TOWARDS A FINER CLASSIFICATION OF STRONGLY MINIMAL SETS

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Zilber's trichotomy conjecture proposed that, in principle, the properties of the algebraic closure geometry of a theory T determine the character of T. More precisely, the geometry can be discrete, locally modular, or something else. And the theory is then 'successor-like', 'module-like' or 'field-like'. Hrushovski's construction refutes this conjecture by producing a family of strongly minimal structures which are not locally modular but admit no associative function.

A *linear space* is collection of points and lines such that two points determine a line, a minimal condition for the word geometry. A linear space is a Steiner k-system if every line (block) has cardinality k. Theorem 1. [BP20]. For every k there is a strongly minimal k-Steiner system (M, R) (R is collinearity). Classical result. If k is a prime power, there is a coordinatization of (M, R) by a quasigroup. Theorem 2 [Bal20]. If k is a prime power there is a stongly minimal quasigroup $(M, R, *)$ inducing a k-Steiner system. Theorem 3 [BV20]. In 'most' cases no such quasigroup is definable in (M, R). More generally, the Steiner systems and Hrushovski's original example a) do not admit a)elimination of imaginaries or (more strongly) b) an $\emptyset$-definable binary function. *In this talk we introduce the notion of a G-decomposition of a finite G-normal set and sketch the proof of this theorem.* Changing the μ-function or adding axioms like linear space yields profoundly different strongly minimal sets. The *ab initio* Hrushovski construction gives **non-trivial flat classes**:

(1) no binary function
(2) definable binary functions exist
   (a) no commutative binary function (elimination of imaginaries fails)
   (b) strongly minimal quasigroups: $(M, R, *)$ [Bal20] and an example of Hrushovski [Hru93, Proposition 18]
   (c) Non-Desarguesian projective planes coordinatized by ternary fields [Bal95]

References

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