Math 251: Calculus III

Fall 2021

With the help of these new field concepts Faraday succeeded in forming a qualitative concept of the whole complex of electromagnetic effects discovered by him and his predecessors. The precise formulation of the time-space laws of those fields was the work of Maxwell. Imagine his feelings when the differential equations he had formulated proved to him that electromagnetic fields spread in the form of polarized waves and with the speed of light! To few... in the world has such an experience been vouchsafed.

— Albert Einstein, 1940

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Course Goals

The Real Goals

From at least the first semester of your calculus education, we have made grandiose promises. Calculus will underlie error analysis, mechanical systems, electromagnetism, fluid flows, and every other system worthy of scientific analysis. Up to now, however, it has done so only in a world of two variables, one a function of the other. In this course, that will change.

The goal of this course is to equip you to conduct mathematical analysis of situations involving more than two dimensions, and more than one independent variable. As it has been said, it's opener there in the wide open air. Not only are there technical details to be dealt with (How do derivatives in different directions interact? How do you specify a line in a space of many dimensions?), but there is room for much more interesting geometry — with interesting implications for the dynamical systems represented.

The Official Goals

The course will acquaint students of engineering, mathematics and science with the concepts and techniques of the calculus for functions of multiple variables and for vector-valued functions.

Upon completion of the course, the student should be able to:

- Compute dot products and cross products, find the angle between two vectors, find equations of lines and planes in space.
- Find derivatives and integrals of vector-valued functions.
- Apply derivatives and integrals to solve problems involving motion in space; find velocity, speed and acceleration given the position vector of a particle; find the position of a particle given information about its initial velocity and acceleration.
- Find the arclength of a given curve.
- Evaluate first and higher order partial derivatives of a function of several variables, using chain rules as appropriate.
- Evaluate directional derivatives of a function of several variables; find and interpret graphically the gradient of a given function of several variables; find the tangent plane and normal line to a given surface at a given point.
- Recognize a critical point for a function of several variables. Determine whether a critical point yields a local extremum or saddle point, or neither; maximize and minimize functions using the method of Lagrange Multipliers.
- Convert double and triple integrals to iterated integrals and vice versa. Use double and/or triple integrals to find areas, volumes, surface areas, moments and centers of mass.
- Use rectangular, cylindrical and spherical coordinates to parametrize standard surfaces in space and to evaluate appropriate integrals.

- Use the Jacobian to find the differential of areas and volumes.
- Determine whether a given vector field is conservative; determine whether a line integral is independent of path; find the curl and divergence of a given vector field.
- Evaluate line integrals along smooth curves; apply line integrals to compute work done by a force along a curve. Evaluate line integrals using Greens Theorem and Stokes Theorem.
- Find the flux of a vector field through a given surface. Use the Divergence Theorem to compute flux.

Course Content

We begin by an overview of the geometry and algebra of vectors, and of figures defined parametrically in 3-dimensional space. A few points of calculus transfer directly from the 2-dimensional case.

As soon as we try to take a derivative, though, there are complications to consider. Small intervals are replaced by small spheres. The derivative can depend on the direction you're going (as, of course, it can, if you walk in rolling hills). And even knowing the derivative in all directions can tell you less about the geometry than it used to. This explains, among other things, an important point of game theory.

Integral multivariable calculus comes in two main installments. One is more or less what you would expect. Instead of the area under a curve, you look at the volume under a surface. There are, of course, technicalities, but the concept will be familiar.

Then there are vector fields. Imagine, if you will, an electromagnetic or gravitational field, or perhaps the function giving, for every point inside the heart, the velocity of blood at that point. This last can, to some precision, be observed by a sonogram. From this, we can calculate the speed of the flow through a certain region, identify whether a region contains a "source" or a "sink" (i.e. blood entering or leaving the chamber there), and many other useful issues. This is the work of vector calculus, the second part of integral multivariable calculus, and the final segment of our course.

