Math 519: Algebraic Structures I
Fall 2012

"In one sense symmetric means something like well-proportioned, well-balanced, and symmetry denotes that sort of concordance of several parts by which they integrate into a whole. *Beauty* is bound up with symmetry. Thus Polykleitos, who wrote a book on proportion, and whom the ancients praised for the harmonious perfection of this sculptures, uses the word, and Dürer follows him in setting down a canon of proportions for the human figure. In this sense the idea is by no means restricted to spatial objects; the synonym 'harmony' points more toward its acoustical and musical than its geometric applications. *Ebenmass* is a good German equivalent for the Greek symmetry; for like this it carries also the connotation of 'middle measure,' the mean toward which the virtuous should strive in their actions according to Aristotle's Nicomachean Ethics, and which Galen in *De temperamentis* describes as that state of mind which is equally removed from both extremes."

— H. Weyl, 1952

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Course Goals

After the first difficult course sequence in abstract algebra, you are probably familiar with the definitions and basic properties of groups, rings, and fields. However, you're entitled at the outset of this course still to be uncomfortable about some of the more "interesting" issues you've seen: Which details in the formulation of the definitions really matter? Which polynomials are irreducible? You're also entitled not to have had your head far enough above the water yet to have asked the "big" questions: What groups are there? In what ways can new groups be built out of old ones? What is the geometric content of a ring?

By the end of this course, you will have begun to answer all of these questions and more like them. A course like this, along with two or three others, is required of practically every mathematician-in-training anywhere in the world, and forms part of the basic language in which mathematicians communicate with one another. The goal of this course is that you will share this basic language, as far as algebra is concerned.

Course Content

It's difficult to express the content of this course without either making it sound like a course you've already had or writing for an audience which has already had the present course. It is true that your prior knowledge of algebra will make many parts of this class seem familiar. The difference is that we can feel free now not to think so much about learning the definition of a group (for instance) as about classifying groups. We can feel free now not to think so much about giving examples of divisibility and irreducibility in a ring as about characterizing them.

The first (and longer) component of this course will cover the classification and action of groups. Along the way, we will take a little extra trouble to develop tools that are more broadly useful (e.g. the language of categories). While an actual classification of all finite groups is, in several mathematically meaningful senses, impossible, you will be able in principal, by the end of this course, to classify groups of any reasonably small group, and also those of certain other well-behaved families.

The second component of the course will cover the theory of rings. Again, we will focus on tools that are more broadly useful. Special emphasis will fall on issues of divisibility, reducibility, and ideal structure. We will also discuss modules, which are a mutual generalization of ideals, vector spaces, and Abelian groups.

Course Activities

Homework will be assigned frequently, and will be due each week on Wednesdays (unless otherwise announced). The most common thing in all of mathematics — I do it myself, as does every other mathematician I know — is to see somebody else doing a problem and say, "Yes, yes, of course. I understand completely," and then walk away and realize that we had no idea at all what was going on. Homework is your guard against this. If you really understand

how to do the homework, you're generally in pretty good shape. If you can't, you've got plenty of time to figure it out, ask me, ask a friend, or take whatever other action you see fit.

Homework will always be due at 4:30 on the appointed day. You are, of course, welcome to turn it in when you come to class. If you wish, though, you may continue to work on it, and may deliver it to my office or my department mailbox.

Cooperation on homework is strongly encouraged. Most people will find that it is usually necessary: more people can have more ideas faster and can find the problems with the wrong ones faster. Besides, it's both more fun and representative of the way mathematical research is done. Talk with each other, talk with me, talk with friends, use any resource. It is important, however, to be sure that you understand the solution you present. In designing the tests, I will assume thorough familiarity with all homework problems due before the date of the exam.

You are also encouraged to visit me in my office (see note on office hours above) or to call or e-mail me. To be more clear: It's a hard class. I'd like to see you do well in it. I'd love to talk with you and to help you in any way that I can.

The homework will often be hard. You will seldom be able to solve all of the problems in one setting. Plan your time accordingly.

Some homework may call on you to use certain freely-available mathematical software. More details will follow.

The class will meet on Monday, Wednesday, and Friday at noon. A typical meeting will begin with a discussion of any questions folks have, with procedural matters treated first. This will be followed by a discussion of new material (often in the form of problems, on which students will work in groups) and typically an assignment of new homework.

You should be in every class meeting, and should make sure that you are actively engaged. It goes without saying that when a problem is assigned for group work, you must do it. If you wait for me to tell you how to do it, then by the time I talk about the solution with the class, everybody else will understand it and will be ready to ask about issues you haven't encountered, and you will be lost. Don't do this. You should be careful to ask any questions you have. You should also feel free to be wrong. We all will be at some point in the class. That's why we gather together, instead of just reading the book on our own: we can help one another understand better, and we can try out ideas on each other, even if we aren't quite sure of them.

Text: Dummit and Foote, *Abstract Algebra*, 3rd Ed, Wiley, 2004, ISBN: 978-047-143-3347

Be warned. The bookstores have been known to offer some other books as "recommended" for math courses. They are recommended by the bookstore, not by the math department, and not by me. I don't particularly recommend against them (since I have little idea what they'll be), but let the buyer be ware.

The text makes a great effort — and a successful one at most points — to be readable. It will provide an important opportunity to get an explanation in a different voice (at times very different) than that of your beloved teacher.

