

Inference After Variable Selection

by

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## AN ABSTRACT OF THE DISSERTATION OF

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This thesis presents inference for the multiple linear regression model  $Y = \beta_1x_1 + \dots + \beta_p x_p + e$  after model or variable selection, including prediction intervals for a future value of the response variable  $Y_f$ , and testing hypotheses with the bootstrap. If  $n$  is the sample size, most results are for  $n/p$  large, but prediction intervals are developed that increase in average length slowly as  $p$  increases for fixed  $n$  if the model is sparse:  $k$  predictors have nonzero coefficients  $\beta_i$  where  $n/k$  is large.

**KEY WORDS:** Bootstrap; Forward Selection; Lasso; Partial Least Squares; Prediction Interval; Principal Components Regression; Relaxed Lasso; Ridge Regression.

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# CHAPTER 1

## INTRODUCTION

Suppose that the response variable  $Y_i$  and at least one predictor variable  $x_{i,j}$  are quantitative with  $x_{i,1} \equiv 1$ . Let  $\mathbf{x}_i^T = (x_{i,1}, \dots, x_{i,p}) = (1 \ \mathbf{u}_i^T)$  and  $\boldsymbol{\beta} = (\beta_1, \dots, \beta_p)^T$  where  $\beta_1$  corresponds to the intercept. Then the multiple linear regression (MLR) model is

$$Y_i = \beta_1 + x_{i,2}\beta_2 + \cdots + x_{i,p}\beta_p + e_i = \mathbf{x}_i^T \boldsymbol{\beta} + e_i \quad (1.1)$$

for  $i = 1, \dots, n$ . This model is also called the full model. Here  $n$  is the sample size and the random variable  $e_i$  is the  $i$ th error. In matrix notation, these  $n$  equations become

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{e}, \quad (1.2)$$

where  $\mathbf{Y}$  is an  $n \times 1$  vector of dependent variables,  $\mathbf{X}$  is an  $n \times p$  matrix of predictors,  $\boldsymbol{\beta}$  is a  $p \times 1$  vector of unknown coefficients, and  $\mathbf{e}$  is an  $n \times 1$  vector of unknown errors. The  $i$ th fitted value  $\hat{Y}_i = \mathbf{x}_i^T \hat{\boldsymbol{\beta}}$  and the  $i$ th residual  $r_i = Y_i - \hat{Y}_i$  where  $\hat{\boldsymbol{\beta}}$  is an estimator of  $\boldsymbol{\beta}$ . Ordinary least squares (OLS) is often used for inference if  $n/p$  is large.

It is often convenient to use the centered response  $\mathbf{Z} = \mathbf{Y} - \bar{\mathbf{Y}}$  where  $\bar{\mathbf{Y}} = \bar{Y}\mathbf{1}$ , and the  $n \times (p-1)$  matrix of standardized nontrivial predictors  $\mathbf{W} = (W_{ij})$ . For  $j = 1, \dots, p-1$ , let  $W_{ij}$  denote the  $(j+1)$ th variable standardized so that  $\sum_{i=1}^n W_{ij} = 0$  and  $\sum_{i=1}^n W_{ij}^2 = n$ . Hence

$$W_{ij} = \frac{x_{i,j+1} - \bar{x}_{j+1}}{\tilde{\sigma}_{j+1}} \quad \text{where} \quad \tilde{\sigma}_{j+1}^2 = \frac{1}{n} \sum_{i=1}^n (x_{i,j+1} - \bar{x}_{j+1})^2.$$

Note that the sample correlation matrix of the nontrivial predictors  $\mathbf{u}_i$  is

$$\mathbf{R}_{\mathbf{u}} = \frac{\mathbf{W}^T \mathbf{W}}{n}.$$

Then regression through the origin is used for the model

$$\mathbf{Z} = \mathbf{W}\boldsymbol{\eta} + \boldsymbol{\epsilon} \quad (1.3)$$

where the vector of fitted values  $\hat{\mathbf{Y}} = \bar{\mathbf{Y}} + \hat{\mathbf{Z}}$ .

There are many alternative methods for estimating  $\boldsymbol{\beta}$ , including forward selection with OLS, principal component regression (PCR), partial least squares (PLS) due to Wold (1975), lasso due to Tibshirani (1996), relaxed lasso due to Meinshausen (2007), and ridge regression (RR): see Hoerl and Kennard (1970). These six methods produce  $M$  models and use a criterion to select the final model (e.g.  $C_p$  or 10-fold cross validation (CV)). The number of models  $M$  depends on the method. The full model is (approximately) fit with OLS. For one of the  $M$  models, some of the methods use  $\hat{\boldsymbol{\eta}} = \mathbf{0}$  and fit the model  $Y_i = \beta_1 + e_i$  with  $\hat{Y} = \bar{Y}$ . Lasso and ridge regression have a parameter  $\lambda$ . When  $\lambda = 0$ , the full OLS model is used. These methods also use a maximum value  $\lambda_M$  of  $\lambda$  and a grid of  $M$   $\lambda$  values  $0 \leq \lambda_1 < \lambda_2 < \dots < \lambda_{M-1} < \lambda_M$  where often  $\lambda_1 = 0$ . For lasso,  $\lambda_M$  is the smallest value of  $\lambda$  such that  $\hat{\boldsymbol{\eta}}_{\lambda_M} = \mathbf{0}$ . Hence  $\hat{\boldsymbol{\eta}}_{\lambda_i} \neq \mathbf{0}$  for  $i < M$ . For forward selection, PCR, and PLS,  $M \leq p$ . See James, Witten, Hastie, and Tibshirani (2013, ch. 6) for more details about these six methods.

## 1.1 INFERENCE FOR RIDGE REGRESSION AND LASSO

Consider choosing  $\hat{\boldsymbol{\eta}}$  to minimize the criterion

$$Q(\boldsymbol{\eta}) = \frac{1}{a}(\mathbf{Z} - \mathbf{W}\boldsymbol{\eta})^T(\mathbf{Z} - \mathbf{W}\boldsymbol{\eta}) + \frac{\lambda_{1,n}}{a} \sum_{i=1}^{p-1} |\eta_i|^j \quad (1.4)$$

where  $\lambda_{1,n} \geq 0$ ,  $a > 0$ , and  $j > 0$  are known constants. Then  $j = 2$  corresponds to ridge regression,  $j = 1$  corresponds to lasso, and  $a = 1, 2, n$ , and  $2n$  are common. The residual

sum of squares  $RSS(\boldsymbol{\eta}) = (\mathbf{Z} - \mathbf{W}\boldsymbol{\eta})^T(\mathbf{Z} - \mathbf{W}\boldsymbol{\eta})$ , and  $\lambda_{1,n} = 0$  corresponds to the OLS estimator  $\hat{\boldsymbol{\eta}}_{OLS} = (\mathbf{W}^T\mathbf{W})^{-1}\mathbf{W}^T\mathbf{Z}$ .

In the following three paragraphs, assume  $p$  is fixed. Knight and Fu (2000) prove i) that  $\hat{\boldsymbol{\eta}}$  is a consistent estimator of  $\boldsymbol{\eta}$  if  $\lambda_{1,n} = o(n)$  so  $\lambda_{1,n}/n \rightarrow 0$  as  $n \rightarrow \infty$ , ii)  $\hat{\boldsymbol{\eta}}_{OLS}$  and  $\hat{\boldsymbol{\eta}}$  are asymptotically equivalent if  $\lambda_{1,n} \rightarrow \infty$  too slowly as  $n \rightarrow \infty$ , iii)  $\hat{\boldsymbol{\eta}}$  is a  $\sqrt{n}$  consistent estimator of  $\boldsymbol{\eta}$  if  $\lambda_{1,n} = O(\sqrt{n})$  (so  $\lambda_{1,n}/\sqrt{n}$  is bounded), and iv) if  $\lambda_{1,n}/\sqrt{n} \rightarrow \tau \geq 0$ , then

$$\sqrt{n}(\hat{\boldsymbol{\eta}}_{RR} - \boldsymbol{\eta}) \xrightarrow{D} N_{p-1}(-\tau \mathbf{V}\boldsymbol{\eta}, \sigma^2 \mathbf{V})$$

where

$$\mathbf{R}_u = \frac{\mathbf{W}^T \mathbf{W}}{n} \xrightarrow{P} \mathbf{V}^{-1} \quad (1.5)$$

as  $n \rightarrow \infty$ . If  $\tau = 0$ , then OLS and ridge regression have the same limiting distribution. Note that  $\mathbf{V}^{-1} = \boldsymbol{\rho}_u$  if the  $\mathbf{u}_i$  are a random sample from a population with a nonsingular population correlation matrix  $\boldsymbol{\rho}_u$ . Under (1.5), if  $\lambda_{1,n}/n \rightarrow 0$  then

$$\frac{\mathbf{W}^T \mathbf{W} + \lambda_{1,n} \mathbf{I}_{p-1}}{n} \xrightarrow{P} \mathbf{V}^{-1}, \quad \text{and} \quad n(\mathbf{W}^T \mathbf{W} + \lambda_{1,n} \mathbf{I}_{p-1})^{-1} \xrightarrow{P} \mathbf{V}.$$

The following identity from Gunst and Mason (1980, p. 342) is useful for ridge regression inference:

$$\begin{aligned} \hat{\boldsymbol{\eta}}_{RR} &= (\mathbf{W}^T \mathbf{W} + \lambda_{1,n} \mathbf{I}_{p-1})^{-1} \mathbf{W}^T \mathbf{Z} = (\mathbf{W}^T \mathbf{W} + \lambda_{1,n} \mathbf{I}_{p-1})^{-1} \mathbf{W}^T \mathbf{W} \hat{\boldsymbol{\eta}}_{OLS} = \mathbf{A}_n \hat{\boldsymbol{\eta}}_{OLS} \\ &= [\mathbf{I}_{p-1} - \lambda_{1,n} (\mathbf{W}^T \mathbf{W} + \lambda_{1,n} \mathbf{I}_{p-1})^{-1}] \hat{\boldsymbol{\eta}}_{OLS} = \mathbf{B}_n \hat{\boldsymbol{\eta}}_{OLS} \end{aligned}$$

since  $\mathbf{A}_n - \mathbf{B}_n = \mathbf{0}$ . If  $\lambda_{1,n}/\sqrt{n} \rightarrow \tau \geq 0$ , then

$$\sqrt{n}(\hat{\boldsymbol{\eta}}_{RR} - \boldsymbol{\eta}) = \sqrt{n}(\hat{\boldsymbol{\eta}}_{RR} - \hat{\boldsymbol{\eta}}_{OLS} + \hat{\boldsymbol{\eta}}_{OLS} - \boldsymbol{\eta}) =$$

$$\sqrt{n}(\hat{\boldsymbol{\eta}}_{OLS} - \boldsymbol{\eta}) - \sqrt{n}\frac{\lambda_{1,n}}{n}n(\mathbf{W}^T\mathbf{W} + \lambda_{1,n}\mathbf{I}_{p-1})^{-1}\hat{\boldsymbol{\eta}}_{OLS}$$

$$\xrightarrow{D} N_{p-1}(\mathbf{0}, \sigma^2 \mathbf{V}) - \tau \mathbf{V} \boldsymbol{\eta} \sim N_{p-1}(-\tau \mathbf{V} \boldsymbol{\eta}, \sigma^2 \mathbf{V}).$$

**Theorem 1.1.** Let  $\hat{\boldsymbol{\eta}}_L$  be the lasso estimator. Then

$$\sqrt{n}(\hat{\boldsymbol{\eta}}_L - \boldsymbol{\eta}) \sim N_{p-1}\left(\frac{-\tau}{2}\mathbf{V}\mathbf{s}, \sigma^2 \mathbf{V}\right).$$

where

$$\mathbf{R}_u = \frac{\mathbf{W}^T \mathbf{W}}{n} \xrightarrow{P} \mathbf{V}^{-1}$$

as  $n \rightarrow \infty$ . If  $\tau = 0$ , then OLS and lasso regression have the same limiting distribution.

*Proof.* The following identity from Efron and Hastie (2016, p. 308), for example, is useful for inference for the lasso estimator  $\hat{\boldsymbol{\eta}}_L$ :

$$\frac{-1}{n} \mathbf{W}^T (\mathbf{Z} - \mathbf{W} \hat{\boldsymbol{\eta}}_L) + \frac{\lambda_{1,n}}{2n} \mathbf{s}_n = \mathbf{0} \quad \text{or} \quad -\mathbf{W}^T (\mathbf{Z} - \mathbf{W} \hat{\boldsymbol{\eta}}_L) + \frac{\lambda_{1,n}}{2} \mathbf{s}_n = \mathbf{0}$$

where  $s_{in} \in [-1, 1]$  and  $s_{in} = \text{sign}(\hat{\eta}_{i,L})$  if  $\hat{\eta}_{i,L} \neq 0$ . Here  $\text{sign}(\eta_i) = 1$  if  $\eta_i > 1$  and  $\text{sign}(\eta_i) = -1$  if  $\eta_i < 1$ . Note that  $\mathbf{s}_n = \mathbf{s}_{n,\hat{\boldsymbol{\eta}}_L}$  depends on  $\hat{\boldsymbol{\eta}}_L$ . Thus

$$\hat{\boldsymbol{\eta}}_L = (\mathbf{W}^T \mathbf{W})^{-1} \mathbf{W}^T \mathbf{Z} - n(\mathbf{W}^T \mathbf{W})^{-1} \frac{\lambda_{1,n}}{2n} \mathbf{s}_n.$$

If  $\lambda_{1,n}/\sqrt{n} \rightarrow \tau \geq 0$  and  $\mathbf{s}_n \xrightarrow{P} \mathbf{s} = \mathbf{s}_{\boldsymbol{\eta}}$ , then

$$\begin{aligned} \sqrt{n}(\hat{\boldsymbol{\eta}}_L - \boldsymbol{\eta}) &= \sqrt{n}(\hat{\boldsymbol{\eta}}_L - \hat{\boldsymbol{\eta}}_{OLS} + \hat{\boldsymbol{\eta}}_{OLS} - \boldsymbol{\eta}) = \\ \sqrt{n}(\hat{\boldsymbol{\eta}}_{OLS} - \boldsymbol{\eta}) - \sqrt{n}\frac{\lambda_{1,n}}{2n}n(\mathbf{W}^T \mathbf{W})^{-1} \mathbf{s}_n &\xrightarrow{D} N_{p-1}(\mathbf{0}, \sigma^2 \mathbf{V}) - \frac{\tau}{2} \mathbf{V} \mathbf{s} \sim N_{p-1}\left(\frac{-\tau}{2} \mathbf{V} \mathbf{s}, \sigma^2 \mathbf{V}\right). \end{aligned}$$

□

If none of the elements of  $\boldsymbol{\eta}$  are zero, and if  $\hat{\boldsymbol{\eta}}_L$  is a consistent estimator of  $\boldsymbol{\eta}$ , then  $\mathbf{s}_n \xrightarrow{P} \mathbf{s} = \mathbf{s}_{\boldsymbol{\eta}}$ . If  $\lambda_{1,n}/\sqrt{n} \rightarrow 0$ , then OLS and lasso are asymptotically equivalent even if  $\mathbf{s}_n$  does not converge to a vector  $\mathbf{s}$  as  $n \rightarrow \infty$  since  $\mathbf{s}_n$  is bounded.

The results in the above three paragraphs hold after model selection if  $\lambda_{1,n}$  is replaced by  $\hat{\lambda}_{1,n}$  and  $o$  and  $O$  are replaced by  $o_P$  and  $O_P$ , e.g.  $\hat{\lambda}_{1,n} = o_P(\sqrt{n})$  makes lasso or ridge regression asymptotically equivalent to OLS. For model selection, the  $M$  values of  $\lambda$  are denoted by  $\lambda_1, \lambda_2, \dots, \lambda_M$  where  $\lambda_i = \lambda_{1,n,i}$  depends on  $n$  for  $i = 1, \dots, M$ . If  $\lambda_s$  corresponds to the model selected, then  $\hat{\lambda}_{1,n} = \lambda_s$ .

## CHAPTER 2

### VARIABLE SELECTION

Variable selection, also called subset or model selection, is the search for a subset of predictor variables that can be deleted without important loss of information. Following Olive and Hawkins (2005), a *model for variable selection* can be described by

$$\mathbf{x}^T \boldsymbol{\beta} = \mathbf{x}_S^T \boldsymbol{\beta}_S + \mathbf{x}_E^T \boldsymbol{\beta}_E = \mathbf{x}_S^T \boldsymbol{\beta}_S \quad (2.1)$$

where  $\mathbf{x} = (\mathbf{x}_S^T, \mathbf{x}_E^T)^T$ ,  $\mathbf{x}_S$  is a  $k_S \times 1$  vector and  $\mathbf{x}_E$  is a  $(p - k_S - 1) \times 1$  vector. Given that  $\mathbf{x}_S$  is in the model,  $\boldsymbol{\beta}_E = \mathbf{0}$  and  $E$  denotes the subset of terms that can be eliminated given that the subset  $S$  is in the model. Let  $\mathbf{x}_I$  be the vector of  $k$  terms from a candidate subset indexed by  $I$ , and let  $\mathbf{x}_O$  be the vector of the remaining predictors (out of the candidate submodel). Suppose that  $S$  is a subset of  $I$  and that model (2.1) holds. Then

$$\mathbf{x}^T \boldsymbol{\beta} = \mathbf{x}_S^T \boldsymbol{\beta}_S = \mathbf{x}_S^T \boldsymbol{\beta}_S + \mathbf{x}_{I/S}^T \boldsymbol{\beta}_{(I/S)} + \mathbf{x}_O^T \mathbf{0} = \mathbf{x}_I^T \boldsymbol{\beta}_I \quad (2.2)$$

where  $\mathbf{x}_{I/S}$  denotes the predictors in  $I$  that are not in  $S$ . Since this is true regardless of the values of the predictors,  $\boldsymbol{\beta}_O = \mathbf{0}$  if  $S \subseteq I$ .

Forward selection forms a sequence of submodels  $I_1, \dots, I_M$  where  $I_j$  uses  $j$  predictors including the constant. Let  $I_1$  use  $x_1^* = x_1 \equiv 1$ : the model has a constant but no nontrivial predictors. To form  $I_2$ , consider all models  $I$  with two predictors including  $x_1^*$ . Compute  $Q_2(I) = SSE(I) = RSS(I) = \mathbf{r}^T(I)\mathbf{r}(I) = \sum_{i=1}^n r_i^2(I) = \sum_{i=1}^n (Y_i - \hat{Y}_i(I))^2$ . Let  $I_2$  minimize  $Q_2(I)$  for the  $p - 1$  models  $I$  that contain  $x_1^*$  and one other predictor. Denote the predictors in  $I_2$  by  $x_1^*, x_2^*$ . In general, to form  $I_j$  consider all models  $I$  with  $j$  predictors

including variables  $x_1^*, \dots, x_{j-1}^*$ . Compute  $Q_j(I) = \mathbf{r}^T(I)\mathbf{r}(I) = \sum_{i=1}^n r_i^2(I) = \sum_{i=1}^n (Y_i - \hat{Y}_i(I))^2$ . Let  $I_j$  minimize  $Q_j(I)$  for the  $p - j + 1$  models  $I$  that contain  $x_1^*, \dots, x_{j-1}^*$  and one other predictor not already selected. Denote the predictors in  $I_j$  by  $x_1^*, \dots, x_j^*$ . Continue in this manner for  $j = 2, \dots, M$ . Often  $M = \min(\lceil n/J \rceil, p)$  for some integer  $J$  such as  $J = 5, 10$ , or  $20$ . Here  $\lceil x \rceil$  is the smallest integer  $\geq x$ , e.g.,  $\lceil 7.7 \rceil = 8$ .

## 2.1 CRITERIA FOR FINAL MODEL SELECTION

When there is a sequence of  $M$  submodels, the final submodel  $I_d$  needs to be selected. Let  $\mathbf{x}_I$  and  $\hat{\beta}_I$  be  $a \times 1$ . Hence the candidate model contains  $a$  terms, including a constant. Suppose the  $e_i$  are independent and identically distributed (iid) with variance  $V(e_i) = \sigma^2$ . Then there are many criteria used to select the final submodel  $I_d$ . A simple method is to take the model that uses  $d = M = \min(\lceil n/J \rceil, p)$  variables. If  $p$  is fixed, the method will use the full OLS model once  $n/J \geq p$ . For a given data set,  $p, n$  and  $\hat{\sigma}^2$  act as constants, and a criterion below may add a constant or be divided by a constant without changing the subset  $I_{\min}$  that minimizes the criterion.

### 2.1.1 Criterion when there is a good estimator for $\sigma^2$

Let criteria  $C_S(I)$  have the form

$$C_S(I) = SSE(I) + aK_n\hat{\sigma}^2.$$

These criteria need a good estimator of  $\sigma^2$ . The criterion  $C_p(I) = AIC_S(I)$  uses  $K_n = 2$  while the  $BIC_S(I)$  criterion uses  $K_n = \log(n)$ . Typically  $\hat{\sigma}^2$  is the full OLS model

$$MSE = \sum_{i=1}^n \frac{r_i^2}{n-p}$$

when  $n/p$  is large. Then  $\hat{\sigma}^2 = MSE$  is a  $\sqrt{n}$  consistent estimator of  $\sigma^2$  under mild conditions by Su and Cook (2012).

### 2.1.2 $AIC(I)$ and $BIC(I)$

It is hard to get a good estimator of  $\sigma^2$  when  $n/p$  is not large. The following criterion are described in Burnham and Anderson (2004), but still need  $n/p$  large.  $AIC$  is due to Akaike (1973) and  $BIC$  to Schwarz (1978).

$$AIC(I) = n \log \left( \frac{SSE(I)}{n} \right) + 2a, \text{ and}$$

$$BIC(I) = n \log \left( \frac{SSE(I)}{n} \right) + a \log(n).$$

Let  $I_{min}$  be the submodel that minimizes the criterion. Following Seber and Lee (2003, p. 448) and Nishi (1984), the probability that model  $I_{min}$  from  $C_p$  or  $AIC$  underfits goes to zero as  $n \rightarrow \infty$ . If  $\hat{\beta}_I$  is  $a \times 1$ , form the  $p \times 1$  vector  $\hat{\beta}_{I,0}$  from  $\hat{\beta}_I$  by adding 0s corresponding to the omitted variables. Since there are a finite number of regression models  $I$  that contain the true model, and each such model gives a  $\sqrt{n}$  consistent estimator  $\hat{\beta}_{I,0}$  of  $\beta$ , the probability that  $I_{min}$  picks one of these models goes to one as  $n \rightarrow \infty$ . Hence  $\hat{\beta}_{I_{min},0}$  is a  $\sqrt{n}$  consistent estimator of  $\beta$  under model (2.1). Olive (2017b: § 5.3.4, 2017c § 3.4.1) showed that  $\hat{\beta}_{I_{min},0}$  is a consistent estimator.

### 2.1.3 EBIC

The EBIC criterion given in Luo and Chen (2012) may work when  $n/p$  is not large.

Let  $0 \leq \gamma \leq 1$  and  $|I| = a \leq \min(n, p)$  if  $\hat{\beta}_I$  is  $a \times 1$ . We may use  $a \leq \min(n/5, p)$ . Then

$$EBIC(I) = n \log \left( \frac{SSE(I)}{n} \right) + a \log(n) + 2\gamma \log \left[ \binom{p}{a} \right] = BIC(I) + 2\gamma \log \left[ \binom{p}{a} \right].$$

This criterion can give good results if  $p = p_n = O(n^k)$  and  $\gamma > 1 - 1/(2k)$ . Hence we will use  $\gamma = 1$ .

The above criteria can be applied to forward selection and relaxed lasso. The  $C_p$  criterion can also be applied to lasso. See, for example, Efron and Hastie (2016, pp. 221, 231).

## 2.2 R FUNCTIONS FOR VARIABLE SELECTION

Many methods for variable selection have been suggested. We will consider several *R* functions including i) forward selection with the minimum  $C_p$  criterion as computed with `regsubsets` function from the `leaps` library. The remaining methods often use 10 fold cross validation (CV) and include ii) principal components regression (PCR) with the `pcr` function from the `pls` library, iii) partial least squares (PLS) with the `plsr` function from the `pls` library, iv) ridge regression with the `cv.glmnet` function from the `glmnet` library, and v) lasso with the `cv.glmnet` function from the `glmnet` library.

### 2.3 OLS SUB MODEL THEOREM

**Theorem 2.1.**

Suppose the usual linear model  $\mathbf{Y} = \mathbf{X}\beta + \mathbf{e}$ , with  $E\mathbf{Y} = \mathbf{X}\beta$  and  $E(\mathbf{e}) = \mathbf{0}$ .

Then  $Cov(\mathbf{Y}) = Cov(\mathbf{e}) = \sigma^2 \mathbf{I}$ .

If we break down  $\mathbf{X}$  and  $\beta$  as follows;

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_I & \mathbf{X}_0 \end{bmatrix}, \beta = \begin{bmatrix} \beta_I \\ \beta_0 \end{bmatrix},$$

$$\mathbf{X}\beta = \mathbf{X}_I\beta_I + \mathbf{X}_0\beta_0$$

where  $\beta_I = [\mathbf{X}_I^T \mathbf{X}_I]^{-1} \mathbf{X}_I^T \mathbf{Y} = \mathbf{A}\mathbf{Y}$ , then

$$E(\hat{\beta}_I) = \beta_I + [\mathbf{X}_I^T \mathbf{X}_I]^{-1} \mathbf{X}_I^T \mathbf{X}_0 \beta_0 \text{ and}$$

$$Cov(\hat{\beta}_I) = \sigma^2 (\mathbf{X}_I^T \mathbf{X}_I)^{-1}.$$

*Proof.* Assume this is an arbitrary submodel. When the submodel contains the set of predictors  $\mathbf{S}$  then  $\hat{\beta}_I$  works well and estimates  $\beta_I$ , but if  $I$  does not contain  $S$  then

$$\begin{aligned} E(\hat{\beta}_I) &= E([\mathbf{X}_I^T \mathbf{X}_I]^{-1} \mathbf{X}_I^T \mathbf{Y}) \\ &= E(\mathbf{A}\mathbf{Y}) = \mathbf{A}E(\mathbf{Y}) = \mathbf{A}\mathbf{X}\beta \\ &= \mathbf{A}(\mathbf{X}_I\beta_I + \mathbf{X}_0\beta_0) = [\mathbf{X}_I^T \mathbf{X}_I]^{-1} \mathbf{X}_I^T (\mathbf{X}_I\beta_I + \mathbf{X}_0\beta_0) = [\mathbf{X}_I^T \mathbf{X}_I]^{-1} \mathbf{X}_I^T \mathbf{X}_I \beta_I + \\ &\quad [\mathbf{X}_I^T \mathbf{X}_I]^{-1} \mathbf{X}_I^T \mathbf{X}_0 \beta_0 = \beta_I + [\mathbf{X}_I^T \mathbf{X}_I]^{-1} \mathbf{X}_I^T \mathbf{X}_0 \beta_0. \end{aligned}$$

When  $\beta_0 = 0$  then  $E(\hat{\beta}_I)$  is equal to  $\hat{\beta}_I$ .

Now consider the  $Cov(\hat{\beta}_I)$ :

$$\begin{aligned} Cov(\hat{\beta}_I) &= Cov(\mathbf{A}\mathbf{Y}) = \mathbf{A}Cov(\mathbf{Y})\mathbf{A}^T = \mathbf{A}\sigma^2 \mathbf{I}\mathbf{A}^T = \sigma^2 \mathbf{A}\mathbf{A}^T = \\ &\sigma^2 ([\mathbf{X}_I^T \mathbf{X}_I]^{-1} \mathbf{X}_I^T) ([\mathbf{X}_I^T \mathbf{X}_I]^{-1} \mathbf{X}_I^T)^T = \sigma^2 [\mathbf{X}_I^T \mathbf{X}_I]^{-1} \mathbf{X}_I^T \mathbf{X}_I ([\mathbf{X}_I^T \mathbf{X}_I]^{-1})^T = \\ &\sigma^2 ([\mathbf{X}_I^T \mathbf{X}_I]^T)^{-1} = \sigma^2 (\mathbf{X}_I^T \mathbf{X}_I)^{-1}. \end{aligned}$$

□

The above results shows why OLS does not work well if the submodel does not contains enough predictors.

## CHAPTER 3

### PREDICTION INTERVALS

Consider predicting a future test response variable  $Y_f$  given a  $p \times 1$  vector of predictors  $\mathbf{x}_f$  and training data  $(\mathbf{x}_1, Y_1), \dots, (\mathbf{x}_n, Y_n)$ . A large sample  $100(1 - \delta)\%$  prediction interval (PI) has the form  $[\hat{L}_n, \hat{U}_n]$  where  $P[\hat{L}_n \leq Y_f \leq \hat{U}_n] \rightarrow 1 - \delta$  as the sample size  $n \rightarrow \infty$ .

#### 3.1 SHORTH PI

The shorth( $c$ ) estimator is useful for making prediction intervals. Let  $Z_{(1)}, \dots, Z_{(n)}$  be the order statistics of  $Z_1, \dots, Z_n$ . Then let the shortest closed interval containing at least  $c$  of the  $Z_i$  be

$$\text{shorth}(c) = [Z_{(s)}, Z_{(s+c-1)}]. \quad (3.1)$$

Let

$$k_n = \lceil n(1 - \delta) \rceil \quad (3.2)$$

where  $\lceil x \rceil$  is the smallest integer  $\geq x$ , e.g.,  $\lceil 7.7 \rceil = 8$ . Frey (2013) showed that for large  $n\delta$  and iid data, the shorth( $k_n$ ) PI has maximum undercoverage  $\approx 1.12\sqrt{\delta/n}$ , and used the shorth( $c$ ) estimator as the large sample  $100(1 - \delta)\%$  PI where

$$c = \min(n, \lceil n[1 - \delta + 1.12\sqrt{\delta/n}] \rceil). \quad (3.3)$$

A problem with the prediction intervals that cover  $\approx 100(1 - \delta)\%$  of the training data cases  $Y_i$  (such as (3.1) using  $c = k_n$  given by (3.2)), is that they have coverage lower than the nominal coverage of  $1 - \delta$  for moderate  $n$ . This result is not surprising since empirically statistical methods perform worse on test data. Increasing  $c$  will improve the coverage for

moderate samples. Let  $df$  be the model degrees of freedom. Then empirically for many models, for  $n \approx 20df$ , prediction intervals such as (3.1) applied to iid data or pseudodata using  $c = k_n$  tend to have undercoverage as high as 5%. The undercoverage decreases rapidly as  $n$  increases. Let  $q_n = \min(1 - \delta + 0.05, 1 - \delta + p/n)$  for  $\delta > 0.1$  and

$$q_n = \min(1 - \delta/2, 1 - \delta + 10\delta p/n), \text{ otherwise.} \quad (3.4)$$

If  $1 - \delta < 0.999$  and  $q_n < 1 - \delta + 0.001$ , set  $q_n = 1 - \delta$ . Using

$$c = \lceil nq_n \rceil \quad (3.5)$$

decreased the undercoverage. For  $p = 1$  and  $n \geq 20$ , the correction factors  $c/n$  for  $c$  given by (3.3) and (3.5) do not differ by much more than 3% for  $0.01 \leq \delta \leq 0.5$ .

### 3.2 OLIVE (2013) PI

Olive (2013) developed prediction intervals for models of the form  $Y_i = m(\mathbf{x}_i) + e_i$ , and variable selection models for (1.1) have this form, as noted by Olive (2017b). Let  $c$  be given by (3.5), and let

$$b_n = \left(1 + \frac{15}{n}\right) \sqrt{\frac{n+2p}{n-p}}. \quad (3.6)$$

Compute the shorth( $c$ ) of the residuals  $= (r_{(d)}, r_{(d+c-1)}) = (\tilde{\xi}_{\delta_1}, \tilde{\xi}_{1-\delta_2})$ . Then a 100  $(1 - \delta)\%$  large sample PI for  $Y_f$  is

$$[\hat{m}(\mathbf{x}_f) + b_n \tilde{\xi}_{\delta_1}, \hat{m}(\mathbf{x}_f) + b_n \tilde{\xi}_{1-\delta_2}], \quad (3.7)$$

### 3.3 TWO NEW PREDICTION INTERVALS

#### 3.3.1 The First new prediction Interval

Results from Hastie, Tibshirani, and Wainwright (2015, pp. 20, 296, ch. 6, ch. 11) suggest that lasso can perform well for sparse models: the subset  $S$  in (2.2) contains  $k_S = a_S$  predictors where  $a_S/n \rightarrow 0$  as  $n \rightarrow \infty$ . Let  $d$  be a crude estimate of the model degrees of freedom. With the exception of ridge regression,  $d$  is the number of “variables” used by the method. Forward selection, lasso, and relaxed lasso use variables  $x_1^*, \dots, x_d^*$  while PCR and PLS use variables that are linear combinations of the predictors  $V_j = \boldsymbol{\gamma}_j^T \mathbf{x}$  for  $j = 1, \dots, d$ . See Efron and Hastie (2016, pp. 221, 222, 231) and Tibshirani (2015) for lasso degrees of freedom.

For  $n/p$  large, Olive (2013) developed prediction intervals for models of the form  $Y_i = m(\mathbf{x}_i) + e_i$ , and variable selection models for (1.1) have this form, as noted by Olive (2017b). The first new PI, that can be useful even if  $n/p$  is not large, is defined below. The PI is similar to the Olive (2013) PI with  $p$  replaced by  $d$ , if  $d$  is not too large.

Let  $q_n = \min(1 - \delta + 0.05, 1 - \delta + d/n)$  for  $\delta > 0.1$  and

$$q_n = \min(1 - \delta/2, 1 - \delta + 10\delta d/n), \quad \text{otherwise.} \quad (3.8)$$

If  $1 - \delta < 0.999$  and  $q_n < 1 - \delta + 0.001$ , set  $q_n = 1 - \delta$ . Let

$$c = \lceil nq_n \rceil, \quad (3.9)$$

and let

$$b_n = \left(1 + \frac{15}{n}\right) \sqrt{\frac{n+2d}{n-d}} \quad (3.10)$$

if  $d \leq 8n/9$ , and

$$b_n = 5 \left( 1 + \frac{15}{n} \right),$$

otherwise. Compute the shorth( $c$ ) of the residuals  $= [r_{(s)}, r_{(s+c-1)}] = [\tilde{\xi}_{\delta_1}, \tilde{\xi}_{1-\delta_2}]$ . Then the first new  $100(1 - \delta)\%$  large sample PI for  $Y_f$  is

$$[\hat{m}(\mathbf{x}_f) + b_n \tilde{\xi}_{\delta_1}, \hat{m}(\mathbf{x}_f) + b_n \tilde{\xi}_{1-\delta_2}]. \quad (3.11)$$

### 3.3.2 The Second new prediction Interval (Validation PI)

The second new PI randomly divides the data into two half sets  $H$  and  $V$  where  $H$  has  $n_H = \lceil n/2 \rceil$  of the cases and  $V$  has the remaining  $n_V = n - n_H$  cases  $i_1, \dots, i_{n_V}$ . The estimator  $\hat{m}_H(\mathbf{x})$  is computed using the training data set  $H$ . Then the validation residuals  $v_j = Y_{i_j} - \hat{m}_H(\mathbf{x}_{i_j})$  are computed for  $j = 1, \dots, n_V$  cases in the validation set  $V$ . Find the Frey PI  $[v_{(s)}, v_{(s+c-1)}]$  of the validation residuals (replacing  $n$  in (3.3) by  $n_V = n - n_H$ ). Then second new  $100(1 - \delta)\%$  large sample PI for  $Y_f$  is

$$[\hat{m}_H(\mathbf{x}_f) + v_{(s)}, \hat{m}_H(\mathbf{x}_f) + v_{(s+c-1)}]. \quad (3.12)$$

The PIs (3.11) and (3.12) are asymptotically equivalent if  $p$  is fixed and  $n \rightarrow \infty$ , but  $\hat{m}_H$  has about half the efficiency of  $\hat{m}$ . When PI (3.11) has severe undercoverage because  $\hat{m}$  is a poor estimator of  $m$ , it is expected that PI (3.12) may have coverage closer to the nominal coverage. For example, if  $\hat{m}$  interpolates the data and  $\hat{m}_H$  interpolates the training data from  $H$ , then the validation residuals will be huge. Hence PI (3.12) will be long compared to PI (3.11).

We can also motivate PI (3.12) by modifying the justification for the Lei, G'Sell, Rinaldo, Tibshirani, and Wasserman (2016) split conformal prediction interval  $[\hat{m}_H(\mathbf{x}_f) -$

$a_q, \hat{m}_H(\mathbf{x}_f) + a_q]$  where  $a_q$  is an appropriate quantile of the absolute validation residuals.

Suppose  $(Y_i, \mathbf{x}_i)$  are iid for  $i = 1, \dots, n, n+1$  where  $(Y_f, \mathbf{x}_f) = (Y_{n+1}, \mathbf{x}_{n+1})$ . Compute  $\hat{m}_H(\mathbf{x})$  from the cases in  $H$ . For example, get  $\hat{\beta}_H$  from the cases in  $H$ . Consider the validation residuals  $v_i$  for  $i = 1, \dots, n_V$  and the validation residual  $v_{n_V+1}$  for case  $(Y_f, \mathbf{x}_f)$ . Since these  $n_V + 1$  cases are iid, the probability that  $v_t$  has rank  $j$  for  $j = 1, \dots, n_V + 1$  is  $1/(n_V + 1)$  for each  $t$ , i.e., the ranks follow the discrete uniform distribution. Let  $t = n_V + 1$  and let  $v_{(i)}$  be the ordered residuals using  $i = 1, \dots, n_V$ . That is, get the order statistics without using the unknown validation residual  $v_{n_V+1}$ . Then  $v_{(i)}$  has rank  $i$  if  $v_{(i)} < v_{n_V+1}$  but rank  $i + 1$  if  $v_{(i)} > v_{n_V+1}$ . Thus

$$P(Y_f \in [\hat{m}_H(\mathbf{x}_f) + v_{(k)}, \hat{m}_H(\mathbf{x}_f) + v_{(k+b-1)}]) = P(v_{(k)} \leq v_{n_V+1} \leq v_{(k+b-1)}) \geq$$

$P(v_{n_V+1} \text{ has rank between } k+1 \text{ and } k+b-1 \text{ and there are no tied ranks}) \geq (b-1)/(n_V + 1) \approx 1 - \delta$  if  $b = \lceil (n_V + 1)(1 - \delta) \rceil + 1$  and  $k+b-1 \leq n_V$ . This probability statement holds for a fixed  $k$  such as  $k = \lceil n_V \delta/2 \rceil$ . The statement is not true when the shorth( $b$ ) estimator is used since the shortest interval using  $k = s$  can have  $s$  change with the data set. That is,  $s$  is not fixed. Hence if PI's were made from  $J$  independent data sets, the PI's with fixed  $k$  would contain  $Y_f$  about  $J(1 - \delta)$  times, but this value would be smaller for the shorth( $b$ ) prediction intervals where  $s$  can change with the data set.

The above argument works if the estimator  $\hat{m}(\mathbf{x})$  is “symmetric in the data.” The assumption of iid cases is stronger than that of iid errors  $e_i$ . The split conformal PI can have good coverage, but PI (3.12) does not need the error distribution to be symmetric to be asymptotically optimal.

The PIs (3.11) and (3.12) can be used with  $\hat{m}(\mathbf{x}) = \hat{Y}_f = \mathbf{x}_{I_d}^T \hat{\beta}_{I_d}$  where  $I_d$  denotes the

index of predictors selected from the model or variable selection method. The PIs (3.11) and (3.12) need the shorth of the residuals to be a consistent estimator of the population shorth of the error distribution. Olive and Hawkins (2003) show that if the  $\|\mathbf{x}_i\|$  are bounded and  $\hat{\boldsymbol{\beta}}$  is a consistent estimator of  $\boldsymbol{\beta}$ , then  $\max_{i=1,\dots,n} |r_i - e_i| \xrightarrow{P} 0$  and the sample quantiles of the residuals estimate the population quantiles of the error distribution. For OLS with fixed  $p$ , each submodel  $I$  produces a  $\sqrt{n}$  consistent estimator provided that  $S \subseteq I$ .

The Cauchy Schwartz inequality says  $|\mathbf{a}^T \mathbf{b}| \leq \|\mathbf{a}\| \|\mathbf{b}\|$ . Suppose  $\sqrt{n}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}) = O_P(1)$  is bounded in probability. This will occur if  $\sqrt{n}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}) \xrightarrow{D} N_p(\mathbf{0}, \boldsymbol{\Sigma})$ , e.g. if  $\hat{\boldsymbol{\beta}}$  is the OLS estimator. Then

$$|r_i - e_i| = |Y_i - \mathbf{x}_i^T \hat{\boldsymbol{\beta}} - (Y_i - \mathbf{x}_i^T \boldsymbol{\beta})| = |\mathbf{x}_i^T (\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})|.$$

Hence

$$\sqrt{n} \max_{i=1,\dots,n} |r_i - e_i| \leq (\max_{i=1,\dots,n} \|\mathbf{x}_i\|) \|\sqrt{n}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})\| = O_P(1)$$

since  $\max \|\mathbf{x}_i\| = O_P(1)$  or there is extrapolation. Hence OLS residuals behave well if the zero mean error distribution of the iid  $e_i$  has a finite variance  $\sigma^2$ .

Note that correction factors  $b_n \rightarrow 1$  are used in large sample confidence intervals and tests if the limiting distribution is  $N(0,1)$  or  $\chi_p^2$ , but a  $t_{d_n}$  or  $pF_{p,d_n}$  cutoff is used:  $t_{d_n,1-\delta}/z_{1-\delta} \rightarrow 1$  and  $pF_{p,d_n,1-\delta}/\chi_{p,1-\delta}^2 \rightarrow 1$  if  $d_n \rightarrow \infty$  as  $n \rightarrow \infty$ . Using correction factors for prediction intervals and bootstrap confidence regions improves the performance for moderate sample size  $n$ .

### 3.3.3 PI after model selection

The PI (3.7) was used for the variable selection estimators.

Heuristically, the other methods consider a small number of models including the full OLS model, and models that beat the full OLS model for 10 fold CV likely fit the data well when  $p$  is fixed and  $n \rightarrow \infty$ . Hence under regularity conditions, the PI (3.7) is likely to perform well for the other methods.

As shown in simulations, lasso and ridge regression tended to have prediction intervals that were too long when  $p \geq 20$ ,  $a = p$  or  $p - 1$ , and the predictor variables were correlated (with  $\psi \geq 0.5$ ). As a possible remedy, consider 10 fold CV where the data set has been randomly divided into 10 groups of approximately equal size. For  $j = 1, \dots, 10$ , compute the estimator when the  $j$ th group is left out and compute the PIs for the  $Y_f = Y_i$  in the left out group  $j$ . After obtaining the  $n$  PIs, one for each  $Y_i$ , compute the proportion of times  $Y_i$  was in its PI and the average length of the PIs. Consider the  $\lambda_i$  where the proportion  $\geq 1 - \delta$ , and use  $\lambda_a$  that had the shortest average PI length as the  $\lambda$  for the variable selection estimator. This technique changes the CV criterion to average PI length, given that the observed coverage was at least as large as the nominal coverage.

## CHAPTER 4

### BOOTSTRAPPING HYPOTHESIS TESTS

We also want to use bootstrap tests. Consider testing  $H_0 : \boldsymbol{\mu} = \mathbf{c}$  versus  $H_1 : \boldsymbol{\mu} \neq \mathbf{c}$  where  $\mathbf{c}$  is a known  $r \times 1$  vector. Given training data  $\mathbf{z}_1, \dots, \mathbf{z}_n$ , a large sample  $100(1 - \delta)\%$  confidence region for  $\boldsymbol{\mu}$  is a set  $\mathcal{A}_n$  such that  $P(\boldsymbol{\mu} \in \mathcal{A}_n) \rightarrow 1 - \delta$  as  $n \rightarrow \infty$ . Then reject  $H_0$  if  $\mathbf{c}$  is not in the confidence region  $\mathcal{A}_n$ . For example, let  $\boldsymbol{\mu} = \mathbf{A}\boldsymbol{\beta}$  where  $\boldsymbol{\beta}$  is a  $p \times 1$  vector of parameters, and  $\mathbf{A}$  is a known full rank  $r \times p$  matrix with  $1 \leq r \leq p$ .

To bootstrap a confidence region, Mahalanobis distances and prediction regions will be useful. Consider predicting a future test value  $\mathbf{z}_f$ , given past training data  $\mathbf{z}_1, \dots, \mathbf{z}_n$  where the  $\mathbf{z}_i$  are  $r \times 1$  random vectors. A *large sample*  $100(1 - \delta)\%$  *prediction region* is a set  $\mathcal{A}_n$  such that  $P(\mathbf{z}_f \in \mathcal{A}_n) \rightarrow 1 - \delta$  as  $n \rightarrow \infty$ . Let the  $r \times 1$  column vector  $T$  be a multivariate location estimator, and let the  $r \times r$  symmetric positive definite matrix  $\mathbf{C}$  be a dispersion estimator. Then the *i*th *squared sample Mahalanobis distance* is the scalar

$$D_i^2 = D_i^2(T, \mathbf{C}) = D_{\mathbf{z}_i}^2(T, \mathbf{C}) = (\mathbf{z}_i - T)^T \mathbf{C}^{-1} (\mathbf{z}_i - T) \quad (4.1)$$

for each observation  $\mathbf{z}_i$ . Notice that the Euclidean distance of  $\mathbf{z}_i$  from the estimate of center  $T$  is  $D_i(T, \mathbf{I}_r)$  where  $\mathbf{I}_r$  is the  $r \times r$  identity matrix. The classical Mahalanobis distance  $D_i$  uses  $(T, \mathbf{C}) = (\bar{\mathbf{z}}, \mathbf{S})$ , the sample mean and sample covariance matrix where

$$\bar{\mathbf{z}} = \frac{1}{n} \sum_{i=1}^n \mathbf{z}_i \quad \text{and} \quad \mathbf{S} = \frac{1}{n-1} \sum_{i=1}^n (\mathbf{z}_i - \bar{\mathbf{z}})(\mathbf{z}_i - \bar{\mathbf{z}})^T. \quad (4.2)$$

Let  $q_n$  and  $c$  be given by (3.4) and (3.5) with  $p$  replaced by  $r$ . Let  $(T, \mathbf{C}) = (\bar{\mathbf{z}}, \mathbf{S})$ , and let  $D_{(U_n)}$  be the  $100q_n$ th sample quantile of the  $D_i$ . Then the Olive (2013) large sample

$100(1 - \delta)\%$  nonparametric prediction region for a future value  $\mathbf{z}_f$  given iid data  $\mathbf{z}_1, \dots, \mathbf{z}_n$

is

$$\{\mathbf{z} : D_{\mathbf{z}}^2(\bar{\mathbf{z}}, \mathbf{S}) \leq D_{(U_n)}^2\}, \quad (4.3)$$

while the classical large sample  $100(1 - \delta)\%$  prediction region is

$$\{\mathbf{z} : D_{\mathbf{z}}^2(\bar{\mathbf{z}}, \mathbf{S}) \leq \chi_{r,1-\delta}^2\}. \quad (4.4)$$

The following theorem is proved in Olive (2017cde) and shows that the hyperellipsoid  $R_c$  centered at the statistic  $T_n$  is a large sample  $100(1 - \delta)\%$  confidence region for  $\boldsymbol{\mu}$ , but the hyperellipsoid centered at known  $\boldsymbol{\mu}$  is a large sample  $100(1 - \delta)\%$  prediction region for a future value of the statistic  $T_{f,n}$ .

