Bifurcations, chaos, and system collapse in a three node power system

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Abstract

A model of the three node power system proposed by Rajesh and Padiyar [Electr. Power Energy Syst. 21 (1999) 375] is studied. As the bifurcation parameter $P_m$ (input power to the generator) is changed, the system including the effects of the non-linearity exhibits complex dynamics emerging from static and dynamic bifurcations which link with the system collapse. The analyses for the model exhibit dynamical bifurcations, including three Hopf bifurcations, cyclic fold bifurcations, torus bifurcations and period-doubling bifurcations, and complex dynamical behaviors, including periodic orbits, period-doubling orbits, quasi-periodic orbits, phase-locked phenomena and two chaotic regions between two Hopf bifurcations, i.e. in the ‘Hopf window’ and intermittency chaos. Moreover, one of the two chaotic regions results from period-doubling bifurcations, and another results from quasi-periodic orbits emerging from a torus bifurcation. Simulations are given to illustrate the various types of dynamic behaviors associated with the power system collapse for the model. In particular, we first shown that the oscillatory transient may play a role in the collapse, and there are different critical points for different dominated state variables. Besides, the hard-limits and increases of the damping factor widen the feasible operating region of the power system, and prevent the torus bifurcation to occur so that some complex dynamical phenomena can be inhibited.

Keywords: Power systems; Chaos; Bifurcations; Hard-limits

1. Introduction

Rajesh and Padiyar investigated dynamical phenomena of the ‘two-axis model’, the ‘one-axis model’ and the classical model for power systems in Refs. [16, 17]. In the choice of bifurcation parameters, they indicated that the reactive power demand at the load bus as a bifurcation parameter is unrealistic and cannot characterize a wide range of operating conditions. Hence, they took input power to the generator ($P_m$), the constant real and reactive powers of the motor ($P_{ld}$, $Q_{ld}$) and the reference voltage to the AVR ($V_{rd}$) as the bifurcation parameters, and found out the conditions leading to chaotic behaviors. In particular, they gave a comprehensive study of bifurcations for two-axis model, and also found a torus bifurcation resulting in the emergence of quasi-periodic solutions after a torus bifurcation (TR) induced by a Hopf bifurcation(HB) (corresponding to TR$^3$ and HB$^5$ here) for the one-axis model. However, there are still no detail analyses on the one-axis model. In particular, some important behaviors involving phase-locked phenomena and chaos are not given in Ref. [17]. Therefore, Section 4 in this paper aims to investigate the dynamical behaviors emerging from dynamic bifurcations for one-axis model in details. Specifically we first present sustained oscillations and the two chaotic regions via period-doubling and quasi-periodic routes, and show phase-locked phenomena in which the quasi-periodic orbit becomes a complex periodic orbit. Moreover this paper indicates that global dynamic phenomena are drastically affected by the bifurcation parameter $P_m$, and the state variables dominating system behavior are different at different critical points. In Section 5 of this paper, the effects of the damping factor on the system are considered. When the damping factor is greater than some value, the Hopf bifurcation and the torus bifurcation will be inhibited so that the complex behaviors resulted from the bifurcations can be prevented. However, this role of the damping factor is not as drastic as that of machine damping reported in Ref. [19].

To tackle the problems of voltage stability, the power industry utilizes an emerging control to set the control gains of the excitation voltage profiles and faster voltage...
responses. However, such excitation control gains also introduce some side-effect problems, and need to be examined so as to better understand the role of excitation voltage controls (especially the excitation hard-limits) in voltage events [11]. Venkatasubramanian made preliminary effort towards developing a formal stability theory for analyzing the various types of hard-limits in Ref. [20]. Recently Ji and Venkatasubramanian [7] studied an interconnected power system modelled by an equivalent single-machine infinite-bus system, and investigated the role of a wind-up type hard-limit saturation [8,20] in the excitation control. They showed that at a subcritical Hopf bifurcation where the saturation model is used.

The rest of the paper is arranged as follows. Section 2 contains a brief description of a power system model. In Section 3, we review some basic bifurcation and chaos theories that are relevant to our studies. The results of bifurcation analyses with some numerical simulation are given in Section 4. The effects of the damping factor on bifurcation can be eliminated when the saturation model is used.

The system model is shown in Fig. 1 where we neglect the damper winding on the q-axis and describe the system by the following set of equations: generator model, excitation system for the generator, dynamic load model, and network model.

The dynamics of the generator are described by the rotor mechanical equations and stator equations [14]. Rotor equations for the generator model and the equation for the field winding on d-axis are

\[ \dot{\delta} = w_B S_m, \]
\[ \dot{S}_m = \frac{-P_e + P_m - dS_m}{2H}, \]
\[ E_q = \frac{E_{fd} - E_fd + i_d(x_d - x_d)}{T_{d0}}, \]

where \( d \) is the damping factor, \( w_B \) is the system frequency in rad/s, the generator slip \( S_m \) is defined by \( S_m = (w - w_B)w_B \), and the power delivered by the generator \( P_e \) is expressed as

\[ P_e = E_fdq_d + (x_d - x_q)idLd. \]

Neglected stator transients and the stator resistance, stator equations are given by two algebraic equations as the following:

\[ v_q = E_q + x_did_q, \]
\[ v_d = -x_qid. \]

The equation for the excitation system is given by

\[ E_{fd} = -E_{fd} + K_A(V_{ref} - V_f)/T_A, \]

where \( V_{ref} \) is the reference voltage to the AVR.

The load model includes a dynamic induction motor and a constant PQ load in parallel. The induction motor model specifies the real and reactive power demands \( P \) and \( Q \) of the motor in terms of load voltage \( V \) and frequency \( \omega \). The combined model for the motor and the PQ load is

\[ P = P_{ld} + P_0 + p_1\delta L + p_2\dot{V}_L + p_3V_L, \]
\[ Q = Q_{ld} + Q_0 + q_1\delta L + q_2\dot{V}_L + q_3V_L^2, \]

where \( P_{ld} \) and \( Q_{ld} \) are the constant real and reactive powers of the motor, and \( P_0 \) and \( Q_0 \) represent the PQ load.