Course Activities

Homework will be assigned daily or almost daily and will be collected weekly, on Wednesdays (unless otherwise announced). There will be a truckload of it, and that's not because I'm mean. The most common thing in all of mathematics — I do it myself, as does every other mathematician I know — is to see somebody else doing a problem and say, "Yes, yes, of course. I understand completely," and then walk away and realize that we had no idea at all what was going on. Homework is your guard against this. If you really understand how to do the homework, you're generally in pretty good shape. If you can't, you've got plenty of time to figure it out, ask me, ask a friend, or take whatever other action you see fit.

Homework will always be due at 4:30 on the appointed day. You are, of course, welcome to turn it in when you come to class. If you wish, though, you may continue to work on it, and may deliver it to my office or my department mailbox.

Cooperation on homework is strongly encouraged. There will almost certainly be problems on which it is necessary. Talk with each other, talk with me, talk with friends, use any resource. It is important, however, to be sure that you understand the solution you present. In designing the tests, I will assume thorough familiarity with all homework problems due before the date of the exam.

You are also encouraged to visit me in my office (see note on office hours above) or to call or e-mail me. To be more clear: It's a hard class. I'd like to see you do well in it. I'd love to talk with you and to help you in any way that I can.

It is wise to work on the homework as it is assigned, for a couple of reasons. First, there will be enough of it that it will not be practical to just sit down and do the whole week's worth in an evening. Second (and more importantly), the material builds on itself, so that a few days without working through at least some of the problems may find you feeling a little lost.

The class will meet on Monday, Wednesday, and Friday at 8:00am. A typical meeting will begin with a discussion of any questions folks have, with procedural matters treated first. This will be followed by a discussion of new material (often in the form of problems, on which students will work in groups) and typically an assignment of new homework.

You should be in every class meeting, and should make sure that you are actively engaged. It goes without saying that when a problem is assigned for group work, you must do it. If you wait for me to tell you how to do it, then by the time I talk about the solution with the class, everybody else will understand it and will be ready to ask about issues you haven't encountered, and you will be lost. Don't do this. You should be careful to ask any questions you

have. You should also feel free to be wrong. We all will be at some point in the class. That's why we gather together, instead of just reading the book on our own: we can help one another understand better, and we can try out ideas on each other, even if we aren't quite sure of them.

Text: Stewart, *Essential Calculus with Early Transcendentals*, Second Edition

The text makes a great effort — and a successful one at most points — to be readable. It will provide an important opportunity to get an explanation in a different voice (at times very different) than that of your beloved teacher. It will also be the source of the bulk of the homework problems. Be careful of this, though: One can easily get the impression from the book that the right way to think about things is to memorize some formula or some procedure. In practice, if you try to do this with everything we will learn in the approximately forty-five hours we have together in class this semester, plus the time spent outside of class, you will likely be overwhelmed and miserable. Better is to try and find the big ideas, and re-build everything else as you need it. You'll do better with this class and with later ones, and you'll not have to memorize nearly as much (i.e. it's easier).

There will also be some exams. Each exam will be preceded by a review sheet indicating *exactly* what material will be covered and an in-class review session. Exams will be given in the regularly scheduled class time and place on September 15, October 20, and November 17. In addition, there will be a final exam at a time to be announced. I will forward more information on the final schedule as soon as I have it. The final will test your ability to do all of the things we have worked on in class.

The general philosophy is that class sessions and homework will be very hard and tests will be pretty easy (assuming, of course, that you've suffered through the class meetings and homework leading up to them). Again, my goal with the homework is to help you to understand the material so well that you're unhappy with me for giving such a boring (easy) test.

In all activities for this class, make sure that you *do something*. It is depressing how often students who probably know something relevant to a problem do absolutely nothing, allowing no opportunity to receive credit on the part they actually know.

Grading

Grades will be calculated from the following sources:

Homework	200
Regular Exams (100 each)	300
Final Exam	200
	700pts

Failure to attend class regularly will certainly adversely affect your grades on each of these factors. For instance, while I do not artificially lower grades for bad attendance, it has consistently held that almost all grades below C- that have been achieved in classes that I have taught have been associated with significant attendance problems.

