Categoricity for the inferential ω-logic and $L_{\omega_1,\omega}$

John T. Baldwin
University of Illinois at Chicago
Constantin C. Brîncuş
Romanian Academy

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Abstract

This lecture, based on [BB] has mathematical and philosophical aspects: **Mathematics** We consider two extensions of first order logic by 'ω-rules'. In each case we characterize the countable structures whose theory in the logic is categorical (has a unique model). In the one-sorted inferential ω-logic, both Robinson's system Q and Peano Arithmetic become categorical. In the two-sorted generalized ω-logic we show each complete $L_{\omega_1,\omega}$ sentence defines the same class of structures as a first-order theory with the appropriate $G-\omega$-rule. In particular we characterize in our ω-logics those structures (necessarily countable) that are categorical: unique model. **Philosophy** These logics are much weaker than second order logic and we argue that they do not appeal to the arithmetical concepts that the categoricity theorems themselves aim to secure. The results depend on proving that the inferential rules for the logics are categorical, i.e. they uniquely determine certain truth-conditions for the logical connectives and quantifiers. We provide an extensive answer to the doxological challenge (on referential determinacy) proposed in [BW17] and we develop a philosophical view of mathematics -which we call *cognitive modelism*- according to which classical mathematics is best understood as a complex process of constructing and developing a distinctive class of concepts, rather than merely describing a fixed pre-existing realm of structures.

References

[BB] John T. Baldwin and Constantin C. Brîncuş. Categoricity for the inferential ω-logic and $l_{\omega_1,\omega}$. `https://arxiv.org/pdf/2602.02854`.

[BW17] T. Button and S. Walsh. *Philosophy and Model Theory*. Oxford University Press, 2017.