Fig. 1. A three-node power system model.
All of the parameters in Eq. (14) will be fixed except \( a = \theta_1 \) by using the following parameter values.

\[
\begin{align*}
\text{and (9), and rearranging Eqs. (8) and (9) so that} \\
\text{The real and reactive powers supplied to the load by the} \\
\text{network are} \\
P &= -V_1^2 Y_1 \cos(\phi_1) + V_1^2 Y_2 \cos(\phi_2) + V_L V_1 \cos(r_1) \\
&\quad + E_b V_L Y_2 \cos(r_2), \\
Q &= V_1^2 Y_1 \sin(\phi_1) + V_1^2 Y_2 \sin(\phi_2) + V_L V_1 \sin(r_1) \\
&\quad + E_b V_L Y_2 \sin(r_2),
\end{align*}
\]

where

\[
\begin{align*}
a &= V_1 Y_1 \cos(\delta_L - \delta - \phi + \phi_1) + E_b Y_3 \cos(\delta + \phi - \phi_3), \\
b &= V_1 Y_1 \sin(\delta_L - \delta - \phi + \phi_1) - E_b Y_3 \sin(\delta + \phi - \phi_3).
\end{align*}
\]

The network model in the D–Q reference frame is written as

\[
\begin{align*}
i_q \cos(\phi) + i_d \sin(\phi) &= V_q - a, \\
i_d \cos(\phi) - i_q \sin(\phi) &= V_d - b,
\end{align*}
\]

where

\[
\begin{align*}
a &= V_1 Y_1 \cos(\delta_L - \delta - \phi_1) + E_b Y_3 \cos(\delta + \phi - \phi_3), \\
b &= V_1 Y_1 \sin(\delta_L - \delta - \phi_1) - E_b Y_3 \sin(\delta + \phi - \phi_3).
\end{align*}
\]

where

\[
\begin{align*}
J &= w_b S_m, \\
\dot{S}_m &= -P_g + P_m - dS_m, \\
\dot{E}_q' &= \frac{E_{fd} - E_q' + i_d(x_d - x_d')}{T_{d0}}, \\
\dot{E}_{fd} &= -\frac{E_{fd} + K_A(V_{ref} - V_t)}{T_A}, \\
\dot{\delta}_L &= (Q - Q_{ld} - Q_0 - g_2 V_L - Q_3 V_1^2)/q_1, \\
V_L &= (P - P_{ld} - P_0 - p_1 \delta - p_3 V_1)/p_2,
\end{align*}
\]

All of the parameters in Eq. (14) will be fixed except \( P_m \), i.e. \( P_m \) will be taken as the bifurcation parameter. We follow Ref. [17] by using the following parameter values.

\[
\begin{align*}
Y_1 &= 4.9752, \quad Y_2 = 1.6584, \quad Y_3 = 1.1056, \quad \phi_1 = \phi_2 = \phi_3 = -1.4711, \quad E_b = 1.0, \quad x_q = 1.79, \quad x_d = 1.71, \quad T_{d0} = 4.3, \\
T_{q0} &= 0.85, \quad x_d' = 0.169, \quad x_q' = 0.23, \quad H = 2.894, \quad w_b = 377, \quad d = 0.05, \quad E_m = 1.0 P_0 = 0.4, \quad Q_0 = 0.8, \quad p_1 = 0.24, \quad q_1 = -0.02, \quad p_2 = 1.7, \quad p_3 = -1.866, \quad q_3 = 0.2, \quad q_3 = 1.4, \quad K_A = 200, \quad T_A = 0.05.
\end{align*}
\]

3. Review of bifurcation and chaos

In this section, we give a brief review of bifurcations and chaos in Refs. [6,12,22] which are examined in a more intensive manner in this paper. By definition, bifurcations occur at those parameter values where there is a change in the system’s qualitative properties.

The power system model of Eq. (14) can be written as an autonomous differential equation of the form with \( \mu \) as a bifurcation parameter

\[
\dot{x} = f(x, \mu), \quad x \in \mathbb{R}^6, \quad \mu \in \mathbb{R}, \quad f \in C^r, \quad r \geq 2,
\]

where generally the state vector \( x \) may consist of generator angle, generator angular velocity, load voltage magnitude, etc. the parameter \( \mu \) may be real, reactive power or input power to the generator, etc.

At a fixed point (a equilibrium point or a steady-state solution) \( x(\mu_0) \), since the right-hand term of Eq. (15) becomes zero, its stability is dominated by the eigenvalues of the Jacobian \( J = \frac{\partial f}{\partial x} \) evaluated at \( x(\mu_0) \). Define the eigenvalues of \( J \) to be \( \lambda_j(\mu_0) \), \( j = 1, 2, \ldots, 6 \), \( x(\mu_0) \) is hyperbolic if \( \det[\lambda_j(\mu_0)] \neq 0 \), for all \( i = 1, 2, \ldots, 6 \); \( x(\mu_0) \) is non-hyperbolic if there exists \( j \) such that \( \det[\lambda_j(\mu_0)] = 0 \).

3.1. Saddle-node bifurcation (SNB)

At a SNB point, two equilibrium points, generally one stable and one unstable, coalesce and become a saddle-node point, and then disappear as the parameter passes through the bifurcation value. For the SNB, \( J \) has a simple zero eigenvalue with the remaining eigenvalues having non-zero real parts. Therefore, the necessary conditions for SNB are given by

\[
f(x_0, \mu_0) = 0, \quad \det J(x_0, \mu_0) = 0.
\]

Not all points satisfying these necessary conditions are SNB points (the points satisfying these conditions can be transcritical or pitchfork bifurcation). To illustrate the nature of sufficient conditions we consider a scalar system or reduce the system to one-dimensional system by the center manifold theorem. For such a system \( f(x, \mu) \), the sufficient conditions for SNB are:

\[
f(x_0, \mu_0) = 0, \quad \frac{\partial f}{\partial x} \bigg|_{(x_0, \mu_0)} = 0, \quad \frac{\partial f}{\partial \mu} \bigg|_{(x_0, \mu_0)} \neq 0,
\]

\[
\dot{\frac{\partial f}{\partial x}} \bigg|_{(x_0, \mu_0)} \neq 0.
\]

SNB is considered as a main reason for dynamic instability of the system (15) and is associated with voltage collapse problems in power systems [3,4,9].

3.2. Hopf bifurcation

Hopf bifurcation (HB) does not yield any changes in the number of equilibrium points. This type of bifurcations are
characterized by a complex conjugate pair of eigenvalues for $x(\mu_0)$ lying on the imaginary axis of the complex plane, and result in the birth or death of a stable or unstable limit cycle.

The Hopf bifurcation theorem [6] states that under certain conditions a branch of periodic solutions splits off from an equilibrium when some parameters change. The crucial hypotheses made in the theorem for Eq. (15) are the Jacobian matrix $J(A)$ crucial hypotheses made in the theorem for Eq. (15) are the Jacobian matrix $J(A) = D_x f(x_0(\mu), \mu)$ has a pair of complex conjugate eigenvalues, $\alpha(\mu) \pm i\beta(\mu)$ and for some value $\mu = \mu_0$

$$\alpha(\mu_0) = 0, \quad \beta(\mu_0) > 0,$$

and

$$\alpha'_\mu(\mu_0) \neq 0,$$

and the remaining eigenvalues of $A(\mu_0)$ have non-zero real parts. Next we summarize the conditions for deciding Hopf bifurcation of the six-order system of Eq. (14) according to Ref. [18].

