Math 419: Introduction to Abstract Algebra II
Spring 2015

"The twenty-three year old Felix Klein in his famous Erlanger Programm proposed to classify geometries by their automorphisms. He hit on something fundamental here: in a sense, *structure is whatever is preserved by automorphisms.*

— W. Hodges, 1993

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Course Goals

In mathematics, we understand things first by abstracting their essential features, and second by looking at the functions that preserve those features. In the case of this class, we will do this with numbers and with polynomial equations.

There are two central goals, and one subsidiary goal. The first central goal is that students will become adept in analyzing mathematical structures by means of their automorphisms. The second is that students will know and be able to use by analogy some of the classical impossibility proofs, notably the insolvability of the general quintic equation and the impossibility of general angle trisection.

A subsidiary, but very important, goal, is that students will get familiar with a good deal of algebraic machinery. Many special classes of rings which imitate one feature or other of the central examples, theorems about these classes, and even the examples themselves are standard points of reference for mathematically educated people.

Course Content

The central elements of study in this course are rings (contexts where one can add, subtract, and multiply). The three central examples are the ring of natural numbers, the ring of polynomials with real coefficients, and the ring of $n \times n$ real matrices. Almost every other ring you will encounter in this class or elsewhere is either an attempt either to generalize things known about these examples to a different context or an attempt to isolate some features of these examples for more detailed study.

We will start by defining rings and looking over several examples. We will then generalize the notion of divisibility to an arbitrary ring in the form of "ideals." We will then discuss several important properties of the three example rings, and see how they play out in the polynomials in detail.

A special class of rings is the class in which one may divide — the fields. In general, any (commutative) ring gives rise to a field of fractions in a way analogous to the construction of the rational numbers or the rational functions. Quite a lot of what happens in algebra — including the major impossibility results — depends only on the field of fractions, and not on the ring itself. There will be a brief review of necessary linear algebra, but if you've never studied it before, you'll want to be especially careful around this part of the course.

We'll look at how fields sit inside other fields. The canonical example here is the reals inside the complex numbers. The most important technique (and the subject of nearly the last third of the course) is the use of the "Galois group," that is, of the group of ring automorphisms, or the group of symmetries of the bigger field that fix the smaller one (complex conjugation is the example to have in mind). We will calculate these groups for several examples, and will use concerns related to them to show the impossibility of some classical geometric constructions (most importantly, angle trisection), and the impossibility of a generalization of the quadratic formula to equations of degree 5.

Course Activities

Homework will be assigned frequently, and will be due each week on Fridays (unless otherwise announced). The most common thing in all of mathematics — I do it myself, as does every other mathematician I know — is to see somebody else doing a problem and say, "Yes, yes, of course. I understand completely," and then walk away and realize that we had no idea at all what was going on. Homework is your guard against this. If you really understand how to do the homework, you're generally in pretty good shape. If you can't, you've got plenty of time to figure it out, ask me, ask a friend, or take whatever other action you see fit.

Homework will always be due at 4:30 on the appointed day. You are, of course, welcome to turn it in when you come to class. If you wish, though, you may continue to work on it, and may deliver it to my office or my department mailbox.

There will be several computer laboratory experiences. Primarily, you will use the software package GAP for the labs. This software is installed on most of the Windows machines in the Math Department computer lab (Neckers 258/278), and is freely available for download. The point of these exercises is twofold: First, that you do some exploratory and experimental work to motivate theorems and proofs you will see (a proof often isn't worth much unless you have an informed opinion on whether a theorem should be true); and second, that you accustom yourself to thinking of math in an experimental and exploratory way, which is fundamental for any mathematical research.

Cooperation on homework and labs is strongly encouraged. There will almost certainly be problems on which it is necessary. Talk with each other, talk with me, talk with friends, use any resource. It is important, however, to be sure that you understand the solution you present. In designing the tests, I will assume thorough familiarity with all homework problems due before the date of the exam.

You are also encouraged to visit me in my office (see note on office hours above) or to call or e-mail me. To be more clear: It's a hard class. I'd like to see you do well in it. I'd love to talk with you and to help you in any way that I can.

The homework will often be hard. You will seldom be able to solve all of the problems in one setting. Plan your time accordingly.

Some homework will call on you to use certain freely-available mathematical software. More details will follow.

The class will meet on Monday, Wednesday, and Friday at 9:00. A typical meeting will begin with a discussion of any questions folks have, with procedural matters treated first. This will be followed by a discussion of new material (often in the form of problems, on which students will work in groups) and typically an assignment of new homework.

