1. Let R be a PID, and S a subset closed under multiplication. Show that $S^{-1}R$ is a PID.

2. Prove the Chinese Remainder Theorem: Let $A_1, \ldots, A_n$ be ideals in a ring R such that $R^2 + A_i = R$ for all i and $A_i + A_j = R$ for all $i \neq j$. If $b_1, \ldots, b_n \in R$, then there is $b \in R$ such that $b \equiv b_i (mod\ A_i)$, and b is unique up to congruence modulo $\bigcap_i A_i$.

3. Let R be the subring $\{a + b\sqrt{10} | a, b \in \mathbb{Z}\}$ of the field of real numbers.

   (a) Show that the map $N : R \to \mathbb{Z}$ given by $a + b\sqrt{10} \mapsto (a + \sqrt{10})(a - \sqrt{10})$ is a multiplicative homomorphism with $N(u) = 0$ if and only if $u = 0$.

   (b) Show that u is a unit in R if and only if $N(U)$ is a unit in $\mathbb{Z}$.

   (c) Show that $2, 3, 4 + \sqrt{10}$, and $4 - \sqrt{10}$ are irreducible but not prime in R.