Theorem: suppose that the differential equation (15) has an equilibrium point $x = x_0(\mu)$, the characteristic equation of the Jacobian matrix $A(\mu) = D_x f(x_0(\mu), \mu)$ is

$$P(\lambda) = \lambda^6 + c_1\lambda^5 + c_2\lambda^4 + c_3\lambda^3 + c_4\lambda^2 + c_5\lambda + c_6 = 0,$$

if the following conditions are satisfied:

(i) there exists $\mu = \mu_0$ such that $c_i(\mu_0) > 0$ for all $i = 1, 2, ..., 6,$ and

$$H_5(\mu_0) = -c_2c_4c_5+c_2c_3^2c_5-c_3^3+c_3c_6-c_3c_6 - c_1c_2^2 + c_3c_4c_6 + 2c_2c_5c_6$$

$$+c_1c_2c_5^2 + c_3c_5c_6 - 2c_5c_6 = 0,$$

(ii)

$$H_4(\mu_0) = -c_2^2c_4 + c_2c_5c_6 - c_2^2c_3c_5 - c_2^2c_6 - c_1^2c_2c_4$$

$$- c_2c_3c_5 + 2c_4c_5 - c_5c_6 \neq 0,$$

and

$$H_3(\mu_0) = -c_2^2 - c_1^2c_4 + c_1c_2c_3 + c_3 \neq 0,$$

(iii)

$$\left. \frac{dH_3(\mu)}{d\mu} \right|_{\mu = \mu_0} \neq 0,$$

then there is a Hopf bifurcation. The equilibrium $x(\mu_0)$ may bifurcate into ‘small amplitude’ periodic solutions as the value $\mu$ passes through $\mu_0$. In other words, the periodic solution will emerge when $\mu < \mu_0$ or $\mu > \mu_0$ is sufficiently close to $\mu_0$.

Remark 1. The characteristic equation has a pair of purely imaginary real roots if the condition (i) of the theorem is satisfied. The conditions (ii) and (iii) are referred to as the transverse condition of Hopf bifurcation theorem [6].

Remark 2. A Hopf bifurcation is said to be subcritical or supercritical if the periodic solutions are unstable or stable, in particular, a subcritical Hopf bifurcation may induce voltage collapse phenomena [1,3,19].

3.3. Cyclic fold, period-doubling bifurcation, torus bifurcation, phase-locked phenomena

The stability of the branch of periodic solutions created from Hopf bifurcations can be determined by Floquet multipliers.

Letting the periodic solution of Eq. (15) at $\mu = \mu_0$ be $x(t, \mu_0)$ and have the minimal period $T$, then the local behavior near $x(t, \mu_0)$ is determined by linearizing Eq. (15) along $x(t, \mu_0)$. The linearized equation is

$$\dot{y} = D_x f(x(t, \mu_0), \mu_0)y.$$

It is a six-dimensional linear system and has six linearly independent solutions $y_i$, where $i = 1, 2, ..., 6$. These solutions can be expressed in the form of an $6 \times 6$ matrix called a fundamental matrix solution as

$$Y(t) = [y_1(t), y_2(t), ..., y_6(t)].$$

Obviously, $Y$ satisfies the matrix equation

$$\dot{Y} = D_x f(x(t, \mu_0), \mu_0)Y.$$

When $\Phi(t)$ satisfies the above matrix equation and the initial condition

$$\Phi(0) = I,$$

where $I$ is the $6 \times 6$ identity matrix, $\Phi(T)$ is called the monodromy matrix, and the eigenvalues of the monodromy matrix are called the Floquet or characteristic multipliers [12], Ref. [15] provides the detail procedure to calculate the Floquet multipliers of a periodic solution.

The monodromy matrix has six Floquet multipliers. One of them associated with a periodic solution $x(t, \mu)$ is always unity and the other five multipliers $\eta_i(\mu)$, $i = 1, 2, ..., 5$, determine the stability of $x(t, \mu)$ by the rule: $x(t, \mu)$ is stable if $|\eta_i(\mu)| < 1$ for all $i = 1, 2, ..., 5$; $x(t, \mu)$ is unstable if $|\eta_i(\mu)| > 1$ for some $i$.

When we change the parameter $\mu$, the multipliers $\eta_i(\mu)$ also vary because the multipliers are the functions of the parameter, therefore depending on the parameter. Generically there exist three different ways in which multipliers cross the unit circle [12].

(i) Only one real multiplier crosses the unit circle along the positive real axis, i.e. there exists $\eta_i(\mu_c) = 1$.

(ii) Only one real multiplier crosses the unit circle along the negative real axis, i.e. there exists $\eta_i(\mu_c) = -1$.

(iii) A pair of complex conjugate multipliers crosses the unit circle with a non-zero imaginary part, i.e. there exists $\text{Im}(\eta_i(\mu_c)) \neq 0$. 
In the first case, a stable periodic solutions and an unstable periodic solutions, coalesce and obliterate each other at the bifurcation point $\mu_c$ which is called cyclic fold bifurcation just like saddle-node bifurcation for fixed points in the Poincaré map. When the function $f$ satisfies some symmetry or other regularity properties, pitchfork or transcritical bifurcation may occur [6,22]. The flip bifurcation of the double period bifurcation occurs in the second case, where a branch of period-doubled solutions is created or destroyed at the critical value $\mu_c$. If the bifurcation is supercritical, a branch of stable period-doubled solutions emerges, and the original branch of stable periodic solutions continues as a branch of unstable periodic solutions at the post-bifurcation. On the other hand, if the bifurcation is subcritical, a branch of unstable period-doubled solutions is destroyed, and the original branch of stable periodic solutions continues as a branch of unstable periodic solutions at the post-bifurcation. In the third case, the bifurcation generates a torus, which is called secondary Hopf bifurcation, Hopf bifurcation of the periodic orbit or torus bifurcation.