**Theorem 4.1.** *Let the  $100(1 - \delta)$ th percentile  $D_{1-\delta}^2$  be a continuity point of the distribution of  $D^2$ . Assume that  $D_{\boldsymbol{\mu}}^2(T_n, \boldsymbol{\Sigma}_T) \xrightarrow{D} D^2$ ,  $D_{\boldsymbol{\mu}}^2(T_n, \hat{\boldsymbol{\Sigma}}_T) \xrightarrow{D} D^2$ , and  $\hat{D}_{1-\delta}^2 \xrightarrow{P} D_{1-\delta}^2$  where  $P(D^2 \leq D_{1-\delta}^2) = 1 - \delta$ . i) Then  $R_c = \{\mathbf{w} : D_{\mathbf{w}}^2(T_n, \hat{\boldsymbol{\Sigma}}_T) \leq \hat{D}_{1-\delta}^2\}$  is a large sample  $100(1 - \delta)\%$  confidence region for  $\boldsymbol{\mu}$ , and if  $\boldsymbol{\mu}$  is known, then  $R_p = \{\mathbf{w} : D_{\mathbf{w}}^2(\boldsymbol{\mu}, \hat{\boldsymbol{\Sigma}}_T) \leq \hat{D}_{1-\delta}^2\}$  is a large sample  $100(1 - \delta)\%$  prediction region for a future value of the statistic  $T_{f,n}$ . ii) Region  $R_c$  contains  $\boldsymbol{\mu}$  iff region  $R_p$  contains  $T_n$ .*

Hence if there was an iid sample  $T_{1,n}, \dots, T_{B,n}$  of the statistic, the prediction region (4.3) for  $T_{f,n}$  contains  $E(T_n) = \boldsymbol{\mu}$  with asymptotic coverage  $\geq 1 - \delta$ . Often the  $n$  is suppressed. To make the asymptotic coverage equal to  $1 - \delta$ , use the large sample  $100(1 - \delta)\%$  confidence region  $\{\mathbf{w} : D_{\mathbf{w}}^2(T_{1,n}, \mathbf{S}_T) \leq D_{(U_B)}^2\}$ . The prediction region method bootstraps this procedure by using a bootstrap sample of the statistic  $T_{1,n}^*, \dots, T_{B,n}^*$ . Let  $\bar{T}^*$  and  $\mathbf{S}_T^*$  be the sample mean and sample covariance matrix of  $T_{1,n}^*, \dots, T_{B,n}^*$ . Centering the region at  $T_{1,n}^*$  instead of  $\bar{T}^*$  is not needed since the bootstrap sample is centered near  $T_n$ : the

distribution of  $T_n - \boldsymbol{\mu}$  is approximated by the distribution of  $T^* - T_n$  or by the distribution of  $T^* - \bar{T}^*$ .

#### 4.1 PREDICTION REGION METHOD

The prediction region method is simple. Let  $\hat{\boldsymbol{\mu}}$  be a consistent estimator of  $\boldsymbol{\mu}$  and make a bootstrap sample  $\mathbf{w}_i = \hat{\boldsymbol{\mu}}_i^* - \mathbf{c}$  for  $i = 1, \dots, B$ . Using the nonparametric prediction region (4.3) for the  $\mathbf{w}_i$  as a large sample  $100(1 - \delta)\%$  confidence region, fail to reject  $H_0$  if  $\mathbf{0}$  is in the confidence region (if  $D_{\mathbf{0}} \leq D_{(U_B)}$ ), and reject  $H_0$  otherwise. The method tends to work well in simulations if  $\sqrt{n}(T_n - \boldsymbol{\mu}) \xrightarrow{D} \mathbf{z}$  where the random vector  $\mathbf{z}$  has a nonsingular covariance matrix.

Following Bickel and Ren (2001), let the vector of parameters  $\boldsymbol{\mu} = T(F)$ , the statistic  $T_n = T(F_n)$ , and  $T^* = T(F_n^*)$  where  $F$  is the cdf of iid  $\mathbf{x}_1, \dots, \mathbf{x}_n$ ,  $F_n$  is the empirical cdf, and  $F_n^*$  is the empirical cdf of  $\mathbf{x}_1^*, \dots, \mathbf{x}_n^*$ , a sample from  $F_n$  using the nonparametric bootstrap. If  $\sqrt{n}(F_n - F) \xrightarrow{D} \mathbf{z}_F$ , a Gaussian random process, and if  $T$  is sufficiently smooth (Hadamard differentiable with a Hadamard derivative  $\dot{T}(F)$ ), then  $\sqrt{n}(T_n - \boldsymbol{\mu}) \xrightarrow{D} \mathbf{X}$  and  $\sqrt{n}(T_i^* - \bar{T}^*) \xrightarrow{D} \mathbf{X}$  with  $\mathbf{X} = \dot{T}(F)\mathbf{z}_F$ . Olive (2017be) used these results to show that if  $\mathbf{X} \sim N_r(\mathbf{0}, \Sigma_T)$ , then  $\sqrt{n}(\bar{T}^* - T_n) \xrightarrow{D} \mathbf{0}$ ,  $\sqrt{n}(\bar{T}^* - \boldsymbol{\mu}) \xrightarrow{D} \mathbf{X}$ , and that the prediction region method large sample  $100(1 - \delta)\%$  confidence region for  $\boldsymbol{\mu}$  is

$$\{\mathbf{w} : (\mathbf{w} - \bar{T}^*)^T [\mathbf{S}_T^*]^{-1} (\mathbf{w} - \bar{T}^*) \leq D_{(U_B)}^2\} = \{\mathbf{w} : D_{\mathbf{w}}^2(\bar{T}^*, \mathbf{S}_T^*) \leq D_{(U_B)}^2\} \quad (4.5)$$

where  $D_{(U_B)}^2$  is computed from  $D_i^2 = (T_i^* - \bar{T}^*)^T [\mathbf{S}_T^*]^{-1} (T_i^* - \bar{T}^*)$  for  $i = 1, \dots, B$ . Note that the corresponding test for  $H_0 : \boldsymbol{\mu} = \boldsymbol{\mu}_0$  rejects  $H_0$  if  $(\bar{T}^* - \boldsymbol{\mu}_0)^T [\mathbf{S}_T^*]^{-1} (\bar{T}^* - \boldsymbol{\mu}_0) > D_{(U_B)}^2$ .

The prediction region method for testing  $H_0 : \boldsymbol{\mu} = \mathbf{c}$  versus  $H_1 : \boldsymbol{\mu} \neq \mathbf{c}$  is simple. Let

$\hat{\mu}$  be a consistent estimator of  $\mu$  and make a bootstrap sample  $\mathbf{w}_i = \hat{\mu}_i^* - \mathbf{c}$  for  $i = 1, \dots, B$ .

Make the nonparametric prediction region (4.3) for the  $\mathbf{w}_i$  and fail to reject  $H_0$  if  $\mathbf{0}$  is in the prediction region (if  $D_{\mathbf{0}} \leq D_{(U_B)}$ ), reject  $H_0$  otherwise.

The Bickel and Ren (2001) hypothesis testing method is equivalent to using confidence region (4.3) with  $\bar{T}^*$  replaced by  $T_n$  and  $U_B$  replaced by  $k_B = \lceil B(1 - \delta) \rceil$ . If region (4.3) or the Bickel and Ren (2001) region is a large sample  $100(1 - \delta)\%$  confidence region, then so is the other region if  $\sqrt{n}(\bar{T}^* - T_n) \xrightarrow{D} \mathbf{0}$ . Hadamard differentiability and asymptotic normality are sufficient conditions for both regions to be large sample confidence regions if  $S_T^* \xrightarrow{P} \Sigma_T$ , but Bickel and Ren (2001) showed that their method can work when Hadamard differentiability fails. For  $r = 1$ , the percentile method uses an interval that contains  $U_B \approx k_B = \lceil B(1 - \delta) \rceil$  of the  $T_{i,n}^*$  from a bootstrap sample  $T_{1,n}^*, \dots, T_{B,n}^*$  where the statistic  $T_n$  is an estimator of  $\mu$  based on a sample of size  $n$ . Note that the squared Mahalanobis distance  $D_\mu^2 = (\mu - \bar{T}^*)^2 / S_T^{2*} \leq D_{(U_B)}^2$  is equivalent to  $\mu \in [\bar{T}^* - S_T^* D_{(U_B)}, \bar{T}^* + S_T^* D_{(U_B)}]$ , which is an interval centered at  $\bar{T}^*$  just long enough to cover  $U_B$  of the  $T_i^*$ . Hence the prediction region method is a special case of the percentile method if  $r = 1$ . Efron (2014) uses a similar large sample  $100(1 - \delta)\%$  confidence interval assuming that  $T_n$  is asymptotically normal. The Frey (2013) shorth( $c$ ) interval applied to the  $T_{i,n}^*$  is recommended since the shorth confidence interval can be much shorter than the Efron (2014) or prediction region method confidence intervals if  $r = 1$ . See Olive (2017e) for more information about the prediction region method.

Following Olive (2017ce), we describe the prediction region method for bootstrapping forward selection where 0s need to be added for omitted variables. Bootstrapping lasso and

ridge regression is similar, but lasso already has the 0s and ridge regression usually does not produce 0 slope estimates. Suppose  $n > 20p$ . If  $\hat{\beta}_I$  is  $(k \times 1) \times 1$ , form  $\hat{\beta}_{I,0}$  from  $\hat{\beta}_I$  by adding 0s corresponding to the omitted variables. Then  $\hat{\beta}_{I,0}$  is a nonlinear estimator of  $\beta$ , and the residual bootstrap method can be applied. Let  $\hat{\beta} = \hat{\beta}_{I_{min},0}$  be formed from the forward selection model  $I_{min}$  that minimizes the  $C_p$  criterion. Instead of computing the least squares estimator from regressing  $\mathbf{Y}_i^*$  on  $\mathbf{X}$ , perform variable selection on  $\mathbf{Y}_i^*$  and  $\mathbf{X}$ , fit the model that minimizes the criterion, and add 0s corresponding to the omitted variables, resulting in estimators  $\hat{\beta}_1^*, \dots, \hat{\beta}_B^*$ . Then test  $\mu = \mathbf{A}\beta = \mathbf{c}$  using the prediction region method for  $r > 1$  and the shorth if  $r = 1$ .

## 4.2 RESIDUAL BOOTSTRAP

For models of form (1.1) with  $n > 20p$ , the residual bootstrap makes sense: the residuals from the full model are sampled with replacement resulting in a bootstrap sample  $r_1^*, \dots, r_n^*$  and  $Y_i^* = \hat{Y}_i + r_i^*$  for  $i = 1, \dots, n$  are collected into a vector  $\mathbf{Y}_j^*$  which is regressed on  $\mathbf{X}$  to get  $\hat{\beta}_j^*$  for  $j = 1, \dots, n$ . The nonparametric bootstrap selects  $n$  cases with replacement to form  $(\mathbf{Y}_j^*, \mathbf{X}_j^*)$ , and regresses  $\mathbf{Y}_j^*$  on  $\mathbf{X}_j^*$  to form  $\hat{\beta}_j^*$ .

Note that if  $S \subseteq I$ , and  $\mathbf{Y} = \mathbf{X}_I\beta_I + \mathbf{e}_I$ , then  $\sqrt{n}(\hat{\beta}_I - \beta_I) \xrightarrow{D} N_{k+1}(\mathbf{0}, \sigma_I^2 \mathbf{W}_I)$  under mild regularity conditions where  $n(\mathbf{X}_I^T \mathbf{X}_I)^{-1} \rightarrow \mathbf{W}_I$ . Hence  $\sqrt{n}(\hat{\beta}_{I,0} - \beta_c) \xrightarrow{D} N_p(\mathbf{0}, \sigma_I^2 \mathbf{W}_{I,0})$  where the  $\mathbf{W}_{I,0}$  has a column and row of zeroes added for each variable not in  $I$ . Note that  $\mathbf{W}_{I,0}$  is singular unless  $I$  corresponds to the full model. For example, if

$p = 3$  and model  $I$  uses a constant and  $x_3$  with

$$\mathbf{W}_I = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix}, \text{ then } \mathbf{W}_{I,0} = \begin{bmatrix} W_{11} & 0 & W_{12} \\ 0 & 0 & 0 \\ W_{21} & 0 & W_{22} \end{bmatrix}.$$

Hence it is reasonable to conjecture that  $\sqrt{n}(\hat{\beta}_{I_{min},0} - \beta_c) \xrightarrow{D} \mathbf{X}$  where

$$\mathbf{X} = \sum_{i=1}^K \pi_i N_p(\mathbf{0}, \sigma_{I_i}^2 \mathbf{W}_{I_i,0}),$$

$0 \leq \pi_i \leq 1$ ,  $\sum_{i=1}^K \pi_i = 1$ , and  $K$  is the number of subsets  $I_i$  that contain  $S$ . Note that the limiting distribution is not an elliptically contoured distribution (unless the full model has  $\pi_k = 1$ ) since the probability of a 0 would be 0.

Prediction intervals and regions can have higher than the nominal coverage  $1 - \delta$  if the distribution is discrete or a mixture of a discrete distribution and some other distribution. In particular, coverage can be high if the  $\mathbf{w}_i$  distribution is a mixture of a point mass at  $\mathbf{0}$  and the method checks whether  $\mathbf{0}$  is in the prediction region. Such a mixture often occurs for forward selection methods and lasso. The bootstrap sample for the  $W_i = \hat{\beta}_{ij}^*$  can contain many zeroes and be highly skewed if the  $j$ th predictor is weak. Then the computer program may fail because  $\mathbf{S}\mathbf{w}$  is singular, but if all or nearly all of the  $\hat{\beta}_{ij}^* = 0$ , then there is strong evidence that the  $j$ th predictor is not needed given that the other predictors are in the variable selection method.

As an extreme simulation case, suppose  $\hat{\beta}_{ij}^* = 0$  for  $i = 1, \dots, B$  and for each run in the simulation. Consider testing  $H_0 : \beta_j = 0$ . Then regardless of the nominal coverage  $1 - \delta$ , the closed interval  $[0,0]$  will contain 0 for each run and the observed coverage will be  $1 > 1 - \delta$ . Using the open interval  $(0,0)$  would give observed coverage 0. Also intervals

$[0, b]$  and  $[a, 0]$  correctly suggest failing to reject  $\beta_j = 0$ , while intervals  $(0, b)$  and  $(a, 0)$  incorrectly suggest rejecting  $H_0 : \beta_j = 0$ . Hence closed regions and intervals make sense.

Following Seber and Lee (2003, p. 448) and Nishi (1984), the probability that model  $I_{min}$  from  $C_p$  or AIC underfits goes to zero as  $n \rightarrow \infty$ . Since there are a finite number of regression models  $I$  that contain the true model, and each model gives a consistent estimator  $\hat{\boldsymbol{\beta}}_{I,0}$  of  $\boldsymbol{\beta}$ , the probability that  $I_{min}$  picks one of these models goes to one as  $n \rightarrow \infty$ . Hence  $\hat{\boldsymbol{\beta}}_{I_{min},0}$  is a consistent estimator of  $\boldsymbol{\beta}$  under model (2.2).

Note that when performing the prediction region method for lasso and ridge regression, we use the residual bootstrap where the residuals are from the full OLS model. Efron (1982, p. 36) notes that for the OLS residual bootstrap for the full OLS model,  $E[\hat{\boldsymbol{\beta}}_j^*] = \hat{\boldsymbol{\beta}}_{OLS}$ , and the sample covariance matrix of the  $\hat{\boldsymbol{\beta}}_j^*$  is estimating the population bootstrap matrix  $\frac{n-p}{n} MSE(\mathbf{X}^T \mathbf{X})^{-1}$  as  $B \rightarrow \infty$ . Hence the residual bootstrap standard error  $SE(\hat{\beta}_i) \approx \sqrt{\frac{n-p}{n}} SE(\hat{\beta}_{i,OLS})$ . Here the expectations are with respect to the bootstrap distribution. Camponovo (2015) suggests that the nonparametric bootstrap does not work for lasso.

# CHAPTER 5

## EXAMPLES AND SIMULATIONS

### 5.1 EXAMPLE AND SIMULATIONS

#### 5.1.1 Example

The Hebbler (1847) data was collected from  $n = 26$  districts in Prussia in 1843. We will study the relationship between  $Y = \text{the number of women married to civilians}$  in the district with the predictors  $x_1 = \text{constant}$ ,  $x_2 = \text{pop} = \text{the population of the district in 1843}$ ,  $x_3 = \text{mmen} = \text{the number married civilian men in the district}$ ,  $x_4 = \text{mmilmen} = \text{number of married men in the military in the district}$ , and  $x_5 = \text{milwmn} = \text{the number of women married to husbands in the military in the district}$ . Sometimes the person conducting the survey would not count a spouse if the spouse was not at home. Hence  $Y$  is highly correlated but not equal to  $x_3$ . Similarly,  $x_4$  and  $x_5$  are highly correlated but not equal. We expect that  $Y = x_3 + e$  is a good model, but  $n/p = 5.2$  is small.

Forward selection selected the model with the minimum  $C_p$  while the other methods used 10-fold CV. PLS and PCR used the OLS full model with PI length 2395.74, forward selection used a constant and  $mmen$  with PI length 2114.72, ridge regression had PI length 20336.58, lasso and relaxed lasso used a constant,  $mmen$ , and  $pop$  with lasso PI length 8482.62 and relaxed lasso PI length 2226.53. Figure 1 shows the response plots for forward selection, ridge regression, lasso, and relaxed lasso. The plots for PLS=PCR=OLS full model were similar to those of forward selection and relaxed lasso. The plots suggest that the MLR model is appropriate since the plotted points scatter about the identity line. The

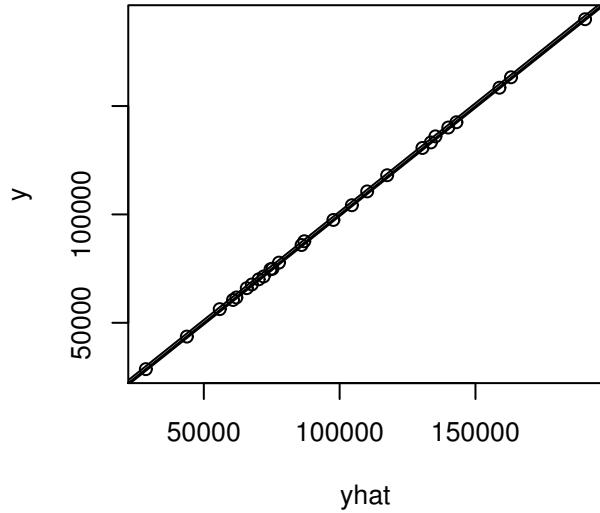
90% pointwise prediction bands are also shown, and consist of two lines parallel to the identity line. These bands are very narrow in Figure 1 a) and d).

### 5.1.2 Simulation

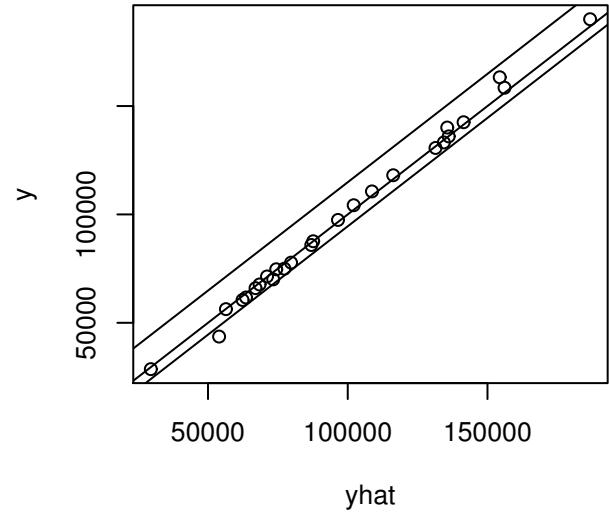
For the simulations, for  $u = 1, \dots, n$ , we generated  $\mathbf{z}_u \sim N_{p-1}(\mathbf{0}, \mathbf{I})$  where the  $m = p - 1$  elements of the vector  $\mathbf{z}_u$  are iid  $N(0,1)$ . Let the  $m \times m$  matrix  $\mathbf{A} = (a_{ij})$  with  $a_{ii} = 1$  and  $a_{ij} = \psi$  where  $0 \leq \psi < 1$  for  $i \neq j$ . Then the vector of predictors  $\mathbf{w}_u = \mathbf{A}\mathbf{z}_u$  so that  $Cov(\mathbf{w}) = \Sigma_{\mathbf{w}} = \mathbf{A}\mathbf{A}^T = (\sigma_{ij})$  where the diagonal entries  $\sigma_{ii} = [1 + (m - 1)\psi^2]$  and the off diagonal entries  $\sigma_{ij} = [2\psi + (m - 2)\psi^2]$ . Hence the correlations are  $cor(x_i, x_j) = (2\psi + (m - 2)\psi^2)/(1 + (m - 1)\psi^2)$  for  $i \neq j$  where  $x_i$  and  $x_j$  are nontrivial predictors. As  $\psi$  gets close to 1, the predictor vectors cluster about the line in the direction of  $(1, \dots, 1)^T$ . The simulation used  $\psi = 0, 0.5$ , and  $0.9$ . Then  $Y_i = 1 + 1w_{i,1} + \dots + 1w_{i,j} + e_i$  for  $i = 1, \dots, n$  with  $a = k + 1$  and  $k = 1, p - 2$ , or  $p - 1$ . Hence  $\beta = (1, \dots, 1, 0, \dots, 0)^T$  with  $a$  ones and  $p - a$  zeros. The zero mean errors  $e_i$  were iid of five types: i)  $N(0,1)$  errors, ii)  $t_3$  errors, iii)  $EXP(1) - 1$  errors, iv) uniform( $-1, 1$ ) errors, and v)  $0.9 N(0,1) + 0.1 N(0,100)$  errors.

The lengths of the asymptotically optimal 95% PIs are i)  $3.92 = 2(1.96)$ , ii)  $6.365$ , iii)  $2.996$ , iv)  $1.90 = 2(0.95)$ , and v)  $13.490$ . The simulation used 5000 runs, so an observed coverage in  $(0.94, 0.96)$  gives no reason to doubt that the PI has the nominal coverage of 0.95. The simulations use  $n = 10p, 20p$  and  $100p$ . Let  $a$  be the number of nonzero coefficients, including the constant, in  $\beta$ . The coverage was often high for  $n = 10p$  and  $20p$ , but close to the nominal coverage of 0.95 for  $n = 100p$ , where the average lengths were slightly longer than the asymptotically optimal lengths, except the lasso and ridge regression PIs were far too long when  $\psi \geq 0.5$  and  $a = p - 1$  or  $a = p$ . Tables 5.1 -5.60

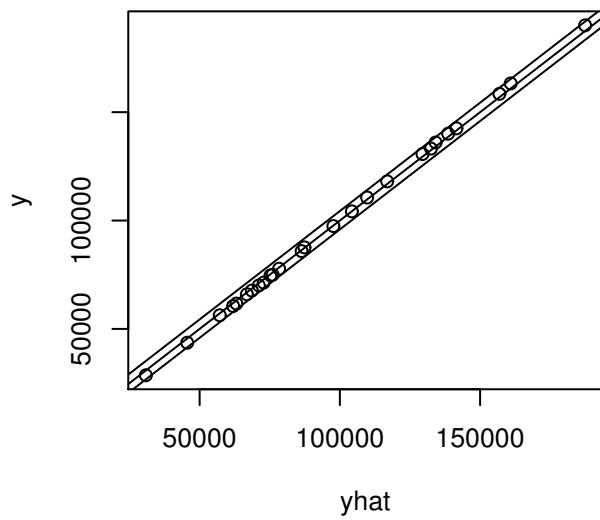
**a) Forward Selection**



**b) Ridge Regression**



**c) Lasso**



**d) Relaxed Lasso**

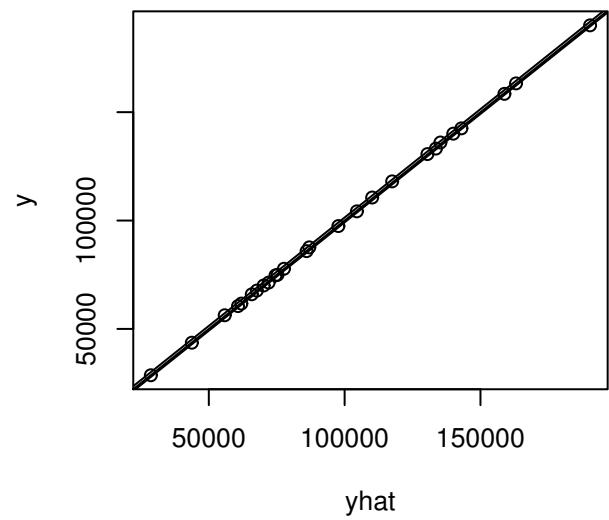


Figure 5.1. Marry Data Response Plots

used 10 fold CV except forward selection used  $C_p$  with PI (3.7). Tables 5.61 -5.300 used PI (3.11) with  $d = \min(\lceil n/J \rceil, p)$ .

### 5.1.3 Error type 1

Table 5.1.  $p = 5$ , error type = 1

|     |   |     | LASSO    |        |         | RIDGE    |        |         |
|-----|---|-----|----------|--------|---------|----------|--------|---------|
| n   | a | psi | coverage | length | penalty | coverage | length | penalty |
| 50  | 1 | 0   | 0.9882   | 5.7506 | 0.1095  | 0.9876   | 5.6775 | 0.1661  |
|     |   | 0.5 | 0.9880   | 5.7155 | 0.0535  | 0.9878   | 5.7767 | 0.1997  |
|     |   | 0.9 | 0.9892   | 5.7663 | 0.0200  | 0.9886   | 5.8401 | 0.2092  |
|     | 4 | 0   | 0.9866   | 5.6600 | 0.0318  | 0.9876   | 5.6825 | 0.1377  |
|     |   | 0.5 | 0.9868   | 5.6629 | 0.0342  | 0.9890   | 5.8135 | 0.4034  |
|     |   | 0.9 | 0.9888   | 5.7097 | 0.0408  | 0.9910   | 5.9038 | 0.6010  |
|     | 5 | 0   | 0.9858   | 5.6417 | 0.0114  | 0.9874   | 5.6908 | 0.1467  |
|     |   | 0.5 | 0.9862   | 5.6438 | 0.0357  | 0.9902   | 5.7452 | 0.5348  |
|     |   | 0.9 | 0.9876   | 5.6858 | 0.0532  | 0.9906   | 5.9346 | 0.8009  |
| 100 | 1 | 0   | 0.9844   | 5.0979 | 0.0778  | 0.9832   | 5.0656 | 0.1131  |
|     |   | 0.5 | 0.9850   | 5.0807 | 0.0376  | 0.9836   | 5.1385 | 0.1482  |
|     |   | 0.9 | 0.9858   | 5.1034 | 0.0155  | 0.9862   | 5.1485 | 0.2017  |
|     | 4 | 0   | 0.9820   | 5.0542 | 0.0216  | 0.9826   | 5.0766 | 0.1258  |
|     |   | 0.5 | 0.9830   | 5.0581 | 0.0281  | 0.9840   | 5.1798 | 0.3944  |
|     |   | 0.9 | 0.9834   | 5.0743 | 0.0397  | 0.9862   | 5.1988 | 0.6034  |
|     | 5 | 0   | 0.9818   | 5.0474 | 0.0083  | 0.9824   | 5.0837 | 0.1334  |
|     |   | 0.5 | 0.9822   | 5.0503 | 0.0342  | 0.9844   | 5.1157 | 0.5195  |
|     |   | 0.9 | 0.9822   | 5.0649 | 0.0529  | 0.9860   | 5.2260 | 0.8043  |
| 500 | 1 | 0   | 0.9510   | 4.0813 | 0.0344  | 0.9510   | 4.0857 | 0.1096  |
|     |   | 0.5 | 0.9514   | 4.0793 | 0.0169  | 0.9514   | 4.1343 | 0.1450  |
|     |   | 0.9 | 0.9506   | 4.0826 | 0.0134  | 0.9520   | 4.1050 | 0.2031  |
|     | 4 | 0   | 0.9484   | 4.0757 | 0.0113  | 0.9482   | 4.0935 | 0.1161  |
|     |   | 0.5 | 0.9484   | 4.0782 | 0.0260  | 0.9482   | 4.1643 | 0.3955  |
|     |   | 0.9 | 0.9490   | 4.0809 | 0.0400  | 0.9490   | 4.1447 | 0.6080  |
|     | 5 | 0   | 0.9488   | 4.0748 | 0.0074  | 0.9492   | 4.0956 | 0.1190  |
|     |   | 0.5 | 0.9488   | 4.0771 | 0.0343  | 0.9492   | 4.1163 | 0.5206  |
|     |   | 0.9 | 0.9486   | 4.0824 | 0.0533  | 0.9502   | 4.1631 | 0.8105  |

Table 5.2.  $p = 5$ , error type = 1

|     |   |     | PLS      |        |         | PCR      |        |         |
|-----|---|-----|----------|--------|---------|----------|--------|---------|
| n   | a | psi | coverage | length | penalty | coverage | length | penalty |
| 50  | 1 | 0   | 0.9854   | 5.6385 | 0.7488  | 0.9842   | 5.8770 | 0.3500  |
|     |   | 0.5 | 0.9860   | 5.6405 | 0.9196  | 0.9868   | 5.7827 | 0.7340  |
|     |   | 0.9 | 0.9854   | 5.6434 | 1.7406  | 0.9878   | 5.7349 | 1.5420  |
|     | 4 | 0   | 0.9852   | 5.6383 | 0.6070  | 0.9848   | 6.1056 | 0.2606  |
|     |   | 0.5 | 0.9856   | 5.6418 | 0.8962  | 0.9872   | 5.7881 | 0.7468  |
|     |   | 0.9 | 0.9854   | 5.6443 | 1.7418  | 0.9886   | 5.7355 | 1.5308  |
|     | 5 | 0   | 0.9854   | 5.6378 | 0.5732  | 0.9828   | 6.2429 | 0.2792  |
|     |   | 0.5 | 0.9856   | 5.6433 | 1.7860  | 0.9876   | 5.7311 | 1.5724  |
|     |   | 0.9 | 0.9852   | 5.6435 | 1.7828  | 0.9882   | 5.7296 | 1.5860  |
| 100 | 1 | 0   | 0.9818   | 5.0468 | 0.5580  | 0.9826   | 5.1911 | 0.2104  |
|     |   | 0.5 | 0.9820   | 5.0469 | 0.5522  | 0.9820   | 5.1302 | 0.4648  |
|     |   | 0.9 | 0.9818   | 5.0480 | 1.6984  | 0.9838   | 5.0919 | 1.4980  |
|     | 4 | 0   | 0.9820   | 5.0467 | 0.5700  | 0.9814   | 5.3452 | 0.1608  |
|     |   | 0.5 | 0.9818   | 5.0467 | 0.5422  | 0.9836   | 5.1363 | 0.4758  |
|     |   | 0.9 | 0.9824   | 5.0475 | 1.7022  | 0.9834   | 5.0926 | 1.5078  |
|     | 5 | 0   | 0.9818   | 5.0466 | 0.5370  | 0.9816   | 5.3863 | 0.1590  |
|     |   | 0.5 | 0.9820   | 5.0480 | 1.7992  | 0.9840   | 5.0879 | 1.5982  |
|     |   | 0.9 | 0.9818   | 5.0481 | 1.8020  | 0.9832   | 5.0880 | 1.6220  |
| 500 | 1 | 0   | 0.9492   | 4.0742 | 0.5430  | 0.9492   | 4.1051 | 0.0474  |
|     |   | 0.5 | 0.9494   | 4.0742 | 0.3270  | 0.9486   | 4.0938 | 0.1132  |
|     |   | 0.9 | 0.9488   | 4.0741 | 1.3090  | 0.9500   | 4.0835 | 1.1276  |
|     | 4 | 0   | 0.9492   | 4.0742 | 0.5502  | 0.9482   | 4.1232 | 0.0246  |
|     |   | 0.5 | 0.9492   | 4.0742 | 0.3132  | 0.9496   | 4.0970 | 0.1328  |
|     |   | 0.9 | 0.9492   | 4.0742 | 1.3044  | 0.9500   | 4.0835 | 1.1314  |
|     | 5 | 0   | 0.9492   | 4.0742 | 0.5288  | 0.9492   | 4.1468 | 0.0296  |
|     |   | 0.5 | 0.9496   | 4.0742 | 1.8126  | 0.9502   | 4.0811 | 1.6176  |
|     |   | 0.9 | 0.9496   | 4.0742 | 1.8058  | 0.9508   | 4.0810 | 1.6340  |

Table 5.3.  $p = 5$ , error type = 1

|     |   |     | Ii       |        |         | I-Min    |        |         |
|-----|---|-----|----------|--------|---------|----------|--------|---------|
| n   | a | psi | coverage | length | penalty | coverage | length | penalty |
| 50  | 1 | 0   | 0.9882   | 5.7505 | 1.2858  | 0.9882   | 5.7180 | 1.5018  |
|     |   | 0.5 | 0.9874   | 5.7500 | 1.2886  | 0.9868   | 5.7195 | 1.4882  |
|     |   | 0.9 | 0.9878   | 5.7295 | 1.1714  | 0.9872   | 5.7069 | 1.3104  |
|     | 4 | 0   | 0.9856   | 5.6764 | 0.0966  | 0.9858   | 5.6653 | 0.1714  |
|     |   | 0.5 | 0.9866   | 5.6820 | 0.0376  | 0.9860   | 5.6684 | 0.1364  |
|     |   | 0.9 | 0.9894   | 5.7542 | -1.9134 | 0.9886   | 5.7300 | -1.7362 |
|     | 5 | 0   | 0.9854   | 5.6380 | 0.0000  | 0.9854   | 5.6380 | 0.0000  |
|     |   | 0.5 | 0.9852   | 5.6512 | -0.0962 | 0.9854   | 5.6430 | -0.0454 |
|     |   | 0.9 | 0.9856   | 5.7561 | -2.6814 | 0.9860   | 5.7127 | -2.3842 |
| 100 | 1 | 0   | 0.9858   | 5.0972 | 1.2546  | 0.9856   | 5.0833 | 1.4626  |
|     |   | 0.5 | 0.9848   | 5.0959 | 1.2678  | 0.9844   | 5.0835 | 1.4686  |
|     |   | 0.9 | 0.9840   | 5.0896 | 1.1480  | 0.9838   | 5.0815 | 1.2820  |
|     | 4 | 0   | 0.9844   | 5.0649 | 0.0788  | 0.9840   | 5.0589 | 0.1520  |
|     |   | 0.5 | 0.9844   | 5.0632 | 0.0812  | 0.9844   | 5.0580 | 0.1516  |
|     |   | 0.9 | 0.9822   | 5.0970 | -1.5394 | 0.9824   | 5.0792 | -1.2490 |
|     | 5 | 0   | 0.9818   | 5.0467 | 0.0000  | 0.9818   | 5.0467 | 0.0000  |
|     |   | 0.5 | 0.9818   | 5.0467 | -0.0002 | 0.9818   | 5.0467 | 0.0000  |
|     |   | 0.9 | 0.9828   | 5.0850 | -2.1236 | 0.9820   | 5.0746 | -1.9622 |
| 500 | 1 | 0   | 0.9496   | 4.0820 | 1.2570  | 0.9498   | 4.0797 | 1.4776  |
|     |   | 0.5 | 0.9502   | 4.0815 | 1.2780  | 0.9504   | 4.0791 | 1.4914  |
|     |   | 0.9 | 0.9502   | 4.0820 | 1.1840  | 0.9506   | 4.0798 | 1.3966  |
|     | 4 | 0   | 0.9502   | 4.0765 | 0.0850  | 0.9500   | 4.0756 | 0.1590  |
|     |   | 0.5 | 0.9490   | 4.0767 | 0.0882  | 0.9488   | 4.0758 | 0.1610  |
|     |   | 0.9 | 0.9486   | 4.0800 | -0.3878 | 0.9486   | 4.0774 | -0.1516 |
|     | 5 | 0   | 0.9492   | 4.0742 | 0.0000  | 0.9492   | 4.0742 | 0.0000  |
|     |   | 0.5 | 0.9492   | 4.0742 | 0.0000  | 0.9492   | 4.0742 | 0.0000  |
|     |   | 0.9 | 0.9478   | 4.0793 | -0.7274 | 0.9476   | 4.0766 | -0.4800 |

Table 5.4.  $p = 10$ , error type = 1

|      |    |     | LASSO    |        |         | RIDGE    |        |         |
|------|----|-----|----------|--------|---------|----------|--------|---------|
| n    | a  | psi | coverage | length | penalty | coverage | length | penalty |
| 100  | 1  | 0   | 0.9884   | 5.4461 | 0.1128  | 0.9836   | 5.3203 | 0.1583  |
|      |    | 0.5 | 0.9876   | 5.4145 | 0.0402  | 0.9860   | 5.5306 | 0.2032  |
|      |    | 0.9 | 0.9892   | 5.4659 | 0.0205  | 0.9910   | 5.5309 | 0.2996  |
|      | 9  | 0   | 0.9828   | 5.2936 | 0.0130  | 0.9834   | 5.3527 | 0.1584  |
|      |    | 0.5 | 0.9846   | 5.3291 | 0.0926  | 0.9866   | 5.6287 | 1.4070  |
|      |    | 0.9 | 0.9856   | 5.4841 | 0.1563  | 0.9888   | 5.7184 | 2.3751  |
|      | 10 | 0   | 0.9824   | 5.2896 | 0.0080  | 0.9832   | 5.3585 | 0.1634  |
|      |    | 0.5 | 0.9832   | 5.3225 | 0.1041  | 0.9876   | 5.4681 | 1.5817  |
|      |    | 0.9 | 0.9852   | 5.5211 | 0.1774  | 0.9900   | 5.7454 | 2.6748  |
| 200  | 1  | 0   | 0.9826   | 4.8221 | 0.0804  | 0.9820   | 4.7659 | 0.1113  |
|      |    | 0.5 | 0.9824   | 4.8081 | 0.0282  | 0.9828   | 4.9321 | 0.1897  |
|      |    | 0.9 | 0.9822   | 4.8304 | 0.0197  | 0.9836   | 4.8713 | 0.2995  |
|      | 9  | 0   | 0.9796   | 4.7554 | 0.0101  | 0.9802   | 4.7976 | 0.1431  |
|      |    | 0.5 | 0.9798   | 4.7822 | 0.0930  | 0.9830   | 5.0233 | 1.4137  |
|      |    | 0.9 | 0.9816   | 4.9088 | 0.1573  | 0.9838   | 5.0357 | 2.3911  |
|      | 10 | 0   | 0.9790   | 4.7539 | 0.0074  | 0.9796   | 4.7997 | 0.1469  |
|      |    | 0.5 | 0.9796   | 4.7826 | 0.1045  | 0.9820   | 4.8718 | 1.5887  |
|      |    | 0.9 | 0.9810   | 4.9487 | 0.1770  | 0.9844   | 5.0567 | 2.6899  |
| 1000 | 1  | 0   | 0.9550   | 4.0503 | 0.0361  | 0.9558   | 4.0489 | 0.1096  |
|      |    | 0.5 | 0.9542   | 4.0488 | 0.0140  | 0.9526   | 4.1681 | 0.1899  |
|      |    | 0.9 | 0.9540   | 4.0523 | 0.0197  | 0.9550   | 4.0734 | 0.2999  |
|      | 9  | 0   | 0.9554   | 4.0390 | 0.0073  | 0.9546   | 4.0636 | 0.1230  |
|      |    | 0.5 | 0.9558   | 4.0576 | 0.0929  | 0.9558   | 4.2358 | 1.4123  |
|      |    | 0.9 | 0.9546   | 4.1698 | 0.1577  | 0.9544   | 4.2067 | 2.3966  |
|      | 10 | 0   | 0.9552   | 4.0390 | 0.0073  | 0.9546   | 4.0647 | 0.1245  |
|      |    | 0.5 | 0.9546   | 4.0616 | 0.1044  | 0.9546   | 4.1137 | 1.5871  |
|      |    | 0.9 | 0.9544   | 4.2122 | 0.1774  | 0.9570   | 4.2250 | 2.6961  |

Table 5.5.  $p = 10$ , error type = 1

|      |    |     | PLS      |        |         | PCR      |        |         |
|------|----|-----|----------|--------|---------|----------|--------|---------|
| n    | a  | psi | coverage | length | penalty | coverage | length | penalty |
| 100  | 1  | 0   | 0.9828   | 5.2875 | 4.6474  | 0.9822   | 5.3909 | 0.4702  |
|      |    | 0.5 | 0.9830   | 5.2902 | 5.2654  | 0.9826   | 5.4084 | 2.0524  |
|      |    | 0.9 | 0.9834   | 5.2963 | 6.8178  | 0.9864   | 5.4339 | 5.7218  |
|      | 9  | 0   | 0.9826   | 5.2873 | 3.7566  | 0.9826   | 5.5023 | 0.1128  |
|      |    | 0.5 | 0.9828   | 5.2894 | 5.3198  | 0.9838   | 5.4030 | 2.0242  |
|      |    | 0.9 | 0.9838   | 5.2976 | 6.8126  | 0.9866   | 5.4317 | 5.7076  |
|      | 10 | 0   | 0.9826   | 5.2872 | 3.7162  | 0.9826   | 5.5007 | 0.0998  |
|      |    | 0.5 | 0.9836   | 5.2982 | 6.8630  | 0.9876   | 5.4293 | 5.8948  |
|      |    | 0.9 | 0.9828   | 5.2974 | 6.8696  | 0.9874   | 5.4296 | 5.9612  |
| 200  | 1  | 0   | 0.9792   | 4.7526 | 4.5584  | 0.9792   | 4.7918 | 0.1616  |
|      |    | 0.5 | 0.9794   | 4.7531 | 4.7764  | 0.9796   | 4.7909 | 0.8830  |
|      |    | 0.9 | 0.9790   | 4.7550 | 6.7832  | 0.9824   | 4.8180 | 5.5524  |
|      | 9  | 0   | 0.9790   | 4.7525 | 4.0604  | 0.9784   | 4.8446 | 0.0442  |
|      |    | 0.5 | 0.9792   | 4.7526 | 4.7374  | 0.9788   | 4.7885 | 0.8588  |
|      |    | 0.9 | 0.9786   | 4.7551 | 6.7756  | 0.9810   | 4.8198 | 5.5346  |
|      | 10 | 0   | 0.9790   | 4.7526 | 4.0734  | 0.9780   | 4.8581 | 0.0452  |
|      |    | 0.5 | 0.9792   | 4.7545 | 6.8928  | 0.9814   | 4.8154 | 5.9610  |
|      |    | 0.9 | 0.9790   | 4.7544 | 6.8956  | 0.9810   | 4.8147 | 5.9848  |
| 1000 | 1  | 0   | 0.9548   | 4.0384 | 4.6970  | 0.9548   | 4.0423 | 0.0128  |
|      |    | 0.5 | 0.9548   | 4.0383 | 4.2100  | 0.9550   | 4.0436 | 0.1088  |
|      |    | 0.9 | 0.9556   | 4.0384 | 6.1100  | 0.9550   | 4.0505 | 3.8516  |
|      | 9  | 0   | 0.9548   | 4.0383 | 4.5476  | 0.9546   | 4.0439 | 0.0018  |
|      |    | 0.5 | 0.9548   | 4.0383 | 4.2138  | 0.9546   | 4.0434 | 0.1122  |
|      |    | 0.9 | 0.9546   | 4.0384 | 6.1430  | 0.9552   | 4.0500 | 3.8570  |
|      | 10 | 0   | 0.9546   | 4.0383 | 4.5270  | 0.9548   | 4.0459 | 0.0022  |
|      |    | 0.5 | 0.9546   | 4.0384 | 6.9048  | 0.9562   | 4.0494 | 6.0390  |
|      |    | 0.9 | 0.9548   | 4.0384 | 6.9100  | 0.9552   | 4.0490 | 6.0674  |

Table 5.6.  $p = 10$ , error type = 1

|      |    |     | Ii       |        |         | I-Min    |        |         |
|------|----|-----|----------|--------|---------|----------|--------|---------|
| n    | a  | psi | coverage | length | penalty | coverage | length | penalty |
| 100  | 1  | 0   | 0.9882   | 5.4153 | 1.7604  | 0.9874   | 5.3799 | 2.2832  |
|      |    | 0.5 | 0.9874   | 5.4108 | 1.8434  | 0.9874   | 5.3843 | 2.3040  |
|      |    | 0.9 | 0.9868   | 5.4056 | 1.5272  | 0.9868   | 5.3796 | 1.9234  |
|      | 9  | 0   | 0.9824   | 5.3027 | 0.0906  | 0.9822   | 5.2982 | 0.1662  |
|      |    | 0.5 | 0.9824   | 5.3047 | 0.0826  | 0.9824   | 5.2986 | 0.1648  |
|      |    | 0.9 | 0.9822   | 5.3947 | -4.5546 | 0.9830   | 5.3584 | -4.0566 |
|      | 10 | 0   | 0.9828   | 5.2871 | 0.0000  | 0.9828   | 5.2871 | 0.0000  |
|      |    | 0.5 | 0.9828   | 5.2876 | -0.0064 | 0.9828   | 5.2872 | -0.0028 |
|      |    | 0.9 | 0.9822   | 5.3896 | -5.2246 | 0.9822   | 5.3557 | -4.7246 |
| 200  | 1  | 0   | 0.9806   | 4.8108 | 1.7230  | 0.9814   | 4.7954 | 2.2646  |
|      |    | 0.5 | 0.9798   | 4.8068 | 1.8110  | 0.9812   | 4.7955 | 2.2814  |
|      |    | 0.9 | 0.9812   | 4.8078 | 1.5274  | 0.9810   | 4.7966 | 1.9322  |
|      | 9  | 0   | 0.9812   | 4.7601 | 0.0810  | 0.9806   | 4.7571 | 0.1586  |
|      |    | 0.5 | 0.9798   | 4.7600 | 0.0814  | 0.9798   | 4.7572 | 0.1562  |
|      |    | 0.9 | 0.9810   | 4.7976 | -3.4344 | 0.9804   | 4.7809 | -2.8882 |
|      | 10 | 0   | 0.9790   | 4.7525 | 0.0000  | 0.9790   | 4.7525 | 0.0000  |
|      |    | 0.5 | 0.9790   | 4.7525 | 0.0000  | 0.9790   | 4.7525 | 0.0000  |
|      |    | 0.9 | 0.9792   | 4.7956 | -3.9636 | 0.9802   | 4.7780 | -3.3968 |
| 1000 | 1  | 0   | 0.9554   | 4.0485 | 1.7170  | 0.9546   | 4.0458 | 2.2628  |
|      |    | 0.5 | 0.9554   | 4.0474 | 1.8002  | 0.9548   | 4.0451 | 2.2880  |
|      |    | 0.9 | 0.9552   | 4.0475 | 1.7916  | 0.9540   | 4.0453 | 2.2736  |
|      | 9  | 0   | 0.9542   | 4.0395 | 0.0880  | 0.9550   | 4.0391 | 0.1608  |
|      |    | 0.5 | 0.9530   | 4.0395 | 0.0940  | 0.9538   | 4.0394 | 0.1628  |
|      |    | 0.9 | 0.9544   | 4.0407 | -0.2906 | 0.9546   | 4.0397 | -0.0368 |
|      | 10 | 0   | 0.9548   | 4.0383 | 0.0000  | 0.9548   | 4.0383 | 0.0000  |
|      |    | 0.5 | 0.9548   | 4.0383 | 0.0000  | 0.9548   | 4.0383 | 0.0000  |
|      |    | 0.9 | 0.9562   | 4.0399 | -0.4550 | 0.9556   | 4.0388 | -0.2402 |