In like manner, you should not underestimate the impact of your homework. Not only does the experience of the homework problems impact your test grades, but the homework itself is a considerable portion of the grade in the class. Moreover, since you can use the book, talk with friends, talk with a tutor, ask me how to do the problem, etc., *everyone should receive a grade of near 100% on the homework*. It is depressing how rarely this happens. Indeed, due largely to negligence in completing and turning in all of the assigned problems, many students find that their homework grade instead brings their grade in the course down. Don't let this happen to you.

In all work done for this class, work is more important than answers. A correct answer without correct work (or worse, with work that does not match the answer) is not worth much at all, while generally correct work with an incorrect answer is almost as good as being completely right. Thus, getting the right answer does not guarantee a good grade on the problem, and getting a wrong answer does not guarantee a bad one.

I will make the following guarantees about letter grades. I may decide to lower these criteria (i.e. give a higher grade than the one shown here, if I see that the questions were hard enough that lower numbers more accurately reflect my true standards), but will never raise them.

Percent of total	Grade
90–100	A
80–89	B
70–79	C
60–69	D
≤ 59	E

Prerequisites

The prerequisites of this course are designed to save you from spending a semester being miserable and failing this course. *I am on your side, and wish you success. That is why I am telling you this.* This course picks up exactly where Math 250 left off, and it does so at a very quick pace. To take this course, you must have a grade of C or better in Math 250.

Any student not meeting these requirements is *strongly* advised to delay taking this class until they are satisfied.

Catalog Description

251 (3 credits) Calculus III (Advanced University Core Curriculum course) [IAI Course: M1 900] Further topics in calculus. Definite integrals over solid regions, applications of partial derivatives, vectors and vector operations, derivatives of vector functions, line integrals. Greens theorem. Prerequisite: 250 with a grade of C or better. Satisfies University Core Curriculum Mathematics requirements in lieu of 110 or 113.

Syllabus Attachment

Fall 2021

MISSION STATEMENT FOR SOUTHERN ILLINOIS UNIVERSITY CARBONDALE

SIU embraces a unique tradition of access and opportunity, inclusive excellence, innovation in research and creativity, and outstanding teaching focused on nurturing student success. As a nationally ranked public research university and regional economic catalyst, we create and exchange knowledge to shape future leaders, improve our communities, and transform lives.

IMPORTANT DATES

Semester Classes Begin:…..……..…….**08/16/2021**
Last day to add full-term course (without Dean's signature): ……**08/22/2021**
Last day to withdraw from the University with a full refund:…….**08/27/2021**
Last day to drop a full-term course for a credit/refund:……...**08/29/2021**
Last day to drop a full-term course (W grade, no refund):……..**10/24/2021**
Final examinations: ……**12/06–12/10/2021**

Note: Please verify the above dates with the Registrar calendar and find more detailed information on deadlines at http://registrar.siu.edu/calendars. For add/drop dates that apply to shorter-than-full-term courses, please look at the Schedule of Classes search results at http://registrar.siu.edu/schedclass/index.php

FALL SEMESTER HOLIDAYS

Labor Day 09/06/2021
Veterans Day 11/11/2021
Thanksgiving Break 11/20-11/28/2021

DIVERSITY

Southern Illinois University Carbondale's goal is to provide a welcoming campus where all of our students, faculty and staff can study and work in a respectful, positive environment free from racism and intimidation. For more information visit: https://diversity.siu.edu. Additional informational flyer: https://diversity.siu.edu/_common/documents/help_contacts.pdf.

DISABILITY SUPPORT SERVICES

SIU Carbondale is committed to providing an inclusive and accessible experience for all students with disabilities. Disability Support Services coordinates the implementation of accommodations. If you think you may be eligible for accommodations but have not yet obtained approval please contact DSS immediately at (618) 453-5738 or https://disabilityservices.siu.edu. You may request accommodations at any time, but timely requests help to insure accommodations are in place when needed. Accommodations and services are determined through an interactive process with students and may involve consideration of specific course design and learning objectives in consultation with faculty.