I will also have the following books on reserve in the library:

- J. Gallian, *Contemporary Abstract Algebra*
- T. Hungerford, *Algebra*
- N. Jacobson, *Basic Algebra*
- S. Lang, *Algebra*

You may find these books helpful, and you are encouraged to use them often. Gallian is at a much lower level than this course, but gives very accessible treatments of much of the prerequisite material. It's good to look at if you're not thoroughly comfortable with the algebra you know from earlier courses, or if I think something should be familiar but you don't find it familiar (this almost always happens at some point).

Hungerford is probably the most common textbook to use for a course like this one, with Lang showing a strong second. Both explain everything in this course excellently. The styles are very different; you should sample both of them to see which is the most natural to you. Jacobson's book is more difficult, but more comprehensive.

You are at the point in your career when you should be thinking about building your professional library. You should, of course, be keeping all of your textbooks after the end of the course. Moreover, you should seriously consider buying one of Hungerford, Jacobson, and Lang. These books are routinely referenced in the research literature of many fields of mathematics when the author wants to "remind" readers of a "well-known" theorem. If you plan to specialize in algebra or a closely-related field, you will probably want to own all three, eventually.

There will also be some exams. Each exam will be preceded by a review sheet indicating *exactly* what material will be covered and a review session. One exam will be given in the regularly scheduled class time and place on October 5. There will be a final exam in a time and place to be announced. I will forward information on the final schedule as soon as I have it. The final will test your ability to do all of the things we have worked on in class, with particular emphasis on material covered since the last exam.

Each student will complete a significant research project over the course of the term, and will give a presentation on the results between November 30 and December 5. More information will be forthcoming.

Everything in this class will make more sense, the project will be easier, and you will feel more at home in the mathematical world if you are an active participant in the mathematical life of the department. You should be going to colloquium every time it happens (usually 3:00 on the first and third Thursday of each month). If you are a Ph.D. student, you should find some seminar that you can attend regularly. I will not grade you on this. You are professionals in training, and professionals strive to broaden their understanding of the profession.

The general philosophy is that class sessions and homework will be very hard and tests will be pretty easy (assuming, of course, that you've suffered through the class meetings and homework leading up to them). Again, my goal with the homework is to help you to understand the material so well that you're unhappy with me for giving such a boring (easy) test.

In all activities for this class, make sure that you *do something*. It is depressing how often students who probably know something relevant to a problem do absolutely nothing, allowing no opportunity to receive credit on the part they actually know.

Grading

Grades will be calculated from the following sources:

Homework	200
Mid-Term Exam	100
Project	200
Final Exam	150
	650 pts

Failure to attend class regularly will certainly adversely affect your grades on each of these factors. For instance, while I do not artificially lower grades for bad attendance, it has consistently held that almost all grades below C- that have been achieved in classes that I have taught have been associated with significant attendance problems.

In like manner, you should not underestimate the impact of your homework. Not only does the experience of the homework problems impact your test grades, but the homework itself is a considerable portion of the grade in the class. *You have time to consult books, collaborate with classmates, ask me, or do whatever else you need to on the homework. Thus, everyone should receive a grade near 100% on the homework.*

In all work done for this class, work is more important than answers. A correct answer without correct work (or worse, with work that does not match the answer) is not worth much at all, while generally correct work with an incorrect answer is almost as good as being completely right. Thus, getting the right answer does not guarantee a good grade on the problem, and getting a wrong answer does not guarantee a bad one.

I will make the following guarantees about letter grades. I may decide to lower these criteria (i.e. give a higher grade than the one shown here, if I see that the questions were hard enough that lower numbers more accurately reflect my true standards), but will never raise them.

Percent of total	Grade
90–100	A
80–89	B
70–79	C
60–69	D
≤ 59	E

Prerequisites

The prerequisites of this course are designed to save you from spending a semester being miserable and failing this course. *I am on your side, and wish you success. That is why I am telling you this.* To take this course, you must have completed Math 419 or have my consent (which roughly corresponds to me being convinced that you would get at least a B on the 419 final exam, were it given today). If you don't know what a group, a ring, or an ideal is, you're in trouble.

Any student not meeting these requirements is *strongly* advised to delay taking this class until they are satisfied.

Catalog Description

Introduction to the basic techniques in the classification of finite groups, including homomorphism theorems, classification of finitely generated Abelian groups, Sylow's theorems and classification of small groups, divisibility theory in rings, especially polynomial rings. Prerequisite: 419 or consent of instructor.

Emergency Procedures

Southern Illinois University Carbondale is committed to providing a safe and healthy environment for study and work. Because some health and safety circumstances are beyond our control, we ask that you become familiar with the SIUC Emergency Response Plan and Building Emergency Response Team (BERT) program. Emergency response information is available on posters in buildings on campus, available on BERTs website at www.bert.siu.edu, Department of Safetys website www.dps.siu.edu (disaster drop down) and in Emergency Response Guideline pamphlet. Know how to respond to each type of emergency.

Instructors will provide guidance and direction to students in the classroom in the event of an emergency affecting your location. It is important that you follow these instructions and stay with your instructor during an evacuation or sheltering emergency. The Building Emergency Response Team will provide assistance to your instructor in evacuating the building or sheltering within the facility.