Math 251: Calculus III

Spring 2013

With the help of these new field concepts Faraday succeeded in forming a qualitative concept of the whole complex of electromagnetic effects discovered by him and his predecessors. The precise formulation of the time-space laws of those fields was the work of Maxwell. Imagine his feelings when the differential equations he had formulated proved to him that electromagnetic fields spread in the form of polarized waves and with the speed of light! To few... in the world has such an experience been vouchsafed.

— Albert Einstein, 1940

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Course Goals

The Real Goals

From at least the first semester of your calculus education, we have made grandiose promises. Calculus will underlie error analysis, mechanical systems, electromagnetism, fluid flows, and every other system worthy of scientific analysis. Up to now, however, it has done so only in a world of two variables, one a function of the other. In this course, that will change.

The goal of this course is to equip you to conduct mathematical analysis of situations involving more than two dimensions, and more than one independent variable. As it has been said, it's opener there in the wide open air. Not only are there technical details to be dealt with (How do derivatives in different directions interact? How do you specify a line in a space of many dimensions?), but there is room for much more interesting geometry — with interesting implications for the dynamical systems represented.

The Official Goals

The course will acquaint students of engineering, mathematics and science with the concepts and techniques of the calculus for functions of multiple variables and for vector-valued functions.

Upon completion of the course, the student should be able to:

- Compute dot products and cross products, find the angle between two vectors, find equations of lines and planes in space.
- Find derivatives and integrals of vector-valued functions.
- Apply derivatives and integrals to solve problems involving motion in space; find velocity, speed and acceleration given the position vector of a particle; find the position of a particle given information about its initial velocity and acceleration.
- Find the arclength of a given curve.
- Evaluate first and higher order partial derivatives of a function of several variables, using chain rules as appropriate.
- Evaluate directional derivatives of a function of several variables; find and interpret graphically the gradient of a given function of several variables; find the tangent plane and normal line to a given surface at a given point.
- Recognize a critical point for a function of several variables. Determine whether a critical point yields a local extremum or saddle point, or neither; maximize and minimize functions using the method of Lagrange Multipliers.
- Convert double and triple integrals to iterated integrals and vice versa. Use double and/or triple integrals to find areas, volumes, surface areas, moments and centers of mass.
- Use rectangular, cylindrical and spherical coordinates to parametrize standard surfaces in space and to evaluate appropriate integrals.

- Use the Jacobian to find the differential of areas and volumes.
- Determine whether a given vector field is conservative; determine whether a line integral is independent of path; find the curl and divergence of a given vector field.
- Evaluate line integrals along smooth curves; apply line integrals to compute work done by a force along a curve. Evaluate line integrals using Greens Theorem and Stokes Theorem.
- Find the flux of a vector field through a given surface. Use the Divergence Theorem to compute flux.

Course Content

We begin by an overview of the geometry and algebra of vectors, and of figures defined parametrically in 3-dimensional space. A few points of calculus transfer directly from the 2-dimensional case.

As soon as we try to take a derivative, though, there are complications to consider. Small intervals are replaced by small spheres. The derivative can depend on the direction you're going (as, of course, it can, if you walk in rolling hills). And even knowing the derivative in all directions can tell you less about the geometry than it used to. This explains, among other things, an important point of game theory.

Integral multivariable calculus comes in two main installments. One is more or less what you would expect. Instead of the area under a curve, you look at the volume under a surface. There are, of course, technicalities, but the concept will be familiar.

Then there are vector fields. Imagine, if you will, an electromagnetic or gravitational field, or perhaps the function giving, for every point inside the heart, the velocity of blood at that point. This last can, to some precision, be observed by a sonogram. From this, we can calculate the speed of the flow through a certain region, identify whether a region contains a "source" or a "sink" (i.e. blood entering or leaving the chamber there), and many other useful issues. This is the work of vector calculus, the second part of integral multivariable calculus, and the final segment of our course.

Course Activities

Homework will be assigned daily or almost daily and will be collected weekly, on Wednesdays (unless otherwise announced). There will be a truckload of it, and that's not because I'm sadistic. The most common thing in all of mathematics — I do it myself, as does every other mathematician I know — is to see somebody else doing a problem and say, "Yes, yes, of course. I understand completely," and then walk away and realize that we had no idea at all what was going on. Homework is your guard against this. If you really understand how to do the homework, you're generally in pretty good shape. If you can't, you've got plenty of time to figure it out, ask me, ask a friend, or take whatever other action you see fit.

Homework will always be due at 4:30 on the appointed day. You are, of course, welcome to turn it in when you come to class. If you wish, though, you may continue to work on it, and may deliver it to my office or my department mailbox.

Cooperation on homework is strongly encouraged. There will almost certainly be problems on which it is necessary. Talk with each other, talk with me, talk with friends, use any resource. It is important, however, to be sure that you understand the solution you present. In designing the tests, I will assume thorough familiarity with all homework problems due before the date of the exam.

You are also encouraged to visit me in my office (see note on office hours above) or to call or e-mail me. To be more clear: It's a hard class. I'd like to see you do well in it. I'd love to talk with you and to help you in any way that I can.

It is wise to work on the homework as it is assigned, for a couple of reasons. First, there will be enough of it that it will not be practical to just sit down and do the whole week's worth in an evening. Second (and more importantly), the material builds on itself, so that a few days without working through at least some of the problems may find you feeling a little lost.

