The Automorphism group of a Hahn field

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Abstract

Let $k$ be a field, $G$ a totally ordered abelian group. The maximal field of generalised power series $k((G))$, endowed with its canonical valuation, plays a fundamental role in the classification of valued fields (Kaplansky, 1942 and 1945). In this talk, we describe the group of valuation preserving automorphisms of any Hahn field $K$, i.e. a subfield of the maximal Hahn field, which contains the minimal Hahn field $k(G)$ (the fraction field of the group ring $k[G]$). Under the assumption that $K$ satisfies two lifting properties we prove a structure theorem decomposing into a 4-factor semi-direct product of notable subgroups. We identify a large class of fields satisfying the two aforementioned lifting properties. We then focus on the group of strongly additive automorphisms of $K$. We give an explicit description of the group of strongly additive internal automorphisms in terms of the groups of homomorphisms of $G$ into $k^{\times}$ and of $G$ into the group of 1-units of the valuation ring of $K$. To illustrate the power of our methods, we apply our results to some special cases, such as the field of Laurent series (Schilling, 1944) and that of Puiseux series (Deschamps, 2005).

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