After the bifurcation from a periodic orbit (with frequency $\omega_1(\mu_0)$) to a torus, there are two frequencies: $\omega_1(\mu_0)$ and $\omega_2(\mu_0)$. The related flow on the torus is called a quasi-periodic solution if $\omega_1$ and $\omega_2$ are incommensurate (i.e. $\omega_1/\omega_2$ is irrational). For some $\mu$, $\omega_1$ and $\omega_2$ are commensurate, then the trajectory on the torus is closed and periodic, which is called phase-locked or frequency-locked phenomenon. Higher dimensional tori have more frequencies. Rotation number and Arnold tongue well describe the nature of the phenomenon [6,12,22].

3.4. Routes to chaos

Chaotic solutions are identified with non-periodicity and sensitive dependence on initial conditions. Hence, chaos is complicated geometrical objects that possess fractal dimensions. We denote the flow generated by Eq. (15) by $\phi(x,t)$ and assume that it exists for all $t > 0$. And assume that $A \subset \mathbb{R}^6$ is a compact set invariant under $\phi(x,t)$, i.e. $\phi(A,t) \subset A$ for all $t > 0$. Then the generally acceptable definitions for the sensitive dependence and chaos are given in Ref. [6].

Definition 1. The flow $\phi(x,t)$ is said to have sensitive dependence on initial conditions on $A$ if there exists $\varepsilon > 0$ such that, for any $\bar{x} \in A$ and any neighborhood $U$ of $\bar{x}$, there exists $\bar{y} \in U$ and $t > 0$ such that $|\phi(x,t) - \phi(y,t)| > \varepsilon$.

Roughly speaking, Definition 1 implies that for any point $x \in A$ there is (at least) one point arbitrarily close to $A$ that diverges from $x$.

Definition 2. $A$ is said to be chaotic if

(i) $\phi(x,t)$ has sensitive dependence on initial conditions on $A$.
(ii) $\phi(x,t)$ is topologically transitive on $A$.
(iii) The periodic orbits of $\phi(x,t)$ are dense in $A$.

Remark. In Ref. [22], the (iii) is not explicitly included in the definition of a chaotic invariant set because a closed invariant set $A$ is topologically transitive if a point $y \in A$ exists such that its orbit is dense in $A$.

Definition 3. Suppose $A \subset \mathbb{R}^6$ is an attractor, then $A$ is called a strange attractor if it is chaos.

Presently, there are many fairly well understood transitions or routes to chaos. In this paper, we only introduce three routes to chaos: periodic doubling bifurcation, intermittency and torus breakdown, which are observed in the dynamical simulation of power systems.

Recall that before a supercritical period-doubling bifurcation, there exists a branch of stable periodic solutions, and after the bifurcation, there exists a branch of stable period-two solutions (period-doubled solutions). If the branch of period two solutions repeat the same bifurcation as the parameter is varied, a branch of period four solutions is created. And so fourth, one can expect a branch of period $2^n$ solution, and finally a branch of chaotic attractors appears. This is called a period-doubling cascade to chaos. There are many researches about it in power systems [2,3,10,13].

Intermittency is also a route to chaos observed frequently in physical experiments. We describe the main features of the route. For values of a parameter $\mu$ less than a critical value $\mu_c$, the system oscillates in a regular mode, and there exists a stable periodic solution for the system. As $\mu$ changes and is slightly more than a threshold value $\mu_c$, the oscillations seems to be regular and closely similar to the behaviors for $\mu < \mu_c$, but they are occasionally interrupted by turbulent bursts of aperiodic oscillations at irregular intervals. With increasing $\mu$, the time intervals (the time regular oscillation) between two consecutive bursts become smaller and smaller and more and more difficult to be distinguished. As $\mu$ is increased further, the periodic oscillations disappear and are completely replaced with chaotic response.

Chaos also can appear following a torus characterized by two oscillatory modes with incommensurate frequencies. As the parameter $\mu$ is varied, if a branch of stable periodic solution undergoes a supercritical secondary Hopf bifurcation, a branch of two period quasi-periodic solution with two incommensurate frequencies appears. With increasing $\mu$, a rich variety of bifurcations of the torus can take place. One of those post-bifurcation states is a chaotic attractor. As the parameter is varied, the two-torus is deformed, destroyed, and chaos emerges at last. In this scenario, the closed curve of the two quasi-periodic solutions in the Poincaré section deforms, wrinkles, and becomes fractal, and finally breaks down. Since the transition to chaos from torus fulfills through the destruction of the closed curve, this transition is often described as chaos via torus breakdown.
3.5. Identifying chaos

To decide whether an oscillation is chaotic, there are several numerical indices such as fractal dimensions, Fourier spectra, and Lyapunov exponents. Just like what we did in Section 4, we can also distinguish among fixed points, periodic oscillations, quasi-periodic solutions and chaotic solutions by using Lyapunov exponents. Chaotic attractor can be seen as the result of an infinite number of stretchings in one or more directions and contractions in another or some other directions, combined with foldings, where Lyapunov exponents just measure these stretchings and contractions.

The linearization of Eq. (15) is the variational equation

$$\dot{y} = f_3(x, \mu)y,$$

where the Jacobian $f_3$ is evaluated along the particular trajectory $x(t, x_0)$. For any initial vector $y(0)$, the solution $y(t)$ can be expressed

$$y(t) = \Phi(t)y(0).$$

Here the fundamental solution matrix $\Phi(t)$ comes from

$$\Phi = f_3(x(t, x_0), \mu)\Phi, \quad \Phi(0) = I.$$

Let the eigenvalues of $\Phi(t)$ be $\lambda_i(t)$, $i = 1, 2, \ldots, 6$, then these eigenvalues represent the stretching of the principal axes. Now we give the definition of Lyapunov exponents:

$$L_k = \lim_{t \to 0} \frac{1}{t} \ln(\lambda_k(t)), \quad k = 1, 2, \ldots, 6.$$

From this definition, we can see that $L_k$ represents the average rate of stretching or contraction of the $i$th direction in the phase space $\mathbb{R}^6$ along the solution $x(t, x_0)$. That is to say, the trajectory will disperse others nearing it in a particular direction along it if a Lyapunov exponent is positive, otherwise, the trajectory will attract others nearing it in a particular direction along it. Hence for dissipative systems, the sum of all Lyapunov exponents must be negative, $\sum L_k < 0$.

For a fixed point, when all Lyapunov exponents are negative, the point is a stable fixed point. For a limit cycle of an autonomous system, one of the Lyapunov exponents is always zero, and for a stable periodic orbit, all Lyapunov exponents except one zero eigenvalue are negative. For a $m$-torus, there are $m$ Lyapunov exponents equal to zero. When all other Lyapunov exponents are negative, the torus is stable. In dissipative systems, chaos are characterized by one or more positive Lyapunov exponents, which correspond to the nature of sensitivity to initial conditions. However, a positive Lyapunov exponents is a necessary but not sufficient condition for chaos, it is still the standard by which we confirm chaotic behaviors in numerical simulations. For a detailed discussion, see Ref. [12].