Table 5.7.  $p = 20$ , error type = 1

|      |    |     | LASSO    |         |         | RIDGE    |         |         |
|------|----|-----|----------|---------|---------|----------|---------|---------|
| n    | a  | psi | coverage | length  | penalty | coverage | length  | penalty |
| 200  | 1  | 0   | 0.9862   | 5.1549  | 0.1001  | 0.9782   | 4.9924  | 0.1565  |
|      |    | 0.5 | 0.9862   | 5.1350  | 0.0257  | 0.9872   | 5.3618  | 0.2565  |
|      |    | 0.9 | 0.9876   | 5.1765  | 0.0284  | 0.9880   | 5.2258  | 0.4599  |
|      | 19 | 0   | 0.9766   | 4.9639  | 0.0077  | 0.9784   | 5.0421  | 0.1763  |
|      |    | 0.5 | 0.9822   | 7.1374  | 0.9469  | 0.9886   | 7.1115  | 15.4018 |
|      |    | 0.9 | 0.9874   | 12.1008 | 1.9016  | 0.9864   | 12.1859 | 41.4036 |
|      | 20 | 0   | 0.9766   | 4.9636  | 0.0071  | 0.9792   | 5.0458  | 0.1793  |
|      |    | 0.5 | 0.9826   | 7.5324  | 1.0472  | 0.9886   | 7.5123  | 19.4674 |
|      |    | 0.9 | 0.9884   | 12.8327 | 2.0441  | 0.9870   | 12.7475 | 43.7748 |
| 400  | 1  | 0   | 0.9794   | 4.7830  | 0.0718  | 0.9754   | 4.7080  | 0.1102  |
|      |    | 0.5 | 0.9794   | 4.7760  | 0.0197  | 0.9790   | 5.0068  | 0.2569  |
|      |    | 0.9 | 0.9798   | 4.7931  | 0.0284  | 0.9804   | 4.8271  | 0.5059  |
|      | 19 | 0   | 0.9752   | 4.6957  | 0.0072  | 0.9772   | 4.7487  | 0.1551  |
|      |    | 0.5 | 0.9814   | 6.5197  | 0.9137  | 0.9840   | 6.4959  | 15.0130 |
|      |    | 0.9 | 0.9844   | 11.1318 | 1.8824  | 0.9880   | 11.2281 | 41.4985 |
|      | 20 | 0   | 0.9756   | 4.6959  | 0.0072  | 0.9760   | 4.7504  | 0.1571  |
|      |    | 0.5 | 0.9822   | 6.8874  | 1.0179  | 0.9858   | 6.8815  | 19.2160 |
|      |    | 0.9 | 0.9848   | 11.6681 | 1.9914  | 0.9866   | 11.7311 | 43.8077 |
| 2000 | 1  | 0   | 0.9590   | 4.0508  | 0.0322  | 0.9570   | 4.0469  | 0.1098  |
|      |    | 0.5 | 0.9580   | 4.0521  | 0.0169  | 0.9546   | 4.2663  | 0.2572  |
|      |    | 0.9 | 0.9590   | 4.0547  | 0.0285  | 0.9584   | 4.0750  | 0.5680  |
|      | 19 | 0   | 0.9568   | 4.0380  | 0.0072  | 0.9542   | 4.0655  | 0.1282  |
|      |    | 0.5 | 0.9518   | 5.4480  | 0.8817  | 0.9516   | 5.4220  | 14.5389 |
|      |    | 0.9 | 0.9540   | 9.4723  | 1.8850  | 0.9530   | 9.4713  | 41.5668 |
|      | 20 | 0   | 0.9568   | 4.0381  | 0.0073  | 0.9552   | 4.0662  | 0.1290  |
|      |    | 0.5 | 0.9522   | 5.7692  | 0.9925  | 0.9544   | 5.7485  | 18.8624 |
|      |    | 0.9 | 0.9538   | 9.9125  | 1.9897  | 0.9514   | 9.8951  | 43.8760 |

Table 5.8.  $p = 20$ , error type = 1

|      |    |     | PLS      |        |         | PCR      |        |         |
|------|----|-----|----------|--------|---------|----------|--------|---------|
| n    | a  | psi | coverage | length | penalty | coverage | length | penalty |
| 200  | 1  | 0   | 0.9790   | 4.9622 | 14.2624 | 0.9800   | 4.9984 | 0.4948  |
|      |    | 0.5 | 0.9786   | 4.9645 | 15.2648 | 0.9788   | 5.0535 | 3.7794  |
|      |    | 0.9 | 0.9794   | 4.9811 | 16.9344 | 0.9876   | 5.1587 | 14.7684 |
|      | 19 | 0   | 0.9794   | 4.9618 | 12.4752 | 0.9786   | 5.0106 | 0.0210  |
|      |    | 0.5 | 0.9786   | 4.9646 | 15.2704 | 0.9796   | 5.0543 | 3.7614  |
|      |    | 0.9 | 0.9804   | 4.9812 | 16.9414 | 0.9840   | 5.1556 | 14.7718 |
|      | 20 | 0   | 0.9786   | 4.9617 | 12.4384 | 0.9792   | 5.0126 | 0.0222  |
|      |    | 0.5 | 0.9802   | 4.9798 | 16.9692 | 0.9868   | 5.1504 | 15.2218 |
|      |    | 0.9 | 0.9806   | 4.9800 | 16.9668 | 0.9860   | 5.1510 | 15.3116 |
| 400  | 1  | 0   | 0.9758   | 4.6933 | 14.1778 | 0.9756   | 4.7062 | 0.1490  |
|      |    | 0.5 | 0.9750   | 4.6938 | 14.6326 | 0.9742   | 4.7151 | 1.2480  |
|      |    | 0.9 | 0.9754   | 4.6976 | 16.9196 | 0.9808   | 4.7857 | 14.3778 |
|      | 19 | 0   | 0.9758   | 4.6935 | 13.1034 | 0.9756   | 4.7086 | 0.0054  |
|      |    | 0.5 | 0.9758   | 4.6937 | 14.5724 | 0.9758   | 4.7158 | 1.2108  |
|      |    | 0.9 | 0.9748   | 4.6979 | 16.9160 | 0.9798   | 4.7852 | 14.4478 |
|      | 20 | 0   | 0.9754   | 4.6935 | 13.0878 | 0.9754   | 4.7091 | 0.0066  |
|      |    | 0.5 | 0.9750   | 4.6976 | 16.9778 | 0.9788   | 4.7809 | 15.3744 |
|      |    | 0.9 | 0.9748   | 4.6975 | 16.9794 | 0.9794   | 4.7808 | 15.4614 |
| 2000 | 1  | 0   | 0.9566   | 4.0360 | 14.4946 | 0.9566   | 4.0369 | 0.0064  |
|      |    | 0.5 | 0.9566   | 4.0360 | 13.8930 | 0.9566   | 4.0376 | 0.1066  |
|      |    | 0.9 | 0.9570   | 4.0361 | 16.2020 | 0.9572   | 4.0501 | 9.3010  |
|      | 19 | 0   | 0.9566   | 4.0360 | 13.9140 | 0.9566   | 4.0360 | 0.0000  |
|      |    | 0.5 | 0.9566   | 4.0360 | 13.8766 | 0.9568   | 4.0375 | 0.0958  |
|      |    | 0.9 | 0.9558   | 4.0361 | 16.1920 | 0.9582   | 4.0504 | 9.4936  |
|      | 20 | 0   | 0.9566   | 4.0360 | 13.9086 | 0.9566   | 4.0360 | 0.0000  |
|      |    | 0.5 | 0.9564   | 4.0362 | 16.9840 | 0.9572   | 4.0509 | 15.6098 |
|      |    | 0.9 | 0.9566   | 4.0362 | 16.9826 | 0.9586   | 4.0509 | 15.6514 |

Table 5.9.  $p = 20$ , error type = 1

|      |    |     | Ii       |        |         | I-Min    |        |         |
|------|----|-----|----------|--------|---------|----------|--------|---------|
| n    | a  | psi | coverage | length | penalty | coverage | length | penalty |
| 200  | 1  | 0   | 0.9838   | 5.0992 | 2.8208  | 0.9826   | 5.0672 | 3.8870  |
|      |    | 0.5 | 0.9840   | 5.0961 | 2.9948  | 0.9834   | 5.0680 | 3.9056  |
|      |    | 0.9 | 0.9842   | 5.0997 | 2.5710  | 0.9830   | 5.0739 | 3.4680  |
|      | 19 | 0   | 0.9780   | 4.9706 | 0.0878  | 0.9786   | 4.9679 | 0.1612  |
|      |    | 0.5 | 0.9778   | 4.9707 | 0.0908  | 0.9784   | 4.9691 | 0.1568  |
|      |    | 0.9 | 0.9744   | 5.0624 | -8.0654 | 0.9746   | 5.0263 | -7.0706 |
|      | 20 | 0   | 0.9788   | 4.9613 | 0.0000  | 0.9788   | 4.9613 | 0.0000  |
|      |    | 0.5 | 0.9788   | 4.9613 | 0.0000  | 0.9788   | 4.9613 | 0.0000  |
|      |    | 0.9 | 0.9768   | 5.0588 | -8.6244 | 0.9758   | 5.0252 | -7.6184 |
| 400  | 1  | 0   | 0.9780   | 4.7551 | 2.7666  | 0.9784   | 4.7393 | 3.8592  |
|      |    | 0.5 | 0.9796   | 4.7540 | 2.9582  | 0.9782   | 4.7402 | 3.8902  |
|      |    | 0.9 | 0.9798   | 4.7553 | 2.6970  | 0.9780   | 4.7415 | 3.6552  |
|      | 19 | 0   | 0.9746   | 4.6967 | 0.0814  | 0.9746   | 4.6954 | 0.1560  |
|      |    | 0.5 | 0.9750   | 4.6966 | 0.0790  | 0.9750   | 4.6958 | 0.1592  |
|      |    | 0.9 | 0.9728   | 4.7318 | -4.9272 | 0.9740   | 4.7161 | -3.8920 |
|      | 20 | 0   | 0.9756   | 4.6934 | 0.0000  | 0.9756   | 4.6934 | 0.0000  |
|      |    | 0.5 | 0.9756   | 4.6934 | 0.0000  | 0.9756   | 4.6934 | 0.0000  |
|      |    | 0.9 | 0.9720   | 4.7301 | -5.2950 | 0.9728   | 4.7144 | -4.2786 |
| 2000 | 1  | 0   | 0.9576   | 4.0465 | 2.7464  | 0.9568   | 4.0439 | 3.8028  |
|      |    | 0.5 | 0.9574   | 4.0461 | 2.9296  | 0.9576   | 4.0437 | 3.8606  |
|      |    | 0.9 | 0.9580   | 4.0459 | 2.9320  | 0.9576   | 4.0437 | 3.8682  |
|      | 19 | 0   | 0.9564   | 4.0366 | 0.0924  | 0.9564   | 4.0365 | 0.1658  |
|      |    | 0.5 | 0.9556   | 4.0367 | 0.0864  | 0.9558   | 4.0365 | 0.1638  |
|      |    | 0.9 | 0.9554   | 4.0368 | 0.0470  | 0.9558   | 4.0366 | 0.1510  |
|      | 20 | 0   | 0.9566   | 4.0360 | 0.0000  | 0.9566   | 4.0360 | 0.0000  |
|      |    | 0.5 | 0.9566   | 4.0360 | 0.0000  | 0.9566   | 4.0360 | 0.0000  |
|      |    | 0.9 | 0.9564   | 4.0361 | -0.0456 | 0.9566   | 4.0360 | -0.0146 |

Table 5.10.  $p = 50$ , error type = 1

|      |    |     | LASSO    |         |         | RIDGE    |         |          |
|------|----|-----|----------|---------|---------|----------|---------|----------|
| n    | a  | psi | coverage | length  | penalty | coverage | length  | penalty  |
| 500  | 1  | 0   | 0.9882   | 5.1527  | 0.0788  | 0.9812   | 4.9571  | 0.1557   |
|      |    | 0.5 | 0.9890   | 5.1630  | 0.0260  | 0.9886   | 5.5414  | 0.3945   |
|      |    | 0.9 | 0.9894   | 5.1823  | 0.0455  | 0.9898   | 5.2125  | 2.3158   |
|      | 49 | 0   | 0.9780   | 4.9323  | 0.0069  | 0.9794   | 5.0308  | 0.1934   |
|      |    | 0.5 | 0.9838   | 29.4716 | 5.4232  | 0.9878   | 29.1538 | 276.2661 |
|      |    | 0.9 | 0.9884   | 51.5459 | 8.8077  | 0.9874   | 50.2634 | 492.4197 |
|      | 50 | 0   | 0.9776   | 4.9325  | 0.0069  | 0.9784   | 5.0323  | 0.1946   |
|      |    | 0.5 | 0.9840   | 30.0982 | 5.5362  | 0.9884   | 29.6323 | 282.0203 |
|      |    | 0.9 | 0.9880   | 52.6117 | 8.9912  | 0.9868   | 51.3007 | 502.6784 |
| 1000 | 1  | 0   | 0.9796   | 4.7745  | 0.0559  | 0.9738   | 4.6880  | 0.1097   |
|      |    | 0.5 | 0.9808   | 4.7813  | 0.0260  | 0.9784   | 5.1593  | 0.3954   |
|      |    | 0.9 | 0.9798   | 4.7936  | 0.0456  | 0.9808   | 4.8177  | 2.3211   |
|      | 49 | 0   | 0.9744   | 4.6794  | 0.0071  | 0.9770   | 4.7408  | 0.1668   |
|      |    | 0.5 | 0.9848   | 27.1686 | 5.4327  | 0.9860   | 26.9734 | 276.7502 |
|      |    | 0.9 | 0.9850   | 47.7251 | 8.8261  | 0.9852   | 46.5427 | 493.4512 |
|      | 50 | 0   | 0.9746   | 4.6795  | 0.0071  | 0.9756   | 4.7414  | 0.1679   |
|      |    | 0.5 | 0.9836   | 27.7414 | 5.5459  | 0.9862   | 27.4171 | 282.5143 |
|      |    | 0.9 | 0.9850   | 48.7126 | 9.0100  | 0.9850   | 47.4989 | 503.7313 |
| 5000 | 1  | 0   | 0.9510   | 4.0554  | 0.0251  | 0.9480   | 4.0489  | 0.1097   |
|      |    | 0.5 | 0.9514   | 4.0575  | 0.0260  | 0.9520   | 4.3926  | 0.3956   |
|      |    | 0.9 | 0.9516   | 4.0617  | 0.0456  | 0.9506   | 4.0814  | 2.3223   |
|      | 49 | 0   | 0.9498   | 4.0435  | 0.0072  | 0.9486   | 4.0726  | 0.1336   |
|      |    | 0.5 | 0.9576   | 22.9195 | 5.4337  | 0.9582   | 22.8305 | 276.8005 |
|      |    | 0.9 | 0.9588   | 40.4261 | 8.8322  | 0.9580   | 39.4116 | 493.7890 |
|      | 50 | 0   | 0.9508   | 4.0437  | 0.0073  | 0.9486   | 4.0728  | 0.1339   |
|      |    | 0.5 | 0.9606   | 23.3987 | 5.5469  | 0.9610   | 23.2087 | 282.5654 |
|      |    | 0.9 | 0.9586   | 41.2620 | 9.0162  | 0.9584   | 40.2207 | 504.0763 |

Table 5.11.  $p = 50$ , error type = 1

|      |    |     | PLS      |        |         | PCR      |        |         |
|------|----|-----|----------|--------|---------|----------|--------|---------|
| n    | a  | psi | coverage | length | penalty | coverage | length | penalty |
| 500  | 1  | 0   | 0.9776   | 4.9243 | 44.1168 | 0.9788   | 4.9360 | 0.5632  |
|      |    | 0.5 | 0.9788   | 4.9304 | 45.6042 | 0.9814   | 4.9969 | 7.5334  |
|      |    | 0.9 | 0.9804   | 4.9498 | 46.9952 | 0.9882   | 5.1708 | 43.8786 |
|      | 49 | 0   | 0.9782   | 4.9237 | 41.0860 | 0.9778   | 4.9273 | 0.0012  |
|      |    | 0.5 | 0.9782   | 4.9296 | 45.5902 | 0.9786   | 4.9979 | 7.5416  |
|      |    | 0.9 | 0.9800   | 4.9493 | 46.9958 | 0.9880   | 5.1702 | 43.8168 |
|      | 50 | 0   | 0.9778   | 4.9237 | 41.1072 | 0.9776   | 4.9277 | 0.0010  |
|      |    | 0.5 | 0.9802   | 4.9471 | 46.9990 | 0.9886   | 5.1583 | 44.7416 |
|      |    | 0.9 | 0.9798   | 4.9474 | 46.9992 | 0.9880   | 5.1580 | 44.7980 |
| 1000 | 1  | 0   | 0.9738   | 4.6734 | 43.9162 | 0.9748   | 4.6763 | 0.1322  |
|      |    | 0.5 | 0.9734   | 4.6734 | 44.5844 | 0.9738   | 4.6831 | 1.6540  |
|      |    | 0.9 | 0.9740   | 4.6787 | 46.9940 | 0.9792   | 4.7906 | 43.0660 |
|      | 49 | 0   | 0.9736   | 4.6734 | 42.2408 | 0.9738   | 4.6734 | 0.0000  |
|      |    | 0.5 | 0.9744   | 4.6739 | 44.5664 | 0.9734   | 4.6829 | 1.7140  |
|      |    | 0.9 | 0.9740   | 4.6785 | 46.9924 | 0.9784   | 4.7923 | 43.1088 |
|      | 50 | 0   | 0.9736   | 4.6734 | 42.2468 | 0.9738   | 4.6734 | 0.0000  |
|      |    | 0.5 | 0.9732   | 4.6780 | 46.9994 | 0.9790   | 4.7790 | 45.0346 |
|      |    | 0.9 | 0.9740   | 4.6780 | 46.9994 | 0.9782   | 4.7787 | 45.0822 |
| 5000 | 1  | 0   | 0.9498   | 4.0386 | 44.2560 | 0.9498   | 4.0387 | 0.0052  |
|      |    | 0.5 | 0.9498   | 4.0386 | 43.7142 | 0.9488   | 4.0391 | 0.0892  |
|      |    | 0.9 | 0.9506   | 4.0388 | 46.3834 | 0.9482   | 4.0561 | 26.7282 |
|      | 49 | 0   | 0.9498   | 4.0386 | 43.5068 | 0.9498   | 4.0386 | 0.0000  |
|      |    | 0.5 | 0.9498   | 4.0386 | 43.6980 | 0.9496   | 4.0389 | 0.0818  |
|      |    | 0.9 | 0.9494   | 4.0387 | 46.4070 | 0.9490   | 4.0562 | 27.1914 |
|      | 50 | 0   | 0.9498   | 4.0386 | 43.5122 | 0.9498   | 4.0386 | 0.0000  |
|      |    | 0.5 | 0.9502   | 4.0386 | 47.0000 | 0.9522   | 4.0565 | 45.4298 |
|      |    | 0.9 | 0.9500   | 4.0387 | 47.0000 | 0.9522   | 4.0563 | 45.4428 |

Table 5.12.  $p = 50$ , error type = 1

|      |    |     | Ii       |        |          | I-Min    |        |         |
|------|----|-----|----------|--------|----------|----------|--------|---------|
| n    | a  | psi | coverage | length | penalty  | coverage | length | penalty |
| 500  | 1  | 0   | 0.9850   | 5.0625 | 6.3984   | 0.9856   | 5.0360 | 8.5492  |
|      |    | 0.5 | 0.9842   | 5.0604 | 6.5596   | 0.9842   | 5.0368 | 8.6302  |
|      |    | 0.9 | 0.9848   | 5.0617 | 6.3518   | 0.9848   | 5.0378 | 8.4302  |
|      | 49 | 0   | 0.9778   | 4.9277 | 0.0742   | 0.9778   | 4.9265 | 0.1498  |
|      |    | 0.5 | 0.9780   | 4.9274 | 0.0752   | 0.9782   | 4.9264 | 0.1522  |
|      |    | 0.9 | 0.9742   | 4.9935 | -10.7068 | 0.9762   | 4.9705 | -8.8512 |
|      | 50 | 0   | 0.9778   | 4.9238 | 0.0000   | 0.9778   | 4.9238 | 0.0000  |
|      |    | 0.5 | 0.9778   | 4.9238 | 0.0000   | 0.9778   | 4.9238 | 0.0000  |
|      |    | 0.9 | 0.9756   | 4.9942 | -11.0666 | 0.9762   | 4.9695 | -9.2146 |
| 1000 | 1  | 0   | 0.9758   | 4.7353 | 6.3868   | 0.9756   | 4.7215 | 8.6396  |
|      |    | 0.5 | 0.9772   | 4.7335 | 6.5390   | 0.9764   | 4.7219 | 8.6216  |
|      |    | 0.9 | 0.9776   | 4.7337 | 6.5002   | 0.9766   | 4.7219 | 8.5936  |
|      | 49 | 0   | 0.9740   | 4.6747 | 0.0836   | 0.9742   | 4.6743 | 0.1532  |
|      |    | 0.5 | 0.9736   | 4.6746 | 0.0844   | 0.9738   | 4.6743 | 0.1580  |
|      |    | 0.9 | 0.9728   | 4.6870 | -2.9844  | 0.9734   | 4.6795 | -1.8396 |
|      | 50 | 0   | 0.9738   | 4.6734 | 0.0000   | 0.9738   | 4.6734 | 0.0000  |
|      |    | 0.5 | 0.9738   | 4.6734 | 0.0000   | 0.9738   | 4.6734 | 0.0000  |
|      |    | 0.9 | 0.9718   | 4.6855 | -3.1798  | 0.9728   | 4.6788 | -2.0432 |
| 5000 | 1  | 0   | 0.9502   | 4.0491 | 6.3052   | 0.9504   | 4.0471 | 8.5714  |
|      |    | 0.5 | 0.9508   | 4.0490 | 6.4562   | 0.9500   | 4.0472 | 8.5294  |
|      |    | 0.9 | 0.9512   | 4.0490 | 6.4586   | 0.9494   | 4.0472 | 8.5278  |
|      | 49 | 0   | 0.9502   | 4.0387 | 0.0828   | 0.9500   | 4.0386 | 0.1592  |
|      |    | 0.5 | 0.9504   | 4.0388 | 0.0848   | 0.9502   | 4.0387 | 0.1586  |
|      |    | 0.9 | 0.9504   | 4.0388 | 0.0852   | 0.9502   | 4.0387 | 0.1584  |
|      | 50 | 0   | 0.9498   | 4.0386 | 0.0000   | 0.9498   | 4.0386 | 0.0000  |
|      |    | 0.5 | 0.9498   | 4.0386 | 0.0000   | 0.9498   | 4.0386 | 0.0000  |
|      |    | 0.9 | 0.9498   | 4.0386 | 0.0000   | 0.9498   | 4.0386 | 0.0000  |

### 5.1.4 Error type 2

Table 5.13.  $p = 5$ , error type = 2

|     |   |     | LASSO    |        |         | RIDGE    |         |         |
|-----|---|-----|----------|--------|---------|----------|---------|---------|
| n   | a | psi | coverage | length | penalty | coverage | length  | penalty |
| 50  | 1 | 0   | 0.9772   | 9.9328 | 0.1824  | 0.9736   | 9.8355  | 8.3426  |
|     |   | 0.5 | 0.9746   | 9.8959 | 0.0921  | 0.9740   | 9.9275  | 1.6133  |
|     |   | 0.9 | 0.9758   | 9.9943 | 0.0294  | 0.9772   | 10.1353 | 0.6892  |
|     | 4 | 0   | 0.9742   | 9.7695 | 0.0568  | 0.9744   | 9.7885  | 1.4834  |
|     |   | 0.5 | 0.9730   | 9.7707 | 0.0527  | 0.9758   | 9.9376  | 0.5609  |
|     |   | 0.9 | 0.9756   | 9.9452 | 0.0458  | 0.9776   | 10.1609 | 0.6115  |
|     | 5 | 0   | 0.9734   | 9.7372 | 0.0219  | 0.9746   | 9.7845  | 1.4030  |
|     |   | 0.5 | 0.9734   | 9.7361 | 0.0451  | 0.9758   | 9.9101  | 0.6962  |
|     |   | 0.9 | 0.9760   | 9.9013 | 0.0566  | 0.9784   | 10.1766 | 0.8057  |
| 100 | 1 | 0   | 0.9714   | 9.1887 | 0.1283  | 0.9702   | 9.1209  | 1.2608  |
|     |   | 0.5 | 0.9718   | 9.1752 | 0.0623  | 0.9710   | 9.1811  | 0.3677  |
|     |   | 0.9 | 0.9728   | 9.2326 | 0.0198  | 0.9726   | 9.2914  | 0.2566  |
|     | 4 | 0   | 0.9704   | 9.1076 | 0.0363  | 0.9704   | 9.1178  | 0.1793  |
|     |   | 0.5 | 0.9700   | 9.1127 | 0.0359  | 0.9714   | 9.2040  | 0.4420  |
|     |   | 0.9 | 0.9712   | 9.1821 | 0.0407  | 0.9726   | 9.3169  | 0.6064  |
|     | 5 | 0   | 0.9700   | 9.0958 | 0.0110  | 0.9702   | 9.1195  | 0.1703  |
|     |   | 0.5 | 0.9702   | 9.0957 | 0.0358  | 0.9710   | 9.1814  | 0.5732  |
|     |   | 0.9 | 0.9714   | 9.1522 | 0.0533  | 0.9732   | 9.3268  | 0.8065  |
| 500 | 1 | 0   | 0.9564   | 6.6879 | 0.0583  | 0.9566   | 6.6803  | 0.1150  |
|     |   | 0.5 | 0.9554   | 6.6851 | 0.0281  | 0.9556   | 6.7018  | 0.1482  |
|     |   | 0.9 | 0.9554   | 6.6912 | 0.0136  | 0.9544   | 6.7046  | 0.2058  |
|     | 4 | 0   | 0.9554   | 6.6784 | 0.0166  | 0.9560   | 6.6837  | 0.1190  |
|     |   | 0.5 | 0.9566   | 6.6805 | 0.0266  | 0.9556   | 6.7236  | 0.3965  |
|     |   | 0.9 | 0.9556   | 6.6825 | 0.0400  | 0.9554   | 6.7227  | 0.6081  |
|     | 5 | 0   | 0.9564   | 6.6768 | 0.0074  | 0.9550   | 6.6837  | 0.1218  |
|     |   | 0.5 | 0.9554   | 6.6768 | 0.0343  | 0.9538   | 6.6995  | 0.5219  |
|     |   | 0.9 | 0.9544   | 6.6803 | 0.0533  | 0.9536   | 6.7347  | 0.8105  |

Table 5.14.  $p = 5$ , error type = 2

|     |   |     | PLS      |        |         | PCR      |         |         |
|-----|---|-----|----------|--------|---------|----------|---------|---------|
| n   | a | psi | coverage | length | penalty | coverage | length  | penalty |
| 50  | 1 | 0   | 0.9734   | 9.7343 | 1.2448  | 0.9748   | 9.9860  | 0.6706  |
|     |   | 0.5 | 0.9726   | 9.7428 | 1.3232  | 0.9752   | 9.9234  | 1.0894  |
|     |   | 0.9 | 0.9742   | 9.7454 | 1.7674  | 0.9758   | 9.9200  | 1.5528  |
|     | 4 | 0   | 0.9732   | 9.7337 | 0.8578  | 0.9754   | 10.1383 | 0.3900  |
|     |   | 0.5 | 0.9722   | 9.7444 | 1.3014  | 0.9746   | 9.9137  | 1.0718  |
|     |   | 0.9 | 0.9738   | 9.7491 | 1.7742  | 0.9752   | 9.9247  | 1.5602  |
|     | 5 | 0   | 0.9728   | 9.7335 | 0.8100  | 0.9736   | 10.2180 | 0.3730  |
|     |   | 0.5 | 0.9738   | 9.7473 | 1.8096  | 0.9754   | 9.9202  | 1.5706  |
|     |   | 0.9 | 0.9736   | 9.7485 | 1.8006  | 0.9750   | 9.9231  | 1.5876  |
| 100 | 1 | 0   | 0.9704   | 9.0976 | 0.9366  | 0.9704   | 9.2133  | 0.3752  |
|     |   | 0.5 | 0.9696   | 9.0949 | 1.0148  | 0.9704   | 9.1838  | 0.8324  |
|     |   | 0.9 | 0.9696   | 9.0997 | 1.7724  | 0.9712   | 9.1874  | 1.5566  |
|     | 4 | 0   | 0.9700   | 9.0968 | 0.7708  | 0.9710   | 9.3033  | 0.2260  |
|     |   | 0.5 | 0.9698   | 9.0993 | 0.9948  | 0.9698   | 9.1934  | 0.8400  |
|     |   | 0.9 | 0.9696   | 9.0992 | 1.7594  | 0.9708   | 9.1909  | 1.5576  |
|     | 5 | 0   | 0.9698   | 9.0966 | 0.7370  | 0.9702   | 9.3822  | 0.2244  |
|     |   | 0.5 | 0.9696   | 9.0988 | 1.8188  | 0.9710   | 9.1862  | 1.6026  |
|     |   | 0.9 | 0.9696   | 9.0992 | 1.8058  | 0.9712   | 9.1877  | 1.6066  |
| 500 | 1 | 0   | 0.9558   | 6.6770 | 0.6646  | 0.9560   | 6.7100  | 0.0962  |
|     |   | 0.5 | 0.9558   | 6.6770 | 0.4510  | 0.9560   | 6.6979  | 0.2624  |
|     |   | 0.9 | 0.9558   | 6.6770 | 1.6196  | 0.9556   | 6.6883  | 1.4342  |
|     | 4 | 0   | 0.9558   | 6.6770 | 0.6704  | 0.9560   | 6.7325  | 0.0490  |
|     |   | 0.5 | 0.9558   | 6.6770 | 0.4456  | 0.9556   | 6.6967  | 0.2498  |
|     |   | 0.9 | 0.9556   | 6.6771 | 1.6180  | 0.9554   | 6.6891  | 1.4276  |
|     | 5 | 0   | 0.9560   | 6.6770 | 0.6410  | 0.9554   | 6.7353  | 0.0454  |
|     |   | 0.5 | 0.9556   | 6.6770 | 1.8196  | 0.9554   | 6.6879  | 1.6340  |
|     |   | 0.9 | 0.9556   | 6.6770 | 1.8100  | 0.9552   | 6.6873  | 1.6378  |

Table 5.15.  $p = 5$ , error type = 2

|     |   |     | Ii       |        |         | I-Min    |        |         |
|-----|---|-----|----------|--------|---------|----------|--------|---------|
| n   | a | psi | coverage | length | penalty | coverage | length | penalty |
| 50  | 1 | 0   | 0.9762   | 9.9737 | 1.2850  | 0.9756   | 9.9096 | 1.4938  |
|     |   | 0.5 | 0.9754   | 9.9523 | 1.2292  | 0.9750   | 9.8985 | 1.4116  |
|     |   | 0.9 | 0.9740   | 9.9293 | 1.2100  | 0.9742   | 9.8714 | 1.3724  |
|     | 4 | 0   | 0.9732   | 9.8293 | 0.0044  | 0.9740   | 9.7976 | 0.1108  |
|     |   | 0.5 | 0.9732   | 9.8391 | -0.4678 | 0.9728   | 9.7962 | -0.2490 |
|     |   | 0.9 | 0.9750   | 9.9112 | -1.9072 | 0.9746   | 9.8848 | -1.8164 |
|     | 5 | 0   | 0.9730   | 9.7616 | -0.1250 | 0.9732   | 9.7456 | -0.0772 |
|     |   | 0.5 | 0.9728   | 9.7794 | -0.8186 | 0.9724   | 9.7506 | -0.6098 |
|     |   | 0.9 | 0.9748   | 9.9013 | -2.9142 | 0.9740   | 9.8778 | -2.7928 |
| 100 | 1 | 0   | 0.9722   | 9.2154 | 1.2606  | 0.9716   | 9.1810 | 1.4812  |
|     |   | 0.5 | 0.9708   | 9.2085 | 1.2518  | 0.9704   | 9.1769 | 1.4584  |
|     |   | 0.9 | 0.9714   | 9.1908 | 1.1746  | 0.9710   | 9.1664 | 1.3160  |
|     | 4 | 0   | 0.9708   | 9.1401 | 0.0742  | 0.9706   | 9.1261 | 0.1482  |
|     |   | 0.5 | 0.9706   | 9.1371 | -0.0608 | 0.9708   | 9.1237 | 0.0614  |
|     |   | 0.9 | 0.9722   | 9.1798 | -1.9200 | 0.9716   | 9.1681 | -1.7926 |
|     | 5 | 0   | 0.9698   | 9.1007 | -0.0144 | 0.9698   | 9.0984 | -0.0088 |
|     |   | 0.5 | 0.9696   | 9.1017 | -0.2460 | 0.9698   | 9.0989 | -0.1474 |
|     |   | 0.9 | 0.9710   | 9.1626 | -2.7926 | 0.9712   | 9.1454 | -2.5192 |
| 500 | 1 | 0   | 0.9564   | 6.6901 | 1.2574  | 0.9566   | 6.6863 | 1.4778  |
|     |   | 0.5 | 0.9564   | 6.6885 | 1.2706  | 0.9560   | 6.6848 | 1.4766  |
|     |   | 0.9 | 0.9560   | 6.6876 | 1.1408  | 0.9558   | 6.6849 | 1.2690  |
|     | 4 | 0   | 0.9554   | 6.6815 | 0.0782  | 0.9554   | 6.6800 | 0.1592  |
|     |   | 0.5 | 0.9558   | 6.6813 | 0.0780  | 0.9558   | 6.6797 | 0.1518  |
|     |   | 0.9 | 0.9570   | 6.6866 | -1.1268 | 0.9564   | 6.6848 | -0.9246 |
|     | 5 | 0   | 0.9558   | 6.6770 | 0.0000  | 0.9558   | 6.6770 | 0.0000  |
|     |   | 0.5 | 0.9558   | 6.6771 | -0.0010 | 0.9558   | 6.6771 | -0.0004 |
|     |   | 0.9 | 0.9566   | 6.6858 | -1.7416 | 0.9558   | 6.6826 | -1.4552 |

Table 5.16.  $p = 10$ , error type = 2

|      |    |     | LASSO    |        |         | RIDGE    |         |         |
|------|----|-----|----------|--------|---------|----------|---------|---------|
| n    | a  | psi | coverage | length | penalty | coverage | length  | penalty |
| 100  | 1  | 0   | 0.9732   | 9.8031 | 0.1885  | 0.9712   | 9.5736  | 2.4373  |
|      |    | 0.5 | 0.9738   | 9.7603 | 0.0678  | 0.9728   | 9.7888  | 0.6444  |
|      |    | 0.9 | 0.9742   | 9.9237 | 0.0239  | 0.9758   | 9.9960  | 0.3979  |
|      | 9  | 0   | 0.9704   | 9.4358 | 0.0202  | 0.9712   | 9.4726  | 0.1962  |
|      |    | 0.5 | 0.9698   | 9.4715 | 0.0933  | 0.9718   | 9.8063  | 1.4780  |
|      |    | 0.9 | 0.9724   | 9.6836 | 0.1567  | 0.9754   | 10.0675 | 2.3817  |
|      | 10 | 0   | 0.9708   | 9.4322 | 0.0107  | 0.9712   | 9.4726  | 0.1953  |
|      |    | 0.5 | 0.9706   | 9.4374 | 0.1046  | 0.9726   | 9.7356  | 1.6951  |
|      |    | 0.9 | 0.9724   | 9.6742 | 0.1763  | 0.9760   | 10.0780 | 2.6794  |
| 200  | 1  | 0   | 0.9726   | 8.6020 | 0.1343  | 0.9712   | 8.4802  | 0.5794  |
|      |    | 0.5 | 0.9724   | 8.5839 | 0.0474  | 0.9716   | 8.5922  | 0.2878  |
|      |    | 0.9 | 0.9736   | 8.6439 | 0.0205  | 0.9738   | 8.6873  | 0.3223  |
|      | 9  | 0   | 0.9712   | 8.4562 | 0.0146  | 0.9702   | 8.4743  | 0.1531  |
|      |    | 0.5 | 0.9704   | 8.4726 | 0.0931  | 0.9706   | 8.6535  | 1.4221  |
|      |    | 0.9 | 0.9710   | 8.5629 | 0.1574  | 0.9734   | 8.7538  | 2.3925  |
|      | 10 | 0   | 0.9702   | 8.4521 | 0.0079  | 0.9716   | 8.4749  | 0.1557  |
|      |    | 0.5 | 0.9702   | 8.4609 | 0.1046  | 0.9720   | 8.5921  | 1.6020  |
|      |    | 0.9 | 0.9718   | 8.5692 | 0.1771  | 0.9728   | 8.7664  | 2.6915  |
| 1000 | 1  | 0   | 0.9544   | 6.6473 | 0.0615  | 0.9550   | 6.6317  | 0.1243  |
|      |    | 0.5 | 0.9542   | 6.6435 | 0.0218  | 0.9540   | 6.6877  | 0.1929  |
|      |    | 0.9 | 0.9540   | 6.6482 | 0.0197  | 0.9546   | 6.6623  | 0.3021  |
|      | 9  | 0   | 0.9542   | 6.6291 | 0.0088  | 0.9560   | 6.6390  | 0.1262  |
|      |    | 0.5 | 0.9560   | 6.6387 | 0.0930  | 0.9540   | 6.7324  | 1.4139  |
|      |    | 0.9 | 0.9532   | 6.6868 | 0.1577  | 0.9542   | 6.7320  | 2.3974  |
|      | 10 | 0   | 0.9540   | 6.6287 | 0.0074  | 0.9556   | 6.6397  | 0.1276  |
|      |    | 0.5 | 0.9556   | 6.6399 | 0.1045  | 0.9548   | 6.6726  | 1.5888  |
|      |    | 0.9 | 0.9538   | 6.7075 | 0.1774  | 0.9538   | 6.7404  | 2.6970  |

Table 5.17.  $p = 10$ , error type = 2

|      |    |     | PLS      |        |         | PCR      |        |         |
|------|----|-----|----------|--------|---------|----------|--------|---------|
| n    | a  | psi | coverage | length | penalty | coverage | length | penalty |
| 100  | 1  | 0   | 0.9704   | 9.4339 | 5.8158  | 0.9702   | 9.6013 | 1.4924  |
|      |    | 0.5 | 0.9702   | 9.4558 | 6.1860  | 0.9738   | 9.7155 | 3.8892  |
|      |    | 0.9 | 0.9710   | 9.4629 | 6.8616  | 0.9728   | 9.7908 | 5.8826  |
|      | 9  | 0   | 0.9702   | 9.4309 | 4.2226  | 0.9712   | 9.6089 | 0.2058  |
|      |    | 0.5 | 0.9700   | 9.4536 | 6.1666  | 0.9698   | 9.7056 | 3.8472  |
|      |    | 0.9 | 0.9708   | 9.4607 | 6.8572  | 0.9724   | 9.7880 | 5.8728  |
|      | 10 | 0   | 0.9700   | 9.4307 | 4.1580  | 0.9710   | 9.6262 | 0.1996  |
|      |    | 0.5 | 0.9710   | 9.4594 | 6.8820  | 0.9730   | 9.7896 | 5.9248  |
|      |    | 0.9 | 0.9710   | 9.4602 | 6.8824  | 0.9722   | 9.7927 | 5.9738  |
| 200  | 1  | 0   | 0.9706   | 8.4503 | 5.1664  | 0.9692   | 8.5011 | 0.5794  |
|      |    | 0.5 | 0.9706   | 8.4517 | 5.6052  | 0.9700   | 8.5243 | 2.5158  |
|      |    | 0.9 | 0.9706   | 8.4586 | 6.8662  | 0.9720   | 8.5923 | 5.8420  |
|      | 9  | 0   | 0.9706   | 8.4510 | 4.3542  | 0.9712   | 8.5289 | 0.0832  |
|      |    | 0.5 | 0.9706   | 8.4511 | 5.5864  | 0.9718   | 8.5308 | 2.5072  |
|      |    | 0.9 | 0.9700   | 8.4582 | 6.8660  | 0.9732   | 8.5946 | 5.8388  |
|      | 10 | 0   | 0.9708   | 8.4511 | 4.3026  | 0.9706   | 8.5380 | 0.0792  |
|      |    | 0.5 | 0.9708   | 8.4580 | 6.9066  | 0.9724   | 8.5976 | 5.9888  |
|      |    | 0.9 | 0.9698   | 8.4583 | 6.9064  | 0.9726   | 8.5968 | 6.0074  |
| 1000 | 1  | 0   | 0.9544   | 6.6284 | 4.8696  | 0.9542   | 6.6372 | 0.0586  |
|      |    | 0.5 | 0.9546   | 6.6285 | 4.6576  | 0.9544   | 6.6360 | 0.4072  |
|      |    | 0.9 | 0.9540   | 6.6286 | 6.7114  | 0.9542   | 6.6464 | 5.2610  |
|      | 9  | 0   | 0.9544   | 6.6284 | 4.7190  | 0.9544   | 6.6365 | 0.0050  |
|      |    | 0.5 | 0.9542   | 6.6285 | 4.6982  | 0.9548   | 6.6362 | 0.4388  |
|      |    | 0.9 | 0.9546   | 6.6286 | 6.6986  | 0.9540   | 6.6463 | 5.2606  |
|      | 10 | 0   | 0.9544   | 6.6284 | 4.6394  | 0.9548   | 6.6380 | 0.0062  |
|      |    | 0.5 | 0.9544   | 6.6285 | 6.9118  | 0.9532   | 6.6443 | 6.0560  |
|      |    | 0.9 | 0.9544   | 6.6285 | 6.9136  | 0.9540   | 6.6444 | 6.0688  |

Table 5.18.  $p = 10$ , error type = 2

|      |    |     | Ii       |        |         | I-Min    |        |         |
|------|----|-----|----------|--------|---------|----------|--------|---------|
| n    | a  | psi | coverage | length | penalty | coverage | length | penalty |
| 100  | 1  | 0   | 0.9740   | 9.7555 | 1.7610  | 0.9744   | 9.6646 | 2.2672  |
|      |    | 0.5 | 0.9722   | 9.7338 | 1.7844  | 0.9728   | 9.6648 | 2.2372  |
|      |    | 0.9 | 0.9730   | 9.7184 | 1.5574  | 0.9728   | 9.6564 | 1.9366  |
|      | 9  | 0   | 0.9702   | 9.4832 | 0.0462  | 0.9696   | 9.4645 | 0.1326  |
|      |    | 0.5 | 0.9694   | 9.4839 | -0.6098 | 0.9694   | 9.4628 | -0.3146 |
|      |    | 0.9 | 0.9716   | 9.6326 | -5.5992 | 0.9716   | 9.5871 | -5.1592 |
|      | 10 | 0   | 0.9704   | 9.4431 | -0.0488 | 0.9704   | 9.4383 | -0.0354 |
|      |    | 0.5 | 0.9694   | 9.4545 | -0.8196 | 0.9688   | 9.4381 | -0.5542 |
|      |    | 0.9 | 0.9718   | 9.6207 | -6.3884 | 0.9718   | 9.5803 | -5.9570 |
| 200  | 1  | 0   | 0.9716   | 8.5842 | 1.7310  | 0.9726   | 8.5490 | 2.2558  |
|      |    | 0.5 | 0.9718   | 8.5798 | 1.8276  | 0.9708   | 8.5498 | 2.2810  |
|      |    | 0.9 | 0.9726   | 8.5704 | 1.5138  | 0.9714   | 8.5467 | 1.9066  |
|      | 9  | 0   | 0.9706   | 8.4701 | 0.0752  | 0.9706   | 8.4645 | 0.1476  |
|      |    | 0.5 | 0.9708   | 8.4721 | -0.0212 | 0.9708   | 8.4645 | 0.0838  |
|      |    | 0.9 | 0.9704   | 8.5072 | -4.9152 | 0.9712   | 8.4919 | -4.4408 |
|      | 10 | 0   | 0.9708   | 8.4538 | -0.0064 | 0.9706   | 8.4530 | -0.0042 |
|      |    | 0.5 | 0.9708   | 8.4577 | -0.1224 | 0.9706   | 8.4544 | -0.0700 |
|      |    | 0.9 | 0.9718   | 8.4975 | -5.6226 | 0.9704   | 8.4841 | -5.1410 |
| 1000 | 1  | 0   | 0.9544   | 6.6434 | 1.7086  | 0.9548   | 6.6399 | 2.2424  |
|      |    | 0.5 | 0.9538   | 6.6426 | 1.7946  | 0.9552   | 6.6393 | 2.2636  |
|      |    | 0.9 | 0.9540   | 6.6437 | 1.5758  | 0.9544   | 6.6404 | 1.9936  |
|      | 9  | 0   | 0.9540   | 6.6307 | 0.0778  | 0.9544   | 6.6303 | 0.1500  |
|      |    | 0.5 | 0.9538   | 6.6303 | 0.0774  | 0.9542   | 6.6298 | 0.1466  |
|      |    | 0.9 | 0.9530   | 6.6365 | -2.2948 | 0.9538   | 6.6329 | -1.7454 |
|      | 10 | 0   | 0.9544   | 6.6287 | -0.0006 | 0.9544   | 6.6287 | -0.0006 |
|      |    | 0.5 | 0.9544   | 6.6288 | -0.0018 | 0.9544   | 6.6285 | -0.0010 |
|      |    | 0.9 | 0.9546   | 6.6364 | -2.6880 | 0.9544   | 6.6327 | -2.1304 |

Table 5.19.  $p = 20$ , error type = 2

|      |    |     | LASSO    |         |         | RIDGE    |         |         |
|------|----|-----|----------|---------|---------|----------|---------|---------|
| n    | a  | psi | coverage | length  | penalty | coverage | length  | penalty |
| 200  | 1  | 0   | 0.9774   | 9.1651  | 0.1693  | 0.9740   | 8.8793  | 1.0859  |
|      |    | 0.5 | 0.9776   | 9.1295  | 0.0435  | 0.9770   | 9.1726  | 0.4242  |
|      |    | 0.9 | 0.9786   | 9.2281  | 0.0285  | 0.9792   | 9.2857  | 0.4653  |
|      | 19 | 0   | 0.9720   | 8.7404  | 0.0096  | 0.9740   | 8.7861  | 0.1840  |
|      |    | 0.5 | 0.9744   | 8.9431  | 0.3434  | 0.9780   | 9.3115  | 4.8382  |
|      |    | 0.9 | 0.9836   | 12.6456 | 1.5176  | 0.9860   | 12.7337 | 33.5835 |
|      | 20 | 0   | 0.9720   | 8.7378  | 0.0076  | 0.9742   | 8.7864  | 0.1864  |
|      |    | 0.5 | 0.9752   | 9.0368  | 0.4167  | 0.9788   | 9.3056  | 6.6668  |
|      |    | 0.9 | 0.9842   | 13.2791 | 1.6803  | 0.9846   | 13.3655 | 37.0735 |
| 400  | 1  | 0   | 0.9760   | 8.6580  | 0.1216  | 0.9736   | 8.4833  | 0.3732  |
|      |    | 0.5 | 0.9760   | 8.6398  | 0.0311  | 0.9756   | 8.7012  | 0.2900  |
|      |    | 0.9 | 0.9760   | 8.6831  | 0.0284  | 0.9764   | 8.7205  | 0.4662  |
|      | 19 | 0   | 0.9736   | 8.4476  | 0.0082  | 0.9728   | 8.4718  | 0.1597  |
|      |    | 0.5 | 0.9736   | 8.5499  | 0.2972  | 0.9748   | 8.7906  | 4.5522  |
|      |    | 0.9 | 0.9736   | 8.4465  | 0.0073  | 0.9734   | 8.4722  | 0.1617  |
|      | 20 | 0   | 0.9722   | 8.5738  | 0.3301  | 0.9752   | 8.7286  | 5.8459  |
|      |    | 0.5 | 0.9722   | 8.5738  | 0.3301  | 0.9752   | 8.7286  | 5.8459  |
|      |    | 0.9 | 0.9776   | 12.1085 | 1.5951  | 0.9796   | 12.1732 | 36.3487 |
| 2000 | 1  | 0   | 0.9518   | 6.6713  | 0.0556  | 0.9502   | 6.6482  | 0.1293  |
|      |    | 0.5 | 0.9500   | 6.6726  | 0.0182  | 0.9482   | 6.7635  | 0.2675  |
|      |    | 0.9 | 0.9508   | 6.6758  | 0.0285  | 0.9502   | 6.6882  | 0.5724  |
|      | 19 | 0   | 0.9490   | 6.6455  | 0.0072  | 0.9498   | 6.6586  | 0.1300  |
|      |    | 0.5 | 0.9500   | 6.7354  | 0.2919  | 0.9492   | 6.8468  | 4.5562  |
|      |    | 0.9 | 0.9524   | 9.1544  | 1.2731  | 0.9512   | 9.2063  | 31.3306 |
|      | 20 | 0   | 0.9500   | 6.6454  | 0.0073  | 0.9496   | 6.6587  | 0.1307  |
|      |    | 0.5 | 0.9494   | 6.7452  | 0.3081  | 0.9504   | 6.7558  | 5.6440  |
|      |    | 0.9 | 0.9518   | 9.6919  | 1.4577  | 0.9530   | 9.7518  | 35.0398 |

