Math 250: Calculus II
Spring 2012

A possible explanation of the physicist's use of mathematics to formulate his laws of nature is that he is a somewhat irresponsible person. As a result, when he finds a connection between two quantities which resembles a connection well-known from mathematics, he will jump at the conclusion that the connection is that discussed in mathematics simply because he does not know of any other similar connection. It is not the intention of the present discussion to refute the charge that the physicist is a somewhat irresponsible person. Perhaps he is. However, it is important to point out that the mathematical formulation of the physicist's often crude experience leads in an uncanny number of cases to an amazingly accurate description of a large class of phenomena. This shows that the mathematical language has more to commend it than being the only language which we can speak; it shows that it is, in a very real sense, the correct language.

— E. Wigner, 1960

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Course Goals

The Real Goals

In Calculus I, you learned about some really amazing and useful concepts: the derivative and the integral. The most important reason, in all scientific and technological fields, to learn about a new concept is that it allows you to express your interesting questions much more exactly, and then answer them. The more exact statements of scientific questions that are made possible by the language of calculus are called *differential equations*, and they are ubiquitous in science.

This course is about answering these questions — that is, about *solving differential equations*. The goal of this course is no less than that you will, by the end, have all the mathematical sophistication you need to solve differential equations, whether exactly or approximately, in the one-dimensional case, and most of those you will need for the general case.

A secondary goal is to develop a certain mental toughness. You are preparing for mentally challenging careers and lives. You will often be called on in your life to do things that are not easy. I'm not going to make this course artificially hard. But I'll also not deny that I hope you'll come out of this natually difficult course better able to think hard for long periods, try things that don't always work, keep with a problem when less-trained people would have given up hope, and, in short, better prepared to meet the intellectual challenges in your future.

The Official Goals

Upon completion of the course, the student should be able to

- Evaluate integrals using appropriate techniques–including substitution, trigonometric substitution, partial fractions, integration by parts.
- Use integration to solve problems involving arc length and surface area, for curves given parametrically or in rectangular coordinates.
- Evaluate limits using l'Hopital's rule where appropriate.
- Recognize and evaluate improper integrals; determine whether an improper integral converges.
- Apply appropriate convergence tests to determine whether a given series is absolutely convergent, conditionally convergent, or divergent. (Tests include: comparison test, ratio tests, nth root test, integral test, Alternating Series test.)
- Use the Binomial Series where appropriate.
- Determine the radius of convergence and the interval of convergence of $\sum_{n=1}^{\infty} c_n(x-a)^n$

- Use Taylor (and MacLaurin) expansions for simple calculations. This includes the series expansions for $exp(x)$, $\sin x$, $\cos x$.
- Sketch a curve given parametrically (including elementary polar graphs).
- Understand and apply polar coordinates.

The instructor may wish to include additional topics in lecture, homework, quizzes and/or class tests. Suggested additional topics may (should) include any of: work, moments and center of mass, hydrostatic pressure, the Squeeze Theorem, convergence tests other than the standard ones (ratio, root, comparison, alternating series test, integral test; nth term test for divergence), estimation of error, use of Taylor polynomials to evaluate limits, differential equations, some multivariate topic

Course Content

There's little getting around the fact that integration is the key tool in solving differential equations. The traditional start of this course is to first make a detailed study of how to integrate several functions (I assume you already know how to integrate most of the easy ones, like polynomials and trig functions), and then to give a collection of motivating examples that show why these efforts are worthwhile. For my taste, this puts the motivation too far back and asks you to swallow some of the hardest material in the undergraduate curriculum while just trusting me that I'll show you a reason later. Instead, we'll intersperse the two. Still, the focus of the first month of the course will be on integrating and on examples of integrals.

One of the key things we learn from studying integration is that we really need a better way to do things. We quickly get nostalgic for the days of integrating polynomials back in Calculus I. With enough work, one can usually replace a function by a power series, which looks a good deal like a polynomial (in the sense of being easy to integrate, at least). The next six weeks of the course will be devoted to developing this viewpoint, and putting together the skills necessary for handling these series.