MILITARY COMMUNITY

There are complexities of being a member of the military community and also a student, and military and veteran related developments can complicate academic life. If you are a member of the military community and in need of accommodations please visit Veterans Services at http://veterans.siu.edu/.

STUDENT MULTICULTURAL RESOURCE CENTER

The Student Multicultural Resource Center serves as a catalyst for inclusion, diversity and innovation. As the Center continues its work, we are here to ensure that you think, grow and succeed. We encourage you to stop by the Center, located in the Student Services Building Room 140, to see the resources available and discover ways you can get involved on the campus. Visit us at https://smrc.siu.edu/.

SALUKI CARES

The purpose of Saluki Cares is to develop, facilitate and coordinate a university-wide program of care and support for students in any type of distress-physical, emotional, financial, or personal. By working closely with faculty, staff, students and their families, SIUC will continue to display a culture of care and demonstrate to our students and their families that they are an important part of the community. For information on Saluki Cares: Call (618) 453-2461, email siucares@siu.edu, or http://salukicares.siu.edu/.

SAFETY AWARENESS FACTS AND EDUCATION

Title IX makes it clear that violence and harassment based on sex and gender is a Civil Rights offense subject to the same kinds of accountability and the same kinds of support applied to offenses against other protected categories such as race, national origin, etc. If you or someone you know has been harassed or assaulted, you can find the appropriate resources here: http://safe.siu.edu

COVID-19

SIUC's policy on face masks and social distancing is consistent with the guidance from the Centers for Disease Control and Prevention and the Illinois Department of Public Health. For up-to-date information, students, faculty, and staff should visit SIUC's COVID website (https://siu.edu/coronavirus), which includes the Saluki Safety Plan. People can also send email to pandemicinfo@siu.edu.

WITHDRAWAL POLICY ~ Undergraduate only

Students who officially register for a session must officially withdraw from that registration in a timely manner to avoid being charged as well as receiving a failing grade for those classes. An official withdrawal must be initiated by the student, or on behalf of the student through the academic unit, and be processed by the Registrar's office. For the proper procedures to follow when dropping courses and when withdrawing from SIUC visit: http://registrar.siu.edu/students/withdrawal.php

SIUC's EARLY WARNING INTERVENTION PROGRAM (EWIP)

Students enrolled in courses participating in SIUC's Early Warning Intervention Program might be contacted by University staff during a semester. More information can be found at the University Core Curriculum's Overview webpage: http://corecurriculum.siu.edu/program-overview/.

EMERGENCY PROCEDURES

We ask that you become familiar with Emergency Preparedness at SIU. Emergency response information is available on posters in buildings on campus, on the Emergency Preparedness at SIU website, and though text and email alerts. To register for alerts visit: http://emergency.siu.edu/

CATALOGS

catalog.siu.edu
gradcatalog.siu.edu

Graduate policies often vary from Undergraduate policies. To view the applicable policies for graduate students, please refer to the graduate catalog.

CENTER FOR LEARNING AND SUPPORT SERVICES

Tutoring: http://tutoring.siu.edu/
Math Labs: http://math.siu.edu/courses/course-help.php

WRITING CENTER: http://write.siu.edu/

PLAGIARISM

See the Student Conduct Code: http://srr.siu.edu/student-conduct-code/

INCOMPLETE POLICY~ Undergraduate only

http://registrar.siu.edu/grades/incomplete.php

REPEAT POLICY: http://registrar.siu.edu/students/repeatclasses.php

MORRIS LIBRARY HOURS: http://libguides.lib.siu.edu/hours

ADVISEMENT: http://advisement.siu.edu/

SIU ONLINE: https://online.siu.edu/

Need additional help with an issue? Visit SALUKI SOLUTION FINDER at http://solutionfinder.siu.edu/