Table 5.20.  $p = 20$ , error type = 2

|      |    |     | PLS      |        |         | PCR      |        |         |
|------|----|-----|----------|--------|---------|----------|--------|---------|
| n    | a  | psi | coverage | length | penalty | coverage | length | penalty |
| 200  | 1  | 0   | 0.9714   | 8.7420 | 15.8412 | 0.9718   | 8.8462 | 2.4950  |
|      |    | 0.5 | 0.9726   | 8.7632 | 16.3288 | 0.9734   | 9.0415 | 9.6008  |
|      |    | 0.9 | 0.9730   | 8.7789 | 16.9664 | 0.9766   | 9.1590 | 15.1610 |
|      | 19 | 0   | 0.9708   | 8.7361 | 13.0184 | 0.9708   | 8.7907 | 0.0614  |
|      |    | 0.5 | 0.9730   | 8.7659 | 16.3194 | 0.9766   | 9.0447 | 9.6424  |
|      |    | 0.9 | 0.9736   | 8.7774 | 16.9692 | 0.9768   | 9.1617 | 15.1982 |
|      | 20 | 0   | 0.9710   | 8.7360 | 12.9582 | 0.9704   | 8.7925 | 0.0590  |
|      |    | 0.5 | 0.9738   | 8.7767 | 16.9748 | 0.9780   | 9.1623 | 15.3854 |
|      |    | 0.9 | 0.9732   | 8.7768 | 16.9746 | 0.9774   | 9.1612 | 15.3770 |
| 400  | 1  | 0   | 0.9732   | 8.4455 | 15.0306 | 0.9712   | 8.4693 | 0.7044  |
|      |    | 0.5 | 0.9728   | 8.4480 | 15.6188 | 0.9736   | 8.5351 | 5.4056  |
|      |    | 0.9 | 0.9736   | 8.4591 | 16.9674 | 0.9754   | 8.6583 | 15.1924 |
|      | 19 | 0   | 0.9732   | 8.4451 | 13.4362 | 0.9732   | 8.4564 | 0.0148  |
|      |    | 0.5 | 0.9728   | 8.4478 | 15.6112 | 0.9748   | 8.5285 | 5.4374  |
|      |    | 0.9 | 0.9732   | 8.4452 | 13.4536 | 0.9730   | 8.4583 | 0.0136  |
|      | 20 | 0   | 0.9740   | 8.4572 | 16.9836 | 0.9756   | 8.6558 | 15.5362 |
|      |    | 0.5 | 0.9740   | 8.4572 | 16.9836 | 0.9756   | 8.6558 | 15.5362 |
|      |    | 0.9 | 0.9740   | 8.4576 | 16.9848 | 0.9762   | 8.6553 | 15.5128 |
| 2000 | 1  | 0   | 0.9494   | 6.6447 | 14.6820 | 0.9494   | 6.6466 | 0.0566  |
|      |    | 0.5 | 0.9496   | 6.6447 | 14.4850 | 0.9492   | 6.6476 | 0.5116  |
|      |    | 0.9 | 0.9496   | 6.6451 | 16.8462 | 0.9510   | 6.6731 | 13.9172 |
|      | 19 | 0   | 0.9496   | 6.6448 | 14.1506 | 0.9494   | 6.6451 | 0.0004  |
|      |    | 0.5 | 0.9498   | 6.6447 | 14.4988 | 0.9494   | 6.6481 | 0.4860  |
|      |    | 0.9 | 0.9494   | 6.6451 | 16.8554 | 0.9508   | 6.6726 | 13.9260 |
|      | 20 | 0   | 0.9494   | 6.6447 | 14.1484 | 0.9494   | 6.6448 | 0.0000  |
|      |    | 0.5 | 0.9496   | 6.6451 | 16.9842 | 0.9500   | 6.6715 | 15.6582 |
|      |    | 0.9 | 0.9494   | 6.6451 | 16.9856 | 0.9512   | 6.6713 | 15.6668 |

Table 5.21.  $p = 20$ , error type = 2

|      |    |     | Ii       |        |          | I-Min    |        |          |
|------|----|-----|----------|--------|----------|----------|--------|----------|
| n    | a  | psi | coverage | length | penalty  | coverage | length | penalty  |
| 200  | 1  | 0   | 0.9770   | 9.0424 | 2.7820   | 0.9756   | 8.9704 | 3.8412   |
|      |    | 0.5 | 0.9760   | 9.0295 | 2.9532   | 0.9754   | 8.9704 | 3.8660   |
|      |    | 0.9 | 0.9750   | 9.0272 | 2.4360   | 0.9760   | 8.9685 | 3.3324   |
|      | 19 | 0   | 0.9722   | 8.7571 | 0.0754   | 0.9722   | 8.7518 | 0.1482   |
|      |    | 0.5 | 0.9726   | 8.7656 | -0.2044  | 0.9726   | 8.7577 | -0.0168  |
|      |    | 0.9 | 0.9734   | 8.8747 | -11.3566 | 0.9728   | 8.8400 | -10.4328 |
|      | 20 | 0   | 0.9708   | 8.7395 | -0.0088  | 0.9708   | 8.7385 | -0.0056  |
|      |    | 0.5 | 0.9706   | 8.7518 | -0.3198  | 0.9708   | 8.7454 | -0.1868  |
|      |    | 0.9 | 0.9732   | 8.8656 | -12.0570 | 0.9732   | 8.8321 | -11.1370 |
| 400  | 1  | 0   | 0.9748   | 8.5994 | 2.7550   | 0.9760   | 8.5623 | 3.7974   |
|      |    | 0.5 | 0.9760   | 8.5913 | 2.9214   | 0.9750   | 8.5609 | 3.8520   |
|      |    | 0.9 | 0.9750   | 8.5914 | 2.4534   | 0.9750   | 8.5628 | 3.3772   |
|      | 19 | 0   | 0.9730   | 8.4561 | 0.0794   | 0.9726   | 8.4524 | 0.1534   |
|      |    | 0.5 | 0.9736   | 8.4571 | 0.0552   | 0.9736   | 8.4523 | 0.1356   |
|      |    | 0.9 | 0.9732   | 8.4465 | -0.0034  | 0.9732   | 8.4463 | -0.0028  |
|      | 20 | 0   | 0.9732   | 8.4490 | -0.0340  | 0.9732   | 8.4463 | -0.0230  |
|      |    | 0.5 | 0.9732   | 8.4490 | -0.0340  | 0.9732   | 8.4463 | -0.0230  |
|      |    | 0.9 | 0.9730   | 8.4777 | -9.7906  | 0.9734   | 8.4661 | -8.7850  |
| 2000 | 1  | 0   | 0.9506   | 6.6646 | 2.7472   | 0.9512   | 6.6593 | 3.8168   |
|      |    | 0.5 | 0.9510   | 6.6633 | 2.9374   | 0.9504   | 6.6594 | 3.8520   |
|      |    | 0.9 | 0.9506   | 6.6631 | 2.8362   | 0.9500   | 6.6592 | 3.7752   |
|      | 19 | 0   | 0.9498   | 6.6462 | 0.0812   | 0.9498   | 6.6456 | 0.1586   |
|      |    | 0.5 | 0.9498   | 6.6464 | 0.0758   | 0.9496   | 6.6461 | 0.1536   |
|      |    | 0.9 | 0.9486   | 6.6497 | -2.3264  | 0.9498   | 6.6478 | -1.5346  |
|      | 20 | 0   | 0.9494   | 6.6448 | -0.0002  | 0.9494   | 6.6448 | 0.0000   |
|      |    | 0.5 | 0.9494   | 6.6455 | -0.0044  | 0.9494   | 6.6453 | -0.0038  |
|      |    | 0.9 | 0.9486   | 6.6490 | -2.5668  | 0.9498   | 6.6470 | -1.7768  |

Table 5.22.  $p = 50$ , error type = 2

|      |    |     | LASSO    |         |         | RIDGE    |         |          |
|------|----|-----|----------|---------|---------|----------|---------|----------|
| n    | a  | psi | coverage | length  | penalty | coverage | length  | penalty  |
| 500  | 1  | 0   | 0.9818   | 9.4225  | 0.1347  | 0.9790   | 9.0358  | 0.7386   |
|      |    | 0.5 | 0.9828   | 9.4364  | 0.0287  | 0.9824   | 9.5243  | 0.4336   |
|      |    | 0.9 | 0.9814   | 9.4715  | 0.0456  | 0.9822   | 9.5027  | 2.3224   |
|      | 49 | 0   | 0.9778   | 8.8799  | 0.0070  | 0.9766   | 8.9282  | 0.1972   |
|      |    | 0.5 | 0.9860   | 30.1081 | 5.3846  | 0.9866   | 30.0153 | 275.5417 |
|      |    | 0.9 | 0.9866   | 52.1355 | 8.8119  | 0.9874   | 50.9115 | 493.0807 |
|      | 50 | 0   | 0.9778   | 8.8791  | 0.0070  | 0.9760   | 8.9278  | 0.1981   |
|      |    | 0.5 | 0.9864   | 30.7435 | 5.5035  | 0.9866   | 30.5126 | 281.7523 |
|      |    | 0.9 | 0.9864   | 53.1942 | 8.9968  | 0.9878   | 51.9359 | 503.3529 |
| 1000 | 1  | 0   | 0.9772   | 8.7520  | 0.0969  | 0.9764   | 8.5333  | 0.4373   |
|      |    | 0.5 | 0.9776   | 8.7618  | 0.0265  | 0.9772   | 8.8804  | 0.4032   |
|      |    | 0.9 | 0.9788   | 8.7811  | 0.0456  | 0.9786   | 8.7970  | 2.3223   |
|      | 49 | 0   | 0.9752   | 8.4963  | 0.0071  | 0.9766   | 8.5251  | 0.1690   |
|      |    | 0.5 | 0.9806   | 27.8260 | 5.4041  | 0.9812   | 27.7226 | 275.7521 |
|      |    | 0.9 | 0.9824   | 48.1866 | 8.8213  | 0.9812   | 47.0304 | 493.3596 |
|      | 50 | 0   | 0.9754   | 8.4962  | 0.0071  | 0.9762   | 8.5253  | 0.1699   |
|      |    | 0.5 | 0.9788   | 28.3990 | 5.5201  | 0.9822   | 28.1825 | 281.8970 |
|      |    | 0.9 | 0.9816   | 49.1633 | 9.0053  | 0.9810   | 47.9782 | 503.6379 |
| 5000 | 1  | 0   | 0.9502   | 6.6946  | 0.0435  | 0.9512   | 6.6661  | 0.1145   |
|      |    | 0.5 | 0.9508   | 6.6966  | 0.0260  | 0.9532   | 6.8585  | 0.3961   |
|      |    | 0.9 | 0.9510   | 6.7011  | 0.0456  | 0.9516   | 6.7109  | 2.3267   |
|      | 49 | 0   | 0.9508   | 6.6652  | 0.0072  | 0.9500   | 6.6785  | 0.1341   |
|      |    | 0.5 | 0.9562   | 23.5115 | 5.4222  | 0.9560   | 23.4344 | 276.2702 |
|      |    | 0.9 | 0.9574   | 40.8167 | 8.8335  | 0.9542   | 39.8174 | 493.8967 |
|      | 50 | 0   | 0.9506   | 6.6652  | 0.0073  | 0.9504   | 6.6787  | 0.1344   |
|      |    | 0.5 | 0.9564   | 23.9868 | 5.5373  | 0.9562   | 23.8197 | 282.2842 |
|      |    | 0.9 | 0.9578   | 41.6460 | 9.0175  | 0.9542   | 40.6225 | 504.1950 |

Table 5.23.  $p = 50$ , error type = 2

|      |    |     | PLS      |        |         | PCR      |        |         |
|------|----|-----|----------|--------|---------|----------|--------|---------|
| n    | a  | psi | coverage | length | penalty | coverage | length | penalty |
| 500  | 1  | 0   | 0.9774   | 8.8801 | 46.1228 | 0.9764   | 8.9420 | 4.2508  |
|      |    | 0.5 | 0.9772   | 8.9121 | 46.4948 | 0.9810   | 9.2933 | 28.9052 |
|      |    | 0.9 | 0.9774   | 8.9250 | 46.9988 | 0.9818   | 9.4350 | 44.5870 |
|      | 49 | 0   | 0.9760   | 8.8757 | 41.6654 | 0.9754   | 8.8804 | 0.0066  |
|      |    | 0.5 | 0.9784   | 8.9189 | 46.4856 | 0.9796   | 9.3059 | 29.0592 |
|      |    | 0.9 | 0.9778   | 8.9264 | 46.9988 | 0.9812   | 9.4346 | 44.6098 |
|      | 50 | 0   | 0.9760   | 8.8758 | 41.6040 | 0.9758   | 8.8808 | 0.0064  |
|      |    | 0.5 | 0.9776   | 8.9238 | 46.9992 | 0.9808   | 9.4307 | 44.9424 |
|      |    | 0.9 | 0.9776   | 8.9238 | 46.9992 | 0.9814   | 9.4307 | 44.9104 |
| 1000 | 1  | 0   | 0.9762   | 8.4919 | 44.9470 | 0.9758   | 8.5020 | 0.9236  |
|      |    | 0.5 | 0.9762   | 8.4947 | 45.8202 | 0.9766   | 8.5784 | 12.7002 |
|      |    | 0.9 | 0.9754   | 8.5071 | 46.9982 | 0.9780   | 8.7624 | 44.5290 |
|      | 49 | 0   | 0.9758   | 8.4921 | 42.5464 | 0.9760   | 8.4921 | 0.0010  |
|      |    | 0.5 | 0.9760   | 8.4957 | 45.7988 | 0.9760   | 8.5793 | 12.7304 |
|      |    | 0.9 | 0.9754   | 8.5071 | 46.9974 | 0.9784   | 8.7611 | 44.5094 |
|      | 50 | 0   | 0.9758   | 8.4921 | 42.5476 | 0.9760   | 8.4921 | 0.0010  |
|      |    | 0.5 | 0.9750   | 8.5063 | 46.9998 | 0.9778   | 8.7597 | 45.0994 |
|      |    | 0.9 | 0.9750   | 8.5064 | 46.9998 | 0.9772   | 8.7590 | 45.0922 |
| 5000 | 1  | 0   | 0.9516   | 6.6625 | 44.5236 | 0.9518   | 6.6629 | 0.0508  |
|      |    | 0.5 | 0.9514   | 6.6625 | 44.2860 | 0.9522   | 6.6639 | 0.5356  |
|      |    | 0.9 | 0.9508   | 6.6630 | 46.9780 | 0.9514   | 6.7008 | 42.2984 |
|      | 49 | 0   | 0.9516   | 6.6625 | 43.6690 | 0.9518   | 6.6625 | 0.0002  |
|      |    | 0.5 | 0.9516   | 6.6625 | 44.3504 | 0.9518   | 6.6639 | 0.5338  |
|      |    | 0.9 | 0.9506   | 6.6630 | 46.9776 | 0.9530   | 6.7003 | 42.3266 |
|      | 50 | 0   | 0.9516   | 6.6625 | 43.6522 | 0.9518   | 6.6625 | 0.0000  |
|      |    | 0.5 | 0.9512   | 6.6630 | 47.0000 | 0.9520   | 6.6963 | 45.4536 |
|      |    | 0.9 | 0.9514   | 6.6630 | 47.0000 | 0.9526   | 6.6960 | 45.4698 |

Table 5.24.  $p = 50$ , error type = 2

|      |    |     | Ii       |        |          | I-Min    |        |          |
|------|----|-----|----------|--------|----------|----------|--------|----------|
| n    | a  | psi | coverage | length | penalty  | coverage | length | penalty  |
| 500  | 1  | 0   | 0.9808   | 9.2020 | 6.3806   | 0.9796   | 9.1381 | 8.5888   |
|      |    | 0.5 | 0.9804   | 9.1973 | 6.5332   | 0.9794   | 9.1387 | 8.5762   |
|      |    | 0.9 | 0.9806   | 9.1981 | 6.0156   | 0.9802   | 9.1392 | 8.0574   |
|      | 49 | 0   | 0.9772   | 8.8841 | 0.0766   | 0.9766   | 8.8817 | 0.1542   |
|      |    | 0.5 | 0.9770   | 8.8886 | 0.0386   | 0.9766   | 8.8840 | 0.1232   |
|      |    | 0.9 | 0.9764   | 8.9375 | -22.4510 | 0.9764   | 8.9170 | -20.4886 |
|      | 50 | 0   | 0.9760   | 8.8770 | -0.0048  | 0.9760   | 8.8767 | -0.0038  |
|      |    | 0.5 | 0.9760   | 8.8810 | -0.0468  | 0.9760   | 8.8792 | -0.0344  |
|      |    | 0.9 | 0.9744   | 8.9390 | -23.0094 | 0.9756   | 8.9174 | -21.0708 |
| 1000 | 1  | 0   | 0.9760   | 8.6466 | 6.3550   | 0.9764   | 8.6159 | 8.6144   |
|      |    | 0.5 | 0.9764   | 8.6463 | 6.5334   | 0.9766   | 8.6162 | 8.6132   |
|      |    | 0.9 | 0.9760   | 8.6466 | 6.1966   | 0.9768   | 8.6160 | 8.2950   |
|      | 49 | 0   | 0.9760   | 8.4981 | 0.0840   | 0.9764   | 8.4966 | 0.1618   |
|      |    | 0.5 | 0.9756   | 8.4983 | 0.0702   | 0.9760   | 8.4965 | 0.1528   |
|      |    | 0.9 | 0.9766   | 8.4991 | -14.5582 | 0.9766   | 8.4936 | -12.5694 |
|      | 50 | 0   | 0.9760   | 8.4937 | -0.0042  | 0.9760   | 8.4934 | -0.0038  |
|      |    | 0.5 | 0.9760   | 8.4943 | -0.0166  | 0.9760   | 8.4938 | -0.0148  |
|      |    | 0.9 | 0.9760   | 8.4964 | -14.9778 | 0.9768   | 8.4911 | -13.0010 |
| 5000 | 1  | 0   | 0.9512   | 6.6821 | 6.3234   | 0.9526   | 6.6785 | 8.5896   |
|      |    | 0.5 | 0.9520   | 6.6815 | 6.4884   | 0.9526   | 6.6785 | 8.6036   |
|      |    | 0.9 | 0.9522   | 6.6817 | 6.4806   | 0.9528   | 6.6785 | 8.5976   |
|      | 49 | 0   | 0.9500   | 6.6628 | 0.0914   | 0.9504   | 6.6627 | 0.1722   |
|      |    | 0.5 | 0.9508   | 6.6632 | 0.0866   | 0.9506   | 6.6629 | 0.1680   |
|      |    | 0.9 | 0.9506   | 6.6636 | -0.3440  | 0.9506   | 6.6630 | -0.0446  |
|      | 50 | 0   | 0.9518   | 6.6625 | 0.0000   | 0.9518   | 6.6625 | 0.0000   |
|      |    | 0.5 | 0.9518   | 6.6627 | -0.0010  | 0.9518   | 6.6626 | -0.0008  |
|      |    | 0.9 | 0.9514   | 6.6630 | -0.4526  | 0.9516   | 6.6627 | -0.2166  |

### 5.1.5 Error type 3

Table 5.25.  $p = 5$ , error type = 3

|     |   |     | LASSO    |        |         | RIDGE    |        |         |
|-----|---|-----|----------|--------|---------|----------|--------|---------|
| n   | a | psi | coverage | length | penalty | coverage | length | penalty |
| 50  | 1 | 0   | 0.9830   | 5.3971 | 0.1074  | 0.9826   | 5.4047 | 0.2590  |
|     |   | 0.5 | 0.9840   | 5.3570 | 0.0537  | 0.9836   | 5.4984 | 0.2100  |
|     |   | 0.9 | 0.9850   | 5.3324 | 0.0201  | 0.9860   | 5.4049 | 0.2122  |
|     | 4 | 0   | 0.9832   | 5.3884 | 0.0313  | 0.9838   | 5.4169 | 0.1401  |
|     |   | 0.5 | 0.9832   | 5.3712 | 0.0340  | 0.9852   | 5.5493 | 0.4079  |
|     |   | 0.9 | 0.9850   | 5.3487 | 0.0408  | 0.9862   | 5.5233 | 0.6000  |
|     | 5 | 0   | 0.9828   | 5.3722 | 0.0112  | 0.9836   | 5.4330 | 0.1472  |
|     |   | 0.5 | 0.9832   | 5.3763 | 0.0358  | 0.9854   | 5.4164 | 0.5373  |
|     |   | 0.9 | 0.9850   | 5.3589 | 0.0532  | 0.9872   | 5.5624 | 0.7997  |
| 100 | 1 | 0   | 0.9792   | 4.6866 | 0.0764  | 0.9792   | 4.7108 | 0.1163  |
|     |   | 0.5 | 0.9794   | 4.6585 | 0.0372  | 0.9790   | 4.8174 | 0.1510  |
|     |   | 0.9 | 0.9804   | 4.6332 | 0.0155  | 0.9796   | 4.6938 | 0.2023  |
|     | 4 | 0   | 0.9790   | 4.7007 | 0.0216  | 0.9792   | 4.7432 | 0.1255  |
|     |   | 0.5 | 0.9786   | 4.6896 | 0.0282  | 0.9782   | 4.8906 | 0.3946  |
|     |   | 0.9 | 0.9794   | 4.6738 | 0.0397  | 0.9794   | 4.8170 | 0.6033  |
|     | 5 | 0   | 0.9790   | 4.6950 | 0.0084  | 0.9792   | 4.7560 | 0.1333  |
|     |   | 0.5 | 0.9790   | 4.7007 | 0.0342  | 0.9792   | 4.7483 | 0.5199  |
|     |   | 0.9 | 0.9788   | 4.6987 | 0.0529  | 0.9808   | 4.8642 | 0.8041  |
| 500 | 1 | 0   | 0.9534   | 3.3387 | 0.0345  | 0.9536   | 3.3994 | 0.1096  |
|     |   | 0.5 | 0.9536   | 3.3281 | 0.0167  | 0.9524   | 3.5598 | 0.1450  |
|     |   | 0.9 | 0.9534   | 3.3110 | 0.0134  | 0.9550   | 3.4118 | 0.2030  |
|     | 4 | 0   | 0.9538   | 3.3576 | 0.0114  | 0.9536   | 3.4362 | 0.1162  |
|     |   | 0.5 | 0.9548   | 3.3542 | 0.0260  | 0.9508   | 3.6422 | 0.3953  |
|     |   | 0.9 | 0.9548   | 3.3682 | 0.0400  | 0.9536   | 3.5558 | 0.6078  |
|     | 5 | 0   | 0.9544   | 3.3588 | 0.0074  | 0.9536   | 3.4470 | 0.1190  |
|     |   | 0.5 | 0.9538   | 3.3702 | 0.0342  | 0.9530   | 3.4966 | 0.5203  |
|     |   | 0.9 | 0.9540   | 3.4032 | 0.0533  | 0.9544   | 3.6134 | 0.8101  |

Table 5.26.  $p = 5$ , error type = 3

|     |   |     | PLS      |        |         | PCR      |        |         |
|-----|---|-----|----------|--------|---------|----------|--------|---------|
| n   | a | psi | coverage | length | penalty | coverage | length | penalty |
| 50  | 1 | 0   | 0.9824   | 5.3711 | 0.7084  | 0.9824   | 5.6776 | 0.3608  |
|     |   | 0.5 | 0.9828   | 5.3725 | 0.8734  | 0.9838   | 5.5524 | 0.7206  |
|     |   | 0.9 | 0.9832   | 5.3700 | 1.7394  | 0.9842   | 5.3832 | 1.5168  |
|     | 4 | 0   | 0.9826   | 5.3697 | 0.6206  | 0.9826   | 5.8634 | 0.2440  |
|     |   | 0.5 | 0.9828   | 5.3714 | 0.8476  | 0.9826   | 5.5521 | 0.7104  |
|     |   | 0.9 | 0.9830   | 5.3688 | 1.7350  | 0.9842   | 5.3866 | 1.5264  |
|     | 5 | 0   | 0.9826   | 5.3695 | 0.5564  | 0.9822   | 5.9911 | 0.2706  |
|     |   | 0.5 | 0.9830   | 5.3691 | 1.8016  | 0.9848   | 5.3706 | 1.5692  |
|     |   | 0.9 | 0.9834   | 5.3694 | 1.8066  | 0.9854   | 5.3603 | 1.5962  |
| 100 | 1 | 0   | 0.9794   | 4.6938 | 0.5902  | 0.9784   | 4.8992 | 0.2132  |
|     |   | 0.5 | 0.9792   | 4.6941 | 0.5570  | 0.9790   | 4.8307 | 0.4576  |
|     |   | 0.9 | 0.9794   | 4.6928 | 1.6934  | 0.9796   | 4.6902 | 1.4826  |
|     | 4 | 0   | 0.9792   | 4.6936 | 0.5748  | 0.9782   | 5.0291 | 0.1586  |
|     |   | 0.5 | 0.9790   | 4.6940 | 0.5618  | 0.9778   | 4.8233 | 0.4476  |
|     |   | 0.9 | 0.9792   | 4.6928 | 1.7156  | 0.9796   | 4.6896 | 1.4842  |
|     | 5 | 0   | 0.9794   | 4.6937 | 0.5370  | 0.9782   | 5.0842 | 0.1586  |
|     |   | 0.5 | 0.9792   | 4.6929 | 1.8034  | 0.9804   | 4.6621 | 1.5820  |
|     |   | 0.9 | 0.9792   | 4.6926 | 1.8004  | 0.9802   | 4.6584 | 1.5930  |
| 500 | 1 | 0   | 0.9538   | 3.3569 | 0.5450  | 0.9528   | 3.4145 | 0.0490  |
|     |   | 0.5 | 0.9540   | 3.3568 | 0.3200  | 0.9536   | 3.4055 | 0.1146  |
|     |   | 0.9 | 0.9538   | 3.3568 | 1.2922  | 0.9538   | 3.3729 | 1.1474  |
|     | 4 | 0   | 0.9538   | 3.3569 | 0.5558  | 0.9538   | 3.4135 | 0.0250  |
|     |   | 0.5 | 0.9536   | 3.3569 | 0.3320  | 0.9536   | 3.4030 | 0.1098  |
|     |   | 0.9 | 0.9538   | 3.3568 | 1.3070  | 0.9538   | 3.3725 | 1.1226  |
|     | 5 | 0   | 0.9538   | 3.3569 | 0.5330  | 0.9528   | 3.4414 | 0.0320  |
|     |   | 0.5 | 0.9540   | 3.3567 | 1.8106  | 0.9546   | 3.3274 | 1.6084  |
|     |   | 0.9 | 0.9540   | 3.3567 | 1.8144  | 0.9554   | 3.3254 | 1.6332  |

Table 5.27.  $p = 5$ , error type = 3

|     |   |     | Ii       |        |         | I-min    |        |         |
|-----|---|-----|----------|--------|---------|----------|--------|---------|
| n   | a | psi | coverage | length | penalty | coverage | length | penalty |
| 50  | 1 | 0   | 0.9850   | 5.3322 | 1.2696  | 0.9842   | 5.3407 | 1.4802  |
|     |   | 0.5 | 0.9844   | 5.3367 | 1.2660  | 0.9840   | 5.3435 | 1.4808  |
|     |   | 0.9 | 0.9842   | 5.3473 | 1.1708  | 0.9840   | 5.3534 | 1.3122  |
|     | 4 | 0   | 0.9836   | 5.3703 | 0.0922  | 0.9832   | 5.3668 | 0.1658  |
|     |   | 0.5 | 0.9824   | 5.3935 | 0.0202  | 0.9820   | 5.3782 | 0.1272  |
|     |   | 0.9 | 0.9834   | 5.5029 | -1.8616 | 0.9834   | 5.4701 | -1.6752 |
|     | 5 | 0   | 0.9826   | 5.3703 | -0.0014 | 0.9826   | 5.3694 | -0.0002 |
|     |   | 0.5 | 0.9820   | 5.4099 | -0.1250 | 0.9826   | 5.3855 | -0.0652 |
|     |   | 0.9 | 0.9832   | 5.5483 | -2.6116 | 0.9838   | 5.4813 | -2.3550 |
| 100 | 1 | 0   | 0.9794   | 4.6305 | 1.2684  | 0.9798   | 4.6495 | 1.4934  |
|     |   | 0.5 | 0.9794   | 4.6325 | 1.2792  | 0.9784   | 4.6481 | 1.4922  |
|     |   | 0.9 | 0.9790   | 4.6588 | 1.1566  | 0.9786   | 4.6676 | 1.2888  |
|     | 4 | 0   | 0.9798   | 4.6810 | 0.0918  | 0.9798   | 4.6839 | 0.1672  |
|     |   | 0.5 | 0.9796   | 4.6789 | 0.0900  | 0.9800   | 4.6822 | 0.1660  |
|     |   | 0.9 | 0.9778   | 4.7955 | -1.5076 | 0.9782   | 4.7520 | -1.2298 |
|     | 5 | 0   | 0.9794   | 4.6935 | 0.0000  | 0.9794   | 4.6935 | 0.0000  |
|     |   | 0.5 | 0.9794   | 4.6939 | -0.0012 | 0.9794   | 4.6937 | -0.0008 |
|     |   | 0.9 | 0.9776   | 4.8079 | -2.1136 | 0.9774   | 4.7774 | -1.9196 |
| 500 | 1 | 0   | 0.9562   | 3.3125 | 1.2588  | 0.9558   | 3.3246 | 1.4698  |
|     |   | 0.5 | 0.9546   | 3.3131 | 1.2860  | 0.9544   | 3.3245 | 1.4844  |
|     |   | 0.9 | 0.9534   | 3.3243 | 1.1884  | 0.9536   | 3.3307 | 1.4024  |
|     | 4 | 0   | 0.9548   | 3.3449 | 0.0854  | 0.9546   | 3.3482 | 0.1610  |
|     |   | 0.5 | 0.9540   | 3.3447 | 0.0880  | 0.9544   | 3.3483 | 0.1636  |
|     |   | 0.9 | 0.9538   | 3.3844 | -0.3814 | 0.9542   | 3.3681 | -0.1452 |
|     | 5 | 0   | 0.9538   | 3.3569 | 0.0000  | 0.9538   | 3.3569 | 0.0000  |
|     |   | 0.5 | 0.9538   | 3.3569 | 0.0000  | 0.9538   | 3.3569 | 0.0000  |
|     |   | 0.9 | 0.9538   | 3.4090 | -0.7266 | 0.9546   | 3.3848 | -0.4884 |

Table 5.28.  $p = 10$ , error type = 3

|      |    |     | LASSO    |        |         | RIDGE    |        |         |
|------|----|-----|----------|--------|---------|----------|--------|---------|
| n    | a  | psi | coverage | length | penalty | coverage | length | penalty |
| 100  | 1  | 0   | 0.9826   | 5.0712 | 0.1104  | 0.9824   | 5.1036 | 0.1668  |
|      |    | 0.5 | 0.9836   | 5.0281 | 0.0397  | 0.9830   | 5.3268 | 0.2090  |
|      |    | 0.9 | 0.9852   | 5.0034 | 0.0205  | 0.9862   | 5.0745 | 0.3013  |
|      | 9  | 0   | 0.9804   | 5.0877 | 0.0130  | 0.9814   | 5.1694 | 0.1590  |
|      |    | 0.5 | 0.9814   | 5.1293 | 0.0929  | 0.9838   | 5.4815 | 1.4125  |
|      |    | 0.9 | 0.9832   | 5.3474 | 0.1613  | 0.9864   | 5.4732 | 2.3956  |
|      | 10 | 0   | 0.9798   | 5.0819 | 0.0081  | 0.9816   | 5.1779 | 0.1644  |
|      |    | 0.5 | 0.9808   | 5.1406 | 0.1044  | 0.9842   | 5.1754 | 1.5884  |
|      |    | 0.9 | 0.9826   | 5.4734 | 0.2028  | 0.9870   | 5.5471 | 2.7749  |
| 200  | 1  | 0   | 0.9762   | 4.2971 | 0.0798  | 0.9748   | 4.3708 | 0.1128  |
|      |    | 0.5 | 0.9764   | 4.2725 | 0.0282  | 0.9756   | 4.6320 | 0.1899  |
|      |    | 0.9 | 0.9766   | 4.2469 | 0.0197  | 0.9780   | 4.3240 | 0.2992  |
|      | 9  | 0   | 0.9734   | 4.3620 | 0.0099  | 0.9718   | 4.4411 | 0.1429  |
|      |    | 0.5 | 0.9732   | 4.4064 | 0.0929  | 0.9760   | 4.7785 | 1.4123  |
|      |    | 0.9 | 0.9754   | 4.6542 | 0.1573  | 0.9774   | 4.7147 | 2.3893  |
|      | 10 | 0   | 0.9732   | 4.3619 | 0.0074  | 0.9734   | 4.4484 | 0.1467  |
|      |    | 0.5 | 0.9738   | 4.4271 | 0.1044  | 0.9760   | 4.4684 | 1.5872  |
|      |    | 0.9 | 0.9738   | 4.7459 | 0.1794  | 0.9774   | 4.7585 | 2.6975  |
| 1000 | 1  | 0   | 0.9548   | 3.2772 | 0.0357  | 0.9562   | 3.3704 | 0.1097  |
|      |    | 0.5 | 0.9550   | 3.2658 | 0.0140  | 0.9548   | 3.7015 | 0.1900  |
|      |    | 0.9 | 0.9552   | 3.2490 | 0.0197  | 0.9546   | 3.3741 | 0.3001  |
|      | 9  | 0   | 0.9544   | 3.3299 | 0.0073  | 0.9550   | 3.4351 | 0.1231  |
|      |    | 0.5 | 0.9540   | 3.4106 | 0.0930  | 0.9540   | 3.8415 | 1.4131  |
|      |    | 0.9 | 0.9542   | 3.7372 | 0.1578  | 0.9586   | 3.7640 | 2.3978  |
|      | 10 | 0   | 0.9552   | 3.3331 | 0.0073  | 0.9548   | 3.4386 | 0.1247  |
|      |    | 0.5 | 0.9544   | 3.4342 | 0.1045  | 0.9574   | 3.5554 | 1.5880  |
|      |    | 0.9 | 0.9558   | 3.8301 | 0.1775  | 0.9590   | 3.8024 | 2.6975  |

Table 5.29.  $p = 10$ , error type = 3

|      |    |     | PLS      |        |         | PCR      |        |         |
|------|----|-----|----------|--------|---------|----------|--------|---------|
| n    | a  | psi | coverage | length | penalty | coverage | length | penalty |
| 100  | 1  | 0   | 0.9804   | 5.0809 | 4.6928  | 0.9814   | 5.2110 | 0.4524  |
|      |    | 0.5 | 0.9808   | 5.0840 | 5.2992  | 0.9824   | 5.2301 | 1.9986  |
|      |    | 0.9 | 0.9816   | 5.0806 | 6.8046  | 0.9862   | 5.0805 | 5.7256  |
|      | 9  | 0   | 0.9802   | 5.0794 | 3.7304  | 0.9792   | 5.2952 | 0.0992  |
|      |    | 0.5 | 0.9804   | 5.0842 | 5.2704  | 0.9814   | 5.2292 | 1.9652  |
|      |    | 0.9 | 0.9814   | 5.0819 | 6.8188  | 0.9846   | 5.0809 | 5.7016  |
|      | 10 | 0   | 0.9804   | 5.0794 | 3.6872  | 0.9800   | 5.2914 | 0.0894  |
|      |    | 0.5 | 0.9814   | 5.0804 | 6.8760  | 0.9846   | 5.0418 | 5.9098  |
|      |    | 0.9 | 0.9818   | 5.0812 | 6.8840  | 0.9858   | 5.0300 | 5.9556  |
| 200  | 1  | 0   | 0.9742   | 4.3602 | 4.5072  | 0.9740   | 4.4271 | 0.1650  |
|      |    | 0.5 | 0.9744   | 4.3601 | 4.7540  | 0.9730   | 4.4286 | 0.8790  |
|      |    | 0.9 | 0.9738   | 4.3581 | 6.7838  | 0.9756   | 4.3359 | 5.5262  |
|      | 9  | 0   | 0.9738   | 4.3600 | 4.0600  | 0.9738   | 4.4630 | 0.0374  |
|      |    | 0.5 | 0.9734   | 4.3605 | 4.7258  | 0.9734   | 4.4273 | 0.8574  |
|      |    | 0.9 | 0.9748   | 4.3582 | 6.7866  | 0.9766   | 4.3355 | 5.5400  |
|      | 10 | 0   | 0.9738   | 4.3599 | 3.9968  | 0.9738   | 4.4561 | 0.0360  |
|      |    | 0.5 | 0.9740   | 4.3584 | 6.8838  | 0.9760   | 4.2779 | 6.0038  |
|      |    | 0.9 | 0.9742   | 4.3584 | 6.8954  | 0.9758   | 4.2720 | 6.0278  |
| 1000 | 1  | 0   | 0.9556   | 3.3283 | 4.6964  | 0.9556   | 3.3351 | 0.0088  |
|      |    | 0.5 | 0.9554   | 3.3283 | 4.2268  | 0.9552   | 3.3425 | 0.1080  |
|      |    | 0.9 | 0.9556   | 3.3284 | 6.1330  | 0.9546   | 3.3436 | 3.8246  |
|      | 9  | 0   | 0.9554   | 3.3283 | 4.4694  | 0.9560   | 3.3380 | 0.0030  |
|      |    | 0.5 | 0.9554   | 3.3283 | 4.1932  | 0.9548   | 3.3453 | 0.1192  |
|      |    | 0.9 | 0.9554   | 3.3283 | 6.1082  | 0.9546   | 3.3436 | 3.8692  |
|      | 10 | 0   | 0.9556   | 3.3282 | 4.4778  | 0.9556   | 3.3367 | 0.0022  |
|      |    | 0.5 | 0.9556   | 3.3278 | 6.9114  | 0.9550   | 3.2683 | 6.0526  |
|      |    | 0.9 | 0.9558   | 3.3279 | 6.9136  | 0.9548   | 3.2649 | 6.0808  |

Table 5.30.  $p = 10$ , error type = 3

|      |    |     | Ii       |        |         | I-min    |        |         |
|------|----|-----|----------|--------|---------|----------|--------|---------|
| n    | a  | psi | coverage | length | penalty | coverage | length | penalty |
| 100  | 1  | 0   | 0.9854   | 5.0381 | 1.7550  | 0.9840   | 5.0599 | 2.2854  |
|      |    | 0.5 | 0.9842   | 5.0401 | 1.8302  | 0.9840   | 5.0561 | 2.3010  |
|      |    | 0.9 | 0.9842   | 5.0676 | 1.5234  | 0.9828   | 5.0783 | 1.9142  |
|      | 9  | 0   | 0.9806   | 5.0846 | 0.0880  | 0.9808   | 5.0831 | 0.1588  |
|      |    | 0.5 | 0.9808   | 5.0876 | 0.0742  | 0.9812   | 5.0842 | 0.1558  |
|      |    | 0.9 | 0.9794   | 5.2621 | -4.4998 | 0.9790   | 5.2020 | -4.0242 |
|      | 10 | 0   | 0.9804   | 5.0793 | 0.0000  | 0.9804   | 5.0793 | 0.0000  |
|      |    | 0.5 | 0.9802   | 5.0834 | -0.0182 | 0.9804   | 5.0809 | -0.0086 |
|      |    | 0.9 | 0.9792   | 5.2740 | -5.1806 | 0.9792   | 5.2123 | -4.6892 |
| 200  | 1  | 0   | 0.9760   | 4.2766 | 1.7322  | 0.9750   | 4.3042 | 2.2608  |
|      |    | 0.5 | 0.9764   | 4.2799 | 1.8004  | 0.9752   | 4.3031 | 2.2700  |
|      |    | 0.9 | 0.9758   | 4.3098 | 1.5284  | 0.9752   | 4.3243 | 1.9450  |
|      | 9  | 0   | 0.9744   | 4.3547 | 0.0868  | 0.9746   | 4.3567 | 0.1616  |
|      |    | 0.5 | 0.9742   | 4.3545 | 0.0882  | 0.9744   | 4.3558 | 0.1624  |
|      |    | 0.9 | 0.9724   | 4.4999 | -3.3820 | 0.9726   | 4.4541 | -2.8298 |
|      | 10 | 0   | 0.9740   | 4.3598 | 0.0000  | 0.9740   | 4.3598 | 0.0000  |
|      |    | 0.5 | 0.9740   | 4.3598 | 0.0000  | 0.9740   | 4.3598 | 0.0000  |
|      |    | 0.9 | 0.9728   | 4.5147 | -3.9214 | 0.9730   | 4.4668 | -3.3652 |
| 1000 | 1  | 0   | 0.9562   | 3.2716 | 1.7262  | 0.9560   | 3.2901 | 2.2778  |
|      |    | 0.5 | 0.9546   | 3.2739 | 1.8216  | 0.9542   | 3.2901 | 2.2988  |
|      |    | 0.9 | 0.9550   | 3.2761 | 1.8126  | 0.9540   | 3.2909 | 2.2830  |
|      | 9  | 0   | 0.9546   | 3.3229 | 0.0816  | 0.9552   | 3.3246 | 0.1616  |
|      |    | 0.5 | 0.9544   | 3.3229 | 0.0856  | 0.9544   | 3.3244 | 0.1576  |
|      |    | 0.9 | 0.9534   | 3.3448 | -0.3118 | 0.9536   | 3.3338 | -0.0584 |
|      | 10 | 0   | 0.9556   | 3.3283 | 0.0000  | 0.9556   | 3.3283 | 0.0000  |
|      |    | 0.5 | 0.9556   | 3.3283 | 0.0000  | 0.9556   | 3.3283 | 0.0000  |
|      |    | 0.9 | 0.9536   | 3.3531 | -0.4700 | 0.9550   | 3.3387 | -0.2510 |

Table 5.31.  $p = 20$ , error type = 3

|      |    |     | LASSO    |         |         | RIDGE    |         |         |
|------|----|-----|----------|---------|---------|----------|---------|---------|
| n    | a  | psi | coverage | length  | penalty | coverage | length  | penalty |
| 200  | 1  | 0   | 0.9824   | 4.6413  | 0.0999  | 0.9774   | 4.7307  | 0.1594  |
|      |    | 0.5 | 0.9824   | 4.6062  | 0.0260  | 0.9806   | 5.1541  | 0.2581  |
|      |    | 0.9 | 0.9832   | 4.5717  | 0.0284  | 0.9836   | 4.6540  | 0.4641  |
|      | 19 | 0   | 0.9778   | 4.7178  | 0.0076  | 0.9792   | 4.8370  | 0.1767  |
|      |    | 0.5 | 0.9800   | 7.3327  | 0.9679  | 0.9846   | 7.3026  | 15.8974 |
|      |    | 0.9 | 0.9840   | 12.3131 | 1.9318  | 0.9870   | 12.3060 | 41.5356 |
|      | 20 | 0   | 0.9780   | 4.7186  | 0.0071  | 0.9786   | 4.8413  | 0.1794  |
|      |    | 0.5 | 0.9820   | 7.7210  | 1.0659  | 0.9844   | 7.7211  | 19.8842 |
|      |    | 0.9 | 0.9836   | 13.0334 | 2.0737  | 0.9868   | 12.9457 | 44.2639 |
| 400  | 1  | 0   | 0.9828   | 4.2143  | 0.0713  | 0.9788   | 4.3344  | 0.1110  |
|      |    | 0.5 | 0.9818   | 4.1890  | 0.0197  | 0.9808   | 4.7985  | 0.2567  |
|      |    | 0.9 | 0.9820   | 4.1562  | 0.0284  | 0.9820   | 4.2628  | 0.5064  |
|      | 19 | 0   | 0.9778   | 4.3287  | 0.0072  | 0.9788   | 4.4366  | 0.1548  |
|      |    | 0.5 | 0.9796   | 6.6807  | 0.9281  | 0.9798   | 6.6507  | 15.3194 |
|      |    | 0.9 | 0.9812   | 11.2254 | 1.8863  | 0.9822   | 11.3035 | 41.4603 |
|      | 20 | 0   | 0.9774   | 4.3311  | 0.0072  | 0.9780   | 4.4395  | 0.1570  |
|      |    | 0.5 | 0.9784   | 7.0554  | 1.0317  | 0.9810   | 7.0420  | 19.4679 |
|      |    | 0.9 | 0.9826   | 11.8164 | 2.0112  | 0.9830   | 11.8246 | 43.8866 |
| 2000 | 1  | 0   | 0.9560   | 3.2424  | 0.0323  | 0.9556   | 3.3633  | 0.1097  |
|      |    | 0.5 | 0.9572   | 3.2297  | 0.0169  | 0.9552   | 3.8902  | 0.2573  |
|      |    | 0.9 | 0.9572   | 3.2177  | 0.0285  | 0.9544   | 3.3652  | 0.5681  |
|      | 19 | 0   | 0.9552   | 3.3305  | 0.0072  | 0.9544   | 3.4476  | 0.1283  |
|      |    | 0.5 | 0.9556   | 5.4232  | 0.8877  | 0.9568   | 5.3949  | 14.6967 |
|      |    | 0.9 | 0.9526   | 9.4912  | 1.8847  | 0.9530   | 9.4901  | 41.5643 |
|      | 20 | 0   | 0.9550   | 3.3331  | 0.0073  | 0.9546   | 3.4494  | 0.1290  |
|      |    | 0.5 | 0.9562   | 5.7651  | 0.9985  | 0.9546   | 5.7452  | 18.9901 |
|      |    | 0.9 | 0.9512   | 9.9318  | 1.9896  | 0.9522   | 9.9133  | 43.8733 |