The final part of the course will be devoted to a change of variables. My old physical chemistry professor once told me that the "right" way to solve any equation was to find the symmetries of the physical situation, and then choose a system of coordinates that took advantage of those symmetries. His favorite example was that the description of a hydrogen atom (which has spherical symmetry), which is nearly impossible in rectangular coordinates, becomes, if not easy, at least tractable in a system of spherical coordinates. Aside from the rectangular coordinates we've used before, the two most common ways to coordinatize things happening in the plane are parametric equations and polar coordinates. In this course, you will become comfortable with both.

Course Activities

Homework will be assigned daily or almost daily and will be collected weekly, on Fridays (unless otherwise announced). There will be a truckload of it, and that's not because I'm sadistic. The most common thing in all of mathematics — I do it myself, as does every other mathematician I know — is to see somebody else doing a problem and say, "Yes, yes, of course. I understand completely," and then walk away and realize that we had no idea at all what was going on. Homework is your guard against this. If you really understand how to do the homework, you're generally in pretty good shape. If you can't, you've got plenty of time to figure it out, ask me, ask a friend, or take whatever other action you see fit.

Homework will always be due at 4:30 on the appointed day. You are, of course, welcome to turn it in when you come to class. If you wish, though, you may continue to work on it, and may deliver it to my office or my department mailbox.

Cooperation on homework is strongly encouraged. There will almost certainly be problems on which it is necessary. Talk with each other, talk with me, talk with friends, use any resource. It is important, however, to be sure that you understand the solution you present. In designing the tests, I will assume thorough familiarity with all homework problems due before the date of the exam.

You are also encouraged to visit me in my office (see note on office hours above) or to call or e-mail me. To be more clear: It's a hard class. I'd like to see you do well in it. I'd love to talk with you and to help you in any way that I can.

It is wise to work on the homework as it is assigned, for a couple of reasons. First, there will be enough of it that it will not be practical to just sit down and do the whole week's worth in an evening. Second (and more importantly), the material builds on itself, so that a few days without working through at least some of the problems may find you feeling a little lost.

The class will meet on Monday, Wednesday, Thursday, and Friday at 8:00am. A typical meeting will begin with a discussion of any questions folks have, with procedural matters treated first. This will be followed by a discussion of new material (often in the form of problems, on which students will work in groups) and typically an assignment of new homework.

You should be in every class meeting, and should make sure that you are actively engaged. It goes without saying that when a problem is assigned for group work, you must do it. If you wait for me to tell you how to do it, then by the time I talk about the solution with the class, everybody else will understand it and will be ready to ask about issues you haven't encountered, and you will be lost. Don't do this. You should be careful to ask any questions you have. You should also feel free to be wrong. We all will be at some point in the class. That's why we gather together, instead of just reading the book on our own: we can help one another understand better, and we can try out ideas on each other, even if we aren't quite sure of them.

Text: Stewart, *Essential Calculus with Early Transcendentals*, Thompson, Brooks-Cole, ISBN 0-4950-14281

Be warned. The bookstores have been known to offer some other books as "recommended" for math courses. They are recommended by the bookstore, not by the math department, and not by me. I don't particularly recommend against them (since I have little idea what they'll be), but let the buyer be ware.

The text makes a great effort — and a successful one at most points — to be readable. It will provide an important opportunity to get an explanation in a different voice (at times very different) than that of your beloved teacher. It will also be the source of the bulk of the homework problems. Be careful of this, though: One can easily get the impression from the book that the right way to think about things is to memorize some formula or some procedure. In practice, if you try to do this with everything we will learn in the approximately sixty hours we have together in class this semester, plus the time spent outside of class, you will likely be overwhelmed and miserable. Better is to try and find the big ideas, and re-build everything else as you need it. You'll do better with this class and with later ones, and you'll not have to memorize nearly as much (i.e. it's easier).

There will also be some exams. Each exam will be preceded by a review sheet indicating *exactly* what material will be covered, an in-class review session, and an out-of-class review session. Exams will be given in the regularly scheduled class time and place on February 10, March 9, and April 13. In addition, there will be a final exam, in a time and place to be announced. I will forward information on the final schedule as soon as I have it. The final will test your ability to do all of the things we have worked on in class, and will be the same final offered to all other students in Math 250 this semester.