4. Bifurcation analysis and chaos

In this section, we analyze the stability and bifurcation for Eq. (14) where $P_m$ is taken as a bifurcation parameter. All the results are obtained by detailed numerical simulations, and preliminary analyses are based on the mathematical theories and methods of Section 3.

First of all, the bifurcation diagram is given in Fig. 2 by using AUTO97 [5]. There are twelve types of bifurcations, which are listed in Table 1.

From Fig. 2, we show that the equilibrium point $x_3(P_m)$ undergoes four bifurcations labelled as HB1, HB2, HB3 and SNB4. The equilibrium point $x_3(P_m)$ is stable for $0 < P_m < P_m^4 = 0.608664$, but as $P_m$ is increased, the equilibrium $x_3(P_m)$ loses its stability at $P_m = P_m^4$ through HB2. With a further increase in $P_m$, the $x_3(P_m)$ gains stability through HB1 at $P_m = P_m^1 = 1.155867$. It remains stable until $P_m = P_m^2 = 1.191285$, where stability is lost through HB3. Further the SNB1 does not influence the stability of $x_3(P_m)$. The movement of six eigenvalues of the Jacobian matrix at $x_3$ is shown in Fig. 3 by numerical simulation as the parameter $P_m$ varies.

At $P_m = P_m^4$, a supercritical Hopf bifurcation HB4 occurs. The Jacobian matrix at the equilibrium point $x_3$ has a pair of pure imaginary eigenvalues and four negative real part eigenvalues: $\{ \pm 33.6677j, -32.19, -7.7426, -1 \pm 6.8525j \}$. Fig. 3(a) shows the pair of conjugate complex eigenvalues transversely crosses the $oE$ axis at $\hat{E}$ and $e$ in Fig. 3(b) and (c) show the four eigenvalues with negative real parts, so a supercritical Hopf bifurcation occurs at $P_m = P_m^4$, and the equilibrium $x_3$ becomes unstable for $P_m > P_m^4$. Due to the pair of pure imaginary eigenvalues, $\pm \omega_0j = \pm 33.6677j$, there is a stable periodic solution with the period approximately equal to $2\pi/\omega_0$ for $P_m$ slightly more than $P_m^4$ by the Hopf theorem. We check that for $P_m = 0.6099$ there exists a stable periodic solution with a period of $T = 2\pi/\omega_0 = 0.18 \text{s}$ by numerical simulation. Hence, at the critical point HB4, system behavior is dominated by $\delta$ and $S_m$.

At the critical points HB1 and HB2, the eigenvalues of the Jacobian at $x_2$ are $\{ \pm 31.2075j, -20.809, -13.085, -0.12 \pm 7.543j \}$ and $\{ -0.3757 \pm 30.88j, -19.73, -13.95, \pm 7.57j \}$. From Fig. 3(a), we can see that the curves re-cross the $oE$ axis at points $A$ and $a$ ($\pm \omega_0j = \pm 31.2075j$) corresponding to $\delta$ and $S_m$ leading to the subcritical Hopf bifurcation HB1 at $P_m = P_m^1$, and other real parts of the eigenvalues are negative (see points $A$ and $a$ in Fig. 3(b) and (c)), so $x_2$ remains stable until $P_m = P_m^3 = 1.191285$. At $P_m = P_m^3$, another pair of complex eigenvalues crosses the $oE$ axis transversely ($\pm \omega_3 = \pm 7.57321j$) corresponding to $\delta_3$ and $V_3$ from the left to the right half plane, which are shown by points $B$ and $b$ in Fig. 3(b), so a subcritical Hopf bifurcation HB2 occurs again, and $x_2$ loses stability due to it and remains unstable until $P_m = P_m^2$. At $P_m^1$, $P_m^2$, HB1 and HB2 are both subcritical. There
are unstable oscillations with periods approximately equal to $2\pi/\omega_1$ and $2\pi/\omega_2$ for $P_m$ slightly more than $P_m^1$ and $P_m$ slightly less than $P_m^2$, respectively. But the periods of the unstable oscillations near them are not checked in this paper. At the critical points HB$^1$ and HB$^2$, system behavior is dominated by $\delta$, $S_m$ and $d_L$, $V_L$, respectively.

At $P_m^3 = 1.960761$, a saddle-node bifurcation SNB$^3$ occurs, at which the Jacobian has a simple zero eigenvalue, a real positive eigenvalue and two pairs of complex eigenvalues with negative real parts: $\{-11.044 \pm 21.9116, -14.2932 \pm 14.7414, 8.38915, 0\}$ (see points C and c in Fig. 3). The system has no operating point for $P_m > P_m^3 = 1.960761$.

Fig. 2 also shows the branches of the periodic solutions emerging from HB$^3$, HB$^1$ and HB$^2$. As HB$^3$ is supercritical, a family of stable periodic orbits emanates from HB$^4$, and the six multipliers of the stable periodic orbits lie inside the unit circle. As $P_m$ increases, the stable orbits lose their stabilities due to a torus bifurcation TR$^5$ at $P_m^5 = 0.840966$, the corresponding six multipliers are $\{1, 0.0005, 0.190515 \pm 0.981684j, 0.23067 \pm 0.156288j\}$, where the pair of complex conjugate multipliers $0.190515 \pm 0.981684j$ passes across the unit circle from the inside to the outside the unit circle with an increase in $P_m$, so that a torus bifurcation occurs at $P_m = P_m^5$ and the periodic oscillation loses its stability for $P_m > P_m^5$. With further increases in $P_m$, the periodic orbit gains it back at TR$^6$ at $P_m = 1.251128$, and it remains stable until TR$^7$. At TR$^6$, a pair of complex conjugate multipliers $-0.543321 \pm 0.83952234j$ goes to the inside the unit circle, and other multipliers lie inside the unit circle. The periodic oscillation loses its stability through TR$^7$ at $P_m^7 = 1.30209$. From $P_m^7$, it has not regained its stability although it goes through TR$^8$. The branch emerging from HB$^4$ is the same as that from HB$^4$.

On the branch emerging from HB$^2$, unstable periodic oscillations emerge from HB$^2$ at $P_m^2 = 1.191285$ where one of the multipliers lie outside the unit circle. As $P_m$ decreases, the multiplier enters the unit circle at $P_m^9 = 0.954709$ from the positive side of the real axis, so that the periodic branch turns to the right and gains its stability through circle fold bifurcation CFB$^9$. However, with increasing $P_m$, one of the multiplier crosses the unit circle at $P_m^10 = 0.960252$, which results in the period-doubling bifurcation PDB$^{10}$. And then the periodic solution undergoes a cascade of period-doubling bifurcations at PDB$^{11}$ and PDB$^{12}$.