Table 5.32.  $p = 20$ , error type = 3

|      |    |     | PLS      |        |         | PCR      |        |         |
|------|----|-----|----------|--------|---------|----------|--------|---------|
| n    | a  | psi | coverage | length | penalty | coverage | length | penalty |
| 200  | 1  | 0   | 0.9770   | 4.7157 | 14.2744 | 0.9768   | 4.7673 | 0.5028  |
|      |    | 0.5 | 0.9780   | 4.7186 | 15.2660 | 0.9780   | 4.8293 | 3.8724  |
|      |    | 0.9 | 0.9794   | 4.7181 | 16.9438 | 0.9830   | 4.6843 | 14.7874 |
|      | 19 | 0   | 0.9772   | 4.7148 | 12.5086 | 0.9768   | 4.7696 | 0.0252  |
|      |    | 0.5 | 0.9776   | 4.7185 | 15.2758 | 0.9788   | 4.8251 | 3.7584  |
|      |    | 0.9 | 0.9784   | 4.7179 | 16.9406 | 0.9824   | 4.6837 | 14.7720 |
|      | 20 | 0   | 0.9774   | 4.7148 | 12.4680 | 0.9766   | 4.7686 | 0.0234  |
|      |    | 0.5 | 0.9784   | 4.7170 | 16.9688 | 0.9828   | 4.6176 | 15.2716 |
|      |    | 0.9 | 0.9790   | 4.7177 | 16.9690 | 0.9826   | 4.6051 | 15.3358 |
| 400  | 1  | 0   | 0.9776   | 4.3263 | 14.1218 | 0.9778   | 4.3503 | 0.1480  |
|      |    | 0.5 | 0.9778   | 4.3267 | 14.5622 | 0.9768   | 4.3626 | 1.2574  |
|      |    | 0.9 | 0.9770   | 4.3227 | 16.9174 | 0.9804   | 4.2811 | 14.4142 |
|      | 19 | 0   | 0.9778   | 4.3264 | 13.1406 | 0.9774   | 4.3487 | 0.0066  |
|      |    | 0.5 | 0.9774   | 4.3264 | 14.5928 | 0.9778   | 4.3649 | 1.2694  |
|      |    | 0.9 | 0.9774   | 4.3230 | 16.9124 | 0.9806   | 4.2819 | 14.3648 |
|      | 20 | 0   | 0.9774   | 4.3263 | 13.1106 | 0.9774   | 4.3453 | 0.0058  |
|      |    | 0.5 | 0.9776   | 4.3232 | 16.9690 | 0.9806   | 4.1915 | 15.4536 |
|      |    | 0.9 | 0.9774   | 4.3230 | 16.9732 | 0.9802   | 4.1844 | 15.4850 |
| 2000 | 1  | 0   | 0.9544   | 3.3221 | 14.4822 | 0.9542   | 3.3237 | 0.0062  |
|      |    | 0.5 | 0.9542   | 3.3221 | 13.9006 | 0.9544   | 3.3276 | 0.0936  |
|      |    | 0.9 | 0.9550   | 3.3220 | 16.2294 | 0.9558   | 3.3373 | 9.2508  |
|      | 19 | 0   | 0.9542   | 3.3221 | 13.9064 | 0.9542   | 3.3221 | 0.0000  |
|      |    | 0.5 | 0.9540   | 3.3221 | 13.8656 | 0.9542   | 3.3282 | 0.0878  |
|      |    | 0.9 | 0.9550   | 3.3221 | 16.2146 | 0.9546   | 3.3374 | 9.2892  |
|      | 20 | 0   | 0.9542   | 3.3221 | 13.8924 | 0.9542   | 3.3221 | 0.0000  |
|      |    | 0.5 | 0.9550   | 3.3217 | 16.9828 | 0.9560   | 3.2323 | 15.5672 |
|      |    | 0.9 | 0.9554   | 3.3217 | 16.9848 | 0.9560   | 3.2287 | 15.6332 |

Table 5.33.  $p = 20$ , error type = 3

|      |    |     | Ii       |        |         | I-min    |        |         |
|------|----|-----|----------|--------|---------|----------|--------|---------|
| n    | a  | psi | coverage | length | penalty | coverage | length | penalty |
| 200  | 1  | 0   | 0.9802   | 4.6653 | 2.7964  | 0.9794   | 4.6904 | 3.8424  |
|      |    | 0.5 | 0.9804   | 4.6672 | 2.9548  | 0.9800   | 4.6899 | 3.8700  |
|      |    | 0.9 | 0.9802   | 4.6928 | 2.5464  | 0.9794   | 4.7055 | 3.4368  |
|      | 19 | 0   | 0.9770   | 4.7157 | 0.0836  | 0.9768   | 4.7152 | 0.1502  |
|      |    | 0.5 | 0.9762   | 4.7157 | 0.0804  | 0.9760   | 4.7153 | 0.1516  |
|      |    | 0.9 | 0.9760   | 4.9224 | -7.9698 | 0.9760   | 4.8610 | -6.9806 |
|      | 20 | 0   | 0.9770   | 4.7144 | 0.0000  | 0.9770   | 4.7144 | 0.0000  |
|      |    | 0.5 | 0.9770   | 4.7144 | 0.0000  | 0.9770   | 4.7144 | 0.0000  |
|      |    | 0.9 | 0.9752   | 4.9290 | -8.5182 | 0.9746   | 4.8658 | -7.5174 |
| 400  | 1  | 0   | 0.9808   | 4.2467 | 2.7842  | 0.9806   | 4.2741 | 3.8372  |
|      |    | 0.5 | 0.9810   | 4.2505 | 2.9346  | 0.9800   | 4.2743 | 3.8554  |
|      |    | 0.9 | 0.9812   | 4.2681 | 2.6724  | 0.9802   | 4.2842 | 3.6174  |
|      | 19 | 0   | 0.9780   | 4.3231 | 0.0928  | 0.9778   | 4.3244 | 0.1696  |
|      |    | 0.5 | 0.9780   | 4.3237 | 0.0932  | 0.9778   | 4.3247 | 0.1716  |
|      |    | 0.9 | 0.9762   | 4.4720 | -4.8410 | 0.9768   | 4.4248 | -3.8356 |
|      | 20 | 0   | 0.9776   | 4.3262 | 0.0000  | 0.9776   | 4.3262 | 0.0000  |
|      |    | 0.5 | 0.9776   | 4.3262 | 0.0000  | 0.9776   | 4.3262 | 0.0000  |
|      |    | 0.9 | 0.9764   | 4.4788 | -5.2336 | 0.9766   | 4.4291 | -4.2234 |
| 2000 | 1  | 0   | 0.9566   | 3.2646 | 2.7564  | 0.9558   | 3.2827 | 3.8554  |
|      |    | 0.5 | 0.9566   | 3.2676 | 2.9378  | 0.9564   | 3.2831 | 3.8916  |
|      |    | 0.9 | 0.9560   | 3.2679 | 2.9424  | 0.9558   | 3.2832 | 3.8938  |
|      | 19 | 0   | 0.9536   | 3.3196 | 0.0808  | 0.9540   | 3.3202 | 0.1516  |
|      |    | 0.5 | 0.9538   | 3.3195 | 0.0796  | 0.9542   | 3.3203 | 0.1536  |
|      |    | 0.9 | 0.9540   | 3.3213 | 0.0380  | 0.9544   | 3.3209 | 0.1352  |
|      | 20 | 0   | 0.9542   | 3.3221 | 0.0000  | 0.9542   | 3.3221 | 0.0000  |
|      |    | 0.5 | 0.9542   | 3.3221 | 0.0000  | 0.9542   | 3.3221 | 0.0000  |
|      |    | 0.9 | 0.9542   | 3.3240 | -0.0488 | 0.9542   | 3.3227 | -0.0190 |

Table 5.34.  $p = 50$ , error type = 3

|      |    |     | LASSO    |         |         | RIDGE    |         |          |
|------|----|-----|----------|---------|---------|----------|---------|----------|
| n    | a  | psi | coverage | length  | penalty | coverage | length  | penalty  |
| 500  | 1  | 0   | 0.9844   | 4.5541  | 0.0791  | 0.9770   | 4.7143  | 0.1568   |
|      |    | 0.5 | 0.9848   | 4.5272  | 0.0261  | 0.9840   | 5.4186  | 0.3949   |
|      |    | 0.9 | 0.9844   | 4.4756  | 0.0455  | 0.9852   | 4.6259  | 2.3180   |
|      | 49 | 0   | 0.9768   | 4.7182  | 0.0069  | 0.9768   | 4.8689  | 0.1937   |
|      |    | 0.5 | 0.9852   | 29.5055 | 5.4280  | 0.9868   | 29.1946 | 276.5110 |
|      |    | 0.9 | 0.9866   | 51.5629 | 8.8153  | 0.9874   | 50.3339 | 492.8452 |
|      | 50 | 0   | 0.9766   | 4.7196  | 0.0069  | 0.9764   | 4.8704  | 0.1948   |
|      |    | 0.5 | 0.9854   | 30.1311 | 5.5411  | 0.9862   | 29.6646 | 282.2699 |
|      |    | 0.9 | 0.9862   | 52.6287 | 8.9989  | 0.9868   | 51.3668 | 503.1126 |
| 1000 | 1  | 0   | 0.9806   | 4.1386  | 0.0564  | 0.9792   | 4.3234  | 0.1098   |
|      |    | 0.5 | 0.9808   | 4.1208  | 0.0260  | 0.9798   | 5.0363  | 0.3952   |
|      |    | 0.9 | 0.9812   | 4.0962  | 0.0455  | 0.9808   | 4.2482  | 2.3198   |
|      | 49 | 0   | 0.9776   | 4.3312  | 0.0071  | 0.9788   | 4.4600  | 0.1668   |
|      |    | 0.5 | 0.9826   | 27.1555 | 5.4301  | 0.9838   | 26.9770 | 276.6152 |
|      |    | 0.9 | 0.9826   | 47.6976 | 8.8219  | 0.9854   | 46.5351 | 493.2139 |
|      | 50 | 0   | 0.9776   | 4.3328  | 0.0071  | 0.9790   | 4.4616  | 0.1677   |
|      |    | 0.5 | 0.9824   | 27.7298 | 5.5432  | 0.9840   | 27.4175 | 282.3773 |
|      |    | 0.9 | 0.9826   | 48.6860 | 9.0057  | 0.9850   | 47.4911 | 503.4892 |
| 5000 | 1  | 0   | 0.9512   | 3.2152  | 0.0252  | 0.9492   | 3.3660  | 0.1098   |
|      |    | 0.5 | 0.9510   | 3.2162  | 0.0260  | 0.9496   | 4.1023  | 0.3957   |
|      |    | 0.9 | 0.9516   | 3.2416  | 0.0456  | 0.9500   | 3.3807  | 2.3230   |
|      | 49 | 0   | 0.9498   | 3.3499  | 0.0072  | 0.9504   | 3.4663  | 0.1335   |
|      |    | 0.5 | 0.9574   | 22.9331 | 5.4351  | 0.9572   | 22.8472 | 276.8707 |
|      |    | 0.9 | 0.9518   | 40.4491 | 8.8344  | 0.9598   | 39.4321 | 493.9118 |
|      | 50 | 0   | 0.9492   | 3.3516  | 0.0073  | 0.9494   | 3.4671  | 0.1339   |
|      |    | 0.5 | 0.9590   | 23.4118 | 5.5483  | 0.9602   | 23.2231 | 282.6369 |
|      |    | 0.9 | 0.9514   | 41.2857 | 9.0184  | 0.9592   | 40.2441 | 504.2016 |

Table 5.35.  $p = 50$ , error type = 3

|      |    |     | PLS      |        |         | PCR      |        |         |
|------|----|-----|----------|--------|---------|----------|--------|---------|
| n    | a  | psi | coverage | length | penalty | coverage | length | penalty |
| 500  | 1  | 0   | 0.9768   | 4.7075 | 44.1672 | 0.9774   | 4.7247 | 0.5576  |
|      |    | 0.5 | 0.9774   | 4.7111 | 45.5572 | 0.9776   | 4.7938 | 7.5190  |
|      |    | 0.9 | 0.9786   | 4.7083 | 46.9962 | 0.9850   | 4.6355 | 43.7766 |
|      | 49 | 0   | 0.9762   | 4.7069 | 41.0926 | 0.9760   | 4.7094 | 0.0008  |
|      |    | 0.5 | 0.9760   | 4.7108 | 45.5942 | 0.9752   | 4.7906 | 7.4866  |
|      |    | 0.9 | 0.9788   | 4.7080 | 46.9976 | 0.9842   | 4.6369 | 43.8716 |
|      | 50 | 0   | 0.9762   | 4.7070 | 41.1226 | 0.9762   | 4.7083 | 0.0006  |
|      |    | 0.5 | 0.9784   | 4.7077 | 46.9996 | 0.9840   | 4.5251 | 44.7806 |
|      |    | 0.9 | 0.9788   | 4.7078 | 46.9996 | 0.9842   | 4.5172 | 44.8132 |
| 1000 | 1  | 0   | 0.9786   | 4.3170 | 43.9064 | 0.9782   | 4.3250 | 0.1548  |
|      |    | 0.5 | 0.9790   | 4.3174 | 44.5994 | 0.9786   | 4.3336 | 1.6738  |
|      |    | 0.9 | 0.9790   | 4.3125 | 46.9950 | 0.9806   | 4.2448 | 43.2748 |
|      | 49 | 0   | 0.9784   | 4.3170 | 42.2222 | 0.9784   | 4.3170 | 0.0000  |
|      |    | 0.5 | 0.9776   | 4.3178 | 44.5306 | 0.9776   | 4.3346 | 1.6270  |
|      |    | 0.9 | 0.9782   | 4.3131 | 46.9930 | 0.9806   | 4.2460 | 43.2466 |
|      | 50 | 0   | 0.9784   | 4.3170 | 42.1958 | 0.9784   | 4.3170 | 0.0000  |
|      |    | 0.5 | 0.9790   | 4.3131 | 47.0000 | 0.9810   | 4.1135 | 45.1078 |
|      |    | 0.9 | 0.9790   | 4.3131 | 47.0000 | 0.9810   | 4.1075 | 45.1146 |
| 5000 | 1  | 0   | 0.9504   | 3.3242 | 44.2402 | 0.9498   | 3.3246 | 0.0060  |
|      |    | 0.5 | 0.9502   | 3.3242 | 43.6968 | 0.9500   | 3.3258 | 0.0856  |
|      |    | 0.9 | 0.9502   | 3.3239 | 46.3626 | 0.9500   | 3.3399 | 26.3572 |
|      | 49 | 0   | 0.9500   | 3.3242 | 43.4412 | 0.9500   | 3.3242 | 0.0000  |
|      |    | 0.5 | 0.9502   | 3.3242 | 43.6484 | 0.9500   | 3.3261 | 0.0838  |
|      |    | 0.9 | 0.9504   | 3.3239 | 46.3680 | 0.9512   | 3.3404 | 26.4844 |
|      | 50 | 0   | 0.9500   | 3.3242 | 43.4170 | 0.9500   | 3.3242 | 0.0000  |
|      |    | 0.5 | 0.9508   | 3.3236 | 47.0000 | 0.9514   | 3.2020 | 45.3678 |
|      |    | 0.9 | 0.9508   | 3.3236 | 47.0000 | 0.9514   | 3.1992 | 45.4166 |

Table 5.36.  $p = 50$ , error type = 3

|      |    |     | Ii       |        |          | I-min    |        |         |
|------|----|-----|----------|--------|----------|----------|--------|---------|
| n    | a  | psi | coverage | length | penalty  | coverage | length | penalty |
| 500  | 1  | 0   | 0.9818   | 4.6698 | 6.39     | 0.9806   | 4.6855 | 8.5948  |
|      |    | 0.5 | 0.9810   | 4.6695 | 6.5712   | 0.9808   | 4.6859 | 8.6494  |
|      |    | 0.9 | 0.9806   | 4.6779 | 6.3504   | 0.9804   | 4.6907 | 8.4494  |
|      | 49 | 0   | 0.9764   | 4.7069 | 0.0822   | 0.9762   | 4.7071 | 0.1616  |
|      |    | 0.5 | 0.9764   | 4.7067 | 0.0842   | 0.9762   | 4.7070 | 0.1566  |
|      |    | 0.9 | 0.9754   | 4.8867 | -10.6280 | 0.9748   | 4.8364 | -8.7944 |
|      | 50 | 0   | 0.9762   | 4.7067 | 0.0000   | 0.9762   | 4.7067 | 0.0000  |
|      |    | 0.5 | 0.9762   | 4.7067 | 0.0000   | 0.9762   | 4.7067 | 0.0000  |
|      |    | 0.9 | 0.9758   | 4.8883 | -11.0102 | 0.9752   | 4.8368 | -9.1566 |
| 1000 | 1  | 0   | 0.9796   | 4.2452 | 6.3632   | 0.9792   | 4.2643 | 8.5738  |
|      |    | 0.5 | 0.9796   | 4.2468 | 6.4996   | 0.9794   | 4.2639 | 8.5818  |
|      |    | 0.9 | 0.9800   | 4.2492 | 6.4520   | 0.9794   | 4.2649 | 8.5514  |
|      | 49 | 0   | 0.9778   | 4.3159 | 0.0822   | 0.9778   | 4.3160 | 0.1578  |
|      |    | 0.5 | 0.9782   | 4.3161 | 0.0820   | 0.9784   | 4.3162 | 0.1566  |
|      |    | 0.9 | 0.9772   | 4.3886 | -3.0014  | 0.9776   | 4.3553 | -1.8548 |
|      | 50 | 0   | 0.9784   | 4.3170 | 0.0000   | 0.9784   | 4.3170 | 0.0000  |
|      |    | 0.5 | 0.9784   | 4.3170 | 0.0000   | 0.9784   | 4.3170 | 0.0000  |
|      |    | 0.9 | 0.9776   | 4.3913 | -3.1898  | 0.9780   | 4.3574 | -2.0602 |
| 5000 | 1  | 0   | 0.9512   | 3.2726 | 6.3130   | 0.9508   | 3.2854 | 8.5656  |
|      |    | 0.5 | 0.9498   | 3.2736 | 6.4766   | 0.9512   | 3.2854 | 8.6114  |
|      |    | 0.9 | 0.9500   | 3.2735 | 6.4768   | 0.9512   | 3.2854 | 8.6062  |
|      | 49 | 0   | 0.9504   | 3.3232 | 0.0808   | 0.9504   | 3.3235 | 0.1552  |
|      |    | 0.5 | 0.9504   | 3.3232 | 0.0790   | 0.9504   | 3.3235 | 0.1560  |
|      |    | 0.9 | 0.9504   | 3.3232 | 0.0796   | 0.9504   | 3.3235 | 0.1560  |
|      | 50 | 0   | 0.9500   | 3.3242 | 0.0000   | 0.9500   | 3.3242 | 0.0000  |
|      |    | 0.5 | 0.9500   | 3.3242 | 0.0000   | 0.9500   | 3.3242 | 0.0000  |
|      |    | 0.9 | 0.9500   | 3.3242 | 0.0000   | 0.9500   | 3.3242 | 0.0000  |

### 5.1.6 Error type 4

Table 5.37.  $p = 5$ , error type = 4

|     |   |     | LASSO    |        |         | RIDGE    |        |         |
|-----|---|-----|----------|--------|---------|----------|--------|---------|
| n   | a | psi | coverage | length | penalty | coverage | length | penalty |
| 50  | 1 | 0   | 0.9982   | 2.8619 | 0.0631  | 0.9976   | 2.8912 | 0.1085  |
|     |   | 0.5 | 0.9978   | 2.8346 | 0.0309  | 0.9964   | 3.0781 | 0.1434  |
|     |   | 0.9 | 0.9994   | 2.8236 | 0.0155  | 0.9996   | 2.9107 | 0.2007  |
|     | 4 | 0   | 0.9972   | 2.8558 | 0.0193  | 0.9952   | 2.9491 | 0.1304  |
|     |   | 0.5 | 0.9976   | 2.8487 | 0.0276  | 0.9968   | 3.1653 | 0.3932  |
|     |   | 0.9 | 0.9992   | 2.8477 | 0.0396  | 0.9978   | 3.0785 | 0.6010  |
|     | 5 | 0   | 0.9978   | 2.8461 | 0.0088  | 0.9950   | 2.9759 | 0.1414  |
|     |   | 0.5 | 0.9972   | 2.8555 | 0.0343  | 0.9976   | 2.9748 | 0.5189  |
|     |   | 0.9 | 0.9980   | 2.8710 | 0.0528  | 0.9984   | 3.1526 | 0.8011  |
| 100 | 1 | 0   | 0.9970   | 2.4096 | 0.0450  | 0.9958   | 2.4693 | 0.1090  |
|     |   | 0.5 | 0.9974   | 2.3956 | 0.0220  | 0.9918   | 2.6638 | 0.1441  |
|     |   | 0.9 | 0.9992   | 2.3863 | 0.0137  | 0.9976   | 2.4742 | 0.2018  |
|     | 4 | 0   | 0.9958   | 2.4162 | 0.0140  | 0.9944   | 2.5128 | 0.1226  |
|     |   | 0.5 | 0.9970   | 2.4119 | 0.0262  | 0.9910   | 2.7427 | 0.3939  |
|     |   | 0.9 | 0.9974   | 2.4230 | 0.0398  | 0.9960   | 2.6368 | 0.6042  |
|     | 5 | 0   | 0.9964   | 2.4130 | 0.0076  | 0.9932   | 2.5294 | 0.1302  |
|     |   | 0.5 | 0.9954   | 2.4229 | 0.0342  | 0.9958   | 2.5569 | 0.5195  |
|     |   | 0.9 | 0.9956   | 2.4509 | 0.0530  | 0.9940   | 2.7085 | 0.8054  |
| 500 | 1 | 0   | 0.9738   | 1.9779 | 0.0204  | 0.9666   | 2.0309 | 0.1097  |
|     |   | 0.5 | 0.9726   | 1.9757 | 0.0110  | 0.9574   | 2.1895 | 0.1451  |
|     |   | 0.9 | 0.9744   | 1.9753 | 0.0134  | 0.9660   | 2.0425 | 0.2030  |
|     | 4 | 0   | 0.9702   | 1.9775 | 0.0082  | 0.9648   | 2.0522 | 0.1152  |
|     |   | 0.5 | 0.9680   | 1.9814 | 0.0260  | 0.9590   | 2.2484 | 0.3949  |
|     |   | 0.9 | 0.9676   | 1.9909 | 0.0400  | 0.9626   | 2.1663 | 0.6078  |
|     | 5 | 0   | 0.9686   | 1.9784 | 0.0074  | 0.9656   | 2.0590 | 0.1179  |
|     |   | 0.5 | 0.9662   | 1.9863 | 0.0342  | 0.9650   | 2.1088 | 0.5199  |
|     |   | 0.9 | 0.9670   | 2.0073 | 0.0533  | 0.9636   | 2.2205 | 0.8101  |

Table 5.38.  $p = 5$ , error type = 4

|     |   |     | PLS      |        |         | PCR      |        |         |
|-----|---|-----|----------|--------|---------|----------|--------|---------|
| n   | a | psi | coverage | length | penalty | coverage | length | penalty |
| 50  | 1 | 0   | 0.9976   | 2.8432 | 0.4028  | 0.9946   | 3.1518 | 0.2378  |
|     |   | 0.5 | 0.9978   | 2.8436 | 0.4414  | 0.9962   | 3.0032 | 0.3734  |
|     |   | 0.9 | 0.9974   | 2.8427 | 1.6148  | 0.9984   | 2.8616 | 1.3886  |
|     | 4 | 0   | 0.9974   | 2.8432 | 0.4030  | 0.9946   | 3.4154 | 0.1970  |
|     |   | 0.5 | 0.9976   | 2.8435 | 0.4172  | 0.9954   | 3.0152 | 0.3924  |
|     |   | 0.9 | 0.9974   | 2.8437 | 1.6324  | 0.9988   | 2.8626 | 1.4060  |
|     | 5 | 0   | 0.9976   | 2.8431 | 0.3638  | 0.9958   | 3.5434 | 0.2222  |
|     |   | 0.5 | 0.9976   | 2.8431 | 1.7366  | 0.9984   | 2.8429 | 1.5034  |
|     |   | 0.9 | 0.9976   | 2.8431 | 1.7592  | 0.9990   | 2.8342 | 1.5602  |
| 100 | 1 | 0   | 0.9942   | 2.4114 | 0.4034  | 0.9928   | 2.6002 | 0.1334  |
|     |   | 0.5 | 0.9946   | 2.4115 | 0.3036  | 0.9932   | 2.5267 | 0.2374  |
|     |   | 0.9 | 0.9948   | 2.4115 | 1.5054  | 0.9964   | 2.4276 | 1.2976  |
|     | 4 | 0   | 0.9944   | 2.4115 | 0.4070  | 0.9930   | 2.7250 | 0.1054  |
|     |   | 0.5 | 0.9942   | 2.4115 | 0.3102  | 0.9934   | 2.5306 | 0.2434  |
|     |   | 0.9 | 0.9946   | 2.4113 | 1.5224  | 0.9966   | 2.4287 | 1.2978  |
|     | 5 | 0   | 0.9942   | 2.4114 | 0.3854  | 0.9930   | 2.8391 | 0.1316  |
|     |   | 0.5 | 0.9956   | 2.4111 | 1.7676  | 0.9972   | 2.4005 | 1.5488  |
|     |   | 0.9 | 0.9956   | 2.4110 | 1.7898  | 0.9980   | 2.3940 | 1.6164  |
| 500 | 1 | 0   | 0.9686   | 1.9769 | 0.4560  | 0.9670   | 2.0133 | 0.0286  |
|     |   | 0.5 | 0.9688   | 1.9769 | 0.1930  | 0.9680   | 2.0075 | 0.0628  |
|     |   | 0.9 | 0.9694   | 1.9769 | 0.7244  | 0.9696   | 1.9874 | 0.6100  |
|     | 4 | 0   | 0.9688   | 1.9769 | 0.4696  | 0.9680   | 2.0390 | 0.0226  |
|     |   | 0.5 | 0.9682   | 1.9769 | 0.1876  | 0.9658   | 2.0123 | 0.0706  |
|     |   | 0.9 | 0.9682   | 1.9769 | 0.7024  | 0.9680   | 1.9876 | 0.6042  |
|     | 5 | 0   | 0.9690   | 1.9769 | 0.4464  | 0.9680   | 2.0512 | 0.0224  |
|     |   | 0.5 | 0.9696   | 1.9768 | 1.7800  | 0.9738   | 1.9761 | 1.5736  |
|     |   | 0.9 | 0.9706   | 1.9768 | 1.8022  | 0.9752   | 1.9755 | 1.6320  |

Table 5.39.  $p = 5$ , error type = 4

|     |   |     | Ii       |        |         | I-Min    |        |         |
|-----|---|-----|----------|--------|---------|----------|--------|---------|
| n   | a | psi | coverage | length | penalty | coverage | length | penalty |
| 50  | 1 | 0   | 0.9986   | 2.8262 | 1.2976  | 0.9986   | 2.8315 | 1.5184  |
|     |   | 0.5 | 0.9984   | 2.8248 | 1.3074  | 0.9980   | 2.8301 | 1.5168  |
|     |   | 0.9 | 0.9986   | 2.8435 | 1.1774  | 0.9982   | 2.8453 | 1.3100  |
|     | 4 | 0   | 0.9986   | 2.8412 | 0.0986  | 0.9984   | 2.8407 | 0.1718  |
|     |   | 0.5 | 0.9980   | 2.8411 | 0.0952  | 0.9978   | 2.8416 | 0.1722  |
|     |   | 0.9 | 0.9964   | 2.9364 | -1.2504 | 0.9970   | 2.9034 | -1.0392 |
|     | 5 | 0   | 0.9976   | 2.8429 | 0.0000  | 0.9976   | 2.8429 | 0.0000  |
|     |   | 0.5 | 0.9976   | 2.8430 | -0.0002 | 0.9976   | 2.8429 | 0.0000  |
|     |   | 0.9 | 0.9950   | 2.9676 | -1.9368 | 0.9962   | 2.9289 | -1.7048 |
| 100 | 1 | 0   | 0.9984   | 2.3876 | 1.2720  | 0.9970   | 2.3943 | 1.4734  |
|     |   | 0.5 | 0.9984   | 2.3874 | 1.2678  | 0.9978   | 2.3935 | 1.4678  |
|     |   | 0.9 | 0.9980   | 2.4057 | 1.1486  | 0.9974   | 2.4076 | 1.2824  |
|     | 4 | 0   | 0.9962   | 2.4037 | 0.0862  | 0.9958   | 2.4056 | 0.1638  |
|     |   | 0.5 | 0.9968   | 2.4044 | 0.0798  | 0.9966   | 2.4062 | 0.1534  |
|     |   | 0.9 | 0.9942   | 2.4786 | -0.8574 | 0.9944   | 2.4518 | -0.5758 |
|     | 5 | 0   | 0.9944   | 2.4114 | 0.0000  | 0.9944   | 2.4114 | 0.0000  |
|     |   | 0.5 | 0.9944   | 2.4114 | 0.0000  | 0.9944   | 2.4114 | 0.0000  |
|     |   | 0.9 | 0.9922   | 2.4960 | -1.2584 | 0.9932   | 2.4668 | -1.0082 |
| 500 | 1 | 0   | 0.9734   | 1.9755 | 1.2582  | 0.9716   | 1.9761 | 1.4788  |
|     |   | 0.5 | 0.9716   | 1.9754 | 1.2778  | 0.9718   | 1.9759 | 1.4820  |
|     |   | 0.9 | 0.9718   | 1.9753 | 1.2750  | 0.9714   | 1.9759 | 1.4852  |
|     | 4 | 0   | 0.9696   | 1.9764 | 0.0846  | 0.9702   | 1.9767 | 0.1700  |
|     |   | 0.5 | 0.9718   | 1.9765 | 0.0900  | 0.9718   | 1.9766 | 0.1674  |
|     |   | 0.9 | 0.9720   | 1.9768 | 0.0810  | 0.9714   | 1.9768 | 0.1632  |
|     | 5 | 0   | 0.9684   | 1.9769 | 0.0000  | 0.9684   | 1.9769 | 0.0000  |
|     |   | 0.5 | 0.9684   | 1.9769 | 0.0000  | 0.9684   | 1.9769 | 0.0000  |
|     |   | 0.9 | 0.9684   | 1.9773 | -0.0150 | 0.9684   | 1.9770 | -0.0062 |

Table 5.40.  $p = 10$ , error type = 4

|      |    |     | LASSO    |        |         | RIDGE    |        |         |
|------|----|-----|----------|--------|---------|----------|--------|---------|
| n    | a  | psi | coverage | length | penalty | coverage | length | penalty |
| 100  | 1  | 0   | 0.9974   | 2.5935 | 0.0644  | 0.9916   | 2.6598 | 0.1091  |
|      |    | 0.5 | 0.9962   | 2.5684 | 0.0232  | 0.9946   | 3.0739 | 0.1883  |
|      |    | 0.9 | 0.9988   | 2.5628 | 0.0196  | 0.9994   | 2.6611 | 0.2974  |
|      | 9  | 0   | 0.9898   | 2.6156 | 0.0091  | 0.9872   | 2.7876 | 0.1557  |
|      |    | 0.5 | 0.9890   | 2.6909 | 0.0927  | 0.9914   | 3.2481 | 1.4077  |
|      |    | 0.9 | 0.9878   | 3.8462 | 0.4142  | 0.9926   | 3.7711 | 3.8525  |
|      | 10 | 0   | 0.9892   | 2.6154 | 0.0074  | 0.9860   | 2.8010 | 0.1612  |
|      |    | 0.5 | 0.9872   | 2.7270 | 0.1095  | 0.9942   | 2.8486 | 1.5857  |
|      |    | 0.9 | 0.9888   | 4.4054 | 0.5466  | 0.9922   | 4.3505 | 5.2355  |
| 200  | 1  | 0   | 0.9922   | 2.2522 | 0.0465  | 0.9862   | 2.3284 | 0.1093  |
|      |    | 0.5 | 0.9944   | 2.2393 | 0.0167  | 0.9846   | 2.7246 | 0.1893  |
|      |    | 0.9 | 0.9970   | 2.2340 | 0.0197  | 0.9950   | 2.3285 | 0.2989  |
|      | 9  | 0   | 0.9876   | 2.2758 | 0.0078  | 0.9830   | 2.4137 | 0.1407  |
|      |    | 0.5 | 0.9870   | 2.3509 | 0.0929  | 0.9874   | 2.8808 | 1.4119  |
|      |    | 0.9 | 0.9784   | 3.3150 | 0.3867  | 0.9842   | 3.2756 | 3.7813  |
|      | 10 | 0   | 0.9876   | 2.2778 | 0.0073  | 0.9838   | 2.4233 | 0.1447  |
|      |    | 0.5 | 0.9842   | 2.3747 | 0.1045  | 0.9870   | 2.5127 | 1.5868  |
|      |    | 0.9 | 0.9766   | 3.8226 | 0.5248  | 0.9842   | 3.7962 | 5.2063  |
| 1000 | 1  | 0   | 0.9662   | 1.9526 | 0.0209  | 0.9634   | 2.0077 | 0.1096  |
|      |    | 0.5 | 0.9706   | 1.9517 | 0.0125  | 0.9614   | 2.3430 | 0.1899  |
|      |    | 0.9 | 0.9698   | 1.9525 | 0.0197  | 0.9634   | 2.0211 | 0.2999  |
|      | 9  | 0   | 0.9628   | 1.9562 | 0.0071  | 0.9610   | 2.0489 | 0.1224  |
|      |    | 0.5 | 0.9616   | 2.0209 | 0.0929  | 0.9576   | 2.4679 | 1.4123  |
|      |    | 0.9 | 0.9534   | 2.7378 | 0.3559  | 0.9564   | 2.7396 | 3.6841  |
|      | 10 | 0   | 0.9620   | 1.9573 | 0.0073  | 0.9616   | 2.0520 | 0.1239  |
|      |    | 0.5 | 0.9598   | 2.0390 | 0.1044  | 0.9604   | 2.1745 | 1.5871  |
|      |    | 0.9 | 0.9524   | 3.1606 | 0.4974  | 0.9538   | 3.1680 | 5.1358  |

Table 5.41.  $p = 10$ , error type = 4

|      |    |     | PLS      |        |         | PCR      |        |         |
|------|----|-----|----------|--------|---------|----------|--------|---------|
| n    | a  | psi | coverage | length | penalty | coverage | length | penalty |
| 100  | 1  | 0   | 0.9904   | 2.6112 | 4.0032  | 0.9900   | 2.6998 | 0.1590  |
|      |    | 0.5 | 0.9902   | 2.6123 | 4.1954  | 0.9898   | 2.6873 | 0.6392  |
|      |    | 0.9 | 0.9934   | 2.6088 | 6.6818  | 0.9958   | 2.6295 | 5.2176  |
|      | 9  | 0   | 0.9900   | 2.6108 | 3.3494  | 0.9882   | 2.8628 | 0.0802  |
|      |    | 0.5 | 0.9906   | 2.6113 | 4.2520  | 0.9880   | 2.6852 | 0.6278  |
|      |    | 0.9 | 0.9914   | 2.6097 | 6.6928  | 0.9948   | 2.6312 | 5.2324  |
|      | 10 | 0   | 0.9902   | 2.6109 | 3.3176  | 0.9890   | 2.8619 | 0.0670  |
|      |    | 0.5 | 0.9932   | 2.6094 | 6.8388  | 0.9966   | 2.5863 | 5.7646  |
|      |    | 0.9 | 0.9932   | 2.6092 | 6.8726  | 0.9976   | 2.5734 | 5.9134  |
| 200  | 1  | 0   | 0.9880   | 2.2732 | 4.2072  | 0.9874   | 2.3166 | 0.0630  |
|      |    | 0.5 | 0.9878   | 2.2731 | 3.9422  | 0.9876   | 2.3095 | 0.2678  |
|      |    | 0.9 | 0.9886   | 2.2731 | 6.5042  | 0.9912   | 2.2939 | 4.6416  |
|      | 9  | 0   | 0.9878   | 2.2731 | 3.7022  | 0.9876   | 2.3885 | 0.0332  |
|      |    | 0.5 | 0.9880   | 2.2733 | 3.9470  | 0.9870   | 2.3106 | 0.2466  |
|      |    | 0.9 | 0.9892   | 2.2732 | 6.4804  | 0.9908   | 2.2945 | 4.6328  |
|      | 10 | 0   | 0.9882   | 2.2731 | 3.7114  | 0.9874   | 2.3893 | 0.0336  |
|      |    | 0.5 | 0.9884   | 2.2720 | 6.8520  | 0.9966   | 2.2491 | 5.8868  |
|      |    | 0.9 | 0.9890   | 2.2720 | 6.8860  | 0.9958   | 2.2427 | 5.9800  |
| 1000 | 1  | 0   | 0.9626   | 1.9538 | 4.6058  | 0.9628   | 1.9573 | 0.0040  |
|      |    | 0.5 | 0.9628   | 1.9538 | 3.9158  | 0.9626   | 1.9587 | 0.0272  |
|      |    | 0.9 | 0.9636   | 1.9538 | 5.1560  | 0.9652   | 1.9611 | 1.4510  |
|      | 9  | 0   | 0.9632   | 1.9538 | 4.2232  | 0.9626   | 1.9563 | 0.0004  |
|      |    | 0.5 | 0.9624   | 1.9538 | 3.9382  | 0.9630   | 1.9572 | 0.0240  |
|      |    | 0.9 | 0.9618   | 1.9538 | 5.1362  | 0.9636   | 1.9610 | 1.4618  |
|      | 10 | 0   | 0.9630   | 1.9538 | 4.2444  | 0.9628   | 1.9580 | 0.0008  |
|      |    | 0.5 | 0.9624   | 1.9538 | 6.8854  | 0.9684   | 1.9524 | 5.9086  |
|      |    | 0.9 | 0.9622   | 1.9538 | 6.8984  | 0.9670   | 1.9517 | 6.0652  |

Table 5.42.  $p = 10$ , error type = 4

|      |    |     | Ii       |        |         | I-Min    |        |         |
|------|----|-----|----------|--------|---------|----------|--------|---------|
| n    | a  | psi | coverage | length | penalty | coverage | length | penalty |
| 100  | 1  | 0   | 0.9964   | 2.5793 | 1.7746  | 0.9944   | 2.5906 | 2.2930  |
|      |    | 0.5 | 0.9952   | 2.5784 | 1.8470  | 0.9940   | 2.5884 | 2.3128  |
|      |    | 0.9 | 0.9956   | 2.6029 | 1.5996  | 0.9940   | 2.6064 | 2.0086  |
|      | 9  | 0   | 0.9916   | 2.6089 | 0.0880  | 0.9914   | 2.6093 | 0.1604  |
|      |    | 0.5 | 0.9926   | 2.6091 | 0.0842  | 0.9918   | 2.6098 | 0.1598  |
|      |    | 0.9 | 0.9864   | 2.7469 | -2.7350 | 0.9884   | 2.7012 | -2.1928 |
|      | 10 | 0   | 0.9904   | 2.6106 | 0.0000  | 0.9904   | 2.6106 | 0.0000  |
|      |    | 0.5 | 0.9904   | 2.6106 | 0.0000  | 0.9904   | 2.6106 | 0.0000  |
|      |    | 0.9 | 0.9836   | 2.7590 | -3.1862 | 0.9878   | 2.7108 | -2.6546 |
| 200  | 1  | 0   | 0.9942   | 2.2467 | 1.7354  | 0.9918   | 2.2547 | 2.2628  |
|      |    | 0.5 | 0.9952   | 2.2466 | 1.8136  | 0.9936   | 2.2543 | 2.2726  |
|      |    | 0.9 | 0.9940   | 2.2572 | 1.6878  | 0.9934   | 2.2591 | 2.1886  |
|      | 9  | 0   | 0.9888   | 2.2701 | 0.0894  | 0.9888   | 2.2712 | 0.1632  |
|      |    | 0.5 | 0.9882   | 2.2702 | 0.0904  | 0.9882   | 2.2712 | 0.1688  |
|      |    | 0.9 | 0.9848   | 2.3344 | -1.2360 | 0.9864   | 2.3071 | -0.7542 |
|      | 10 | 0   | 0.9882   | 2.2731 | 0.0000  | 0.9882   | 2.2731 | 0.0000  |
|      |    | 0.5 | 0.9882   | 2.2731 | 0.0000  | 0.9882   | 2.2731 | 0.0000  |
|      |    | 0.9 | 0.9828   | 2.3433 | -1.5260 | 0.9840   | 2.3132 | -1.0618 |
| 1000 | 1  | 0   | 0.9648   | 1.9518 | 1.7274  | 0.9634   | 1.9524 | 2.2726  |
|      |    | 0.5 | 0.9666   | 1.9519 | 1.8254  | 0.9642   | 1.9522 | 2.2970  |
|      |    | 0.9 | 0.9666   | 1.9518 | 1.8290  | 0.9646   | 1.9522 | 2.2966  |
|      | 9  | 0   | 0.9640   | 1.9536 | 0.0832  | 0.9642   | 1.9536 | 0.1546  |
|      |    | 0.5 | 0.9636   | 1.9535 | 0.0836  | 0.9634   | 1.9535 | 0.1556  |
|      |    | 0.9 | 0.9634   | 1.9535 | 0.0824  | 0.9630   | 1.9536 | 0.1560  |
|      | 10 | 0   | 0.9628   | 1.9538 | 0.0000  | 0.9628   | 1.9538 | 0.0000  |
|      |    | 0.5 | 0.9628   | 1.9538 | 0.0000  | 0.9628   | 1.9538 | 0.0000  |
|      |    | 0.9 | 0.9628   | 1.9538 | 0.0000  | 0.9628   | 1.9538 | 0.0000  |

Table 5.43.  $p = 20$ , error type = 4

|      |    |     | LASSO    |         |         | RIDGE    |         |         |
|------|----|-----|----------|---------|---------|----------|---------|---------|
| n    | a  | psi | coverage | length  | penalty | coverage | length  | penalty |
| 200  | 1  | 0   | 0.9982   | 2.4205  | 0.0576  | 0.9866   | 2.5036  | 0.1094  |
|      |    | 0.5 | 0.9978   | 2.4080  | 0.0183  | 0.9886   | 3.1587  | 0.2565  |
|      |    | 0.9 | 0.9992   | 2.3978  | 0.0284  | 0.9978   | 2.5048  | 0.5142  |
|      | 19 | 0   | 0.9856   | 2.4677  | 0.0070  | 0.9834   | 2.6787  | 0.1746  |
|      |    | 0.5 | 0.9842   | 7.0022  | 1.1892  | 0.9872   | 7.0821  | 21.6156 |
|      |    | 0.9 | 0.9866   | 12.1538 | 2.0630  | 0.9870   | 12.4390 | 45.4908 |
|      | 20 | 0   | 0.9856   | 2.4703  | 0.0071  | 0.9834   | 2.6859  | 0.1778  |
|      |    | 0.5 | 0.9836   | 7.4428  | 1.2763  | 0.9862   | 7.5363  | 24.9747 |
|      |    | 0.9 | 0.9860   | 12.8003 | 2.1776  | 0.9866   | 13.0792 | 48.0181 |
| 400  | 1  | 0   | 0.9964   | 2.1784  | 0.0414  | 0.9850   | 2.2766  | 0.1097  |
|      |    | 0.5 | 0.9976   | 2.1710  | 0.0171  | 0.9822   | 2.9235  | 0.2568  |
|      |    | 0.9 | 0.9986   | 2.1667  | 0.0284  | 0.9922   | 2.2744  | 0.5332  |
|      | 19 | 0   | 0.9850   | 2.2293  | 0.0071  | 0.9794   | 2.4025  | 0.1538  |
|      |    | 0.5 | 0.9782   | 6.4046  | 1.1795  | 0.9816   | 6.5021  | 21.5611 |
|      |    | 0.9 | 0.9792   | 11.2415 | 2.0654  | 0.9786   | 11.4380 | 45.5442 |
|      | 20 | 0   | 0.9850   | 2.2314  | 0.0072  | 0.9810   | 2.4071  | 0.1559  |
|      |    | 0.5 | 0.9776   | 6.7623  | 1.2569  | 0.9796   | 6.9301  | 24.9894 |
|      |    | 0.9 | 0.9792   | 11.8362 | 2.1801  | 0.9788   | 12.0315 | 48.0743 |
| 2000 | 1  | 0   | 0.9686   | 1.9424  | 0.0186  | 0.9596   | 1.9998  | 0.1097  |
|      |    | 0.5 | 0.9686   | 1.9432  | 0.0169  | 0.9586   | 2.5376  | 0.2571  |
|      |    | 0.9 | 0.9676   | 1.9465  | 0.0285  | 0.9624   | 2.0167  | 0.5691  |
|      | 19 | 0   | 0.9630   | 1.9517  | 0.0072  | 0.9584   | 2.0545  | 0.1278  |
|      |    | 0.5 | 0.9574   | 5.4004  | 1.1782  | 0.9558   | 5.4893  | 21.5697 |
|      |    | 0.9 | 0.9598   | 9.5338  | 2.0682  | 0.9558   | 9.6498  | 45.6055 |
|      | 20 | 0   | 0.9638   | 1.9528  | 0.0073  | 0.9598   | 2.0558  | 0.1287  |
|      |    | 0.5 | 0.9574   | 5.6501  | 1.2436  | 0.9578   | 5.8439  | 24.9855 |
|      |    | 0.9 | 0.9580   | 10.0399 | 2.1831  | 0.9566   | 10.1477 | 48.1391 |

Table 5.44.  $p = 20$ , error type = 4

|      |    |     | PLS      |        |         | PCR      |        |         |
|------|----|-----|----------|--------|---------|----------|--------|---------|
| n    | a  | psi | coverage | length | penalty | coverage | length | penalty |
| 200  | 1  | 0   | 0.9862   | 2.4621 | 13.3200 | 0.9860   | 2.4862 | 0.1076  |
|      |    | 0.5 | 0.9874   | 2.4630 | 13.8342 | 0.9838   | 2.4944 | 0.8248  |
|      |    | 0.9 | 0.9890   | 2.4612 | 16.8248 | 0.9960   | 2.4840 | 13.5116 |
|      | 19 | 0   | 0.9868   | 2.4616 | 11.9970 | 0.9862   | 2.5079 | 0.0128  |
|      |    | 0.5 | 0.9866   | 2.4624 | 13.8370 | 0.9856   | 2.4954 | 0.8518  |
|      |    | 0.9 | 0.9888   | 2.4622 | 16.8392 | 0.9950   | 2.4849 | 13.5948 |
|      | 20 | 0   | 0.9864   | 2.4616 | 11.9792 | 0.9862   | 2.5082 | 0.0138  |
|      |    | 0.5 | 0.9898   | 2.4606 | 16.9512 | 0.9976   | 2.4205 | 15.0250 |
|      |    | 0.9 | 0.9896   | 2.4606 | 16.9618 | 0.9986   | 2.4111 | 15.2100 |
| 400  | 1  | 0   | 0.9864   | 2.2220 | 13.6014 | 0.9864   | 2.2345 | 0.0334  |
|      |    | 0.5 | 0.9868   | 2.2220 | 13.4870 | 0.9866   | 2.2368 | 0.2674  |
|      |    | 0.9 | 0.9880   | 2.2208 | 16.6384 | 0.9926   | 2.2440 | 11.9354 |
|      | 19 | 0   | 0.9868   | 2.2219 | 12.7944 | 0.9864   | 2.2409 | 0.0042  |
|      |    | 0.5 | 0.9872   | 2.2221 | 13.4604 | 0.9846   | 2.2373 | 0.2860  |
|      |    | 0.9 | 0.9872   | 2.2209 | 16.6488 | 0.9906   | 2.2440 | 11.9068 |
|      | 20 | 0   | 0.9864   | 2.2219 | 12.7938 | 0.9864   | 2.2388 | 0.0028  |
|      |    | 0.5 | 0.9878   | 2.2201 | 16.9692 | 0.9964   | 2.1778 | 15.2684 |
|      |    | 0.9 | 0.9878   | 2.2199 | 16.9754 | 0.9976   | 2.1725 | 15.4520 |
| 2000 | 1  | 0   | 0.9634   | 1.9447 | 14.2694 | 0.9630   | 1.9452 | 0.0008  |
|      |    | 0.5 | 0.9626   | 1.9447 | 13.5912 | 0.9626   | 1.9461 | 0.0184  |
|      |    | 0.9 | 0.9634   | 1.9448 | 15.1278 | 0.9634   | 1.9495 | 2.3234  |
|      | 19 | 0   | 0.9630   | 1.9447 | 13.6926 | 0.9630   | 1.9447 | 0.0000  |
|      |    | 0.5 | 0.9628   | 1.9447 | 13.6106 | 0.9628   | 1.9460 | 0.0162  |
|      |    | 0.9 | 0.9620   | 1.9447 | 15.1040 | 0.9626   | 1.9495 | 2.2778  |
|      | 20 | 0   | 0.9626   | 1.9447 | 13.6742 | 0.9630   | 1.9447 | 0.0000  |
|      |    | 0.5 | 0.9624   | 1.9447 | 16.9770 | 0.9672   | 1.9427 | 15.5024 |
|      |    | 0.9 | 0.9626   | 1.9447 | 16.9836 | 0.9664   | 1.9421 | 15.7158 |

