Math 302: Mathematical Communication and the Transition to Higher Mathematics

Spring 2016

Mathematics as an expression of the human mind reflects the active will, the contemplative reason, and the desire for aesthetic perfection. Its basic elements are logic and intuition, analysis and construction, generality and individuality. Though different traditions may emphasize different aspects, it is only the interplay of these antithetic forces and the struggle for their synthesis that constitute the life, usefulness, and supreme value of mathematical science.

— Richard Courant and Herbert Robbins, *What is Mathematics*, 1941.

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Office Hours: Official (guaranteed) hours: Wednesday 1–3, Thursday 10-11; Friday 1:30–4:30; also make an appointment or come see me.

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Course Goals

The Real Goals

This course is intended to teach you to think like a mathematician. This is valuable even if you don't plan to spend the rest of your life proving theorems. For prospective teachers, this promotes such capabilities as posing tasks that call for mathematical reasoning, and promoting strong classroom discourse. For those who work in areas where mathematics is applied, this course is the gateway to the kind of thinking that makes mathematics so valuable in applications — the precision, reproducibility, and all-around "unreasonable effectiveness" of mathematics. If you are studying mathematics at this level, you are training to be, in some sense, a mathematician.

So what do mathematicians do?

First, and above all, we solve problems. Most importantly, we solve hard problems whose answers are not known, where there is no obvious direction of approach to the problem, where multiple solutions are often possible, and where a solution must be known with a high (or at least specified) level of certainty. If you are interested in applications, add to this that the problem is often only vaguely stated and that information necessary to the solution may be only partially available. In this course, you will solve many problems. We will talk about general heuristics for problem solving, and will develop ways to "reduce" one problem to another (maybe easier) one.

Secondly, and usually in service of solving problems, we prove theorems. For a physicist, a proof is suggestive and an empirical observation settles the matter. In mathematics, the opposite is true, and there's no getting around it at this level. If you don't have a proof, you don't have mathematical reasoning. In this course, you will prove lots of theorems. We will talk about strategies for proving things. You will learn to troubleshoot proofs — sometimes by fixing the proof, and sometimes by changing what you claim to be able to prove.

Thirdly, we communicate with other mathematicians. If two mathematicians meet anywhere in the world, there is an excellent chance they can (and will) tell one another about their work. I have done this in cases where we did not share a language. This is possible because mathematicians share a rather large library of ideas and symbols throughout the world. In this course, you will gain access to a large part of this library, and so will become part of this community.

The Official Goals

The emphasis is on writing and presenting proofs. Upon completion of this course, the student should be able to:

- Determine the truth value of a proposition using truth tables
- Construct proofs involving quantifiers, negations, and logical connectives, using appropriate proof techniques (direct proof, proof by contradiction, contrapositive, induction, etc.)
- Perform basic set operations, including extended set operations on indexed families of sets.
- Apply appropriate counting principles to finite sets
- Recognize whether or not a given relation is an equivalence relation, order relation, function

- Construct the partition determined by an equivalence relation and conversely
- Construct the graph of a relation
- Construct new functions from old by various techniques
- Determine whether or not a given function is injective, surjective, bijective
- Demonstrate an understanding of equivalent sets and a rudimentary knowledge of cardinality
- Present correct proofs in writing and orally
- Critique proofs, recognizing flaws in the logic and offering corrections as needed.

Course Content

We will start with a collection of puzzles and problems. Later we will have problems to illustrate particular techniques, but the first few days we will set the tone by just working on fun problems.

As time goes on, we will discuss the particular kind of truth claims mathematicians make. To give proofs with the certainty the world expects from us, we have to pay careful attention to what it means for a statement to be true, as well as to what specific kinds of explanation can make that truth so clear that it "compels belief." Could you convince somebody who doesn't already believe it that every natural number is either even or else one more than an even number?

We will then begin applying these skills across a range of mathematical topics. We'll see the theory of numbers — prime numbers, some neat things about division with remainder (including a secret code), and how to prove that some particular numbers like $\sqrt{2}$ and e are irrational.

After a bit more tooling up (some of the details around functions), we'll see a bit about the weirdness of infinity. How weird is it? Well, it's so weird that only a mathematician (like you're becoming) has the language to say just how weird. As a taste, though, there are just as many even integers as there are integers, and just as many integers as rational numbers. But there are strictly *more* real numbers.

The next section of the course, though, will be strictly finite. We will talk about systematic ways to count things. How many people do you need in a room before there's at least an even chance that two of them share a birthday?

Course Activities

The class will meet on Monday, Wednesday, and Friday at 10:00am. A typical meeting will begin with a discussion of any questions folks have, with procedural matters treated first. This will be followed by a discussion of new material (often in the form of problems, on which students will work together) and typically an assignment of new homework.

You should be in every class meeting, and should make sure that you are actively engaged. It goes without saying that when a problem is assigned for group work, you must do it. If you wait for me to tell you how to do it, then by the time I talk about the solution with the class, everybody else will understand it and will be ready to ask about issues you haven't encountered, and you will be lost. Worse, you won't be practicing your mathematical thinking. Don't do this. You should be careful to ask any questions you have. You should also feel free to be wrong. We all will be at some point in the class. That's why we gather together, instead of just reading the book on our own: we can help one another understand better, and we can try out ideas on each other, even if we aren't quite sure of them.

Homework will be assigned daily or almost daily and will be collected weekly, on Fridays (unless otherwise announced). There will be a truckload of it, and that's not because I'm mean. The most common thing in all of mathematics — I do it myself, as does every other mathematician I know — is to see somebody else doing a problem and say, "Yes, yes, of course. I understand completely," and then walk away and realize that we had no idea at all what was going on. Homework is your guard against this. If you really understand how to do the homework, you're generally in pretty good shape. If you can't, you've got plenty of time to figure it out, ask me, ask a friend, or take whatever other action you see fit.