Next, we further analyze the complex dynamical properties of the system emerging from HB$^4$ and HB$^2$ by using the detailed numerical analysis based on the computations of the largest Lyapunov exponent, the projections and the time domain plot of the trajectories.

<table>
<thead>
<tr>
<th>Table 1</th>
<th></th>
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<tbody>
<tr>
<td>All bifurcation points in Fig. 2 and their $P_m$ values for $d = 0.05$</td>
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<tr>
<td>$P_m$</td>
<td>$P_m$</td>
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<tr>
<td>---------------------------</td>
<td>---------------------------</td>
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<tr>
<td>HB$^1$</td>
<td>1.155867</td>
</tr>
<tr>
<td>HB$^2$</td>
<td>1.191285</td>
</tr>
<tr>
<td>SNB$^3$</td>
<td>1.960761</td>
</tr>
<tr>
<td>HB$^4$</td>
<td>0.608664</td>
</tr>
<tr>
<td>TR$^3$</td>
<td>0.840966</td>
</tr>
<tr>
<td>TR$^6$</td>
<td>1.251128</td>
</tr>
</tbody>
</table>

Fig. 2. Bifurcation diagram of $E_{fd} - P_m$, the solid and the dotted line represent the stable and the unstable stationary points (or periodic solutions), respectively.

Fig. 3. As $P_m$ increases, the eigenvalues cross the points $E \rightarrow A \rightarrow B \rightarrow C \rightarrow D$ or $e \rightarrow a \rightarrow b \rightarrow c \rightarrow d$. (a)–(c) show the movements of three pairs of conjugate eigenvalues.
4.1. Dynamics emerging from HB^4

On the continuation of HB^4, we first find a family of stable periodic solutions which are shown in Fig. 4. The periodic solution encounters the torus bifurcation TR^5 at $P_m^5 = 0.840966$, and the bifurcation results in the emergence of quasi-periodic solution so that the periodic branch loses its stability. For $P_m^5 < P_m \leq 1.043$, the trajectories starting from the neighborhood of the stationary point tend to a quasi-periodic oscillation. Fig. 5 shows that the toroidal attractor expands in the sense of the slowly increasing amplitude of the oscillations while $P_m$ is built up in the interval. The approximate frequencies of the oscillations in Fig. 5 roughly consist of $\omega_1 = 33.39$ and $\omega_2 = 7.13$ for $P_m = 0.851$. We also give the approximate frequencies of the oscillations for quasi-periodic trajectory are $\omega_1 = 33.151$ and $\omega_2 = 7.196$ for $P_m = 0.901$, $\omega_1 = 32.17069$ and $\omega_2 = 7.40735$ for $P_m = 1.043$, and drew the projections of quasi-periodic trajectory for $P_m = 0.901$ and $1.043$, which have analogy to Fig. 5, we omitted these. Those frequencies, respectively, come from the imaginary parts of the complex eigenvalues at the corresponding $P_m$ values.

When we carefully check the variation of the quasi-periodic oscillation further, it is of interest to note that phase-locked phenomena suddenly emerge. When $P_m$ is in the little right neighborhood of 1.043, the toroidal attractor has already been deformed. As a result of the deformation, the unlocking phenomena suddenly occur when $P_m$ is slightly more than 1.043, and the toroidal attractor completely changes into a complex periodic orbit. The projections of the periodic orbits are shown in Fig. 6 for $P_m = 1.0432$ and $P_m = 1.04547$. The periodic orbits last until $P_m$ is approximately equal to 1.09.

When $P_m = 1.09706$, we find a chaotic attractor, whose projection of the chaotic attractor is shown in Fig. 7, and the solution is interrupted by bursts. By carefully checking the bursts of the solutions, it has been found that the time between two bursts are almost the same, and the largest Lyapunov exponent ($L_1$) of the trajectory in Fig. 7 for $P_m = 1.09706$ is positive and approximate to 0.004. As $P_m$ continues to increase slowly, the turbulent bursts occur more and more frequently and the largest Lyapunov exponent is 0.0022 for $P_m = 1.105$, we drew the figure, but omitted it here.

When $P_m$ reaches up to 1.117, the largest Lyapunov exponent of the strange attractor is 0.1437. With a further increase in $P_m$, where the largest Lyapunov exponent of the chaotic attractors is 0.1447. The approximate frequencies of the chaotic oscillations are related to the imaginary parts of the complex eigenvalues of the Jacobian matrix linearized at the corresponding fixed points.

When $P_m$ continuously increases, we find a group of trajectories which are shown in Figs. 7 and 8. Carefully comparing the projections of these trajectories, a cascade of period-doubling bifurcations can be identified. Fig. 7 shows a period orbit with a long period, while Fig. 8 shows the doubled orbits which finally merge in to a chaotic orbit at $P_m = 1.12923$. We show that there is an ‘burst’ in chaotic behavior which has an ‘intermittency’ mechanism. We drew the projection and the corresponding behaviors of $E_{fsd}$ and $V_L$, but omitted it. The largest Lyapunov exponent is 0.146 for $P_m = 1.12923$.

![Fig. 4. The projections of the periodic trajectory for $P_m = 0.74603072$.](image-url)
The chaotic oscillating modes last until \( P_m \) is approximately equal to 1.135. Fig. 9 exemplifies the mode. From Fig. 9, we can see the trajectory is burst frequently by turbulent contractions. The behavior of the system becomes to be quite complex for \( P_m = 1.1295 \) and 1.135. As \( P_m \) grows up further, the turbulent contractions occurs more and more frequently. At last, the trajectory diverges for the variables \( \delta \) and \( \delta_L \).