Table 5.45.  $p = 20$ , error type = 4

|      |    |     | Ii       |        |         | I-Min    |        |         |
|------|----|-----|----------|--------|---------|----------|--------|---------|
| n    | a  | psi | coverage | length | penalty | coverage | length | penalty |
| 200  | 1  | 0   | 0.9956   | 2.4331 | 2.8434  | 0.9932   | 2.4429 | 3.8994  |
|      |    | 0.5 | 0.9948   | 2.4346 | 2.9980  | 0.9924   | 2.4434 | 3.9148  |
|      |    | 0.9 | 0.9930   | 2.4448 | 2.8534  | 0.9910   | 2.4480 | 3.8098  |
|      | 19 | 0   | 0.9880   | 2.4605 | 0.0832  | 0.9876   | 2.4603 | 0.1640  |
|      |    | 0.5 | 0.9864   | 2.4603 | 0.0814  | 0.9860   | 2.4602 | 0.1618  |
|      |    | 0.9 | 0.9830   | 2.5714 | -3.2118 | 0.9840   | 2.5302 | -2.3506 |
|      | 20 | 0   | 0.9864   | 2.4616 | 0.0000  | 0.9864   | 2.4616 | 0.0000  |
|      |    | 0.5 | 0.9864   | 2.4616 | 0.0000  | 0.9864   | 2.4616 | 0.0000  |
|      |    | 0.9 | 0.9830   | 2.5764 | -3.5258 | 0.9832   | 2.5336 | -2.6620 |
| 400  | 1  | 0   | 0.9930   | 2.1908 | 2.7674  | 0.9912   | 2.1998 | 3.8244  |
|      |    | 0.5 | 0.9934   | 2.1923 | 2.9510  | 0.9918   | 2.1998 | 3.8662  |
|      |    | 0.9 | 0.9942   | 2.1931 | 2.9484  | 0.9924   | 2.2003 | 3.8608  |
|      | 19 | 0   | 0.9874   | 2.2204 | 0.0820  | 0.9876   | 2.2207 | 0.1548  |
|      |    | 0.5 | 0.9866   | 2.2206 | 0.0804  | 0.9868   | 2.2209 | 0.1594  |
|      |    | 0.9 | 0.9842   | 2.2457 | -0.5718 | 0.9858   | 2.2313 | -0.1816 |
|      | 20 | 0   | 0.9864   | 2.2219 | 0.0000  | 0.9864   | 2.2219 | 0.0000  |
|      |    | 0.5 | 0.9864   | 2.2219 | 0.0000  | 0.9864   | 2.2219 | 0.0000  |
|      |    | 0.9 | 0.9840   | 2.2486 | -0.7088 | 0.9858   | 2.2331 | -0.3698 |
| 2000 | 1  | 0   | 0.9642   | 1.9426 | 2.7344  | 0.9672   | 1.9430 | 3.8056  |
|      |    | 0.5 | 0.9660   | 1.9426 | 2.9172  | 0.9660   | 1.9430 | 3.8524  |
|      |    | 0.9 | 0.9670   | 1.9426 | 2.9142  | 0.9666   | 1.9430 | 3.8568  |
|      | 19 | 0   | 0.9628   | 1.9446 | 0.0844  | 0.9628   | 1.9446 | 0.1654  |
|      |    | 0.5 | 0.9634   | 1.9446 | 0.0856  | 0.9632   | 1.9447 | 0.1662  |
|      |    | 0.9 | 0.9634   | 1.9446 | 0.0862  | 0.9632   | 1.9447 | 0.1656  |
|      | 20 | 0   | 0.9630   | 1.9447 | 0.0000  | 0.9630   | 1.9447 | 0.0000  |
|      |    | 0.5 | 0.9630   | 1.9447 | 0.0000  | 0.9630   | 1.9447 | 0.0000  |
|      |    | 0.9 | 0.9630   | 1.9447 | 0.0000  | 0.9630   | 1.9447 | 0.0000  |

Table 5.46.  $p = 50$ , error type = 4

|      |    |     | LASSO    |         |         | RIDGE    |         |          |
|------|----|-----|----------|---------|---------|----------|---------|----------|
| n    | a  | psi | coverage | length  | penalty | coverage | length  | penalty  |
| 500  | 1  | 0   | 0.9996   | 2.3296  | 0.0458  | 0.9882   | 2.4529  | 0.1095   |
|      |    | 0.5 | 1.0000   | 2.3252  | 0.0260  | 0.9916   | 3.4228  | 0.3951   |
|      |    | 0.9 | 1.0000   | 2.3312  | 0.0455  | 0.9992   | 2.4534  | 2.3189   |
|      | 49 | 0   | 0.9870   | 2.4297  | 0.0069  | 0.9840   | 2.6907  | 0.1931   |
|      |    | 0.5 | 0.9864   | 29.2537 | 5.4303  | 0.9886   | 28.9000 | 276.6280 |
|      |    | 0.9 | 0.9862   | 51.4575 | 8.8190  | 0.9888   | 50.1937 | 493.0514 |
|      | 50 | 0   | 0.9868   | 2.4315  | 0.0069  | 0.9850   | 2.6943  | 0.1943   |
|      |    | 0.5 | 0.9860   | 29.8794 | 5.5434  | 0.9890   | 29.3848 | 282.3888 |
|      |    | 0.9 | 0.9868   | 52.5310 | 9.0027  | 0.9888   | 51.2332 | 503.3230 |
| 1000 | 1  | 0   | 0.9986   | 2.1316  | 0.0328  | 0.9814   | 2.2520  | 0.1096   |
|      |    | 0.5 | 0.9992   | 2.1318  | 0.0260  | 0.9858   | 3.1824  | 0.3954   |
|      |    | 0.9 | 0.9988   | 2.1451  | 0.0456  | 0.9948   | 2.2599  | 2.3214   |
|      | 49 | 0   | 0.9822   | 2.2194  | 0.0071  | 0.9778   | 2.4154  | 0.1664   |
|      |    | 0.5 | 0.9820   | 26.9042 | 5.4342  | 0.9820   | 26.7112 | 276.8280 |
|      |    | 0.9 | 0.9824   | 47.5853 | 8.8286  | 0.9834   | 46.3997 | 493.5889 |
|      | 50 | 0   | 0.9806   | 2.2209  | 0.0071  | 0.9772   | 2.4174  | 0.1672   |
|      |    | 0.5 | 0.9828   | 27.4816 | 5.5474  | 0.9822   | 27.1549 | 282.5934 |
|      |    | 0.9 | 0.9828   | 48.5766 | 9.0125  | 0.9832   | 47.3586 | 503.8719 |
| 5000 | 1  | 0   | 0.9650   | 1.9374  | 0.0146  | 0.9622   | 1.9962  | 0.1097   |
|      |    | 0.5 | 0.9692   | 1.9406  | 0.0260  | 0.9604   | 2.7593  | 0.3956   |
|      |    | 0.9 | 0.9684   | 1.9500  | 0.0456  | 0.9650   | 2.0249  | 2.3225   |
|      | 49 | 0   | 0.9620   | 1.9600  | 0.0072  | 0.9644   | 2.0639  | 0.1332   |
|      |    | 0.5 | 0.9566   | 22.6848 | 5.4339  | 0.9596   | 22.5948 | 276.8110 |
|      |    | 0.9 | 0.9592   | 40.3054 | 8.8325  | 0.9584   | 39.2754 | 493.8069 |
|      | 50 | 0   | 0.9608   | 1.9607  | 0.0073  | 0.9642   | 2.0645  | 0.1336   |
|      |    | 0.5 | 0.9576   | 23.1715 | 5.5471  | 0.9596   | 22.9717 | 282.5758 |
|      |    | 0.9 | 0.9598   | 41.1435 | 9.0165  | 0.9586   | 40.0886 | 504.0944 |

Table 5.47.  $p = 50$ , error type = 4

|      |    |     | PLS      |        |         | PCR      |        |         |
|------|----|-----|----------|--------|---------|----------|--------|---------|
| n    | a  | psi | coverage | length | penalty | coverage | length | penalty |
| 500  | 1  | 0   | 0.9878   | 2.4099 | 42.8396 | 0.9864   | 2.4159 | 0.0874  |
|      |    | 0.5 | 0.9880   | 2.4105 | 43.7510 | 0.9856   | 2.4233 | 1.0630  |
|      |    | 0.9 | 0.9916   | 2.4082 | 46.9714 | 0.9982   | 2.4248 | 41.2944 |
|      | 49 | 0   | 0.9876   | 2.4098 | 40.6902 | 0.9878   | 2.4160 | 0.0008  |
|      |    | 0.5 | 0.9880   | 2.4104 | 43.7492 | 0.9850   | 2.4233 | 1.0614  |
|      |    | 0.9 | 0.9920   | 2.4083 | 46.9704 | 0.9986   | 2.4250 | 41.2552 |
|      | 50 | 0   | 0.9876   | 2.4097 | 40.6740 | 0.9878   | 2.4143 | 0.0006  |
|      |    | 0.5 | 0.9908   | 2.4071 | 46.9982 | 0.9994   | 2.3277 | 44.7296 |
|      |    | 0.9 | 0.9916   | 2.4071 | 46.9994 | 0.9994   | 2.3219 | 44.8650 |
| 1000 | 1  | 0   | 0.9804   | 2.1960 | 43.1884 | 0.9804   | 2.1981 | 0.0212  |
|      |    | 0.5 | 0.9804   | 2.1960 | 43.2666 | 0.9816   | 2.2005 | 0.2622  |
|      |    | 0.9 | 0.9820   | 2.1953 | 46.8534 | 0.9892   | 2.2216 | 36.2592 |
|      | 49 | 0   | 0.9800   | 2.1960 | 41.8898 | 0.9802   | 2.1960 | 0.0000  |
|      |    | 0.5 | 0.9798   | 2.1961 | 43.2806 | 0.9804   | 2.2004 | 0.2738  |
|      |    | 0.9 | 0.9794   | 2.1949 | 46.8586 | 0.9876   | 2.2220 | 36.2768 |
|      | 50 | 0   | 0.9804   | 2.1960 | 41.8580 | 0.9802   | 2.1960 | 0.0000  |
|      |    | 0.5 | 0.9828   | 2.1940 | 46.9996 | 0.9996   | 2.1296 | 45.0832 |
|      |    | 0.9 | 0.9822   | 2.1940 | 46.9998 | 0.9990   | 2.1265 | 45.2190 |
| 5000 | 1  | 0   | 0.9586   | 1.9402 | 43.8850 | 0.9584   | 1.9402 | 0.0004  |
|      |    | 0.5 | 0.9586   | 1.9402 | 43.3556 | 0.9586   | 1.9404 | 0.0114  |
|      |    | 0.9 | 0.9586   | 1.9402 | 45.1974 | 0.9592   | 1.9427 | 3.2642  |
|      | 49 | 0   | 0.9584   | 1.9402 | 43.2248 | 0.9584   | 1.9402 | 0.0000  |
|      |    | 0.5 | 0.9584   | 1.9402 | 43.3654 | 0.9580   | 1.9404 | 0.0114  |
|      |    | 0.9 | 0.9574   | 1.9402 | 45.2090 | 0.9574   | 1.9426 | 3.2202  |
|      | 50 | 0   | 0.9582   | 1.9402 | 43.2212 | 0.9584   | 1.9402 | 0.0000  |
|      |    | 0.5 | 0.9578   | 1.9402 | 46.9996 | 0.9680   | 1.9378 | 45.3298 |
|      |    | 0.9 | 0.9590   | 1.9402 | 46.9998 | 0.9660   | 1.9375 | 45.4490 |

Table 5.48.  $p = 50$ , error type = 4

|      |    |     | Ii       |        |         | I-Min    |        |         |
|------|----|-----|----------|--------|---------|----------|--------|---------|
| n    | a  | psi | coverage | length | penalty | coverage | length | penalty |
| 500  | 1  | 0   | 0.9954   | 2.3762 | 6.2940  | 0.9938   | 2.3864 | 8.4840  |
|      |    | 0.5 | 0.9960   | 2.3769 | 6.4626  | 0.9934   | 2.3863 | 8.5248  |
|      |    | 0.9 | 0.9958   | 2.3774 | 6.4550  | 0.9940   | 2.3867 | 8.5156  |
|      | 49 | 0   | 0.9880   | 2.4094 | 0.0836  | 0.9880   | 2.4095 | 0.1596  |
|      |    | 0.5 | 0.9874   | 2.4095 | 0.0842  | 0.9878   | 2.4094 | 0.1576  |
|      |    | 0.9 | 0.9876   | 2.4359 | -0.8516 | 0.9868   | 2.4204 | -0.3274 |
|      | 50 | 0   | 0.9878   | 2.4097 | 0.0000  | 0.9878   | 2.4097 | 0.0000  |
|      |    | 0.5 | 0.9878   | 2.4097 | 0.0000  | 0.9878   | 2.4097 | 0.0000  |
|      |    | 0.9 | 0.9868   | 2.4374 | -0.9982 | 0.9862   | 2.4209 | -0.5070 |
| 1000 | 1  | 0   | 0.9904   | 2.1635 | 6.3182  | 0.9870   | 2.1714 | 8.5726  |
|      |    | 0.5 | 0.9884   | 2.1640 | 6.4940  | 0.9872   | 2.1715 | 8.5782  |
|      |    | 0.9 | 0.9886   | 2.1641 | 6.4928  | 0.9870   | 2.1716 | 8.5754  |
|      | 49 | 0   | 0.9794   | 2.1954 | 0.0864  | 0.9796   | 2.1955 | 0.1598  |
|      |    | 0.5 | 0.9788   | 2.1954 | 0.0884  | 0.9796   | 2.1956 | 0.1584  |
|      |    | 0.9 | 0.9788   | 2.1955 | 0.0834  | 0.9794   | 2.1956 | 0.1570  |
|      | 50 | 0   | 0.9802   | 2.1960 | 0.0000  | 0.9802   | 2.1960 | 0.0000  |
|      |    | 0.5 | 0.9802   | 2.1960 | 0.0000  | 0.9802   | 2.1960 | 0.0000  |
|      |    | 0.9 | 0.9802   | 2.1962 | -0.0054 | 0.9802   | 2.1960 | -0.0016 |
| 5000 | 1  | 0   | 0.9624   | 1.9375 | 6.3512  | 0.9610   | 1.9379 | 8.6178  |
|      |    | 0.5 | 0.9632   | 1.9376 | 6.5162  | 0.9600   | 1.9379 | 8.6114  |
|      |    | 0.9 | 0.9636   | 1.9376 | 6.5148  | 0.9598   | 1.9379 | 8.6112  |
|      | 49 | 0   | 0.9588   | 1.9401 | 0.0838  | 0.9588   | 1.9402 | 0.1594  |
|      |    | 0.5 | 0.9606   | 1.9401 | 0.0840  | 0.9604   | 1.9402 | 0.1582  |
|      |    | 0.9 | 0.9608   | 1.9401 | 0.0840  | 0.9606   | 1.9402 | 0.1588  |
|      | 50 | 0   | 0.9584   | 1.9402 | 0.0000  | 0.9584   | 1.9402 | 0.0000  |
|      |    | 0.5 | 0.9584   | 1.9402 | 0.0000  | 0.9584   | 1.9402 | 0.0000  |
|      |    | 0.9 | 0.9584   | 1.9402 | 0.0000  | 0.9584   | 1.9402 | 0.0000  |

### 5.1.7 Error type 5

Table 5.49.  $p = 5$ , error type = 5

|     |   |     | LASSO    |         |         | RIDGE    |         |         |
|-----|---|-----|----------|---------|---------|----------|---------|---------|
| n   | a | psi | coverage | length  | penalty | coverage | length  | penalty |
| 50  | 1 | 0   | 0.9616   | 21.1852 | 0.4107  | 0.9616   | 21.1415 | 94.5727 |
|     |   | 0.5 | 0.9628   | 21.1068 | 0.2307  | 0.9632   | 21.1888 | 32.1884 |
|     |   | 0.9 | 0.9640   | 21.2154 | 0.0650  | 0.9648   | 21.4821 | 9.2305  |
|     | 4 | 0   | 0.9638   | 20.8359 | 0.2330  | 0.9626   | 20.9342 | 25.8287 |
|     |   | 0.5 | 0.9622   | 20.7961 | 0.1294  | 0.9626   | 21.0793 | 1.4247  |
|     |   | 0.9 | 0.9630   | 21.2134 | 0.0638  | 0.9640   | 21.4777 | 0.6886  |
|     | 5 | 0   | 0.9632   | 20.7620 | 0.1540  | 0.9618   | 20.9019 | 16.8077 |
|     |   | 0.5 | 0.9630   | 20.7220 | 0.1036  | 0.9626   | 21.0683 | 1.2830  |
|     |   | 0.9 | 0.9636   | 21.1677 | 0.0716  | 0.9646   | 21.4902 | 0.8334  |
| 100 | 1 | 0   | 0.9650   | 21.4990 | 0.2657  | 0.9658   | 21.4402 | 29.0310 |
|     |   | 0.5 | 0.9644   | 21.4650 | 0.1358  | 0.9654   | 21.5059 | 7.0181  |
|     |   | 0.9 | 0.9648   | 21.5914 | 0.0339  | 0.9646   | 21.7482 | 1.2401  |
|     | 4 | 0   | 0.9648   | 21.2468 | 0.0924  | 0.9646   | 21.3013 | 3.2694  |
|     |   | 0.5 | 0.9644   | 21.2939 | 0.0716  | 0.9644   | 21.4854 | 0.8890  |
|     |   | 0.9 | 0.9646   | 21.5915 | 0.0476  | 0.9640   | 21.7711 | 0.6182  |
|     | 5 | 0   | 0.9646   | 21.2121 | 0.0372  | 0.9648   | 21.2779 | 1.3419  |
|     |   | 0.5 | 0.9642   | 21.2529 | 0.0546  | 0.9646   | 21.4794 | 0.9794  |
|     |   | 0.9 | 0.9650   | 21.5430 | 0.0576  | 0.9644   | 21.7756 | 0.8080  |
| 500 | 1 | 0   | 0.9482   | 13.8010 | 0.1131  | 0.9478   | 13.7766 | 0.3864  |
|     |   | 0.5 | 0.9472   | 13.8006 | 0.0532  | 0.9478   | 13.7919 | 0.3562  |
|     |   | 0.9 | 0.9480   | 13.8216 | 0.0165  | 0.9486   | 13.8375 | 0.2711  |
|     | 4 | 0   | 0.9472   | 13.7743 | 0.0294  | 0.9474   | 13.7744 | 0.1510  |
|     |   | 0.5 | 0.9480   | 13.7777 | 0.0308  | 0.9472   | 13.8039 | 0.4269  |
|     |   | 0.9 | 0.9474   | 13.8011 | 0.0401  | 0.9476   | 13.8466 | 0.6088  |
|     | 5 | 0   | 0.9482   | 13.7717 | 0.0078  | 0.9476   | 13.7748 | 0.1395  |
|     |   | 0.5 | 0.9478   | 13.7716 | 0.0345  | 0.9480   | 13.7992 | 0.5571  |
|     |   | 0.9 | 0.9470   | 13.7921 | 0.0534  | 0.9476   | 13.8471 | 0.8113  |

Table 5.50.  $p = 5$ , error type = 5

|     |   |     | PLS      |         |         | PCR      |         |         |
|-----|---|-----|----------|---------|---------|----------|---------|---------|
| n   | a | psi | coverage | length  | penalty | coverage | length  | penalty |
| 50  | 1 | 0   | 0.9626   | 20.6040 | 2.0248  | 0.9644   | 21.1752 | 1.4570  |
|     |   | 0.5 | 0.9624   | 20.7253 | 1.7814  | 0.9630   | 21.0889 | 1.5424  |
|     |   | 0.9 | 0.9630   | 20.6704 | 1.8904  | 0.9646   | 21.0557 | 1.6786  |
|     | 4 | 0   | 0.9630   | 20.5924 | 1.4584  | 0.9630   | 21.1641 | 0.8686  |
|     |   | 0.5 | 0.9630   | 20.5986 | 1.6350  | 0.9636   | 21.0224 | 1.3652  |
|     |   | 0.9 | 0.9628   | 20.6030 | 1.8334  | 0.9636   | 21.0327 | 1.6280  |
|     | 5 | 0   | 0.9628   | 20.5899 | 1.3242  | 0.9630   | 21.1984 | 0.7400  |
|     |   | 0.5 | 0.9626   | 20.6038 | 1.8662  | 0.9640   | 21.0446 | 1.6440  |
|     |   | 0.9 | 0.9628   | 20.6032 | 1.8492  | 0.9640   | 21.0336 | 1.6346  |
| 100 | 1 | 0   | 0.9640   | 21.2176 | 1.6702  | 0.9642   | 21.4440 | 1.0586  |
|     |   | 0.5 | 0.9640   | 21.2380 | 1.5566  | 0.9646   | 21.4646 | 1.3176  |
|     |   | 0.9 | 0.9640   | 21.2234 | 1.8296  | 0.9648   | 21.4877 | 1.6088  |
|     | 4 | 0   | 0.9644   | 21.2133 | 1.1508  | 0.9650   | 21.4358 | 0.5352  |
|     |   | 0.5 | 0.9640   | 21.2200 | 1.5150  | 0.9640   | 21.4530 | 1.2700  |
|     |   | 0.9 | 0.9642   | 21.2196 | 1.8250  | 0.9646   | 21.4845 | 1.6070  |
|     | 5 | 0   | 0.9646   | 21.2127 | 1.0744  | 0.9650   | 21.4320 | 0.4628  |
|     |   | 0.5 | 0.9642   | 21.2188 | 1.8472  | 0.9644   | 21.4893 | 1.6310  |
|     |   | 0.9 | 0.9642   | 21.2192 | 1.8294  | 0.9646   | 21.4871 | 1.6224  |
| 500 | 1 | 0   | 0.9484   | 13.7716 | 0.9818  | 0.9484   | 13.7991 | 0.2594  |
|     |   | 0.5 | 0.9486   | 13.7717 | 0.8778  | 0.9470   | 13.7927 | 0.7214  |
|     |   | 0.9 | 0.9482   | 13.7719 | 1.7668  | 0.9484   | 13.8022 | 1.5770  |
|     | 4 | 0   | 0.9484   | 13.7716 | 0.9292  | 0.9486   | 13.8056 | 0.1138  |
|     |   | 0.5 | 0.9484   | 13.7716 | 0.8598  | 0.9478   | 13.7893 | 0.7270  |
|     |   | 0.9 | 0.9484   | 13.7718 | 1.7584  | 0.9484   | 13.8029 | 1.5890  |
|     | 5 | 0   | 0.9484   | 13.7715 | 0.8982  | 0.9490   | 13.8062 | 0.1054  |
|     |   | 0.5 | 0.9484   | 13.7717 | 1.8330  | 0.9486   | 13.8040 | 1.6486  |
|     |   | 0.9 | 0.9482   | 13.7717 | 1.8182  | 0.9488   | 13.8034 | 1.6514  |

Table 5.51.  $p = 5$ , error type = 5

|     |   |     | Ii       |         |         | I-Min    |         |         |
|-----|---|-----|----------|---------|---------|----------|---------|---------|
| n   | a | psi | coverage | length  | penalty | coverage | length  | penalty |
| 50  | 1 | 0   | 0.9628   | 21.0589 | 1.2008  | 0.9618   | 20.9289 | 1.4026  |
|     |   | 0.5 | 0.9632   | 21.0298 | 1.1654  | 0.9626   | 20.9318 | 1.3078  |
|     |   | 0.9 | 0.9632   | 21.0426 | 1.2368  | 0.9630   | 20.9125 | 1.4088  |
|     | 4 | 0   | 0.9640   | 20.8381 | -0.7648 | 0.9636   | 20.7258 | -0.5028 |
|     |   | 0.5 | 0.9630   | 20.9102 | -1.3090 | 0.9626   | 20.8074 | -1.0726 |
|     |   | 0.9 | 0.9616   | 21.0270 | -1.8634 | 0.9614   | 20.9349 | -1.7362 |
|     | 5 | 0   | 0.9644   | 20.7346 | -1.2090 | 0.9630   | 20.6329 | -0.9272 |
|     |   | 0.5 | 0.9628   | 20.7937 | -1.8984 | 0.9630   | 20.7173 | -1.6654 |
|     |   | 0.9 | 0.9628   | 21.0285 | -2.8950 | 0.9622   | 20.9477 | -2.7872 |
| 100 | 1 | 0   | 0.9644   | 21.5258 | 1.2320  | 0.9642   | 21.4415 | 1.4484  |
|     |   | 0.5 | 0.9650   | 21.4847 | 1.1688  | 0.9652   | 21.4264 | 1.3294  |
|     |   | 0.9 | 0.9646   | 21.4986 | 1.2218  | 0.9646   | 21.4171 | 1.3872  |
|     | 4 | 0   | 0.9646   | 21.2671 | -0.2582 | 0.9646   | 21.2491 | -0.0598 |
|     |   | 0.5 | 0.9652   | 21.2711 | -0.9186 | 0.9646   | 21.2435 | -0.6788 |
|     |   | 0.9 | 0.9640   | 21.4819 | -1.8932 | 0.9644   | 21.4263 | -1.7830 |
|     | 5 | 0   | 0.9638   | 21.1280 | -0.4822 | 0.9640   | 21.1528 | -0.3118 |
|     |   | 0.5 | 0.9650   | 21.1595 | -1.4208 | 0.9648   | 21.1774 | -1.1628 |
|     |   | 0.9 | 0.9634   | 21.4642 | -2.9166 | 0.9640   | 21.4192 | -2.8222 |
| 500 | 1 | 0   | 0.9476   | 13.8139 | 1.2686  | 0.9474   | 13.8024 | 1.4698  |
|     |   | 0.5 | 0.9478   | 13.8135 | 1.2590  | 0.9472   | 13.8046 | 1.4598  |
|     |   | 0.9 | 0.9486   | 13.8085 | 1.1642  | 0.9488   | 13.8012 | 1.2988  |
|     | 4 | 0   | 0.9482   | 13.7848 | 0.0878  | 0.9482   | 13.7814 | 0.1550  |
|     |   | 0.5 | 0.9488   | 13.7841 | 0.0362  | 0.9484   | 13.7805 | 0.1302  |
|     |   | 0.9 | 0.9472   | 13.7972 | -1.9176 | 0.9478   | 13.7918 | -1.7438 |
|     | 5 | 0   | 0.9484   | 13.7716 | 0.0000  | 0.9484   | 13.7716 | 0.0000  |
|     |   | 0.5 | 0.9486   | 13.7663 | -0.0880 | 0.9484   | 13.7707 | -0.0450 |
|     |   | 0.9 | 0.9472   | 13.7909 | -2.6966 | 0.9474   | 13.7825 | -2.3714 |

Table 5.52.  $p = 10$ , error type = 5

|      |    |     | LASSO    |         |         | RIDGE    |         |         |
|------|----|-----|----------|---------|---------|----------|---------|---------|
| n    | a  | psi | coverage | length  | penalty | coverage | length  | penalty |
| 100  | 1  | 0   | 0.9688   | 23.0022 | 0.3910  | 0.9672   | 22.7614 | 61.3493 |
|      |    | 0.5 | 0.9682   | 22.9416 | 0.1426  | 0.9670   | 22.9726 | 3.4476  |
|      |    | 0.9 | 0.9682   | 23.2572 | 0.0344  | 0.9682   | 23.4394 | 0.8377  |
|      | 9  | 0   | 0.9652   | 21.8971 | 0.0530  | 0.9664   | 22.0767 | 0.7007  |
|      |    | 0.5 | 0.9654   | 22.0803 | 0.0999  | 0.9672   | 22.8498 | 2.0811  |
|      |    | 0.9 | 0.9662   | 22.8714 | 0.1569  | 0.9680   | 23.4812 | 2.3839  |
|      | 10 | 0   | 0.9648   | 21.8723 | 0.0304  | 0.9656   | 22.0559 | 0.6261  |
|      |    | 0.5 | 0.9648   | 21.9827 | 0.1079  | 0.9674   | 22.8336 | 2.3908  |
|      |    | 0.9 | 0.9666   | 22.8178 | 0.1765  | 0.9676   | 23.4841 | 2.6816  |
| 200  | 1  | 0   | 0.9616   | 20.6542 | 0.2600  | 0.9630   | 20.4510 | 11.2444 |
|      |    | 0.5 | 0.9624   | 20.6325 | 0.0935  | 0.9626   | 20.6288 | 1.3254  |
|      |    | 0.9 | 0.9632   | 20.8426 | 0.0253  | 0.9632   | 20.9093 | 0.5184  |
|      | 9  | 0   | 0.9620   | 20.1871 | 0.0256  | 0.9624   | 20.2363 | 0.3207  |
|      |    | 0.5 | 0.9624   | 20.2409 | 0.0936  | 0.9622   | 20.6187 | 1.6834  |
|      |    | 0.9 | 0.9624   | 20.5703 | 0.1573  | 0.9634   | 20.9428 | 2.3901  |
|      | 10 | 0   | 0.9618   | 20.1815 | 0.0103  | 0.9620   | 20.2300 | 0.2949  |
|      |    | 0.5 | 0.9618   | 20.1980 | 0.1048  | 0.9624   | 20.6116 | 1.9633  |
|      |    | 0.9 | 0.9630   | 20.5431 | 0.1769  | 0.9636   | 20.9468 | 2.6887  |
| 1000 | 1  | 0   | 0.9542   | 14.0529 | 0.1182  | 0.9532   | 14.0004 | 0.3985  |
|      |    | 0.5 | 0.9538   | 14.0516 | 0.0414  | 0.9540   | 14.0343 | 0.2524  |
|      |    | 0.9 | 0.9550   | 14.0778 | 0.0198  | 0.9548   | 14.0950 | 0.3223  |
|      | 9  | 0   | 0.9534   | 13.9912 | 0.0127  | 0.9534   | 13.9963 | 0.1308  |
|      |    | 0.5 | 0.9542   | 13.9990 | 0.0931  | 0.9542   | 14.0662 | 1.4146  |
|      |    | 0.9 | 0.9544   | 14.0307 | 0.1577  | 0.9552   | 14.1142 | 2.3973  |
|      | 10 | 0   | 0.9534   | 13.9913 | 0.0074  | 0.9534   | 13.9961 | 0.1324  |
|      |    | 0.5 | 0.9538   | 13.9935 | 0.1046  | 0.9554   | 14.0574 | 1.5895  |
|      |    | 0.9 | 0.9548   | 14.0280 | 0.1774  | 0.9552   | 14.1146 | 2.6968  |

Table 5.53.  $p = 10$ , error type = 5

|      |    |     | PLS      |         |         | PCR      |         |         |
|------|----|-----|----------|---------|---------|----------|---------|---------|
| n    | a  | psi | coverage | length  | penalty | coverage | length  | penalty |
| 100  | 1  | 0   | 0.9652   | 21.9321 | 7.0690  | 0.9672   | 22.6288 | 4.1094  |
|      |    | 0.5 | 0.9656   | 21.9690 | 6.7280  | 0.9664   | 22.8001 | 5.2690  |
|      |    | 0.9 | 0.9656   | 21.9532 | 6.8950  | 0.9682   | 22.9066 | 5.9764  |
|      | 9  | 0   | 0.9656   | 21.8612 | 5.1690  | 0.9650   | 22.0720 | 0.6694  |
|      |    | 0.5 | 0.9654   | 21.9508 | 6.7256  | 0.9674   | 22.8270 | 5.2474  |
|      |    | 0.9 | 0.9656   | 21.9493 | 6.8956  | 0.9684   | 22.9111 | 5.9692  |
|      | 10 | 0   | 0.9650   | 21.8591 | 5.0980  | 0.9652   | 22.0583 | 0.6076  |
|      |    | 0.5 | 0.9660   | 21.9501 | 6.9110  | 0.9678   | 22.9143 | 5.9986  |
|      |    | 0.9 | 0.9656   | 21.9506 | 6.9016  | 0.9684   | 22.9119 | 6.0130  |
| 200  | 1  | 0   | 0.9622   | 20.1904 | 6.5072  | 0.9620   | 20.3792 | 2.6322  |
|      |    | 0.5 | 0.9614   | 20.2010 | 6.5388  | 0.9618   | 20.5360 | 4.7006  |
|      |    | 0.9 | 0.9616   | 20.2032 | 6.8946  | 0.9620   | 20.6588 | 6.0014  |
|      | 9  | 0   | 0.9620   | 20.1813 | 4.9562  | 0.9624   | 20.2534 | 0.2872  |
|      |    | 0.5 | 0.9624   | 20.2053 | 6.5580  | 0.9620   | 20.5609 | 4.8004  |
|      |    | 0.9 | 0.9620   | 20.2035 | 6.8994  | 0.9620   | 20.6537 | 5.9770  |
|      | 10 | 0   | 0.9620   | 20.1814 | 4.9038  | 0.9624   | 20.2575 | 0.2632  |
|      |    | 0.5 | 0.9618   | 20.2033 | 6.9132  | 0.9626   | 20.6570 | 6.0030  |
|      |    | 0.9 | 0.9618   | 20.2029 | 6.9108  | 0.9626   | 20.6584 | 6.0196  |
| 1000 | 1  | 0   | 0.9536   | 13.9910 | 5.4596  | 0.9534   | 13.9986 | 0.3132  |
|      |    | 0.5 | 0.9536   | 13.9910 | 5.3722  | 0.9526   | 14.0087 | 1.9826  |
|      |    | 0.9 | 0.9530   | 13.9919 | 6.8546  | 0.9542   | 14.0552 | 5.9000  |
|      | 9  | 0   | 0.9536   | 13.9911 | 4.8674  | 0.9532   | 13.9967 | 0.0292  |
|      |    | 0.5 | 0.9534   | 13.9914 | 5.3514  | 0.9538   | 14.0105 | 1.9626  |
|      |    | 0.9 | 0.9526   | 13.9921 | 6.8584  | 0.9536   | 14.0577 | 5.8950  |
|      | 10 | 0   | 0.9536   | 13.9910 | 4.8930  | 0.9536   | 14.0003 | 0.0254  |
|      |    | 0.5 | 0.9530   | 13.9920 | 6.9086  | 0.9542   | 14.0593 | 6.1110  |
|      |    | 0.9 | 0.9530   | 13.9921 | 6.9054  | 0.9536   | 14.0583 | 6.1140  |

Table 5.54.  $p = 10$ , error type = 5

|      |    |     | Ii       |         |         | I-Min    |         |         |
|------|----|-----|----------|---------|---------|----------|---------|---------|
| n    | a  | psi | coverage | length  | penalty | coverage | length  | penalty |
| 100  | 1  | 0   | 0.9678   | 22.7774 | 1.6922  | 0.9668   | 22.5406 | 2.2324  |
|      |    | 0.5 | 0.9668   | 22.7174 | 1.6012  | 0.9668   | 22.5324 | 2.0204  |
|      |    | 0.9 | 0.9668   | 22.6763 | 1.6590  | 0.9670   | 22.4918 | 2.0252  |
|      | 9  | 0   | 0.9662   | 21.7953 | -0.9800 | 0.9656   | 21.8130 | -0.5560 |
|      |    | 0.5 | 0.9654   | 21.8909 | -2.9946 | 0.9650   | 21.8722 | -2.4834 |
|      |    | 0.9 | 0.9662   | 22.6257 | -6.4296 | 0.9664   | 22.4837 | -5.9826 |
|      | 10 | 0   | 0.9662   | 21.6594 | -1.2246 | 0.9650   | 21.7153 | -0.8224 |
|      |    | 0.5 | 0.9636   | 21.7640 | -3.4942 | 0.9642   | 21.7783 | -2.9766 |
|      |    | 0.9 | 0.9660   | 22.5859 | -7.3136 | 0.9666   | 22.4553 | -6.8800 |
| 200  | 1  | 0   | 0.9624   | 20.5941 | 1.7412  | 0.9624   | 20.4819 | 2.2662  |
|      |    | 0.5 | 0.9622   | 20.5524 | 1.6896  | 0.9614   | 20.4705 | 2.1600  |
|      |    | 0.9 | 0.9622   | 20.5496 | 1.6246  | 0.9620   | 20.4677 | 1.9946  |
|      | 9  | 0   | 0.9620   | 20.2052 | -0.0402 | 0.9622   | 20.2081 | 0.0948  |
|      |    | 0.5 | 0.9620   | 20.1091 | -1.7028 | 0.9618   | 20.1363 | -1.2178 |
|      |    | 0.9 | 0.9614   | 20.4462 | -6.0360 | 0.9612   | 20.4087 | -5.6732 |
|      | 10 | 0   | 0.9614   | 20.1461 | -0.1378 | 0.9618   | 20.1647 | -0.0716 |
|      |    | 0.5 | 0.9616   | 20.0419 | -2.0414 | 0.9616   | 20.0809 | -1.5562 |
|      |    | 0.9 | 0.9608   | 20.4246 | -6.9036 | 0.9614   | 20.3902 | -6.5434 |
| 1000 | 1  | 0   | 0.9536   | 14.0490 | 1.7164  | 0.9540   | 14.0323 | 2.2568  |
|      |    | 0.5 | 0.9538   | 14.0469 | 1.7952  | 0.9542   | 14.0350 | 2.2782  |
|      |    | 0.9 | 0.9548   | 14.0459 | 1.4790  | 0.9546   | 14.0339 | 1.9036  |
|      | 9  | 0   | 0.9542   | 13.9977 | 0.0880  | 0.9542   | 13.9959 | 0.1612  |
|      |    | 0.5 | 0.9532   | 13.9982 | 0.0812  | 0.9536   | 13.9958 | 0.1612  |
|      |    | 0.9 | 0.9526   | 14.0002 | -4.5872 | 0.9530   | 14.0021 | -4.1018 |
|      | 10 | 0   | 0.9536   | 13.9910 | 0.0000  | 0.9536   | 13.9910 | 0.0000  |
|      |    | 0.5 | 0.9534   | 13.9904 | -0.0070 | 0.9536   | 13.9909 | -0.0026 |
|      |    | 0.9 | 0.9542   | 13.9953 | -5.2558 | 0.9540   | 13.9935 | -4.7612 |

Table 5.55.  $p = 20$ , error type = 5

|      |    |     | LASSO    |         |         | RIDGE    |         |         |
|------|----|-----|----------|---------|---------|----------|---------|---------|
| n    | a  | psi | coverage | length  | penalty | coverage | length  | penalty |
| 200  | 1  | 0   | 0.9638   | 22.0391 | 0.3307  | 0.9616   | 21.6416 | 26.0212 |
|      |    | 0.5 | 0.9634   | 22.0000 | 0.0871  | 0.9628   | 22.0063 | 1.7339  |
|      |    | 0.9 | 0.9656   | 22.3324 | 0.0307  | 0.9664   | 22.4041 | 0.6712  |
|      | 19 | 0   | 0.9618   | 20.6565 | 0.0155  | 0.9612   | 20.7638 | 0.3602  |
|      |    | 0.5 | 0.9602   | 20.7313 | 0.2929  | 0.9638   | 21.9347 | 4.7459  |
|      |    | 0.9 | 0.9640   | 22.1453 | 0.5604  | 0.9664   | 22.6021 | 11.8142 |
|      | 20 | 0   | 0.9620   | 20.6513 | 20.0098 | 0.9610   | 20.7601 | 0.3487  |
|      |    | 0.5 | 0.9608   | 20.6857 | 0.3096  | 0.9646   | 21.9098 | 5.1415  |
|      |    | 0.9 | 0.9642   | 22.2160 | 0.6168  | 0.9660   | 22.6621 | 13.2291 |
| 400  | 1  | 0   | 0.9706   | 22.1061 | 0.2366  | 0.9702   | 21.7341 | 3.5332  |
|      |    | 0.5 | 0.9706   | 22.0767 | 0.0605  | 0.9710   | 22.0290 | 0.9725  |
|      |    | 0.9 | 0.9710   | 22.2438 | 0.0287  | 0.9704   | 22.2935 | 0.5437  |
|      | 19 | 0   | 0.9686   | 21.3931 | 0.0114  | 0.9684   | 21.4252 | 0.1989  |
|      |    | 0.5 | 0.9694   | 21.4013 | 0.2919  | 0.9716   | 22.0499 | 4.4646  |
|      |    | 0.9 | 0.9730   | 22.1035 | 0.5130  | 0.9714   | 22.3348 | 10.4036 |
|      | 20 | 0   | 0.9688   | 21.3913 | 0.0077  | 0.9684   | 21.4246 | 0.1958  |
|      |    | 0.5 | 0.9696   | 21.3779 | 0.3081  | 0.9708   | 22.0374 | 4.8456  |
|      |    | 0.9 | 0.9724   | 22.1231 | 0.5444  | 0.9716   | 22.3435 | 11.1567 |
| 2000 | 1  | 0   | 0.9536   | 14.4745 | 0.1058  | 0.9520   | 14.3888 | 0.4079  |
|      |    | 0.5 | 0.9526   | 14.4709 | 0.0280  | 0.9532   | 14.4555 | 0.2606  |
|      |    | 0.9 | 0.9524   | 14.4924 | 0.0285  | 0.9532   | 14.5023 | 0.5404  |
|      | 19 | 0   | 0.9522   | 14.3787 | 0.0077  | 0.9526   | 14.3805 | 0.1332  |
|      |    | 0.5 | 0.9512   | 14.3881 | 0.2919  | 0.9508   | 14.4837 | 4.5254  |
|      |    | 0.9 | 0.9512   | 14.6074 | 0.5124  | 0.9510   | 14.5354 | 10.2952 |
|      | 20 | 0   | 0.9518   | 14.3781 | 0.0073  | 0.9516   | 14.3811 | 0.1340  |
|      |    | 0.5 | 0.9510   | 0.9516  | 0.9522  | 0.9516   | 0.9518  | 0.9518  |
|      |    | 0.9 | 0.9500   | 0.9502  | 0.9520  | 0.9518   | 0.9508  | 0.9516  |

Table 5.56.  $p = 20$ , error type = 5

|      |    |     | PLS      |         |         | PCR      |         |         |
|------|----|-----|----------|---------|---------|----------|---------|---------|
| n    | a  | psi | coverage | length  | penalty | coverage | length  | penalty |
| 200  | 1  | 0   | 0.9626   | 20.7535 | 17.2850 | 0.9622   | 21.5083 | 9.7214  |
|      |    | 0.5 | 0.9620   | 20.7995 | 16.8774 | 0.9650   | 21.9071 | 13.8680 |
|      |    | 0.9 | 0.9612   | 20.7884 | 16.9784 | 0.9640   | 22.0386 | 15.3514 |
|      | 19 | 0   | 0.9618   | 20.6527 | 13.9186 | 0.9624   | 20.7186 | 0.2850  |
|      |    | 0.5 | 0.9612   | 20.7913 | 16.8766 | 0.9628   | 21.8958 | 13.8872 |
|      |    | 0.9 | 0.9614   | 20.7874 | 16.9740 | 0.9642   | 22.0336 | 15.3400 |
|      | 20 | 0   | 0.9620   | 20.6519 | 13.8592 | 0.9624   | 20.7156 | 0.2666  |
|      |    | 0.5 | 0.9612   | 20.7871 | 16.9810 | 0.9642   | 22.0372 | 15.3776 |
|      |    | 0.9 | 0.9614   | 20.7875 | 16.9806 | 0.9644   | 22.0420 | 15.3904 |
| 400  | 1  | 0   | 0.9684   | 21.3969 | 16.6310 | 0.9702   | 21.5813 | 5.2444  |
|      |    | 0.5 | 0.9686   | 21.4260 | 16.7588 | 0.9694   | 21.9336 | 12.5974 |
|      |    | 0.9 | 0.9682   | 21.4241 | 16.9818 | 0.9706   | 22.0992 | 15.4044 |
|      | 19 | 0   | 0.9688   | 21.3903 | 14.0690 | 0.9686   | 21.4066 | 0.0936  |
|      |    | 0.5 | 0.9686   | 21.4270 | 16.7698 | 0.9690   | 21.9374 | 12.6138 |
|      |    | 0.9 | 0.9680   | 21.4251 | 16.9822 | 0.9698   | 22.1002 | 15.3944 |
|      | 20 | 0   | 0.9688   | 21.3904 | 14.0038 | 0.9686   | 21.4003 | 0.0868  |
|      |    | 0.5 | 0.9682   | 21.4242 | 16.9848 | 0.9710   | 22.1093 | 15.5170 |
|      |    | 0.9 | 0.9680   | 21.4245 | 16.9828 | 0.9712   | 22.1092 | 15.5130 |
| 2000 | 1  | 0   | 0.9518   | 14.3782 | 15.2284 | 0.9512   | 14.3817 | 0.3224  |
|      |    | 0.5 | 0.9518   | 14.3785 | 15.2266 | 0.9524   | 14.3955 | 3.4098  |
|      |    | 0.9 | 0.9518   | 14.3784 | 16.9608 | 0.9516   | 14.4749 | 15.2692 |
|      | 19 | 0   | 0.9518   | 14.3782 | 14.4894 | 0.9518   | 14.3782 | 0.0030  |
|      |    | 0.5 | 0.9518   | 14.3781 | 15.3130 | 0.9518   | 14.3933 | 3.4486  |
|      |    | 0.9 | 0.9522   | 14.3784 | 16.9690 | 0.9520   | 14.4758 | 15.2606 |
|      | 20 | 0   | 0.9518   | 14.3782 | 14.4422 | 0.9518   | 14.3789 | 0.0036  |
|      |    | 0.5 | 14.3839  | 14.4814 | 14.3784 | 14.4791  | 14.3782 | 14.3782 |
|      |    | 0.9 | 14.6314  | 14.5370 | 14.3783 | 14.4789  | 14.3592 | 14.3643 |

Table 5.57.  $p = 20$ , error type = 5

|      |    |     | Ii       |         |          | I-Min    |         |          |
|------|----|-----|----------|---------|----------|----------|---------|----------|
| n    | a  | psi | coverage | length  | penalty  | coverage | length  | penalty  |
| 200  | 1  | 0   | 0.9644   | 21.6298 | 2.7798   | 0.9650   | 21.3919 | 3.8216   |
|      |    | 0.5 | 0.9638   | 21.5698 | 2.7520   | 0.9632   | 21.3876 | 3.6764   |
|      |    | 0.9 | 0.9636   | 21.5948 | 2.4558   | 0.9632   | 21.4090 | 3.2536   |
|      | 19 | 0   | 0.9612   | 20.6266 | -0.2444  | 0.9616   | 20.6523 | -0.0214  |
|      |    | 0.5 | 0.9604   | 20.4381 | -4.1976  | 0.9606   | 20.5022 | -3.2844  |
|      |    | 0.9 | 0.9636   | 21.2594 | -13.9578 | 0.9634   | 21.1754 | -13.1410 |
|      | 20 | 0   | 0.9616   | 20.5702 | -0.3570  | 0.9618   | 20.6109 | -0.1888  |
|      |    | 0.5 | 0.9592   | 20.3813 | -4.5462  | 0.9612   | 20.4453 | -3.6432  |
|      |    | 0.9 | 0.9634   | 21.2391 | -14.8162 | 0.9630   | 21.1496 | -13.9928 |
| 400  | 1  | 0   | 0.9706   | 21.9028 | 2.7658   | 0.9702   | 21.7759 | 3.8260   |
|      |    | 0.5 | 0.9708   | 21.8732 | 2.8916   | 0.9696   | 21.7700 | 3.8158   |
|      |    | 0.9 | 0.9706   | 21.8695 | 2.3990   | 0.9704   | 21.7653 | 3.2520   |
|      | 19 | 0   | 0.9688   | 21.4190 | 0.0742   | 0.9686   | 21.4099 | 0.1498   |
|      |    | 0.5 | 0.9682   | 21.2799 | -1.3842  | 0.9682   | 21.3293 | -0.8050  |
|      |    | 0.9 | 0.9702   | 21.5845 | -12.9152 | 0.9690   | 21.5619 | -12.0424 |
|      | 20 | 0   | 0.9688   | 21.3898 | -0.0028  | 0.9688   | 21.3904 | -0.0010  |
|      |    | 0.5 | 0.9678   | 21.2476 | -1.5662  | 0.9686   | 21.3043 | -1.0174  |
|      |    | 0.9 | 0.9690   | 21.5634 | -13.7016 | 0.9694   | 21.5489 | -12.8310 |
| 2000 | 1  | 0   | 0.9518   | 14.4469 | 2.7858   | 0.9524   | 14.4292 | 3.8858   |
|      |    | 0.5 | 0.9512   | 14.4434 | 2.9658   | 0.9526   | 14.4303 | 3.9016   |
|      |    | 0.9 | 0.9514   | 14.4416 | 2.5276   | 0.9522   | 14.4300 | 3.4498   |
|      | 19 | 0   | 0.9514   | 14.3821 | 0.0850   | 0.9514   | 14.3809 | 0.1588   |
|      |    | 0.5 | 0.9518   | 14.3828 | 0.0830   | 0.9518   | 14.3817 | 0.1576   |
|      |    | 0.9 | 0.9512   | 14.3644 | -8.1498  | 0.9510   | 14.3684 | -7.0936  |
|      | 20 | 0   | 0.9518   | 14.3782 | 0.0000   | 0.9518   | 14.3782 | 0.0000   |
|      |    | 0.5 | 0.3081   | 5.6141  | 16.9846  | 15.6618  | 0.0000  | 0.0000   |
|      |    | 0.9 | 0.5409   | 10.8671 | 16.9848  | 15.6486  | -8.6810 | -7.6342  |

