Math 421: Linear Algebra
Spring 2018

It is a common fallacy to suppose that mathematics is important for physics only because it is a useful tool for making computations. Actually, mathematics plays a more subtle role which in the long run is more important. When a successful mathematical model is created for a physical phenomenon, that is, a model which can be used for accurate computations and predictions, the mathematical structure of the model itself provides a new way of thinking about the phenomenon.

— M. Reed and B. Simon, 1980

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Course Goals

At its most basic level, linear algebra is a well-established theory, not too difficult to understand, that facilitates simple calculation for a small but important family of multivariate functions. When its essential features are axiomatized, though, it provides a deep model for the analysis of a stunning range of mathematical, physical, and computational problems.

In the present course, you will learn this larger form of linear algebra. You will learn ways of representing and decomposing vector spaces (in maximal generality) and functions defined on them into useful parts. These decompositions can facilitate calculations, but they can also help us recognize important subsystems of systems that interest us.

A second goal is that you will learn to think mathematically. Linear algebra is one of the most well-understood parts of mathematics. Because we can delve deeply into it, it is a good place to see what a full mathematical understanding of a problem looks like.

Course Content

We will begin with a quick review of the basic definitions of vector spaces, linear transformations, and their properties. They will be both real and complex vector spaces, but much of what you see here will be familiar. We will consider polynomials as an example of a vector space.

Afterward, we will look at subspaces of a vector space which are fixed by a given linear transformation, and how such subspaces give rise to eigenvalues. This allows a multivariate function to be understood as a combination of lower-dimensional functions. We will look at stronger notions of independence, and several other useful decomposition theorems, as well as traces and determinants.

Course Activities

Homework will be assigned frequently, and will be due each week on Wednesdays (unless otherwise announced). The most common thing in all of mathematics — I do it myself, as does every other mathematician I know — is to see somebody else doing a problem and say, "Yes, yes, of course. I understand completely," and then walk away and realize that we had no idea at all what was going on. Homework is your guard against this. If you really understand how to do the homework, you're generally in pretty good shape. If you can't, you've got plenty of time to figure it out, ask me, ask a friend, or take whatever other action you see fit.

Homework will always be due at 4:30 on the appointed day. You are, of course, welcome to turn it in when you come to class. If you wish, though, you may continue to work on it, and may deliver it to my office or my department mailbox.

Cooperation on homework is strongly encouraged. There will almost certainly be problems on which it is necessary. Talk with each other, talk with me, talk with friends, use any resource. It is important, however, to be sure that you understand the solution you present. In designing the tests, I will assume thorough familiarity with all homework problems due before the date of the exam.

You are also encouraged to visit me in my office (see note on office hours above) or to call or e-mail me. To be more clear: It's a hard class. I'd like to see you do well in it. I'd love to talk with you and to help you in any way that I can.

The homework will often be hard. You will seldom be able to solve all of the problems in one setting. Plan your time accordingly.

The class will meet on Monday, Wednesday, and Friday at noon. A typical meeting will begin with a discussion of any questions folks have, with procedural matters treated first. This will be followed by a discussion of new material (often in the form of problems, on which students will work in groups) and typically an assignment of new homework.

You should be in every class meeting, and should make sure that you are actively engaged. It goes without saying that when a problem is assigned for group work, you must do it. If you wait for me to tell you how to do it, then by the time I talk about the solution with the class, everybody else will understand it and will be ready to ask about issues you haven't encountered, and you will be lost. Don't do this. You should be careful to ask any questions you have. You should also feel free to be wrong. We all will be at some point in the class. That's why we gather together, instead of just reading the book on our own: we can help one another understand better, and we can try out ideas on each other, even if we aren't quite sure of them.

Text: S. Axler, *Linear Algebra Done Right*, 3rd Edition, Springer, 2015

Last time I checked, this book was available in electronic form through the University library. One seems to be able to download a PDF of the full book.

