

1. Prove that the semidirect product of H and K with respect to φ
 - (a) Is a group of order $|H||K|$
 - (b) Has subgroups isomorphic to H and K , where
 - i. H is normal
 - ii. $H \cap K = 1$, and
 - iii. For all $h \in H$ and $k \in K$ we have $khk^{-1} = k \cdot h = \varphi(k)(h)$.
2. Classify all groups of order 12, up to isomorphism.
3. Let R be a principal ideal domain, and S a subset closed under multiplication. Show that $S^{-1}R := \{s^{-1}a : s \in S, a \in R\}$ is a principal ideal domain.