winner exists:

**Every candidate is matched on a one-on-one basis** with every other candidate.

> Drawback: there may not exist a Head-to-Head winner.

## Example – Head-to-Head Winner

**Example:** Suppose that three candidates, *A*, *B*, and *C* are ranked as follows:

Number of Votes	4	3	2
First Choice	A	В	С
Second Choice	В	С	В
Third Choice	С	A	A

**A vs. B:** B wins 5 to 4

A vs. C: C wins 5 to 4 B is the Head-to-Head winner.

*B* **vs. C:** *B* wins 7 to 2

Note: if *C* were to drop out, the result is unchanged; the Plurality winner is not the Head-to-Head winner.

# Monotonicity

When a candidate wins an election and, in a reelection, the only changes are changes that favor that candidate, then that same candidate should win the reelection.

Number of votes	\$ 6	* 3
First choice	A	В
Second choice	В	A

Majority rule is monotone and is the only method for two-candidate elections that is monotone, treats voters equally, and treats both candidates equally.

#### Plurality-with-Elimination is Not Monotone

**Monotonicity:** When a candidate wins an election and, in a reelection, the only changes are changes that favor that candidate, then that same candidate should win the reelection.

Number of Votes	7	6	5	3
First choice	А	B	С	₿
Second choice	B	А	凇	C
Third choice	С	С	А	8
Fourth choice	$\aleph$	₿	×	A

D is eliminated.

B is eliminated.

A is the Winner.

Number of votes	7	6	5	3
First choice	А	В	С	D
Second choice	В	A	В	C
Third choice	С	C	A	В
Fourth choice	D	D	D	Α

A is the winner, so now suppose the voters in the last column raise A to first place.

Number of Votes	7	6	5	3
First choice	A	В	×	A
Second choice	В	A	В	$\mathbf{N}$
Third choice	×	×	А	8
Fourth choice	₿	$\mathbf{N}$	Ø	В

Eliminate D.

Eliminate C.

B wins!