

1. Let $G = GL_2(\mathbb{Q})$. Let

$$a = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}; b = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}.$$

- (a) Show that a has order 4, and that b has order 3.
 - (b) Show that ab has infinite order.
 - (c) Show that the additive group $\mathbb{Z}_2 \oplus \mathbb{Z}$ has nonzero elements x, y with infinite order such that $x + y$ has finite order.
2. Show that if G is an Abelian group, then the set of all elements of G with finite order is a subgroup of G .
3. Dummit–Foote, p. 86, #12.
4. Dummit–Foote, p. 88, #33.
5. Dummit–Foote, p. 96, #16.