Be warned. The bookstores have been known to offer some other books as "recommended" for math courses. They are recommended by the bookstore, not by the math department, and not by me. I don't particularly recommend against them (since I have little idea what they'll be), but let the buyer be ware.

The text makes a great effort — and a successful one at most points — to be readable. It will provide an important opportunity to get an explanation in a different voice (at times very different) than that of your beloved teacher.

There will also be some exams. Each exam will be preceded by a review sheet indicating *exactly* what material will be covered, an in-class review session, and an out-of-class review session. Exams will be given in the regularly scheduled class time and place on February 21 and March 30. In addition, there will be a final exam at a time to be announced. I will forward more information on the final schedule as soon as I have it. The final will test your ability to do all of the things we have worked on in class, with particular emphsis on material covered since the last exam.

Each student will complete a significant research project over the course of the term, and will give a presentation on the results near the end of the term. More information will be forthcoming.

The general philosophy is that class sessions and homework will be very hard and tests will be pretty easy (assuming, of course, that you've suffered through the class meetings and homework leading up to them). Again, my goal with the homework is to help you to understand the material so well that you're unhappy with me for giving such a boring (easy) test.

In all activities for this class, make sure that you *do something*. It is depressing how often students who probably know something relevant to a problem do absolutely nothing, allowing no opportunity to receive credit on the part they actually know.

Grading

Grades will be calculated from the following sources:

Homework	100
In-class exams (total of 2)	200
Project	150
Final Exam	150
	600 pts

Failure to attend class regularly will certainly adversely affect your grades on each of these factors. For instance, while I do not artificially lower grades for bad attendance, it has consistently held that almost all grades below C- that have been achieved in classes that I have taught have been associated with significant attendance problems.

In like manner, you should not underestimate the impact of your homework. Not only does the experience of the homework problems impact your test grades, but the homework itself is a considerable portion of the grade in the class. *When you submit a correct solution to a problem, you will get full credit for that problem. Thus, everyone should receive a grade of 100% on the homework.* No credit will be given for a problem whose correct solution is never submitted.

In all work done for this class, work is more important than answers. A correct answer without correct work (or worse, with work that does not match the answer) is not worth much at all, while generally correct work with an incorrect answer is almost as good as being completely right. Thus, getting the right answer does not guarantee a good grade on the problem, and getting a wrong answer does not guarantee a bad one.

I will make the following guarantees about letter grades. I may decide to lower these criteria (i.e. give a higher grade than the one shown here, if I see that the questions were hard enough that lower numbers more accurately reflect my true standards), but will never raise them.

Percent of total	Grade
90–100	A
80–89	B
70–79	C
60–69	D
≤ 59	E

Prerequisites

The prerequisites of this course are designed to save you from spending a semester being miserable and failing this course. *I am on your side, and wish you success. That is why I am telling you this.* To take this course, you must have completed MATH 221 with a C or better.

Any student not meeting these requirements is *strongly* advised to delay taking this class until they are satisfied.

Catalog Description

The extension of basic linear algebra to arbitrary scalars. The theory and computation of Jordan forms of matrices (as needed e.g., for certain diffusion equations). Inner products, quadratic forms and Sylvester's Law of Inertia. Prerequisite: MATH 221 with C or better.

Emergency Procedures

Southern Illinois University Carbondale is committed to providing a safe and healthy environment for study and work. Because some health and safety circumstances are beyond our control, we ask that you become familiar with the SIUC Emergency Response Plan and Building Emergency Response Team (BERT) program. Emergency response information is available on posters in buildings on campus, available on BERTs website at www.bert.siu.edu, Department of Safetys website www.dps.siu.edu (disaster drop down) and in Emergency Response Guideline pamphlet. Know how to respond to each type of emergency.

Instructors will provide guidance and direction to students in the classroom in the event of an emergency affecting your location. It is important that you follow these instructions and stay with your instructor during an evacuation or sheltering emergency. The Building Emergency Response Team will provide assistance to your instructor in evacuating the building or sheltering within the facility.