The class will meet on Monday, Wednesday, and Friday at noon. A typical meeting will begin with a discussion of any questions folks have, with procedural matters treated first. This will be followed by a discussion of new material (often in the form of problems, on which students will work in groups) and typically an assignment of new homework.

You should be in every class meeting, and should make sure that you are actively engaged. It goes without saying that when a problem is assigned for group work, you must do it. If you wait for me to tell you how to do it, then by the time I talk about the solution with the class, everybody else will understand it and will be ready to ask about issues you haven't encountered, and you will be lost. Don't do this. You should be careful to ask any questions you have. You should also feel free to be wrong. We all will be at some point in the class. That's why we gather together, instead of just reading the book on our own: we can help one another understand better, and we can try out ideas on each other, even if we aren't quite sure of them.

Text: Stewart, *Essential Calculus with Early Transcendentals*, Thompson, Brooks-Cole, ISBN 0-4950-14281

Be warned. The bookstores have been known to offer some other books as "recommended" for math courses. They are recommended by the bookstore, not by the math department, and not by me. I don't particularly recommend against them (since I have little idea what they'll be), but let the buyer be ware.

The text makes a great effort — and a successful one at most points — to be readable. It will provide an important opportunity to get an explanation in a different voice (at times very different) than that of your beloved teacher. It will also be the source of the bulk of the homework problems. Be careful of this, though: One can easily get the impression from the book that the right way to think about things is to memorize some formula or some procedure. In practice, if you try to do this with everything we will learn in the approximately forty-five hours we have together in class this semester, plus the time spent outside of class, you will likely be overwhelmed and miserable. Better is to try and find the big ideas, and re-build everything else as you need it. You'll do better with this class and with later ones, and you'll not have to memorize nearly as much (i.e. it's easier).

There will also be some exams. Each exam will be preceded by a review sheet indicating *exactly* what material will be covered, an in-class review session, and an out-of-class review session. Exams will be given in the regularly scheduled class time and place on February 8, March 8, and April 12. In addition, there will be a final exam, in a time and place to be announced. I will forward information on the final schedule as soon as I have it. The final will test your ability to do all of the things we have worked on in class.

The general philosophy is that class sessions and homework will be very hard and tests will be pretty easy (assuming, of course, that you've suffered through the class meetings and homework leading up to them). Again, my goal with the homework is to help you to understand the material so well that you're unhappy with me for giving such a boring (easy) test.

In all activities for this class, make sure that you *do something.* It is depressing how often students who probably know something relevant to a problem do absolutely nothing, allowing no opportunity to receive credit on the part they actually know.

Grading

Grades will be calculated from the following sources:

Homework	200
Regular Exams (100 each)	300
Final Exam	200
	700pts

Failure to attend class regularly will certainly adversely affect your grades on each of these factors. For instance, while I do not artificially lower grades for bad attendance, it has consistently held that almost all grades below C- that have been achieved in classes that I have taught have been associated with significant attendance problems.

In like manner, you should not underestimate the impact of your homework. Not only does the experience of the homework problems impact your test grades, but the homework itself is a considerable portion of the grade in the class. Moreover, since you can use the book, talk with friends, talk with a tutor, ask me how to do the problem, etc., *everyone should receive a grade of near 100% on the homework.* It is depressing how rarely this happens. Indeed, due largely to negligence in completing and turning in all of the assigned problems, many students find that their homework grade instead brings their grade in the course down. Don't let this happen to you.

In all work done for this class, work is more important than answers. A correct answer without correct work (or worse, with work that does not match the answer) is not worth much at all, while generally correct work with an incorrect answer is almost as good as being completely right. Thus, getting the right answer does not guarantee a good grade on the problem, and getting a wrong answer does not guarantee a bad one.

I will make the following guarantees about letter grades. I may decide to lower these criteria (i.e. give a higher grade than the one shown here, if I see that the questions were hard enough that lower numbers more accurately reflect my true standards), but will never raise them.

Percent of total	Grade
90–100	A
80–89	B
70–79	C
60–69	D
≤ 59	E

Prerequisites

The prerequisites of this course are designed to save you from spending a semester being miserable and failing this course. *I am on your side, and wish you success. That is why I am telling you this.* This course picks up exactly where Math 250 left off, and it does so at a very quick pace. To take this course, you must have a grade of C or better in Math 250.

Any student not meeting these requirements is *strongly* advised to delay taking this class until they are satisfied.

Catalog Description

251 (3 credits) Calculus III (Advanced University Core Curriculum course) [IAI Course: M1 900] Further topics in calculus. Definite integrals over solid regions, applications of partial derivatives, vectors and vector operations, derivatives of vector functions, line integrals. Greens theorem. Prerequisite: 250 with a grade of C or better. Satisfies University Core Curriculum Mathematics requirements in lieu of 110 or 113.

Emergency Procedures

Southern Illinois University Carbondale is committed to providing a safe and healthy environment for study and work. Because some health and safety circumstances are beyond our control, we ask that you become familiar with the SIUC Emergency Response Plan and Building Emergency Response Team (BERT) program. Emergency response information is available on posters in buildings on campus, available on BERTs website at www.bert.siu.edu, Department of Safetys website www.dps.siu.edu (disaster drop down) and in Emergency Response Guideline pamphlet. Know how to respond to each type of emergency.

Instructors will provide guidance and direction to students in the classroom in the event of an emergency affecting your location. It is important that you follow these instructions and stay with your instructor during an evacuation or sheltering emergency. The Building Emergency Response Team will provide assistance to your instructor in evacuating the building or sheltering within the facility.