Table 5.58.  $p = 50$ , error type = 5

|      |    |     | LASSO    |         |         | RIDGE    |         |          |
|------|----|-----|----------|---------|---------|----------|---------|----------|
| n    | a  | psi | coverage | length  | penalty | coverage | length  | penalty  |
| 500  | 1  | 0   | 0.9758   | 24.4032 | 0.2589  | 0.9730   | 23.6871 | 7.4834   |
|      |    | 0.5 | 0.9762   | 24.3648 | 0.0434  | 0.9758   | 24.3581 | 1.2459   |
|      |    | 0.9 | 0.9758   | 24.6002 | 0.0456  | 0.9764   | 24.6575 | 2.1344   |
|      | 49 | 0   | 0.9698   | 22.4348 | 0.0076  | 0.9704   | 22.4989 | 0.2220   |
|      |    | 0.5 | 0.9808   | 30.8675 | 4.5726  | 0.9820   | 31.4847 | 230.3264 |
|      |    | 0.9 | 0.9886   | 52.3307 | 8.2588  | 0.9886   | 53.3699 | 491.2959 |
|      | 50 | 0   | 0.9698   | 22.4328 | 0.0072  | 0.9700   | 22.4986 | 0.2217   |
|      |    | 0.5 | 0.9786   | 31.4094 | 4.7065  | 0.9818   | 32.0013 | 239.0520 |
|      |    | 0.9 | 0.9884   | 53.4085 | 8.4634  | 0.9888   | 54.3898 | 502.2058 |
| 1000 | 1  | 0   | 0.9694   | 23.1464 | 0.1855  | 0.9688   | 22.6177 | 2.2190   |
|      |    | 0.5 | 0.9694   | 23.1346 | 0.0328  | 0.9688   | 23.0961 | 0.6114   |
|      |    | 0.9 | 0.9700   | 23.2336 | 0.0456  | 0.9698   | 23.2634 | 2.3809   |
|      | 49 | 0   | 0.9684   | 22.2102 | 0.0072  | 0.9682   | 22.2374 | 0.1738   |
|      |    | 0.5 | 0.9710   | 28.5856 | 4.4955  | 0.9738   | 28.9385 | 228.1795 |
|      |    | 0.9 | 0.9816   | 47.8869 | 8.1421  | 0.9810   | 49.3925 | 492.7493 |
|      | 50 | 0   | 0.9686   | 22.2091 | 0.0072  | 0.9682   | 22.2369 | 0.1748   |
|      |    | 0.5 | 0.9726   | 29.0773 | 4.6300  | 0.9740   | 29.4207 | 236.8647 |
|      |    | 0.9 | 0.9816   | 48.8570 | 8.3412  | 0.9808   | 50.3181 | 503.4000 |
| 5000 | 1  | 0   | 0.9542   | 14.8694 | 0.0834  | 0.9546   | 14.7622 | 0.4135   |
|      |    | 0.5 | 0.9552   | 14.8746 | 0.0261  | 0.9542   | 14.8697 | 0.3959   |
|      |    | 0.9 | 0.9548   | 14.8844 | 0.0456  | 0.9542   | 14.8858 | 2.5505   |
|      | 49 | 0   | 0.9544   | 14.7483 | 0.0072  | 0.9540   | 14.7515 | 0.1360   |
|      |    | 0.5 | 0.9544   | 22.8692 | 4.4705  | 0.956    | 22.8481 | 227.795  |
|      |    | 0.9 | 0.9506   | 39.9074 | 8.0520  | 0.9538   | 41.5863 | 494.0449 |
|      | 50 | 0   | 0.9540   | 14.7478 | 0.0073  | 0.9538   | 14.7510 | 0.1363   |
|      |    | 0.5 | 0.9534   | 23.3133 | 4.5913  | 0.9558   | 23.2098 | 234.3730 |
|      |    | 0.9 | 0.9506   | 40.6480 | 8.2208  | 0.9538   | 42.3589 | 504.3374 |

Table 5.59.  $p = 50$ , error type = 5

|      |    |     | PLS      |         |         | PCR      |         |         |
|------|----|-----|----------|---------|---------|----------|---------|---------|
| n    | a  | psi | coverage | length  | penalty | coverage | length  | penalty |
| 500  | 1  | 0   | 0.9700   | 22.5805 | 47.4532 | 0.9730   | 23.6569 | 28.1072 |
|      |    | 0.5 | 0.9708   | 22.6512 | 46.9796 | 0.9758   | 24.3090 | 42.0884 |
|      |    | 0.9 | 0.9708   | 22.6268 | 46.9996 | 0.9756   | 24.4456 | 44.8080 |
|      | 49 | 0   | 0.9694   | 22.4335 | 42.5238 | 0.9694   | 22.4347 | 0.0542  |
|      |    | 0.5 | 0.9712   | 22.6565 | 46.9800 | 0.9758   | 24.3149 | 42.0892 |
|      |    | 0.9 | 0.9708   | 22.6268 | 46.9992 | 0.9764   | 24.4510 | 44.8520 |
|      | 50 | 0   | 0.9694   | 22.4332 | 42.4880 | 0.9690   | 22.4335 | 0.0562  |
|      |    | 0.5 | 0.9706   | 22.6253 | 46.9996 | 0.9760   | 24.4552 | 44.8904 |
|      |    | 0.9 | 0.9706   | 22.6253 | 46.9994 | 0.9758   | 24.4533 | 44.8832 |
| 1000 | 1  | 0   | 0.9692   | 22.2150 | 46.8178 | 0.9672   | 22.4034 | 11.3746 |
|      |    | 0.5 | 0.9684   | 22.2686 | 46.9288 | 0.9698   | 23.0245 | 39.0052 |
|      |    | 0.9 | 0.9688   | 22.2598 | 46.9992 | 0.9700   | 23.1708 | 45.0028 |
|      | 49 | 0   | 0.9682   | 22.2096 | 43.0464 | 0.9682   | 22.2097 | 0.0106  |
|      |    | 0.5 | 0.9692   | 22.2727 | 46.9288 | 0.9686   | 23.0158 | 38.8416 |
|      |    | 0.9 | 0.9692   | 22.2606 | 46.9994 | 0.9704   | 23.1692 | 45.0360 |
|      | 50 | 0   | 0.9682   | 22.2096 | 43.0038 | 0.9682   | 22.2100 | 0.0108  |
|      |    | 0.5 | 0.9692   | 22.2591 | 46.9994 | 0.9702   | 23.1707 | 45.1392 |
|      |    | 0.9 | 0.9692   | 22.2591 | 46.9994 | 0.9702   | 23.1720 | 45.1586 |
| 5000 | 1  | 0   | 0.9546   | 14.7486 | 45.0298 | 0.9550   | 14.7492 | 0.3450  |
|      |    | 0.5 | 0.9546   | 14.7485 | 45.3106 | 0.9552   | 14.7594 | 5.7424  |
|      |    | 0.9 | 0.9552   | 14.7496 | 46.9996 | 0.9546   | 14.8731 | 44.9304 |
|      | 49 | 0   | 0.9546   | 14.7486 | 43.8572 | 0.9546   | 14.7486 | 0.0000  |
|      |    | 0.5 | 0.9546   | 14.7486 | 45.31   | 0.955    | 14.7578 | 5.7258  |
|      |    | 0.9 | 0.9556   | 14.7500 | 46.9998 | 0.9548   | 14.8722 | 44.8754 |
|      | 50 | 0   | 0.9546   | 14.7486 | 43.8502 | 0.9546   | 14.7486 | 0.0002  |
|      |    | 0.5 | 0.9556   | 14.7496 | 47.0000 | 0.9546   | 14.8750 | 45.4536 |
|      |    | 0.9 | 0.9554   | 14.7496 | 47.0000 | 0.9548   | 14.8754 | 45.4920 |

Table 5.60.  $p = 50$ , error type = 5

|      |    |     | Ii       |         |          | I-Min    |         |          |
|------|----|-----|----------|---------|----------|----------|---------|----------|
| n    | a  | psi | coverage | length  | penalty  | coverage | length  | penalty  |
| 500  | 1  | 0   | 0.9738   | 23.6191 | 6.3784   | 0.9728   | 23.3927 | 8.5734   |
|      |    | 0.5 | 0.9738   | 23.6044 | 6.5390   | 0.9732   | 23.3900 | 8.5750   |
|      |    | 0.9 | 0.9736   | 23.6133 | 5.8324   | 0.9724   | 23.4058 | 7.8870   |
|      | 49 | 0   | 0.9704   | 22.4600 | 0.0916   | 0.9700   | 22.4512 | 0.1714   |
|      |    | 0.5 | 0.9708   | 22.2286 | -2.4042  | 0.9706   | 22.3165 | -1.4500  |
|      |    | 0.9 | 0.9716   | 22.7959 | -33.2134 | 0.9706   | 22.7720 | -31.4164 |
|      | 50 | 0   | 0.9692   | 22.4328 | -0.0012  | 0.9692   | 22.4329 | -0.0004  |
|      |    | 0.5 | 0.9698   | 22.1960 | -2.5964  | 0.9694   | 22.2966 | -1.6686  |
|      |    | 0.9 | 0.9716   | 22.7763 | -33.9796 | 0.9710   | 22.7567 | -32.1914 |
| 1000 | 1  | 0   | 0.9692   | 22.7667 | 6.3366   | 0.9692   | 22.6584 | 8.5568   |
|      |    | 0.5 | 0.9700   | 22.7658 | 6.4900   | 0.9692   | 22.6609 | 8.5340   |
|      |    | 0.9 | 0.9696   | 22.7649 | 5.8338   | 0.9692   | 22.6602 | 7.9064   |
|      | 49 | 0   | 0.9686   | 22.2227 | 0.0896   | 0.9682   | 22.2191 | 0.1646   |
|      |    | 0.5 | 0.9686   | 22.2136 | -0.0078  | 0.9682   | 22.2161 | 0.1256   |
|      |    | 0.9 | 0.9682   | 22.2265 | -28.2904 | 0.9684   | 22.2407 | -26.3920 |
|      | 50 | 0   | 0.9682   | 22.2098 | 0.0000   | 0.9682   | 22.2098 | 0.0000   |
|      |    | 0.5 | 0.9682   | 22.1999 | -0.1024  | 0.9682   | 22.2065 | -0.0386  |
|      |    | 0.9 | 0.9690   | 22.2135 | -28.9516 | 0.9682   | 22.2288 | -27.0778 |
| 5000 | 1  | 0   | 0.9542   | 14.8215 | 6.2834   | 0.9540   | 14.8076 | 8.5384   |
|      |    | 0.5 | 0.9552   | 14.8224 | 6.4320   | 0.9546   | 14.8071 | 8.5568   |
|      |    | 0.9 | 0.9546   | 14.8208 | 6.2042   | 0.9546   | 14.8067 | 8.3478   |
|      | 49 | 0   | 0.9546   | 14.7505 | 0.0818   | 0.9546   | 14.7500 | 0.1582   |
|      |    | 0.5 | 0.9548   | 14.7506 | 0.0818   | 0.9548   | 14.7499 | 0.1610   |
|      |    | 0.9 | 0.9534   | 14.7183 | -10.7004 | 0.9530   | 14.7254 | -8.7588  |
|      | 50 | 0   | 0.9546   | 14.7486 | 0.0000   | 0.9546   | 14.7486 | 0.0000   |
|      |    | 0.5 | 0.9546   | 14.7486 | 0.0000   | 0.9546   | 14.7486 | 0.0000   |
|      |    | 0.9 | 0.9534   | 14.7163 | -11.0292 | 0.9532   | 14.7244 | -9.1016  |

## 5.2 D PREDICTION REGION SIMULATION

Table 5.61. Etype = 1, J=5, k=1,  $\psi = 0$

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0      | Cov | 0.9134 | 0.9476 | 0.9134 | 0.9134 | 0.9134 |
|      |     |        | Len | 4.0255 | 4.2500 | 4.0256 | 4.0256 | 4.0256 |
| 100  | 40  | 0      | Cov | 0.9200 | 0.9226 | 0.8166 | 0.9170 | 0.8462 |
|      |     |        | Len | 3.8556 | 4.0637 | 3.4933 | 4.6253 | 3.6050 |
| 100  | 100 | 0      | Cov | 0.9192 | 0.8902 | 0.1500 | 0.9304 | 0.7008 |
|      |     |        | Len | 3.7609 | 3.7709 | 0.9522 | 5.0993 | 2.9472 |
| 100  | 200 | 0      | Cov | 0.9096 | 0.8646 | 0.0008 | 0.9350 | 0.5944 |
|      |     |        | Len | 3.7652 | 3.6008 | 0.0027 | 5.2770 | 2.4815 |
| 400  | 20  | 0      | Cov | 0.9416 | 0.9476 | 0.9414 | 0.9414 | 0.9414 |
|      |     |        | Len | 3.9175 | 3.9632 | 3.9174 | 3.9174 | 3.9174 |
| 400  | 40  | 0      | Cov | 0.9214 | 0.9344 | 0.9202 | 0.9202 | 0.9202 |
|      |     |        | Len | 3.8100 | 3.9102 | 3.8099 | 3.8099 | 3.8099 |
| 400  | 100 | 0      | Cov | 0.8888 | 0.9136 | 0.8644 | 0.8794 | 0.8660 |
|      |     |        | Len | 3.5166 | 3.7596 | 3.4823 | 3.7696 | 3.4851 |
| 400  | 200 | 0      | Cov | 0.8906 | 0.8932 | 0.6904 | 0.8964 | 0.7694 |
|      |     |        | Len | 3.3802 | 3.5772 | 2.8376 | 4.2251 | 3.0424 |
| 1000 | 20  | 0      | Cov | 0.9456 | 0.9462 | 0.9458 | 0.9458 | 0.9458 |
|      |     |        | Len | 3.9033 | 3.9203 | 3.9033 | 3.9033 | 3.9033 |
| 1000 | 40  | 0      | Cov | 0.9380 | 0.9430 | 0.9380 | 0.9380 | 0.9380 |
|      |     |        | Len | 3.8623 | 3.9003 | 3.8621 | 3.8621 | 3.8621 |
| 1000 | 100 | 0      | Cov | 0.9202 | 0.9358 | 0.9180 | 0.9180 | 0.9180 |
|      |     |        | Len | 3.7383 | 3.8389 | 3.7381 | 3.7381 | 3.7381 |
| 1000 | 200 | 0      | Cov | 0.8804 | 0.9220 | 0.8780 | 0.8780 | 0.8780 |
|      |     |        | Len | 3.5249 | 3.7369 | 3.5248 | 3.5248 | 3.5248 |

Table 5.62. Etype = 1, J=5, k=1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0.2236 | Cov | 0.9208 | 0.9584 | 0.9174 | 0.9174 | 0.9174 |
|      |     |        | Len | 4.0240 | 4.3435 | 4.0237 | 4.0237 | 4.0237 |
| 100  | 40  | 0.1581 | Cov | 0.9178 | 0.9540 | 0.8170 | 0.9208 | 0.8464 |
|      |     |        | Len | 3.8452 | 4.3344 | 3.4831 | 4.4528 | 3.5992 |
| 100  | 100 | 0.1    | Cov | 0.9116 | 0.9476 | 0.1444 | 0.9238 | 0.7058 |
|      |     |        | Len | 3.7663 | 4.2949 | 0.9797 | 4.9182 | 2.9537 |
| 100  | 200 | 0.07   | Cov | 0.9174 | 0.9496 | 0.0014 | 0.9280 | 0.6012 |
|      |     |        | Len | 3.7683 | 4.2563 | 0.0040 | 5.1525 | 2.4942 |
| 400  | 20  | 0.2236 | Cov | 0.9374 | 0.9468 | 0.9372 | 0.9372 | 0.9372 |
|      |     |        | Len | 3.9153 | 3.9857 | 3.9151 | 3.9151 | 3.9151 |
| 400  | 40  | 0.1581 | Cov | 0.9328 | 0.9472 | 0.9334 | 0.9334 | 0.9334 |
|      |     |        | Len | 3.8133 | 3.9643 | 3.8132 | 3.8132 | 3.8132 |
| 400  | 100 | 0.1    | Cov | 0.8866 | 0.9406 | 0.8582 | 0.8824 | 0.8592 |
|      |     |        | Len | 3.5177 | 3.9206 | 3.4824 | 3.7397 | 3.4846 |
| 400  | 200 | 0.07   | Cov | 0.8826 | 0.9422 | 0.6842 | 0.8950 | 0.7514 |
|      |     |        | Len | 3.3844 | 3.9288 | 2.8392 | 4.1530 | 3.0441 |
| 1000 | 20  | 0.2236 | Cov | 0.9394 | 0.9432 | 0.9388 | 0.9388 | 0.9388 |
|      |     |        | Len | 3.9037 | 3.9304 | 3.9035 | 3.9035 | 3.9035 |
| 1000 | 40  | 0.1581 | Cov | 0.9414 | 0.9472 | 0.9402 | 0.9402 | 0.9402 |
|      |     |        | Len | 3.8612 | 3.9212 | 3.8611 | 3.8611 | 3.8611 |
| 1000 | 100 | 0.1    | Cov | 0.9278 | 0.9474 | 0.9260 | 0.9260 | 0.9260 |
|      |     |        | Len | 3.7375 | 3.8954 | 3.7372 | 3.7372 | 3.7372 |
| 1000 | 200 | 0.07   | Cov | 0.8850 | 0.9426 | 0.8822 | 0.8822 | 0.8822 |
|      |     |        | Len | 3.5252 | 3.8465 | 3.5250 | 3.5250 | 3.5250 |

Table 5.63. Etype = 1, J=5, k=1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9642 | 0.9650 | 0.9226 | 0.9226 | 0.9226 |
|      |     |        | Len | 4.4113 | 4.4064 | 4.0222 | 4.0222 | 4.0222 |
| 100  | 40  | 0.9    | Cov | 0.9638 | 0.9626 | 0.8072 | 0.9288 | 0.8374 |
|      |     |        | Len | 4.4138 | 4.3972 | 3.4852 | 4.0367 | 3.5954 |
| 100  | 100 | 0.9    | Cov | 0.9666 | 0.9592 | 0.1518 | 0.9370 | 0.7088 |
|      |     |        | Len | 4.4616 | 4.3785 | 0.9773 | 4.0615 | 2.9440 |
| 100  | 200 | 0.9    | Cov | 0.9700 | 0.9588 | 0.0016 | 0.9438 | 0.6036 |
|      |     |        | Len | 4.5267 | 4.3484 | 0.0035 | 4.0767 | 2.4870 |
| 400  | 20  | 0.9    | Cov | 0.9562 | 0.9538 | 0.9446 | 0.9446 | 0.9446 |
|      |     |        | Len | 3.9962 | 3.9927 | 3.9136 | 3.9136 | 3.9136 |
| 400  | 40  | 0.9    | Cov | 0.9516 | 0.9516 | 0.9318 | 0.9318 | 0.9318 |
|      |     |        | Len | 4.0012 | 3.9892 | 3.8130 | 3.8130 | 3.8130 |
| 400  | 100 | 0.9    | Cov | 0.9516 | 0.9488 | 0.8738 | 0.9028 | 0.8724 |
|      |     |        | Len | 4.0143 | 3.9821 | 3.4755 | 3.5899 | 3.4787 |
| 400  | 200 | 0.9    | Cov | 0.9570 | 0.9508 | 0.6740 | 0.9036 | 0.7546 |
|      |     |        | Len | 4.0203 | 3.9735 | 2.8373 | 3.6016 | 3.0397 |
| 1000 | 20  | 0.9    | Cov | 0.9512 | 0.9492 | 0.9442 | 0.9442 | 0.9442 |
|      |     |        | Len | 3.9383 | 3.9349 | 3.9047 | 3.9047 | 3.9047 |
| 1000 | 40  | 0.9    | Cov | 0.9516 | 0.9490 | 0.9376 | 0.9376 | 0.9376 |
|      |     |        | Len | 3.9406 | 3.9315 | 3.8620 | 3.8620 | 3.8620 |
| 1000 | 100 | 0.9    | Cov | 0.9496 | 0.9492 | 0.9226 | 0.9226 | 0.9226 |
|      |     |        | Len | 3.9431 | 3.9260 | 3.7372 | 3.7372 | 3.7372 |
| 1000 | 200 | 0.9    | Cov | 0.9512 | 0.9520 | 0.8860 | 0.8860 | 0.8860 |
|      |     |        | Len | 3.9522 | 3.9268 | 3.5275 | 3.5275 | 3.5275 |

Table 5.64. Etype = 1, J=5, k=19,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS     |
|------|-----|--------|-----|---------|---------|--------|---------|--------|
| 100  | 20  | 0      | Cov | 0.9828  | 0.9832  | 0.9832 | 0.9832  | 0.9832 |
|      |     |        | Len | 5.7131  | 5.7094  | 5.7094 | 5.7094  | 5.7094 |
| 100  | 40  | 0      | Cov | 0.9854  | 0.9782  | 0.9364 | 0.9740  | 0.9796 |
|      |     |        | Len | 12.7635 | 6.2476  | 4.9307 | 15.0372 | 5.9440 |
| 100  | 100 | 0      | Cov | 0.9792  | 0.9656  | 0.2104 | 0.9814  | 0.9680 |
|      |     |        | Len | 15.8699 | 8.4696  | 1.4720 | 20.0538 | 6.6593 |
| 100  | 200 | 0      | Cov | 0.9788  | 0.9542  | 0.0006 | 0.9792  | 0.9154 |
|      |     |        | Len | 17.1839 | 10.2816 | 0.0080 | 21.3345 | 7.4098 |
| 400  | 20  | 0      | Cov | 0.9760  | 0.9758  | 0.9758 | 0.9758  | 0.9758 |
|      |     |        | Len | 4.6951  | 4.6929  | 4.6929 | 4.6929  | 4.6929 |
| 400  | 40  | 0      | Cov | 0.9678  | 0.9716  | 0.9670 | 0.9670  | 0.9670 |
|      |     |        | Len | 4.5804  | 4.6395  | 4.5788 | 4.5788  | 4.5788 |
| 400  | 100 | 0      | Cov | 0.9420  | 0.9606  | 0.9248 | 0.8946  | 0.9260 |
|      |     |        | Len | 4.2338  | 4.4501  | 4.1750 | 7.3308  | 4.1804 |
| 400  | 200 | 0      | Cov | 0.9438  | 0.9416  | 0.7628 | 0.9160  | 0.8462 |
|      |     |        | Len | 4.1931  | 4.2513  | 3.4107 | 12.4192 | 3.7043 |
| 1000 | 20  | 0      | Cov | 0.9610  | 0.9604  | 0.9604 | 0.9604  | 0.9604 |
|      |     |        | Len | 4.1756  | 4.1742  | 4.1742 | 4.1742  | 4.1742 |
| 1000 | 40  | 0      | Cov | 0.9566  | 0.9576  | 0.9578 | 0.9578  | 0.9578 |
|      |     |        | Len | 4.1407  | 4.1606  | 4.1388 | 4.1388  | 4.1388 |
| 1000 | 100 | 0      | Cov | 0.9406  | 0.9512  | 0.9386 | 0.9386  | 0.9386 |
|      |     |        | Len | 4.0089  | 4.0956  | 4.0071 | 4.0071  | 4.0071 |
| 1000 | 200 | 0      | Cov | 0.9082  | 0.9392  | 0.9072 | 0.9072  | 0.9072 |
|      |     |        | Len | 3.7783  | 3.9847  | 3.7761 | 3.7761  | 3.7761 |

Table 5.65. Etype = 1, J=5, k=19,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 20  | 0.2236 | Cov | 0.9876  | 0.9862  | 0.9862 | 0.9862  | 0.9862  |
|      |     |        | Len | 5.7789  | 5.7056  | 5.7056 | 5.7056  | 5.7056  |
| 100  | 40  | 0.1581 | Cov | 0.9948  | 0.9706  | 0.9324 | 0.9720  | 0.9706  |
|      |     |        | Len | 46.0662 | 9.2893  | 4.9413 | 10.2913 | 6.7200  |
| 100  | 100 | 0.1    | Cov | 0.9916  | 0.9436  | 0.2166 | 0.9796  | 0.9302  |
|      |     |        | Len | 58.3789 | 14.1499 | 1.4920 | 16.7052 | 8.7784  |
| 100  | 200 | 0.07   | Cov | 0.9910  | 0.9242  | 0.0006 | 0.9824  | 0.8756  |
|      |     |        | Len | 64.5434 | 16.7618 | 0.0113 | 19.6203 | 10.2351 |
| 400  | 20  | 0.2236 | Cov | 0.9768  | 0.9792  | 0.9792 | 0.9792  | 0.9792  |
|      |     |        | Len | 4.7434  | 4.6887  | 4.6887 | 4.6887  | 4.6887  |
| 400  | 40  | 0.1581 | Cov | 0.9704  | 0.9680  | 0.9642 | 0.9642  | 0.9642  |
|      |     |        | Len | 4.7036  | 4.6516  | 4.5786 | 4.5786  | 4.5786  |
| 400  | 100 | 0.1    | Cov | 0.9640  | 0.9654  | 0.9212 | 0.8980  | 0.9230  |
|      |     |        | Len | 4.6206  | 4.5712  | 4.1806 | 6.4729  | 4.1855  |
| 400  | 200 | 0.07   | Cov | 0.9626  | 0.9582  | 0.7728 | 0.9154  | 0.8486  |
|      |     |        | Len | 4.5386  | 4.4916  | 3.4093 | 11.2157 | 3.7047  |
| 1000 | 20  | 0.2236 | Cov | 0.9632  | 0.9662  | 0.9662 | 0.9662  | 0.9662  |
|      |     |        | Len | 4.2243  | 4.1791  | 4.1791 | 4.1791  | 4.1791  |
| 1000 | 40  | 0.1581 | Cov | 0.9556  | 0.9560  | 0.9516 | 0.9516  | 0.9516  |
|      |     |        | Len | 4.2093  | 4.1645  | 4.1394 | 4.1394  | 4.1394  |
| 1000 | 100 | 0.1    | Cov | 0.9508  | 0.9522  | 0.9400 | 0.9400  | 0.9400  |
|      |     |        | Len | 4.1809  | 4.1370  | 4.0073 | 4.0073  | 4.0073  |
| 1000 | 200 | 0.07   | Cov | 0.9534  | 0.9524  | 0.9040 | 0.9040  | 0.9040  |
|      |     |        | Len | 4.1529  | 4.1109  | 3.7798 | 3.7798  | 3.7798  |

Table 5.66. Etype = 1, J=5, k=19,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|---------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9832  | 0.9708 | 0.9624 | 0.9624 | 0.9624 |
|      |     |        | Len | 12.2210 | 5.2090 | 4.7893 | 4.7893 | 4.7893 |
| 100  | 40  | 0.9    | Cov | 0.9898  | 0.9684 | 0.8774 | 0.9636 | 0.9042 |
|      |     |        | Len | 17.4320 | 5.5680 | 4.1075 | 4.8189 | 4.2839 |
| 100  | 100 | 0.9    | Cov | 0.9872  | 0.9674 | 0.1718 | 0.9642 | 0.7864 |
|      |     |        | Len | 27.5161 | 5.5268 | 1.1419 | 4.9168 | 3.5842 |
| 100  | 200 | 0.9    | Cov | 0.9842  | 0.9522 | 0.0012 | 0.9620 | 0.6722 |
|      |     |        | Len | 39.9833 | 5.2621 | 0.0043 | 4.9780 | 3.0999 |
| 400  | 20  | 0.9    | Cov | 0.9710  | 0.9638 | 0.9682 | 0.9682 | 0.9682 |
|      |     |        | Len | 10.2394 | 4.6843 | 4.3824 | 4.3824 | 4.3824 |
| 400  | 40  | 0.9    | Cov | 0.9684  | 0.9558 | 0.9466 | 0.9466 | 0.9466 |
|      |     |        | Len | 14.2887 | 5.0905 | 4.1862 | 4.1862 | 4.1862 |
| 400  | 100 | 0.9    | Cov | 0.9728  | 0.9576 | 0.8916 | 0.9174 | 0.8984 |
|      |     |        | Len | 23.0034 | 5.1294 | 3.7885 | 3.9407 | 3.7928 |
| 400  | 200 | 0.9    | Cov | 0.9718  | 0.9630 | 0.7248 | 0.9240 | 0.7980 |
|      |     |        | Len | 31.8755 | 5.0455 | 3.0778 | 4.0212 | 3.3279 |
| 1000 | 20  | 0.9    | Cov | 0.9610  | 0.9596 | 0.9612 | 0.9612 | 0.9612 |
|      |     |        | Len | 9.6668  | 4.4550 | 4.1652 | 4.1652 | 4.1652 |
| 1000 | 40  | 0.9    | Cov | 0.9642  | 0.9548 | 0.9552 | 0.9552 | 0.9552 |
|      |     |        | Len | 13.5612 | 4.9118 | 4.0767 | 4.0767 | 4.0767 |
| 1000 | 100 | 0.9    | Cov | 0.9620  | 0.9560 | 0.9426 | 0.9426 | 0.9426 |
|      |     |        | Len | 21.9234 | 4.9980 | 3.9335 | 3.9335 | 3.9335 |
| 1000 | 200 | 0.9    | Cov | 0.9572  | 0.9532 | 0.8978 | 0.8978 | 0.8978 |
|      |     |        | Len | 30.2794 | 5.0038 | 3.6956 | 3.6956 | 3.6956 |

Table 5.67. Etype = 1, J=5, k=p-1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 40  | 0      | Cov | 0.9824  | 0.9302  | 0.9330 | 0.9636  | 0.9044  |
|      |     |        | Len | 26.9963 | 18.0634 | 4.9227 | 20.6143 | 17.3405 |
| 100  | 100 | 0      | Cov | 0.9270  | 0.7920  | 0.1574 | 0.9176  | 0.6908  |
|      |     |        | Len | 35.0791 | 24.5700 | 1.3012 | 32.7191 | 22.2295 |
| 100  | 200 | 0      | Cov | 0.9206  | 0.7478  | 0.0004 | 0.9236  | 0.5814  |
|      |     |        | Len | 49.9075 | 34.7351 | 0.0171 | 49.1150 | 28.9191 |
| 400  | 40  | 0      | Cov | 0.9784  | 0.9792  | 0.9792 | 0.9792  | 0.9792  |
|      |     |        | Len | 4.9012  | 4.8971  | 4.8971 | 4.8971  | 4.8971  |
| 400  | 100 | 0      | Cov | 0.9814  | 0.9556  | 0.9710 | 0.9428  | 0.9252  |
|      |     |        | Len | 33.2923 | 18.2836 | 5.1260 | 17.4122 | 17.7642 |
| 400  | 200 | 0      | Cov | 0.8970  | 0.7744  | 0.6930 | 0.8588  | 0.7040  |
|      |     |        | Len | 40.7452 | 28.1340 | 2.9547 | 32.9458 | 25.6157 |
| 1000 | 40  | 0      | Cov | 0.9722  | 0.9696  | 0.9696 | 0.9696  | 0.9696  |
|      |     |        | Len | 4.4963  | 4.4919  | 4.4919 | 4.4919  | 4.4919  |
| 1000 | 100 | 0      | Cov | 0.9772  | 0.9764  | 0.9764 | 0.9764  | 0.9764  |
|      |     |        | Len | 4.8863  | 4.8752  | 4.8752 | 4.8752  | 4.8752  |
| 1000 | 200 | 0      | Cov | 0.9830  | 0.9826  | 0.9826 | 0.9826  | 0.9826  |
|      |     |        | Len | 5.2989  | 5.2696  | 5.2696 | 5.2696  | 5.2696  |

Table 5.68. Etype = 1, J=5, k=p-1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL       | PLS    | PCR    | FS      |
|------|-----|--------|-----|----------|----------|--------|--------|---------|
| 100  | 40  | 0.1581 | Cov | 0.9920   | 0.9480   | 0.9336 | 0.9876 | 0.8858  |
|      |     |        | Len | 112.1507 | 23.6966  | 4.9464 | 5.7811 | 19.4947 |
| 100  | 100 | 0.1    | Cov | 0.9934   | 0.9204   | 0.1986 | 0.9882 | 0.8004  |
|      |     |        | Len | 332.4712 | 71.8206  | 1.3872 | 6.1844 | 50.1935 |
| 100  | 200 | 0.07   | Cov | 0.9890   | 0.9034   | 0.0018 | 0.9850 | 0.7194  |
|      |     |        | Len | 713.1969 | 150.8966 | 0.0056 | 7.0242 | 92.6591 |
| 400  | 40  | 0.1581 | Cov | 0.9806   | 0.9824   | 0.9824 | 0.9824 | 0.9824  |
|      |     |        | Len | 6.9788   | 4.9038   | 4.9038 | 4.9038 | 4.9038  |
| 400  | 100 | 0.1    | Cov | 0.9896   | 0.9622   | 0.9708 | 0.9808 | 0.9212  |
|      |     |        | Len | 119.7160 | 20.7583  | 5.1276 | 5.3146 | 17.8330 |
| 400  | 200 | 0.07   | Cov | 0.9898   | 0.9382   | 0.8584 | 0.9868 | 0.8320  |
|      |     |        | Len | 346.0919 | 67.4367  | 4.1871 | 5.4675 | 48.1354 |
| 1000 | 40  | 0.1581 | Cov | 0.9678   | 0.9710   | 0.9710 | 0.9710 | 0.9710  |
|      |     |        | Len | 6.1985   | 4.4908   | 4.4908 | 4.4908 | 4.4908  |
| 1000 | 100 | 0.1    | Cov | 0.9836   | 0.9748   | 0.9748 | 0.9748 | 0.9748  |
|      |     |        | Len | 16.3194  | 4.8825   | 4.8825 | 4.8825 | 4.8825  |
| 1000 | 200 | 0.07   | Cov | 0.9908   | 0.9800   | 0.9800 | 0.9800 | 0.9800  |
|      |     |        | Len | 36.3881  | 5.2731   | 5.2731 | 5.2731 | 5.2731  |

Table 5.69. Etype = 1, J=5, k=p-1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL      | PLS    | PCR    | FS      |
|------|-----|--------|-----|----------|---------|--------|--------|---------|
| 100  | 40  | 0.9    | Cov | 0.9918   | 0.9712  | 0.9010 | 0.9778 | 0.9210  |
|      |     |        | Len | 38.6312  | 8.1128  | 4.4702 | 5.1697 | 4.7812  |
| 100  | 100 | 0.9    | Cov | 0.9964   | 0.9780  | 0.1990 | 0.9874 | 0.8204  |
|      |     |        | Len | 182.9808 | 18.4703 | 1.3644 | 5.6596 | 6.6923  |
| 100  | 200 | 0.9    | Cov | 0.9958   | 0.9712  | 0.0012 | 0.9904 | 0.7228  |
|      |     |        | Len | 887.4072 | 31.6135 | 0.0050 | 5.6928 | 10.6633 |
| 400  | 40  | 0.9    | Cov | 0.9812   | 0.9734  | 0.9672 | 0.9672 | 0.9672  |
|      |     |        | Len | 33.5145  | 7.6971  | 4.6522 | 4.6522 | 4.6522  |
| 400  | 100 | 0.9    | Cov | 0.9904   | 0.9824  | 0.9538 | 0.9710 | 0.9480  |
|      |     |        | Len | 150.3718 | 17.9883 | 4.7249 | 4.8803 | 4.7920  |
| 400  | 200 | 0.9    | Cov | 0.9952   | 0.9904  | 0.8548 | 0.9874 | 0.8638  |
|      |     |        | Len | 447.9935 | 35.5057 | 4.1642 | 5.2721 | 6.4100  |
| 1000 | 40  | 0.9    | Cov | 0.9760   | 0.9706  | 0.9708 | 0.9708 | 0.9708  |
|      |     |        | Len | 31.4592  | 7.3848  | 4.4890 | 4.4890 | 4.4890  |
| 1000 | 100 | 0.9    | Cov | 0.9874   | 0.9850  | 0.9790 | 0.9790 | 0.9790  |
|      |     |        | Len | 141.7333 | 17.3959 | 4.8763 | 4.8763 | 4.8763  |
| 1000 | 200 | 0.9    | Cov | 0.9958   | 0.9910  | 0.9826 | 0.9826 | 0.9826  |
|      |     |        | Len | 448.5471 | 36.6703 | 5.2712 | 5.2712 | 5.2712  |

Table 5.70. Etype = 1, J=10, k=1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0      | Cov | 0.9440 | 0.9468 | 0.9162 | 0.9386 | 0.9244 |
|      |     |        | Len | 4.1888 | 4.2620 | 4.0218 | 5.0190 | 4.0739 |
| 100  | 40  | 0      | Cov | 0.9516 | 0.9424 | 0.8198 | 0.9510 | 0.9056 |
|      |     |        | Len | 4.1679 | 4.1531 | 3.4835 | 5.4312 | 3.8646 |
| 100  | 100 | 0      | Cov | 0.9428 | 0.9168 | 0.2762 | 0.9464 | 0.8506 |
|      |     |        | Len | 4.1594 | 3.9978 | 1.4048 | 5.7042 | 3.5653 |
| 100  | 200 | 0      | Cov | 0.9462 | 0.9148 | 0.0340 | 0.9502 | 0.8078 |
|      |     |        | Len | 4.1878 | 3.9064 | 0.1381 | 5.8263 | 3.3569 |
| 400  | 20  | 0      | Cov | 0.9396 | 0.9420 | 0.9390 | 0.9390 | 0.9390 |
|      |     |        | Len | 3.9155 | 3.9629 | 3.9155 | 3.9155 | 3.9155 |
| 400  | 40  | 0      | Cov | 0.9304 | 0.9438 | 0.9304 | 0.9304 | 0.9304 |
|      |     |        | Len | 3.8128 | 3.9135 | 3.8127 | 3.8127 | 3.8127 |
| 400  | 100 | 0      | Cov | 0.9284 | 0.9264 | 0.8724 | 0.9204 | 0.8892 |
|      |     |        | Len | 3.7120 | 3.7927 | 3.4769 | 4.5815 | 3.5604 |
| 400  | 200 | 0      | Cov | 0.9230 | 0.9068 | 0.6780 | 0.9226 | 0.8332 |
|      |     |        | Len | 3.6776 | 3.6872 | 2.8430 | 4.8829 | 3.3347 |
| 1000 | 20  | 0      | Cov | 0.9484 | 0.9486 | 0.9470 | 0.9470 | 0.9470 |
|      |     |        | Len | 3.9029 | 3.9209 | 3.9024 | 3.9024 | 3.9024 |
| 1000 | 40  | 0      | Cov | 0.9450 | 0.9514 | 0.9438 | 0.9438 | 0.9438 |
|      |     |        | Len | 3.8616 | 3.8992 | 3.8615 | 3.8615 | 3.8615 |
| 1000 | 100 | 0      | Cov | 0.9252 | 0.9396 | 0.9248 | 0.9248 | 0.9248 |
|      |     |        | Len | 3.7383 | 3.8389 | 3.7383 | 3.7383 | 3.7383 |
| 1000 | 200 | 0      | Cov | 0.9248 | 0.9314 | 0.8868 | 0.9162 | 0.8956 |
|      |     |        | Len | 3.6537 | 3.7575 | 3.5263 | 4.3486 | 3.5590 |

Table 5.71. Etype = 1, J=10, k=1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0.2236 | Cov | 0.9464 | 0.9600 | 0.9114 | 0.9440 | 0.9224 |
|      |     |        | Len | 4.1943 | 4.3740 | 4.0244 | 4.7389 | 4.0838 |
| 100  | 40  | 0.1581 | Cov | 0.9502 | 0.9624 | 0.8076 | 0.9518 | 0.9016 |
|      |     |        | Len | 4.1757 | 4.3547 | 3.4863 | 5.1220 | 3.8685 |
| 100  | 100 | 0.1    | Cov | 0.9504 | 0.9544 | 0.3054 | 0.9522 | 0.8532 |
|      |     |        | Len | 4.1731 | 4.3061 | 1.4898 | 5.4600 | 3.5917 |
| 100  | 200 | 0.07   | Cov | 0.9444 | 0.9482 | 0.0506 | 0.9540 | 0.8180 |
|      |     |        | Len | 4.1890 | 4.2468 | 0.1974 | 5.6412 | 3.3618 |
| 400  | 20  | 0.2236 | Cov | 0.9474 | 0.9504 | 0.9462 | 0.9462 | 0.9462 |
|      |     |        | Len | 3.9164 | 3.9829 | 3.9158 | 3.9158 | 3.9158 |
| 400  | 40  | 0.1581 | Cov | 0.9290 | 0.9480 | 0.9272 | 0.9272 | 0.9272 |
|      |     |        | Len | 3.8162 | 3.9670 | 3.8160 | 3.8160 | 3.8160 |
| 400  | 100 | 0.1    | Cov | 0.9300 | 0.9498 | 0.8714 | 0.9196 | 0.8862 |
|      |     |        | Len | 3.7121 | 3.9625 | 3.4785 | 4.4436 | 3.5612 |
| 400  | 200 | 0.07   | Cov | 0.9200 | 0.9430 | 0.6846 | 0.9216 | 0.8368 |
|      |     |        | Len | 3.6744 | 3.9545 | 2.8388 | 4.7549 | 3.3313 |
| 1000 | 20  | 0.2236 | Cov | 0.9370 | 0.9406 | 0.9358 | 0.9358 | 0.9358 |
|      |     |        | Len | 3.9034 | 3.9297 | 3.9031 | 3.9031 | 3.9031 |
| 1000 | 40  | 0.1581 | Cov | 0.9454 | 0.9460 | 0.9460 | 0.9460 | 0.9460 |
|      |     |        | Len | 3.8613 | 3.9215 | 3.8611 | 3.8611 | 3.8611 |
| 1000 | 100 | 0.1    | Cov | 0.9234 | 0.9448 | 0.9230 | 0.9230 | 0.9230 |
|      |     |        | Len | 3.7374 | 3.8952 | 3.7372 | 3.7372 | 3.7372 |
| 1000 | 200 | 0.07   | Cov | 0.9188 | 0.9398 | 0.8868 | 0.9178 | 0.8866 |
|      |     |        | Len | 3.6540 | 3.9039 | 3.5263 | 4.2717 | 3.5587 |

Table 5.72. Etype = 1, J=10, k=1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9638 | 0.9624 | 0.9102 | 0.9530 | 0.9198 |
|      |     |        | Len | 4.4110 | 4.4052 | 4.0230 | 4.2754 | 4.0707 |
| 100  | 40  | 0.9    | Cov | 0.9632 | 0.9620 | 0.8164 | 0.9546 | 0.8972 |
|      |     |        | Len | 4.4206 | 4.4006 | 3.4918 | 4.2898 | 3.8545 |
| 100  | 100 | 0.9    | Cov | 0.9622 | 0.9534 | 0.2912 | 0.9544 | 0.8392 |
|      |     |        | Len | 4.8641 | 4.3694 | 1.4668 | 4.3063 | 3.5606 |
| 100  | 200 | 0.9    | Cov | 0.9710 | 0.9572 | 0.0528 | 0.9654 | 0.8084 |
|      |     |        | Len | 6.4575 | 4.3565 | 0.1735 | 4.3384 | 3.3489 |
| 400  | 20  | 0.9    | Cov | 0.9510 | 0.9532 | 0.9396 | 0.9396 | 0.9396 |
|      |     |        | Len | 4.0018 | 3.9980 | 3.9191 | 3.9191 | 3.9191 |
| 400  | 40  | 0.9    | Cov | 0.9506 | 0.9516 | 0.9272 | 0.9272 | 0.9272 |
|      |     |        | Len | 4.0035 | 3.9919 | 3.8110 | 3.8110 | 3.8110 |
| 400  | 100 | 0.9    | Cov | 0.9514 | 0.9482 | 0.8652 | 0.9324 | 0.8832 |
|      |     |        | Len | 4.0116 | 3.9785 | 3.4758 | 3.8157 | 3.5584 |
| 400  | 200 | 0.9    | Cov | 0.9532 | 0.9474 | 0.6806 | 0.9328 | 0.8314 |
|      |     |        | Len | 4.0226 | 3.9769 | 2.8386 | 3.8246 | 3.3312 |
| 1000 | 20  | 0.9    | Cov | 0.9484 | 0.9486 | 0.9436 | 0.9436 | 0.9436 |
|      |     |        | Len | 3.9367 | 3.9331 | 3.9030 | 3.9030 | 3.9030 |
| 1000 | 40  | 0.9    | Cov | 0.9488 | 0.9476 | 0.9376 | 0.9376 | 0.9376 |
|      |     |        | Len | 3.9389 | 3.9298 | 3.8618 | 3.8618 | 3.8618 |
| 1000 | 100 | 0.9    | Cov | 0.9574 | 0.9542 | 0.9262 | 0.9262 | 0.9262 |
|      |     |        | Len | 3.9480 | 3.9313 | 3.7410 | 3.7410 | 3.7410 |
| 1000 | 200 | 0.9    | Cov | 0.9512 | 0.9494 | 0.8922 | 0.9300 | 0.8940 |
|      |     |        | Len | 3.9494 | 3.9239 | 3.5252 | 3.7451 | 3.5572 |

Table 5.73. Etype = 1, J=10, k=19,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 20  | 0      | Cov | 0.9828  | 0.9520  | 0.9666 | 0.9672  | 0.9430  |
|      |     |        | Len | 19.6154 | 13.2498 | 4.9831 | 15.0115 | 13.8313 |
| 100  | 40  | 0      | Cov | 0.9800  | 0.9560  | 0.8970 | 0.9722  | 0.9398  |
|      |     |        | Len | 19.6622 | 13.3625 | 4.2952 | 18.7955 | 13.8378 |
| 100  | 100 | 0      | Cov | 0.9824  | 0.9492  | 0.3466 | 0.9790  | 0.9384  |
|      |     |        | Len | 19.5710 | 13.4861 | 2.1035 | 20.7464 | 13.6517 |
| 100  | 200 | 0      | Cov | 0.9780  | 0.9374  | 0.0408 | 0.9740  | 0.9070  |
|      |     |        | Len | 19.1486 | 13.5431 | 0.3431 | 20.6873 | 13.0348 |
| 400  | 20  | 0      | Cov | 0.9764  | 0.9772  | 0.9772 | 0.9772  | 0.9772  |
|      |     |        | Len | 4.7007  | 4.6990  | 4.6990 | 4.6990  | 4.6990  |
| 400  | 40  | 0      | Cov | 0.9708  | 0.9742  | 0.9700 | 0.9700  | 0.9700  |
|      |     |        | Len | 4.5742  | 4.6335  | 4.5725 | 4.5725  | 4.5725  |
| 400  | 100 | 0      | Cov | 0.9608  | 0.9608  | 0.9222 | 0.9422  | 0.9426  |
|      |     |        | Len | 4.6381  | 4.5222  | 4.1752 | 14.0265 | 4.3326  |
| 400  | 200 | 0      | Cov | 0.9654  | 0.9546  | 0.7672 | 0.9596  | 0.9248  |
|      |     |        | Len | 4.7627  | 4.4426  | 3.4101 | 16.6155 | 4.1621  |
| 1000 | 20  | 0      | Cov | 0.9552  | 0.9554  | 0.9554 | 0.9554  | 0.9554  |
|      |     |        | Len | 4.1784  | 4.1766  | 4.1766 | 4.1766  | 4.1766  |
| 1000 | 40  | 0      | Cov | 0.9562  | 0.9560  | 0.9560 | 0.9560  | 0.9560  |
|      |     |        | Len | 4.1414  | 4.1605  | 4.1392 | 4.1392  | 4.1392  |
| 1000 | 100 | 0      | Cov | 0.9442  | 0.9520  | 0.9416 | 0.9416  | 0.9416  |
|      |     |        | Len | 4.0090  | 4.0946  | 4.0067 | 4.0067  | 4.0067  |
| 1000 | 200 | 0      | Cov | 0.9428  | 0.9432  | 0.9044 | 0.9100  | 0.9132  |
|      |     |        | Len | 3.9423  | 4.0049  | 3.7761 | 11.0840 | 3.8189  |

