In [1], Hjorth proved that for every countable ordinal $\alpha$, there exists a complete $\mathcal{L}_{\omega_1, \omega}$-sentence $\phi_\alpha$ that has models of all cardinalities less than or equal to $\aleph_\alpha$, but no models of cardinality $\aleph_\alpha + 1$. Unfortunately, his solution yields not one $\mathcal{L}_{\omega_1, \omega}$-sentence $\phi_\alpha$, but a set of $\mathcal{L}_{\omega_1, \omega}$-sentences, one of which is guaranteed to work.

The following is new: It is independent of the axioms of ZFC which of the Hjorth sentences works. More specifically, we isolate a diagonalization principle for functions from $\omega_1$ to $\omega_1$ which is a consequence of the Bounded Proper Forcing Axiom (BPFA) and then we use this principle to prove that Hjorth’s solution to characterizing $\aleph_2$ in models of BPFA is different than in models of CH.

This raises the question whether Hjorth’s result can be proved in an absolute way and what exactly this means, which we will discuss at the end of the talk.

This is joint work with Philipp Lücke.

REFERENCES