4.2. Dynamics emerging from \( HB^2 \)

As \( HB^2 \) is subcritical, the periodic branch from \( HB^2 \) gains the stability through \( CFB^9 \). With increases of \( P_m \), the stable periodic solution encounters \( PDB^{10} \), and bifurcates to a period-two branch which bifurcates to a new period four branch when it is subjected to \( PDB^{11} \). With this pattern, the periodic branch finally reaches chaos at \( P_m = 0.963 \). Fig. 10(a)–(d) show the projection of a period-one solution and its sample time simulations at \( P_m = 0.9554 \), and the projection of period-two for \( P_m = 0.961 \), four for \( P_m = 0.9627 \) and eight for \( P_m = 0.9623 \), and their sample time simulations are, respectively, found, we only give the plot of period-two for \( P_m = 0.961 \) in Fig. 11. Fig. 12 shows the projection of the chaotic trajectory whose largest Lyapunov exponent is 0.3465 which verifies the chaotic feature at \( P_m = 0.965 \). Fig. 12 shows the sample time simulation. Chaos resulted from cascades of period-doubling bifurcations is observed in the region around \( P_m \in \)
5. The effects of damping factor on bifurcations and dynamics

In this section, we investigate the loci of bifurcation points for system (14) when both $P_m$ and $d$ are varied. From Eq. (14), we know that the damping factor $d$ does not affect the locations of equilibrium points, which implies the damping factor $d$ does not have any effect on the saddle-node bifurcation where the determinant of the linearized Jacobian matrix must be zero. Hence the saddle-node bifurcation is a vertical wall in the $d$ direction in the parameter space. For Hopf bifurcation and torus bifurcation, both $P_m$ and $d$ do play important roles in the bifurcation computation. The bifurcation loci can be computed numerically and the results are shown in Fig. 13 for variation in $P_m$ and $d$. The curve 1, 2 and 4 are the loci of three Hopf bifurcations, and curve 5 is the torus bifurcation locus and the bifurcation points $\text{HB}^1, \text{HB}^2, \text{SNB}^3, \text{HB}^4, \text{and TR}^{5–8}$ in Section 4 are on the curves 1–5 in Fig. 13, respectively.

We observe from Fig. 13 that the bifurcation loci for Eq. (14) divide the parameter space into six regions: (i) the left region of the curve 4 is the operating region of the system where the operating point ($x_s$) is locally stable; (ii) the region between curve 1 and 4 and above curve 5 where [0.963, 0.965], and when $P_m > 0.966$, the chaotic trajectory becomes a quasi-periodic solution.
the equilibrium point $x_s$ is locally unstable and a stable periodic solution exists (Fig. 4); (iii) the region surrounded by curve 1–3 where $x_s$ is locally stable; (iv) the region surrounded by curve 2, 3 and $P_m$-axis, in which $x_s$ is unstable; (v) the right region of curve 3 in which there is no equilibrium points, and (vi) the region surrounded by curve 5 and $P_m$-axis, where the dynamic behavior is the most complex one, there exist a quasi-periodic solution, chaos, phase-locked phenomena period-doubled orbits (see Section 4).

We will show that $d$ can prevent the occurrence of torus bifurcation and stabilize the system. If we take a horizontal cross-section of the parameter plane in Fig. 13 by varying $P_m$ while keeping $d$ at a constant values, the cross-section will have four intersections with curve 5 for small $d$. In other

Fig. 9. The time simulations of the components $E_{phr}$, $V_L$, $\delta$ and $\delta_L$ for $P_m = 1.135$, and the sensitive dependence of $V_L$ on the initial conditions.
words, four torus bifurcations will occur just like the case of 
$d = 0.05$ in Section 4. While the cross-section moves 
upward slowly, the number of the intersections decreases, 
and at last there is no intersection when $d$ is greater than the 
largest value of $d$ on curve 5 (approximately equal to 8.043). 
Therefore, torus bifurcations can be inhibited by $d$ so that it 
can prevent complex phenomena resulted from torus 
bifurcation, and enhance the stability of the power system. 
In order to illustrate the inhibition, we present the 
projections of trajectories in Fig. 14 for $d = 10$. Parameters 
of the trajectories in the figures are the same as those in Fig. 5 except $d = 0.05$. By numerical simulations (we got the 
results, but deleted these because the file is too large), we 
observe that the quasi-periodic orbit in Fig. 5, the chaos and 
the diverging trajectories all become the periodic solutions 
in Fig. 14 when $d$ changes from 0.05 to 10. Hence, the 
increasing in $d$ contributes to the stability of the system.

Besides, note the stable region of $x_s$ will increase when the 
cross-section goes up, i.e. the increase in the damping factor $d$ 
contributes to the stable region for $x_s$ or the feasibility region 
of the system.

In the simulation of this paper, the damping $d$ is set from 
$d = 0$ to 200. According to Ref. [7], the $d$ typically ranges 
from $d = -10$ to 20 p.u. and the values above $d = 100$ are 
mainly of academic interest. Here we just want to display 
the locus of bifurcation points completely. For the curve 2 of 
Fig. 13, i.e. the locus of HB$_2^2$, meets with the saddle-node 
bifurcation locus (curve 3) at last. Therefore, increases in $d$ 
can deter the emergence of HB$_2^2$ to the saddle-node 
bifurcation. This result is similar to that about machine 
damping in the four-order model of power systems, in which 
machine damping can inhibit the emergence of Hopf 
bifurcation so that it can prevent the chaos resulted from 
period-doubling bifurcations [19]. However, the effect of 
the damping factor $d$ is not as significant as that of the 
machine damping.

6. The role of hard-limits in the system

This section investigates the role of hard-limits in 
stability and bifurcation of the system. First assume that
there is an excitation field control in the generator which is simply represented by the single-time-constant transfer function shown in Fig. 15. Note that the wind-up type hard-limit on the signal $E_{fd}$ limits the control output $E_{fd}$ strictly between the values $E_{fd_{\text{min}}}$ and $E_{fd_{\text{max}}}$. With the assumptions, instead of Eq. (7), the equation for $E_{fd}$ in system (14) is replaced by

$$E_{fd} = E_{fd0} - E_{fd} + K_A(V_{ref} - V),$$

and the system (14) becomes into a new system (14'), where $E_{fd}$ is the output of the wind-up limiter or saturation:

$$E_{fd} = \begin{cases} E_{fd_{\text{min}}} & \text{if } E_{fd} > E_{fd_{\text{max}}}, \\ E_{fd} & \text{if } E_{fd_{\text{min}}} \leq E_{fd} \leq E_{fd_{\text{max}}}, \\ E_{fd_{\text{max}}} & \text{if } E_{fd} < E_{fd_{\text{min}}}. \end{cases}$$

(14')

In system (14), $P_m$ is a bifurcation parameter, and all other parameters are chosen to be the same as in Section 2 except: $E_{fd_{\text{min}}} = 0$, $E_{fd_{\text{max}}} = 5$. Before we start to solve for the equilibrium points and bifurcation points of the system (14'), we also assume that the field control output $E_{fd}$ at the operating point must lie within the linear region of the hard-limit for small $P_m$, i.e. between $E_{fd_{\text{min}}}$ and $E_{fd_{\text{max}}}$. In fact, the steady-state value of $E_{fd}$ should always lie within these limiting values in a well-designed system even under load variations. By using AUTO97, the bifurcation diagram for fixed $d = 0.05$ is drawn in Fig. 16, and $P_m$ values of all bifurcation points are listed as the following: HB$^1$ for $P_m^1 = 1.15586$; HB$^2$ for $P_m^2 = 1.1913$; SNB$^3$ for $P_m^3 = 1.9365$; HB$^4$ for $P_m^4 = 0.60866$; CFB$^5$ for $P_m^5 = 1.164358$.