You should be in every class meeting, and should make sure that you are actively engaged. It goes without saying that when a problem is assigned for group work, you must do it. If you wait for me to tell you how to do it, then by the time I talk about the solution with the class, everybody else will understand it and will be ready to ask about issues you haven't encountered, and you will be lost. Don't do this. You should be careful to ask any questions you have. You should also feel free to be wrong. We all will be at some point in the class. That's why we gather together, instead of just reading the book on our own: we can help one another understand better, and we can try out ideas on each other, even if we aren't quite sure of them.

Text: Fraleigh, *A First Course in Abstract Algebra*, 7th edition, Addison Wesley, 2003

Be warned. The bookstores have been known to offer some other books as "recommended" for math courses. They are recommended by the bookstore, not by the math department, and not by me. I don't particularly recommend against them (since I have little idea what they'll be), but let the buyer be ware.

The text makes a great effort — and a successful one at most points — to be readable. It will provide an important opportunity to get an explanation in a different voice (at times very different) than that of your beloved teacher.

There will also be some exams. Each exam will be preceded by a review sheet indicating *exactly* what material will be covered, an in-class review session, and an out-of-class review session. Exams will be given in the regularly scheduled class time and place on February 27 and April 12. In addition, there will be a final exam on Friday, May 15, from 8:00 to 10:00 in the morning. I will forward more information on the final schedule as soon as I have it. The final will test your ability to do all of the things we have worked on in class, with particular emphsis on material covered since the last exam.

Each student will complete a significant research project over the course of the term, and will give a presentation on the results on either May 4 or May 6. More information will be forthcoming.

The general philosophy is that class sessions and homework will be very hard and tests will be pretty easy (assuming, of course, that you've suffered through the class meetings and homework leading up to them). Again, my goal with the homework is to help you to understand the material so well that you're unhappy with me for giving such a boring (easy) test.

In all activities for this class, make sure that you *do something.* It is depressing how often students who probably know something relevant to a problem do absolutely nothing, allowing no opportunity to receive credit on the part they actually know.

Grading

Grades will be calculated from the following sources:

Homework	100
In-class exams (total of 2)	200
Laboratory	100
Project	150
Final Exam	150
	700 pts

Failure to attend class regularly will certainly adversely affect your grades on each of these factors. For instance, while I do not artificially lower grades for bad attendance, it has consistently held that almost all grades below C- that have been achieved in classes that I have taught have been associated with significant attendance problems.

In like manner, you should not underestimate the impact of your homework. Not only does the experience of the homework problems impact your test grades, but the homework itself is a considerable portion of the grade in the class. *When you submit a correct solution to a problem, you will get full credit for that problem. Thus, everyone should receive a grade of 100% on the homework.* No credit will be given for a problem whose correct solution is never submitted.

In all work done for this class, work is more important than answers. A correct answer without correct work (or worse, with work that does not match the answer) is not worth much at all, while generally correct work with an incorrect answer is almost as good as being completely right. Thus, getting the right answer does not guarantee a good grade on the problem, and getting a wrong answer does not guarantee a bad one.

I will make the following guarantees about letter grades. I may decide to lower these criteria (i.e. give a higher grade than the one shown here, if I see that the questions were hard enough that lower numbers more accurately reflect my true standards), but will never raise them.

Percent of total	Grade
90–100	A
80–89	B
70–79	C
60–69	D
≤ 59	E

Prerequisites

The prerequisites of this course are designed to save you from spending a semester being miserable and failing this course. *I am on your side, and wish you success. That is why I am telling you this.* To take this course, you must have completed Math 319 or have my consent (which roughly corresponds to me being convinced that you would get at least a B on the 319 final exam, were it given today). If you don't know what a group is, you're in trouble.

Any student not meeting these requirements is *strongly* advised to delay taking this class until they are satisfied.

Catalog Description

A detailed study of polynomial equations in one variable. Solvable groups and the Galois theory of field extensions are developed and applied to extensions of the quadratic formula, proving the impossibility of trisecting an angle with only a straight-edge and a compass, and to the basic facts about finite fields as needed in coding theory and computer science. Prerequisite: 319 or consent.

Emergency Procedures

Southern Illinois University Carbondale is committed to providing a safe and healthy environment for study and work. Because some health and safety circumstances are beyond our control, we ask that you become familiar with the SIUC Emergency Response Plan and Building Emergency Response Team (BERT) program. Emergency response information is available on posters in buildings on campus, available on BERTs website at www.bert.siu.edu, Department of Safetys website www.dps.siu.edu (disaster drop down) and in Emergency Response Guideline pamphlet. Know how to respond to each type of emergency.

Instructors will provide guidance and direction to students in the classroom in the event of an emergency affecting your location. It is important that you follow these instructions and stay with your instructor during an evacuation or sheltering emergency. The Building Emergency Response Team will provide assistance to your instructor in evacuating the building or sheltering within the facility.