The general philosophy is that class sessions and homework will be very hard and tests will be pretty easy (assuming, of course, that you've suffered through the class meetings and homework leading up to them). Again, my goal with the homework is to help you to understand the material so well that you're unhappy with me for giving such a boring (easy) test.

In all activities for this class, make sure that you *do something*. It is depressing how often students who probably know something relevant to a problem do absolutely nothing, allowing no opportunity to receive credit on the part they actually know.

Calculators

You will need a TI-30 calculator for this course. This calculator, and no other, will be permitted on the final exam. I will be less particular in enforcing the rule on earlier exams, but remember that you don't want to go into the final exam with a calculator you're not already comfortable using.

Grading

Grades will be calculated from the following sources:

Homework	200
Regular Exams (100 each)	300
Final Exam	200
	700pts

I regret that I will not be able to provide a detailed reading of every problem I ask you to submit. The truth is, to learn the material, you need to do more homework than I could possibly read. On each assignment, I will grade a small but representative sample of the problems. If you would like more detailed feedback on another problem, I would be glad to give it.

Failure to attend class regularly will certainly adversely affect your grades on each of these factors. For instance, while I do not artificially lower grades for bad attendance, it has consistently held that almost all grades below C- that have been achieved in classes that I have taught have been associated with significant attendance problems.

In like manner, you should not underestimate the impact of your homework. Not only does the experience of the homework problems impact your test grades, but the homework itself is a considerable portion of the grade in the class. Moreover, since you can use the book, talk with friends, talk with a tutor, ask me how to do the problem, etc., *everyone should receive a grade of near 100% on the homework.* It is depressing how rarely this happens. Indeed, due largely to negligence in completing and turning in all of the assigned problems, many students find that their homework grade instead brings their grade in the course down. Don't let this happen to you.

In all work done for this class, work is more important than answers. A correct answer without correct work (or worse, with work that does not match the answer) is not worth much at all, while generally correct work with an incorrect answer is almost as good as being completely right. Thus, getting the right answer does not guarantee a good grade on the problem, and getting a wrong answer does not guarantee a bad one.

I will make the following guarantees about letter grades. I may decide to lower these criteria (i.e. give a higher grade than the one shown here, if I see that the questions were hard enough that lower numbers more accurately reflect my true standards), but will never raise them.

Percent of total	Grade
90–100	A
80–89	B
70–79	C
60–69	D
≤ 59	E

Prerequisites

The prerequisites of this course are designed to save you from spending a semester being miserable and failing this course. *I am on your side, and wish you success. That is why I am telling you this.* This course picks up exactly where Math 150 left off, and it does so at a very quick pace. To take this course, you must have a grade of C or better in Math 150.

Any student not meeting these requirements is *strongly* advised to delay taking this class until they are satisfied.

Catalog Description

(Advanced University Core Curriculum course) [IAI Course: M1 900] Develops the techniques of single-variable calculus begun in Calculus I and extends the concepts of function, limit, derivative and integral to functions of more than one variable. The treatment is intuitive, as in Calculus I. Techniques of integration, introduction to multivariate calculus, elements of finite series. Prerequisite: 150 with a grade of C or better. Students must present satisfactory placement score or obtain the permission of the department. Satisfies University Core Curriculum Mathematics requirement in lieu of 110 or 113.

Emergency Procedures

Southern Illinois University Carbondale is committed to providing a safe and healthy environment for study and work. Because some health and safety circumstances are beyond our control, we ask that you become familiar with the SIUC Emergency Response Plan and Building Emergency Response Team (BERT) program. Emergency response information is available on posters in buildings on campus, available on BERTs website at www.bert.siu.edu, Department of Safetys website www.dps.siu.edu (disaster drop down) and in Emergency Response Guideline pamphlet. Know how to respond to each type of emergency.

Instructors will provide guidance and direction to students in the classroom in the event of an emergency affecting your location. It is important that you follow these instructions and stay with your instructor during an evacuation or sheltering emergency. The Building Emergency Response Team will provide assistance to your instructor in evacuating the building or sheltering within the facility.