From Fig. 16, it can be observed that the stable periodic solution jumping from the supercritical Hopf
bifurcation point HB$^4$ only undergoes a cyclic fold bifurcation at CFB$^5$, and then arrives at HB$^1$, a subcritical Hopf bifurcation. This means that the hard-limits inhibit torus bifurcations, which do occur for the case without hard-limits (Fig. 2). Therefore, a series complex dynamic behaviors resulted from torus bifurcations are all prevented. Moreover, it seems that the role of the hard-limits is more effective than the role of damping factor $d$ in the sense of inhibiting torus bifurcation. To clarify this point we examine several time-plots of three trajectories when the hard-limits are introduced and find the system in the sustained oscillations and the quasi-periodic trajectory, chaos, and the diverging trajectory will converge to a stable limit cycle due to hard-limits. In other words, the hard-limits somehow introduce positive damping in a global sense so that torus bifurcations are prevented and the trajectories are eventually stabilized into sustained oscillation.

HB$^2$ is a subcritical bifurcation, where a branch of unstable periodic solutions emerge from it, but the branch is always unstable although it also undergoes a cyclic fold bifurcation CFB$^5$. From Figs. 2 and 16, clearly the period-doubling bifurcation point in Fig. 2 are inhibited by the hard-limits so that the chaos resulted from period-doubling bifurcation is also prevented by the hard-limits. The numerical results further confirm that the hard-limits contribute to the stability of the system. When $P_m > P_m^{\text{cr}}$, the transients diverge away quickly.

Fig. 12. The numerical simulations for $P_m = 0.965 : (a)$, the projection of the chaotic trajectory and (b) the corresponding sample time behaviors of $V_L$ and $E_{\mu}$. 
Fig. 13. The locus of bifurcation points in the parameter space $P_m - d$.

Fig. 14. The projection of the trajectory with the same parameters in Fig. 5 for $P_m = 0.851$, but $d = 10$.

Fig. 15. The simplified excitation model.
The preceding paragraph discusses the role of hard-limits while the damping factor $d = 0.05$. A natural question is whether or not this role exists persistently robustly. We carry out the same work but substituting $d = 10$ for $d = 0.05$. The result is shown in Fig. 17. Surprising similarities exist between Figs. 16 and 17, which demonstrate the role of hard-limits is identical for different $d$ values.

Besides, note that the points CFB in Figs. 16 and 17 are between HB$^1$ and HB$^2$, and the operating point for small $P_m$ can move along the branch of stable periodic orbits, and then arrive at another feasibility region for larger $P_m$. Therefore, the two operating regions are connected by the branch of periodic solutions. The cyclic fold bifurcation is contributed to the hard-limits, where $E_{fd}$ arrives at $E_{fd_{max}}$. 

Fig. 16. The bifurcation diagram for the system with hard-limits for $d = 0.05$.

Fig. 17. The bifurcation diagram for the system with hard-limits for $d = 10$. 
7. Conclusion

We found many dynamical bifurcations, emerging from the Hopf bifurcations in the model (14), including cyclic fold bifurcation, period-doubling bifurcations and torus bifurcations which all occur prior to the saddle-node bifurcation. We also observed the complex dynamical behaviors emerging from the dynamical bifurcations including periodic orbits, quasi-periodic orbits, phase-locking phenomena and two different routes to chaos and the intermittency mechanism of chaos. The stable periodic orbit emerges from HB$^4$ at $P^m_m = 0.608664$. The three chaotic regions are also numerically observed: the chaos resulted from period-doubling bifurcations approximately lies in the interval $P_m \in [0.963, 0.965]$ and $P_m \in [1.1292, 1.135]$, and we only give the chaotic attractor is shown in Fig. 12; another chaotic attractor resulted from the quasi-periodic orbits approximately lies in the region $P_m \in [1.09706, 1.122]$, and the phase-locked phenomena approximately occurs in the region $P_m \in [1.0454, 1.09]$; and the corresponding chaotic attractors are obtained, but we deleted these. The above complex dynamical phenomena emphasize the importance of the consideration of non-linearity.

In general, static bifurcations (saddle-node bifurcations) can be related to system collapse, and the oscillations of the quasi-periodic orbits or weak chaos affect the quality of electric power by distorting the voltage and current wave forms and can also cause the power system collapse in certain circumstances. But they are considered as one of the clues for the collapse of complicated power systems or for stability margin narrowing in parameter space.

Besides, we also find that at different Hopf bifurcation points, the system behavior is dominated by different state variables for instance $\delta$ and $\delta_m$ at HB$^4$ and HB$^4$, and $\delta_L$ and $V_L$ at HB$^2$. This fact suggests that different factors of power systems reduce the system stability at different critical points. In other words, we should design different control systems or control different parameters near the critical points.

When the effects of the damping factor $d$ on the system’s dynamics are considered, the torus bifurcation can be inhibited by the increases in $d$, which implies positive damping is beneficial for the stability of the system. Once the hard-limits are introduced to the system, the effects are similar to those of a great increase in the damping factor, i.e. the torus bifurcation is prevented. In Section 5, we do not investigate the loci of the cyclic bifurcation and period-doubling bifurcation like CFB$^9$, PDB$^{10}$ for variations in both $P_m$ and $d$. However, from the analyses of Sections 4 and 5, we can reasonably infer that the damping factor $d$ also exists the largest values along the loci of the cyclic bifurcation and period-doubling bifurcation (they may be smaller than the largest value along the torus bifurcation locus). Therefore $d$ also prevents the cyclic bifurcation and period-doubling bifurcation.

Remark. The size of the original file is too large (> 33M), present file near 2M after deleted a lot of Figures, for examples: the projections of quasi-periodic trajectory and chaotic trajectories and corresponding time behaviors of $V_L$ for $P_m = 0.901, 1.043, 1.09706, 1.105, 1.117, 1.122$; the projections and time simulations of components of $E_{eff}$ and $V_L$ of period-four, period-eight, chaotic behaviors for $P_m = 0.9623, 0.9627, 1.12923, 1.1295, 1.135, 1.145$; the time-plots of the trajectories with hard-limits for $P_m = 0.851, 1.12923, 1.145$.

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References


