A NONLOCAL REACTION-DIFFUSION POPULATION MODEL WITH STAGE STRUCTURE

Dedicated to Professor Paul Waltman on the occasion of his retirement

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ABSTRACT. A threshold dynamics and global attractivity of positive steady state are established in terms of principal eigenvalues for a nonlocal reaction-diffusion population model with stage structure, and the effects of spatial dispersal and maturation period on the evolutionary behavior are also discussed in two specific cases.

1 Introduction

Recently, an increasing attention has been paid to nonlocal and time-delayed population models in order to study the effects of spatial diffusion and time delay on the evolutionary behavior of biological systems (see, e.g., [16, 13, 3, 1, 19, 17]). In the reality, species may drift from one spatial point at a time to another spatial point at another time, and may disperse from a domain to a larger domain. Moreover, the environment is often spatially heterogeneous. To describe the growth of a single species in a multi-patchy environment, certain delay differential equation models were proposed and analyzed in [12, 14, 20]. [13, 1, 3] also formulated the nonlocal and delayed reaction-diffusion models for a single species with stage structure, and established the existence of a family of traveling wave fronts for these models.

In order to obtain a general nonlocal and time delayed model for a single species in a bounded domain, we let $u(t,a,x)$ be the density of individuals with age $a$ at a point $x$ at time $t$, $\tau$ be the length of the juvenile period. Denote by $u_m(t,x)$ the density of mature adults. Then

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