Homework will always be due at 4:30 on the appointed day. You are, of course, welcome to turn it in when you come to class. If you wish, though, you may continue to work on it, and may deliver it to my office or my department mailbox.

Cooperation on homework is strongly encouraged. There will almost certainly be problems on which it is necessary. Talk with each other, talk with me, talk with friends, use any resource. It is important, however, to be sure that you understand the solution you present. In designing the tests, I will assume thorough familiarity with all homework problems due before the date of the exam.

You are also encouraged to visit me in my office (see note on office hours above) or to call or e-mail me. To be more clear: It's a hard class. I'd like to see you do well in it. I'd love to talk with you and to help you in any way that I can.

It is wise to work on the homework as it is assigned, for a couple of reasons. First, there will be enough of it that it will not be practical to just sit down and do the whole week's worth in an evening. Second (and more importantly), the material builds on itself, so that a few days without working through at least some of the problems may find you feeling a little lost.

Text: Edward Burger, *Extending the Frontiers of Mathematics: Inquiries into proof and argumentation*, any edition.

There is an old edition of the book from Key College Publishing that may have some used copies out there. I have both editions, and can't tell the difference between them. You'll be safe with either.

Be warned. The bookstores have been known to offer some other books as "recommended" for math courses. They are recommended by the bookstore, not by the math department, and not by me. I don't particularly recommend against them (since I have little idea what they'll be), but let the buyer be ware.

The text is different from others you have used in some important ways. The first thing that you will probably notice is that it consists almost entirely of problems. The point of this class isn't so much that you learn to apply a specific collection of formulae, but that you think and write like a mathematician. The book is structured to help you with this.

There will be several writing assignments, which will focus on deep thought and clear expression. Each writing assignment will be given with a detailed writing guide and the criteria on which it will be evaluated. Again, cooperation is encouraged, but the work you submit must be your own.

There will also be some exams. Each exam (except the final) will be preceded by a review sheet indicating *exactly* what material will be covered, and by an in-class review session. Exams will be given in the regularly scheduled class time and place on February 19 and March 25. In addition, there will be a final exam, in a time and place to be announced. I will forward information on the final schedule as soon as I have it. The final will test your ability to do all of the things we have worked on in class.

The general philosophy is that class sessions, homework, and writing will be very hard and tests will be pretty easy (assuming, of course, that you've suffered through the class meetings and homework leading up to them). Again, my goal with the homework is to help you to understand the material so well that you're unhappy with me for giving such a boring (easy) test.

In all activities for this class, make sure that you *do something*. It is depressing how often students who probably know something relevant to a problem do absolutely nothing, allowing no opportunity to receive credit on the part they actually know.

Grading

Grades will be calculated from the following sources:

Homework	300
Writing Assignments	100
Regular Exams (100 each)	200
Final Exam	200
	800pts

Failure to attend class regularly will certainly adversely affect your grades on each of these factors. For instance, while I do not artificially lower grades for bad attendance, it has consistently held that almost all grades below C- that have been achieved in classes that I have taught have been associated with significant attendance problems.

In like manner, you should not underestimate the impact of your homework. Not only does the experience of the homework problems impact your test grades, but the homework itself is a considerable portion of the grade in the class.

In all work done for this class, work is more important than answers. A correct answer without correct work (or worse, with work that does not match the answer) is not worth much at all, while generally correct work with an incorrect answer is almost as good as being completely right. Moreover, in this class on communication, *exposition matters deeply*. I will not go out of my way to pick on language issues in homework or tests (although writing assignments should be flawless), but if writing problems of any kind impede my efforts to understand your work, you will lose points on it. This is real-world writing for an audience: If I can't understand why your solution was right,

then you failed to communicate that your solution was right, and it will not get full credit. Thus, getting the right answer does not guarantee a good grade on the problem, and getting a wrong answer does not guarantee a bad one.

I will make the following guarantees about letter grades. I may decide to lower these criteria (i.e. give a higher grade than the one shown here, if I see that the questions were hard enough that lower numbers more accurately reflect my true standards), but will never raise them.

Percent of total	Grade
90–100	A
80–89	B
70–79	C
60–69	D
≤ 59	E

Prerequisites

The prerequisites of this course are designed to save you from spending a semester being miserable and failing this course. *I am on your side, and wish you success. That is why I am telling you this.* To take this course, you must have a grade of C or better in both Math 221 and Math 250.

Any student not meeting these requirements is *strongly* advised to delay taking this class until they are satisfied.

Catalog Description

MATH 302-3 Mathematical Communication and the Transition to Higher Mathematics. A course in communicating mathematical ideas with a special emphasis on reading, writing, and critiquing mathematical proofs. Topics covered include logic, proofs, set theory, relations, functions. Additional illustratory topics will be drawn from linear algebra, number theory, complex variables, and geometry. Prerequisite: MATH 221 and MATH 250 with a grade of C or better.

Emergency Procedures

Southern Illinois University Carbondale is committed to providing a safe and healthy environment for study and work. Because some health and safety circumstances are beyond our control, we ask that you become familiar with the SIUC Emergency Response Plan and Building Emergency Response Team (BERT) program. Emergency response information is available on posters in buildings on campus, available on BERTs website at www.bert.siu.edu, Department of Safetys website www.dps.siu.edu (disaster drop down) and in Emergency Response Guideline pamphlet. Know how to respond to each type of emergency.

Instructors will provide guidance and direction to students in the classroom in the event of an emergency affecting your location. It is important that you follow these instructions and stay with your instructor during an evacuation or sheltering emergency. The Building Emergency Response Team will provide assistance to your instructor in evacuating the building or sheltering within the facility.