1. Prove that if G is a semigroup with left inverses and a left identity, then G is a group.

2. For $a, b \in \mathbb{Q}$, we write $a \sim b$ if and only if $a - b \in \mathbb{Z}$. Show that the set of $\sim$ equivalence classes of rationals forms an infinite Abelian group in which every element has finite order.

3. The center C_G of a group G is defined by

$$C_G = \{a \in G | ax = xa \text{ for all } x \in G\}.$$

   (a) Show that if n is odd, the center of D_{2n} is trivial.

   (b) Show that if n is even, the center of D_{2n} has exactly two elements.

4. Show that if G is a finite group of even order, then G contains an element of order 2.