Table 5.74. Etype = 1, J=10, k=19,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 20  | 0.2236 | Cov | 0.9900  | 0.9612  | 0.9652 | 0.9846  | 0.9378  |
|      |     |        | Len | 74.0058 | 15.0151 | 4.9754 | 5.3109  | 14.6745 |
| 100  | 40  | 0.1581 | Cov | 0.9868  | 0.9574  | 0.9008 | 0.9808  | 0.9310  |
|      |     |        | Len | 76.6257 | 17.6423 | 4.3069 | 12.2907 | 16.1301 |
| 100  | 100 | 0.1    | Cov | 0.9844  | 0.9338  | 0.3936 | 0.9808  | 0.9008  |
|      |     |        | Len | 78.5815 | 20.1753 | 2.1790 | 17.4904 | 17.6116 |
| 100  | 200 | 0.07   | Cov | 0.9864  | 0.9262  | 0.0604 | 0.9822  | 0.8792  |
|      |     |        | Len | 82.8101 | 22.4902 | 0.4862 | 19.6893 | 18.1515 |
| 400  | 20  | 0.2236 | Cov | 0.9734  | 0.9750  | 0.9750 | 0.9750  | 0.9750  |
|      |     |        | Len | 4.7504  | 4.6969  | 4.6969 | 4.6969  | 4.6969  |
| 400  | 40  | 0.1581 | Cov | 0.9748  | 0.9750  | 0.9702 | 0.9702  | 0.9702  |
|      |     |        | Len | 4.7065  | 4.6548  | 4.5788 | 4.5788  | 4.5788  |
| 400  | 100 | 0.1    | Cov | 0.9732  | 0.9682  | 0.9320 | 0.9546  | 0.9490  |
|      |     |        | Len | 4.7559  | 4.5721  | 4.1774 | 11.7475 | 4.3342  |
| 400  | 200 | 0.07   | Cov | 0.9718  | 0.9636  | 0.7636 | 0.9542  | 0.9232  |
|      |     |        | Len | 6.5855  | 4.5206  | 3.4146 | 14.9415 | 4.1719  |
| 1000 | 20  | 0.2236 | Cov | 0.9612  | 0.9598  | 0.9598 | 0.9598  | 0.9598  |
|      |     |        | Len | 4.2192  | 4.1744  | 4.1744 | 4.1744  | 4.1744  |
| 1000 | 40  | 0.1581 | Cov | 0.9518  | 0.9524  | 0.9484 | 0.9484  | 0.9484  |
|      |     |        | Len | 4.2074  | 4.1633  | 4.1367 | 4.1367  | 4.1367  |
| 1000 | 100 | 0.1    | Cov | 0.9554  | 0.9542  | 0.9392 | 0.9392  | 0.9392  |
|      |     |        | Len | 4.1762  | 4.1344  | 4.0038 | 4.0038  | 4.0038  |
| 1000 | 200 | 0.07   | Cov | 0.9516  | 0.9474  | 0.9034 | 0.9150  | 0.9096  |
|      |     |        | Len | 4.1528  | 4.1093  | 3.7772 | 10.0048 | 3.8203  |

Table 5.75. Etype = 1, J=10, k=19,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|----------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9842   | 0.9710 | 0.9608 | 0.9780 | 0.9626 |
|      |     |        | Len | 45.5575  | 5.2322 | 4.7840 | 5.0756 | 4.9286 |
| 100  | 40  | 0.9    | Cov | 0.9866   | 0.9712 | 0.8760 | 0.9814 | 0.9492 |
|      |     |        | Len | 42.4335  | 5.5499 | 4.0903 | 5.1879 | 4.7386 |
| 100  | 100 | 0.9    | Cov | 0.9836   | 0.9604 | 0.3412 | 0.9796 | 0.9088 |
|      |     |        | Len | 123.8216 | 5.4754 | 1.7120 | 5.2963 | 4.4809 |
| 100  | 200 | 0.9    | Cov | 0.9890   | 0.9564 | 0.0578 | 0.9786 | 0.8778 |
|      |     |        | Len | 335.8509 | 5.1968 | 0.2096 | 5.3283 | 4.2461 |
| 400  | 20  | 0.9    | Cov | 0.9720   | 0.9602 | 0.9602 | 0.9602 | 0.9602 |
|      |     |        | Len | 10.2370  | 4.6849 | 4.3789 | 4.3789 | 4.3789 |
| 400  | 40  | 0.9    | Cov | 0.9698   | 0.9616 | 0.9536 | 0.9536 | 0.9536 |
|      |     |        | Len | 14.2814  | 5.0807 | 4.1792 | 4.1792 | 4.1792 |
| 400  | 100 | 0.9    | Cov | 0.9696   | 0.9582 | 0.8886 | 0.9468 | 0.9040 |
|      |     |        | Len | 22.9928  | 5.1238 | 3.7880 | 4.2913 | 3.9122 |
| 400  | 200 | 0.9    | Cov | 0.9670   | 0.9564 | 0.7188 | 0.9508 | 0.8748 |
|      |     |        | Len | 31.8672  | 5.0395 | 3.0756 | 4.3547 | 3.7014 |
| 1000 | 20  | 0.9    | Cov | 0.9628   | 0.9580 | 0.9590 | 0.9590 | 0.9590 |
|      |     |        | Len | 9.6734   | 4.4579 | 4.1692 | 4.1692 | 4.1692 |
| 1000 | 40  | 0.9    | Cov | 0.9606   | 0.9546 | 0.9444 | 0.9444 | 0.9444 |
|      |     |        | Len | 13.5609  | 4.9206 | 4.0799 | 4.0799 | 4.0799 |
| 1000 | 100 | 0.9    | Cov | 0.9580   | 0.9576 | 0.9376 | 0.9376 | 0.9376 |
|      |     |        | Len | 21.9414  | 4.9994 | 3.9317 | 3.9317 | 3.9317 |
| 1000 | 200 | 0.9    | Cov | 0.9576   | 0.9546 | 0.8958 | 0.9366 | 0.9028 |
|      |     |        | Len | 30.2701  | 4.9982 | 3.7022 | 4.0430 | 3.7452 |

Table 5.76. Etype = 1, J=10, k=p-1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 40  | 0      | Cov | 0.9788  | 0.9320  | 0.8792 | 0.9714  | 0.9222  |
|      |     |        | Len | 28.7166 | 21.4549 | 4.2040 | 25.6768 | 21.6615 |
| 100  | 100 | 0      | Cov | 0.9510  | 0.8676  | 0.2784 | 0.9492  | 0.8402  |
|      |     |        | Len | 40.4687 | 30.9453 | 2.6627 | 39.3382 | 30.9364 |
| 100  | 200 | 0      | Cov | 0.9500  | 0.8434  | 0.0304 | 0.9442  | 0.7964  |
|      |     |        | Len | 56.7788 | 43.8308 | 0.8607 | 56.3640 | 42.2023 |
| 400  | 40  | 0      | Cov | 0.9770  | 0.9786  | 0.9786 | 0.9786  | 0.9786  |
|      |     |        | Len | 4.9127  | 4.9071  | 4.9071 | 4.9071  | 4.9071  |
| 400  | 100 | 0      | Cov | 0.9786  | 0.9398  | 0.9408 | 0.9670  | 0.9118  |
|      |     |        | Len | 41.0935 | 29.3346 | 4.4795 | 32.7964 | 28.7049 |
| 400  | 200 | 0      | Cov | 0.9300  | 0.8498  | 0.6936 | 0.9140  | 0.8118  |
|      |     |        | Len | 49.5575 | 37.7931 | 2.9462 | 44.8739 | 36.5594 |
| 1000 | 40  | 0      | Cov | 0.9760  | 0.9756  | 0.9756 | 0.9756  | 0.9756  |
|      |     |        | Len | 4.4901  | 4.4868  | 4.4868 | 4.4868  | 4.4868  |
| 1000 | 100 | 0      | Cov | 0.9820  | 0.9816  | 0.9816 | 0.9816  | 0.9816  |
|      |     |        | Len | 4.8973  | 4.8849  | 4.8849 | 4.8849  | 4.8849  |
| 1000 | 200 | 0      | Cov | 0.9772  | 0.9422  | 0.9562 | 0.9528  | 0.9296  |
|      |     |        | Len | 56.0230 | 38.0741 | 4.6018 | 41.0338 | 37.1150 |

Table 5.77. Etype = 1, J=10, k=p-1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL       | PLS    | PCR    | FS       |
|------|-----|--------|-----|----------|----------|--------|--------|----------|
| 100  | 40  | 0.1581 | Cov | 0.9872   | 0.9470   | 0.8962 | 0.9844 | 0.9044   |
|      |     |        | Len | 160.2461 | 36.4689  | 4.3119 | 5.4261 | 33.5587  |
| 100  | 100 | 0.1    | Cov | 0.9868   | 0.9280   | 0.3576 | 0.9824 | 0.8650   |
|      |     |        | Len | 411.7656 | 97.0153  | 1.8100 | 5.8605 | 85.7560  |
| 100  | 200 | 0.07   | Cov | 0.9872   | 0.9166   | 0.0642 | 0.9854 | 0.8184   |
|      |     |        | Len | 871.3292 | 210.0977 | 0.2412 | 6.7370 | 166.1195 |
| 400  | 40  | 0.1581 | Cov | 0.9780   | 0.9774   | 0.9774 | 0.9774 | 0.9774   |
|      |     |        | Len | 6.9829   | 4.9084   | 4.9084 | 4.9084 | 4.9084   |
| 400  | 100 | 0.1    | Cov | 0.9888   | 0.9556   | 0.9450 | 0.9790 | 0.8966   |
|      |     |        | Len | 258.3051 | 45.6545  | 4.4795 | 4.9824 | 37.9356  |
| 400  | 200 | 0.07   | Cov | 0.9854   | 0.9408   | 0.7880 | 0.9800 | 0.8570   |
|      |     |        | Len | 567.3311 | 104.8690 | 3.6487 | 5.1686 | 80.8306  |
| 1000 | 40  | 0.1581 | Cov | 0.9756   | 0.9694   | 0.9694 | 0.9694 | 0.9694   |
|      |     |        | Len | 6.1908   | 4.4878   | 4.4878 | 4.4878 | 4.4878   |
| 1000 | 100 | 0.1    | Cov | 0.9830   | 0.9772   | 0.9772 | 0.9772 | 0.9772   |
|      |     |        | Len | 16.3258  | 4.8815   | 4.8815 | 4.8815 | 4.8815   |
| 1000 | 200 | 0.07   | Cov | 0.9838   | 0.9606   | 0.9572 | 0.9814 | 0.9028   |
|      |     |        | Len | 362.0462 | 56.1382  | 4.6009 | 4.9373 | 46.3527  |

Table 5.78. Etype = 1, J=10, k=p-1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso     | RL      | PLS    | PCR    | FS      |
|------|-----|--------|-----|-----------|---------|--------|--------|---------|
| 100  | 40  | 0.9    | Cov | 0.9918    | 0.9686  | 0.8908 | 0.9846 | 0.9304  |
|      |     |        | Len | 191.3079  | 7.7374  | 4.3038 | 5.2792 | 5.8268  |
| 100  | 100 | 0.9    | Cov | 0.9904    | 0.9624  | 0.3562 | 0.9870 | 0.8748  |
|      |     |        | Len | 1635.0250 | 15.6466 | 1.7977 | 5.2822 | 10.5333 |
| 100  | 200 | 0.9    | Cov | 0.9902    | 0.9532  | 0.0578 | 0.9868 | 0.8394  |
|      |     |        | Len | 7603.4220 | 28.5576 | 0.2127 | 5.2776 | 19.0490 |
| 400  | 40  | 0.9    | Cov | 0.9866    | 0.9760  | 0.9708 | 0.9708 | 0.9708  |
|      |     |        | Len | 33.5314   | 7.6866  | 4.6538 | 4.6538 | 4.6538  |
| 400  | 100 | 0.9    | Cov | 0.9900    | 0.9766  | 0.9446 | 0.9820 | 0.9274  |
|      |     |        | Len | 142.0953  | 16.9348 | 4.4710 | 4.9001 | 5.8303  |
| 400  | 200 | 0.9    | Cov | 0.9870    | 0.9754  | 0.7922 | 0.9784 | 0.8648  |
|      |     |        | Len | 392.9536  | 31.1084 | 3.6533 | 4.9087 | 9.4875  |
| 1000 | 40  | 0.9    | Cov | 0.9794    | 0.9704  | 0.9680 | 0.9680 | 0.9680  |
|      |     |        | Len | 31.4676   | 7.3765  | 4.4822 | 4.4822 | 4.4822  |
| 1000 | 100 | 0.9    | Cov | 0.9914    | 0.9822  | 0.9762 | 0.9762 | 0.9762  |
|      |     |        | Len | 141.6904  | 17.3494 | 4.8835 | 4.8835 | 4.8835  |
| 1000 | 200 | 0.9    | Cov | 0.9898    | 0.9840  | 0.9584 | 0.9842 | 0.9222  |
|      |     |        | Len | 391.2974  | 32.1430 | 4.6027 | 4.8814 | 6.4369  |

Table 5.79. Etype = 1, J=20, k=1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0      | Cov | 0.9636 | 0.9582 | 0.9246 | 0.9616 | 0.9514 |
|      |     |        | Len | 4.3655 | 4.3185 | 4.0200 | 5.6917 | 4.2138 |
| 100  | 40  | 0      | Cov | 0.9576 | 0.9482 | 0.8270 | 0.9564 | 0.9344 |
|      |     |        | Len | 4.3824 | 4.2639 | 3.5121 | 5.9077 | 4.1394 |
| 100  | 100 | 0      | Cov | 0.9660 | 0.9506 | 0.5148 | 0.9654 | 0.9306 |
|      |     |        | Len | 4.3973 | 4.1839 | 2.0984 | 6.0502 | 4.0367 |
| 100  | 200 | 0      | Cov | 0.9608 | 0.9402 | 0.2444 | 0.9666 | 0.9084 |
|      |     |        | Len | 4.4489 | 4.1472 | 0.9038 | 6.1067 | 3.9495 |
| 400  | 20  | 0      | Cov | 0.9450 | 0.9498 | 0.9452 | 0.9452 | 0.9452 |
|      |     |        | Len | 3.9106 | 3.9567 | 3.9105 | 3.9105 | 3.9105 |
| 400  | 40  | 0      | Cov | 0.9386 | 0.9402 | 0.9228 | 0.9326 | 0.9272 |
|      |     |        | Len | 3.8763 | 3.9176 | 3.8127 | 4.6402 | 3.8320 |
| 400  | 100 | 0      | Cov | 0.9396 | 0.9300 | 0.8608 | 0.9388 | 0.9064 |
|      |     |        | Len | 3.8534 | 3.8475 | 3.4790 | 5.0949 | 3.6930 |
| 400  | 200 | 0      | Cov | 0.9384 | 0.9200 | 0.6912 | 0.9378 | 0.8890 |
|      |     |        | Len | 3.8423 | 3.7854 | 2.8380 | 5.2707 | 3.5843 |
| 1000 | 20  | 0      | Cov | 0.9468 | 0.9480 | 0.9466 | 0.9466 | 0.9466 |
|      |     |        | Len | 3.9037 | 3.9213 | 3.9036 | 3.9036 | 3.9036 |
| 1000 | 40  | 0      | Cov | 0.9432 | 0.9444 | 0.9426 | 0.9426 | 0.9426 |
|      |     |        | Len | 3.8625 | 3.8998 | 3.8625 | 3.8625 | 3.8625 |
| 1000 | 100 | 0      | Cov | 0.9398 | 0.9430 | 0.9250 | 0.9326 | 0.9284 |
|      |     |        | Len | 3.7993 | 3.8472 | 3.7393 | 4.5290 | 3.7559 |
| 1000 | 200 | 0      | Cov | 0.9362 | 0.9294 | 0.8834 | 0.9266 | 0.9066 |
|      |     |        | Len | 3.7762 | 3.7919 | 3.5250 | 4.9037 | 3.6493 |

Table 5.80. Etype = 1, J=20, k=1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0.2236 | Cov | 0.9556 | 0.9566 | 0.9132 | 0.9560 | 0.9382 |
|      |     |        | Len | 4.3817 | 4.3818 | 4.0329 | 5.2342 | 4.2314 |
| 100  | 40  | 0.1581 | Cov | 0.9598 | 0.9564 | 0.8392 | 0.9588 | 0.9326 |
|      |     |        | Len | 4.4050 | 4.3522 | 3.5507 | 5.5215 | 4.1499 |
| 100  | 100 | 0.1    | Cov | 0.9618 | 0.9556 | 0.5930 | 0.9566 | 0.9254 |
|      |     |        | Len | 4.4492 | 4.3167 | 2.3602 | 5.7991 | 4.0499 |
| 100  | 200 | 0.07   | Cov | 0.9578 | 0.9480 | 0.3504 | 0.9602 | 0.9130 |
|      |     |        | Len | 4.4902 | 4.2651 | 1.3051 | 5.9033 | 3.9610 |
| 400  | 20  | 0.2236 | Cov | 0.9370 | 0.9454 | 0.9362 | 0.9362 | 0.9362 |
|      |     |        | Len | 3.9130 | 3.9828 | 3.9127 | 3.9127 | 3.9127 |
| 400  | 40  | 0.1581 | Cov | 0.9318 | 0.9442 | 0.9162 | 0.9312 | 0.9210 |
|      |     |        | Len | 3.8762 | 3.9817 | 3.8122 | 4.4414 | 3.8300 |
| 400  | 100 | 0.1    | Cov | 0.9362 | 0.9498 | 0.8662 | 0.9292 | 0.9070 |
|      |     |        | Len | 3.8543 | 3.9728 | 3.4801 | 4.9007 | 3.6956 |
| 400  | 200 | 0.07   | Cov | 0.9382 | 0.9466 | 0.6798 | 0.9376 | 0.8898 |
|      |     |        | Len | 3.8512 | 3.9693 | 2.8409 | 5.1079 | 3.5880 |
| 1000 | 20  | 0.2236 | Cov | 0.9456 | 0.9490 | 0.9452 | 0.9452 | 0.9452 |
|      |     |        | Len | 3.9015 | 3.9277 | 3.9012 | 3.9012 | 3.9012 |
| 1000 | 40  | 0.1581 | Cov | 0.9382 | 0.9440 | 0.9370 | 0.9370 | 0.9370 |
|      |     |        | Len | 3.8656 | 3.9233 | 3.8655 | 3.8655 | 3.8655 |
| 1000 | 100 | 0.1    | Cov | 0.9390 | 0.9460 | 0.9234 | 0.9370 | 0.9264 |
|      |     |        | Len | 3.7986 | 3.9170 | 3.7381 | 4.4121 | 3.7537 |
| 1000 | 200 | 0.07   | Cov | 0.9354 | 0.9478 | 0.8776 | 0.9342 | 0.9048 |
|      |     |        | Len | 3.7753 | 3.9135 | 3.5232 | 4.7754 | 3.6469 |

Table 5.81. Etype = 1, J=20, k=1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|---------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9682  | 0.9652 | 0.9182 | 0.9624 | 0.9440 |
|      |     |        | Len | 4.7043  | 4.4076 | 4.0274 | 4.4018 | 4.2083 |
| 100  | 40  | 0.9    | Cov | 0.9640  | 0.9616 | 0.8378 | 0.9596 | 0.9330 |
|      |     |        | Len | 6.6393  | 4.3988 | 3.5224 | 4.4132 | 4.1176 |
| 100  | 100 | 0.9    | Cov | 0.9676  | 0.9640 | 0.5678 | 0.9660 | 0.9202 |
|      |     |        | Len | 17.0619 | 4.3750 | 2.2187 | 4.4227 | 3.9871 |
| 100  | 200 | 0.9    | Cov | 0.9690  | 0.9534 | 0.3468 | 0.9612 | 0.8962 |
|      |     |        | Len | 33.1604 | 4.3551 | 1.1268 | 4.4456 | 3.9040 |
| 400  | 20  | 0.9    | Cov | 0.9486  | 0.9482 | 0.9378 | 0.9378 | 0.9378 |
|      |     |        | Len | 3.9942  | 3.9910 | 3.9139 | 3.9139 | 3.9139 |
| 400  | 40  | 0.9    | Cov | 0.9578  | 0.9570 | 0.9316 | 0.9454 | 0.9330 |
|      |     |        | Len | 4.0023  | 3.9886 | 3.8105 | 3.9230 | 3.8290 |
| 400  | 100 | 0.9    | Cov | 0.9486  | 0.9450 | 0.8656 | 0.9426 | 0.9058 |
|      |     |        | Len | 4.0133  | 3.9815 | 3.4800 | 3.9293 | 3.6947 |
| 400  | 200 | 0.9    | Cov | 0.9506  | 0.9456 | 0.6854 | 0.9384 | 0.8914 |
|      |     |        | Len | 4.0205  | 3.9751 | 2.8387 | 3.9315 | 3.5790 |
| 1000 | 20  | 0.9    | Cov | 0.9478  | 0.9478 | 0.9426 | 0.9426 | 0.9426 |
|      |     |        | Len | 3.9359  | 3.9329 | 3.9015 | 3.9015 | 3.9015 |
| 1000 | 40  | 0.9    | Cov | 0.9488  | 0.9472 | 0.9404 | 0.9404 | 0.9404 |
|      |     |        | Len | 3.9386  | 3.9300 | 3.8618 | 3.8618 | 3.8618 |
| 1000 | 100 | 0.9    | Cov | 0.9492  | 0.9456 | 0.9256 | 0.9390 | 0.9246 |
|      |     |        | Len | 3.9427  | 3.9265 | 3.7388 | 3.8473 | 3.7543 |
| 1000 | 200 | 0.9    | Cov | 0.9532  | 0.9518 | 0.8834 | 0.9452 | 0.9082 |
|      |     |        | Len | 3.9505  | 3.9253 | 3.5258 | 3.8541 | 3.6502 |

Table 5.82. Etype = 1, J=20, k=19,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 20  | 0      | Cov | 0.9798  | 0.9542  | 0.9542 | 0.9750  | 0.9590  |
|      |     |        | Len | 20.5362 | 15.6045 | 4.6638 | 18.8798 | 17.3232 |
| 100  | 40  | 0      | Cov | 0.9814  | 0.9516  | 0.8834 | 0.9782  | 0.9576  |
|      |     |        | Len | 20.4335 | 15.5573 | 4.2912 | 20.3879 | 17.2213 |
| 100  | 100 | 0      | Cov | 0.9752  | 0.9480  | 0.5440 | 0.9728  | 0.9428  |
|      |     |        | Len | 20.0507 | 15.3012 | 3.6685 | 20.9439 | 16.8594 |
| 100  | 200 | 0      | Cov | 0.9708  | 0.9412  | 0.2478 | 0.9710  | 0.9362  |
|      |     |        | Len | 20.0902 | 15.7641 | 2.1720 | 20.7360 | 16.4131 |
| 400  | 20  | 0      | Cov | 0.9804  | 0.9804  | 0.9804 | 0.9804  | 0.9804  |
|      |     |        | Len | 4.6983  | 4.6963  | 4.6963 | 4.6963  | 4.6963  |
| 400  | 40  | 0      | Cov | 0.9748  | 0.9746  | 0.9668 | 0.9592  | 0.9758  |
|      |     |        | Len | 5.3919  | 4.6784  | 4.5683 | 13.7161 | 4.6932  |
| 400  | 100 | 0      | Cov | 0.9796  | 0.9784  | 0.9310 | 0.9708  | 0.9808  |
|      |     |        | Len | 5.8983  | 4.6743  | 4.1687 | 17.6364 | 4.6941  |
| 400  | 200 | 0      | Cov | 0.9762  | 0.9752  | 0.7690 | 0.9714  | 0.9770  |
|      |     |        | Len | 6.2529  | 4.6714  | 3.3990 | 18.9708 | 4.6946  |
| 1000 | 20  | 0      | Cov | 0.9598  | 0.9584  | 0.9584 | 0.9584  | 0.9584  |
|      |     |        | Len | 4.1780  | 4.1761  | 4.1761 | 4.1761  | 4.1761  |
| 1000 | 40  | 0      | Cov | 0.9612  | 0.9612  | 0.9612 | 0.9612  | 0.9612  |
|      |     |        | Len | 4.1388  | 4.1584  | 4.1368 | 4.1368  | 4.1368  |
| 1000 | 100 | 0      | Cov | 0.9576  | 0.9556  | 0.9468 | 0.9346  | 0.9476  |
|      |     |        | Len | 4.1109  | 4.1084  | 4.0097 | 11.9266 | 4.0364  |
| 1000 | 200 | 0      | Cov | 0.9592  | 0.9530  | 0.9110 | 0.9408  | 0.9418  |
|      |     |        | Len | 4.1392  | 4.0661  | 3.7787 | 14.8248 | 3.9515  |

Table 5.83. Etype = 1, J=20, k=19,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 20  | 0.2236 | Cov | 0.9858  | 0.9612  | 0.9550 | 0.9810  | 0.9462  |
|      |     |        | Len | 80.6278 | 18.4959 | 4.6486 | 5.1200  | 25.3737 |
| 100  | 40  | 0.1581 | Cov | 0.9830  | 0.9482  | 0.8938 | 0.9792  | 0.9318  |
|      |     |        | Len | 79.7466 | 19.9719 | 4.3527 | 13.2362 | 27.2703 |
| 100  | 100 | 0.1    | Cov | 0.9814  | 0.9228  | 0.6400 | 0.9772  | 0.9186  |
|      |     |        | Len | 78.0582 | 20.9891 | 4.1651 | 17.8431 | 28.7491 |
| 100  | 200 | 0.07   | Cov | 0.9816  | 0.9292  | 0.3858 | 0.9770  | 0.9112  |
|      |     |        | Len | 88.9965 | 27.3598 | 3.1568 | 19.7105 | 29.2283 |
| 400  | 20  | 0.2236 | Cov | 0.9774  | 0.9756  | 0.9756 | 0.9756  | 0.9756  |
|      |     |        | Len | 4.7517  | 4.6967  | 4.6967 | 4.6967  | 4.6967  |
| 400  | 40  | 0.1581 | Cov | 0.9850  | 0.9768  | 0.9678 | 0.9626  | 0.9760  |
|      |     |        | Len | 41.0099 | 5.3323  | 4.5689 | 9.1753  | 4.7545  |
| 400  | 100 | 0.1    | Cov | 0.9772  | 0.9696  | 0.9284 | 0.9710  | 0.9698  |
|      |     |        | Len | 55.1185 | 7.9266  | 4.1663 | 14.6509 | 4.8947  |
| 400  | 200 | 0.07   | Cov | 0.9800  | 0.9626  | 0.7786 | 0.9708  | 0.9752  |
|      |     |        | Len | 60.0520 | 10.4504 | 3.4020 | 17.0005 | 5.1311  |
| 1000 | 20  | 0.2236 | Cov | 0.9608  | 0.9604  | 0.9604 | 0.9604  | 0.9604  |
|      |     |        | Len | 4.2203  | 4.1755  | 4.1755 | 4.1755  | 4.1755  |
| 1000 | 40  | 0.1581 | Cov | 0.9556  | 0.9550  | 0.9526 | 0.9526  | 0.9526  |
|      |     |        | Len | 4.2079  | 4.1634  | 4.1369 | 4.1369  | 4.1369  |
| 1000 | 100 | 0.1    | Cov | 0.9584  | 0.9554  | 0.9386 | 0.9300  | 0.9454  |
|      |     |        | Len | 4.1787  | 4.1355  | 4.0047 | 10.0153 | 4.0323  |
| 1000 | 200 | 0.07   | Cov | 0.9538  | 0.9530  | 0.9104 | 0.9348  | 0.9328  |
|      |     |        | Len | 4.1845  | 4.1105  | 3.7762 | 13.2975 | 3.9488  |

Table 5.84. Etype = 1, J=20, k=19,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|----------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9838   | 0.9660 | 0.9520 | 0.9832 | 0.9582 |
|      |     |        | Len | 230.6439 | 5.5814 | 4.6429 | 5.0520 | 5.5359 |
| 100  | 40  | 0.9    | Cov | 0.9860   | 0.9626 | 0.8934 | 0.9796 | 0.9468 |
|      |     |        | Len | 303.6384 | 5.5955 | 4.0591 | 5.2586 | 5.4741 |
| 100  | 100 | 0.9    | Cov | 0.9838   | 0.9640 | 0.6450 | 0.9830 | 0.9344 |
|      |     |        | Len | 518.9842 | 5.4708 | 2.5886 | 5.4129 | 5.3440 |
| 100  | 200 | 0.9    | Cov | 0.9840   | 0.9522 | 0.3760 | 0.9796 | 0.9196 |
|      |     |        | Len | 939.1129 | 5.3774 | 1.3383 | 5.4572 | 5.2256 |
| 400  | 20  | 0.9    | Cov | 0.9708   | 0.9614 | 0.9632 | 0.9632 | 0.9632 |
|      |     |        | Len | 10.2416  | 4.6872 | 4.3818 | 4.3818 | 4.3818 |
| 400  | 40  | 0.9    | Cov | 0.9668   | 0.9602 | 0.9554 | 0.9674 | 0.9552 |
|      |     |        | Len | 14.2817  | 5.0883 | 4.1901 | 4.3907 | 4.2475 |
| 400  | 100 | 0.9    | Cov | 0.9660   | 0.9618 | 0.8990 | 0.9578 | 0.9362 |
|      |     |        | Len | 22.9640  | 5.1276 | 3.7880 | 4.4828 | 4.1318 |
| 400  | 200 | 0.9    | Cov | 0.9650   | 0.9568 | 0.7234 | 0.9608 | 0.9264 |
|      |     |        | Len | 31.8642  | 5.0423 | 3.0691 | 4.5267 | 4.0473 |
| 1000 | 20  | 0.9    | Cov | 0.9650   | 0.9600 | 0.9616 | 0.9616 | 0.9616 |
|      |     |        | Len | 9.6803   | 4.4626 | 4.1730 | 4.1730 | 4.1730 |
| 1000 | 40  | 0.9    | Cov | 0.9612   | 0.9548 | 0.9546 | 0.9546 | 0.9546 |
|      |     |        | Len | 13.5621  | 4.9165 | 4.0820 | 4.0820 | 4.0820 |
| 1000 | 100 | 0.9    | Cov | 0.9582   | 0.9550 | 0.9366 | 0.9506 | 0.9410 |
|      |     |        | Len | 21.9229  | 4.9974 | 3.9317 | 4.1596 | 3.9579 |
| 1000 | 200 | 0.9    | Cov | 0.9626   | 0.9558 | 0.8948 | 0.9472 | 0.9236 |
|      |     |        | Len | 30.2809  | 5.0038 | 3.6985 | 4.2454 | 3.8672 |

Table 5.85. Etype = 1, J=20, k=p-1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|----------|---------|--------|---------|---------|
| 100  | 40  | 0      | Cov | 0.9804   | 0.9342  | 0.8636 | 0.9720  | 0.9466  |
|      |     |        | Len | 28.7404  | 22.9431 | 4.4476 | 27.9954 | 25.1014 |
| 400  | 40  | 0      | Cov | 0.9718   | 0.9544  | 0.9688 | 0.9584  | 0.9476  |
|      |     |        | Len | 25.6733  | 17.2843 | 4.5671 | 19.0186 | 18.0978 |
| 1000 | 40  | 0      | Cov | 0.9648   | 0.9648  | 0.9648 | 0.9648  | 0.9648  |
|      |     |        | Len | 4.4943   | 4.4897  | 4.4897 | 4.4897  | 4.4897  |
| 100  | 100 | 0      | Cov | 0.9592   | 0.8914  | 0.4858 | 0.9586  | 0.9076  |
|      |     |        | Len | 42.1415  | 34.1534 | 6.2730 | 42.6780 | 37.3070 |
| 400  | 100 | 0      | Cov | 0.9744   | 0.9390  | 0.9298 | 0.9674  | 0.9316  |
|      |     |        | Len | 43.1610  | 34.3478 | 4.1657 | 39.0673 | 34.8319 |
| 1000 | 100 | 0      | Cov | 0.9914   | 0.9822  | 0.9762 | 0.9762  | 0.9762  |
|      |     |        | Len | 141.6904 | 17.3494 | 4.8835 | 4.8835  | 4.8835  |
| 100  | 200 | 0      | Cov | 0.9580   | 0.9020  | 0.2258 | 0.9582  | 0.9058  |
|      |     |        | Len | 60.3672  | 50.2021 | 5.9042 | 60.3441 | 51.8363 |
| 400  | 200 | 0      | Cov | 0.9484   | 0.8884  | 0.6972 | 0.9368  | 0.8738  |
|      |     |        | Len | 53.9373  | 44.4654 | 2.9420 | 51.5629 | 44.8718 |
| 1000 | 200 | 0      | Cov | 0.9762   | 0.9456  | 0.9422 | 0.9678  | 0.9290  |
|      |     |        | Len | 60.1349  | 46.8948 | 4.2862 | 52.5603 | 47.1483 |

Table 5.86. Etype = 1, J=20, k=p-1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL       | PLS    | PCR    | FS       |
|------|-----|--------|-----|----------|----------|--------|--------|----------|
| 100  | 40  | 0.1581 | Cov | 0.9846   | 0.9392   | 0.8892 | 0.9856 | 0.9298   |
|      |     |        | Len | 162.2781 | 39.4733  | 4.0674 | 5.2259 | 55.3481  |
| 100  | 100 | 0.1    | Cov | 0.9798   | 0.9152   | 0.6492 | 0.9796 | 0.9128   |
|      |     |        | Len | 399.3837 | 98.7107  | 2.6146 | 5.6978 | 142.7038 |
| 100  | 200 | 0.07   | Cov | 0.9852   | 0.9206   | 0.3892 | 0.9816 | 0.8976   |
|      |     |        | Len | 912.6472 | 257.4309 | 1.4661 | 6.5425 | 282.1866 |
| 400  | 40  | 0.1581 | Cov | 0.9792   | 0.9626   | 0.9668 | 0.9762 | 0.9348   |
|      |     |        | Len | 130.5781 | 21.5725  | 4.5712 | 4.7171 | 20.8497  |
| 400  | 100 | 0.1    | Cov | 0.9778   | 0.9504   | 0.9288 | 0.9778 | 0.9142   |
|      |     |        | Len | 354.8799 | 68.6769  | 4.1742 | 4.8146 | 62.8941  |
| 400  | 200 | 0.07   | Cov | 0.9766   | 0.9442   | 0.7756 | 0.9804 | 0.8826   |
|      |     |        | Len | 709.1189 | 142.3295 | 3.4075 | 5.0190 | 129.3459 |
| 1000 | 40  | 0.1581 | Cov | 0.9734   | 0.9702   | 0.9702 | 0.9702 | 0.9702   |
|      |     |        | Len | 6.1980   | 4.4889   | 4.4889 | 4.4889 | 4.4889   |
| 1000 | 100 | 0.1    | Cov | 0.9764   | 0.9652   | 0.9692 | 0.9776 | 0.9324   |
|      |     |        | Len | 264.0684 | 37.3751  | 4.5489 | 4.7003 | 34.1218  |
| 1000 | 200 | 0.07   | Cov | 0.9796   | 0.9582   | 0.9440 | 0.9756 | 0.9088   |
|      |     |        | Len | 584.6784 | 92.4509  | 4.2860 | 4.7714 | 79.6519  |

Table 5.87. Etype = 1, J=20, k=p-1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso      | RL      | PLS    | PCR    | FS      |
|------|-----|--------|-----|------------|---------|--------|--------|---------|
| 100  | 40  | 0.9    | Cov | 0.9860     | 0.9622  | 0.8800 | 0.9780 | 0.9284  |
|      |     |        | Len | 696.5069   | 7.9673  | 4.0548 | 5.0565 | 8.0655  |
| 400  | 40  | 0.9    | Cov | 0.9802     | 0.9748  | 0.9672 | 0.9730 | 0.9636  |
|      |     |        | Len | 32.9674    | 7.5491  | 4.5757 | 4.7003 | 4.9483  |
| 1000 | 40  | 0.9    | Cov | 0.9812     | 0.9676  | 0.9704 | 0.9704 | 0.9704  |
|      |     |        | Len | 31.4679    | 7.4021  | 4.4863 | 4.4863 | 4.4863  |
| 100  | 100 | 0.9    | Cov | 0.9834     | 0.9604  | 0.6280 | 0.9810 | 0.9162  |
|      |     |        | Len | 3033.0770  | 16.4428 | 2.5458 | 5.0460 | 17.5271 |
| 400  | 100 | 0.9    | Cov | 0.9830     | 0.9698  | 0.9164 | 0.9694 | 0.9190  |
|      |     |        | Len | 136.2556   | 15.8455 | 4.1673 | 4.6925 | 8.0764  |
| 1000 | 100 | 0.9    | Cov | 0.9822     | 0.9776  | 0.9674 | 0.9786 | 0.9430  |
|      |     |        | Len | 131.9932   | 16.1500 | 4.5487 | 4.6726 | 5.6318  |
| 100  | 200 | 0.9    | Cov | 0.9860     | 0.9552  | 0.3830 | 0.9804 | 0.8886  |
|      |     |        | Len | 10602.9900 | 32.2699 | 1.2882 | 5.0439 | 33.3948 |
| 400  | 200 | 0.9    | Cov | 0.9834     | 0.9698  | 0.7620 | 0.9784 | 0.8976  |
|      |     |        | Len | 479.7499   | 28.6809 | 3.4005 | 4.6915 | 14.6696 |
| 1000 | 200 | 0.9    | Cov | 0.9836     | 0.9758  | 0.9414 | 0.9782 | 0.9136  |
|      |     |        | Len | 364.8946   | 29.9144 | 4.2867 | 4.6716 | 9.4535  |

Table 5.88. Etype = 1, J=50, k=1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0      | Cov | 0.9652 | 0.9616 | 0.9332 | 0.9664 | 0.9670 |
|      |     |        | Len | 4.5257 | 4.3837 | 4.3410 | 6.1232 | 4.4394 |
| 100  | 40  | 0      | Cov | 0.9618 | 0.9574 | 0.9124 | 0.9632 | 0.9624 |
|      |     |        | Len | 4.5434 | 4.3558 | 4.1779 | 6.1877 | 4.4328 |
| 100  | 100 | 0      | Cov | 0.9672 | 0.9612 | 0.8404 | 0.9656 | 0.9706 |
|      |     |        | Len | 4.5921 | 4.3451 | 3.7400 | 6.2295 | 4.4459 |
| 100  | 200 | 0      | Cov | 0.9646 | 0.9556 | 0.7570 | 0.9692 | 0.9656 |
|      |     |        | Len | 4.6350 | 4.3430 | 3.2088 | 6.2590 | 4.4495 |
| 400  | 20  | 0      | Cov | 0.9532 | 0.9492 | 0.9448 | 0.9462 | 0.9462 |
|      |     |        | Len | 3.9669 | 3.9718 | 3.9174 | 4.9557 | 3.9376 |
| 400  | 40  | 0      | Cov | 0.9500 | 0.9452 | 0.9282 | 0.9424 | 0.9420 |
|      |     |        | Len | 3.9619 | 3.9460 | 3.8103 | 5.2597 | 3.8993 |
| 400  | 100 | 0      | Cov | 0.9440 | 0.9374 | 0.8650 | 0.9432 | 0.9294 |
|      |     |        | Len | 3.9595 | 3.9117 | 3.4765 | 5.4498 | 3.8501 |
| 400  | 200 | 0      | Cov | 0.9504 | 0.9424 | 0.6928 | 0.9404 | 0.9296 |
|      |     |        | Len | 3.9576 | 3.8855 | 2.8455 | 5.5121 | 3.8129 |
| 1000 | 20  | 0      | Cov | 0.9466 | 0.9464 | 0.9464 | 0.9464 | 0.9464 |
|      |     |        | Len | 3.9043 | 3.9212 | 3.9041 | 3.9041 | 3.9041 |
| 1000 | 40  | 0      | Cov | 0.9444 | 0.9420 | 0.9388 | 0.9362 | 0.9366 |
|      |     |        | Len | 3.8865 | 3.9032 | 3.8622 | 4.6786 | 3.8694 |
| 1000 | 100 | 0      | Cov | 0.9440 | 0.9418 | 0.9230 | 0.9454 | 0.9354 |
|      |     |        | Len | 3.8781 | 3.8766 | 3.7407 | 5.1360 | 3.8190 |
| 1000 | 200 | 0      | Cov | 0.9448 | 0.9382 | 0.8840 | 0.9408 | 0.9268 |
|      |     |        | Len | 3.8714 | 3.8474 | 3.5248 | 5.2935 | 3.7735 |

Table 5.89. Etype = 1, J=50, k=1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0.2236 | Cov | 0.9646 | 0.9578 | 0.9650 | 0.9654 | 0.9654 |
|      |     |        | Len | 4.7119 | 4.3822 | 5.4855 | 5.5580 | 4.4388 |
| 100  | 40  | 0.1581 | Cov | 0.9622 | 0.9570 | 0.9608 | 0.9612 | 0.9638 |
|      |     |        | Len | 4.7381 | 4.3578 | 5.6994 | 5.7663 | 4.4311 |
| 100  | 100 | 0.1    | Cov | 0.9612 | 0.9524 | 0.9606 | 0.9614 | 0.9596 |
|      |     |        | Len | 4.7805 | 4.3368 | 5.9016 | 5.9624 | 4.4376 |
| 100  | 200 | 0.07   | Cov | 0.9664 | 0.9546 | 0.9658 | 0.9666 | 0.9658 |
|      |     |        | Len | 4.8425 | 4.3153 | 5.9951 | 6.0556 | 4.4356 |
| 400  | 20  | 0.2236 | Cov | 0.9466 | 0.9478 | 0.9400 | 0.9394 | 0.9400 |
|      |     |        | Len | 3.9624 | 3.9892 | 3.9159 | 4.6016 | 3.9345 |
| 400  | 40  | 0.1581 | Cov | 0.9478 | 0.9494 | 0.9306 | 0.9482 | 0.9390 |
|      |     |        | Len | 3.9622 | 3.9843 | 3.8125 | 4.9325 | 3.8998 |
| 400  | 100 | 0.1    | Cov | 0.9452 | 0.9466 | 0.8628 | 0.9484 | 0.9306 |
|      |     |        | Len | 3.9633 | 3.9729 | 3.4786 | 5.2054 | 3.8548 |
| 400  | 200 | 0.07   | Cov | 0.9482 | 0.9480 | 0.6918 | 0.9492 | 0.9262 |
|      |     |        | Len | 3.9693 | 3.9678 | 2.8595 | 5.3412 | 3.8186 |
| 1000 | 20  | 0.2236 | Cov | 0.9452 | 0.9490 | 0.9462 | 0.9462 | 0.9462 |
|      |     |        | Len | 3.9022 | 3.9296 | 3.9021 | 3.9021 | 3.9021 |
| 1000 | 40  | 0.1581 | Cov | 0.9490 | 0.9518 | 0.9432 | 0.9424 | 0.9418 |
|      |     |        | Len | 3.8878 | 3.9283 | 3.8622 | 4.4673 | 3.8695 |
| 1000 | 100 | 0.1    | Cov | 0.9414 | 0.9444 | 0.9200 | 0.9428 | 0.9322 |
|      |     |        | Len | 3.8777 | 3.9250 | 3.7379 | 4.9233 | 3.8167 |
| 1000 | 200 | 0.07   | Cov | 0.9492 | 0.9528 | 0.8894 | 0.9434 | 0.9310 |
|      |     |        | Len | 3.8741 | 3.9218 | 3.5239 | 5.1364 | 3.7738 |

Table 5.90. Etype = 1, J=50, k=1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|---------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9638  | 0.9628 | 0.9652 | 0.9652 | 0.9640 |
|      |     |        | Len | 10.9498 | 4.3812 | 4.4506 | 4.4507 | 4.3859 |
| 100  | 40  | 0.9    | Cov | 0.9678  | 0.9554 | 0.9612 | 0.9612 | 0.9568 |
|      |     |        | Len | 16.4822 | 4.3762 | 4.4567 | 4.4567 | 4.3819 |
| 100  | 100 | 0.9    | Cov | 0.9742  | 0.9552 | 0.9592 | 0.9592 | 0.9602 |
|      |     |        | Len | 27.3490 | 4.3471 | 4.4616 | 4.4616 | 4.3695 |
| 100  | 200 | 0.9    | Cov | 0.9666  | 0.9512 | 0.9642 | 0.9642 | 0.9600 |
|      |     |        | Len | 46.5845 | 4.3114 | 4.4650 | 4.4650 | 4.3609 |
| 400  | 20  | 0.9    | Cov | 0.9462  | 0.9446 | 0.9340 | 0.9444 | 0.9400 |
|      |     |        | Len | 4.0125  | 3.9973 | 3.9182 | 3.9911 | 3.9380 |
| 400  | 40  | 0.9    | Cov | 0.9438  | 0.9434 | 0.9196 | 0.9428 | 0.9318 |
|      |     |        | Len | 4.2330  | 3.9890 | 3.8141 | 3.9935 | 3.9005 |
| 400  | 100 | 0.9    | Cov | 0.9514  | 0.9450 | 0.8648 | 0.9456 | 0.9278 |
|      |     |        | Len | 5.5124  | 3.9802 | 3.4781 | 3.9931 | 3.8461 |
| 400  | 200 | 0.9    | Cov | 0.9462  | 0.9414 | 0.6824 | 0.9428 | 0.9196 |
|      |     |        | Len | 8.4912  | 3.9793 | 2.8470 | 3.9989 | 3.8091 |
| 1000 | 20  | 0.9    | Cov | 0.9516  | 0.9498 | 0.9466 | 0.9466 | 0.9466 |
|      |     |        | Len | 3.9342  | 3.9302 | 3.8999 | 3.8999 | 3.8999 |
| 1000 | 40  | 0.9    | Cov | 0.9472  | 0.9466 | 0.9370 | 0.9460 | 0.9396 |
|      |     |        | Len | 3.9365  | 3.9277 | 3.8591 | 3.9080 | 3.8658 |
| 1000 | 100 | 0.9    | Cov | 0.9504  | 0.9496 | 0.9234 | 0.9458 | 0.9354 |
|      |     |        | Len | 3.9435  | 3.9272 | 3.7396 | 3.9167 | 3.8177 |
| 1000 | 200 | 0.9    | Cov | 0.9480  | 0.9488 | 0.8798 | 0.9462 | 0.9312 |
|      |     |        | Len | 3.9492  | 3.9236 | 3.5241 | 3.9171 | 3.7724 |

Table 5.91. Etype = 1, J=50, k=19,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 20  | 0      | Cov | 0.9570  | 0.9182  | 0.9300 | 0.9588  | 0.9502  |
|      |     |        | Len | 18.0833 | 14.2769 | 7.4242 | 18.9945 | 18.3207 |
| 100  | 40  | 0      | Cov | 0.9550  | 0.9204  | 0.8914 | 0.9606  | 0.9494  |
|      |     |        | Len | 18.0864 | 14.2151 | 8.5679 | 19.3867 | 18.3136 |
| 100  | 100 | 0      | Cov | 0.9578  | 0.9208  | 0.8302 | 0.9656  | 0.9494  |
|      |     |        | Len | 18.0698 | 14.2752 | 9.1437 | 19.5983 | 18.2962 |
| 100  | 200 | 0      | Cov | 0.9622  | 0.9334  | 0.7432 | 0.9644  | 0.9542  |
|      |     |        | Len | 19.1499 | 16.1620 | 8.5866 | 19.7274 | 18.3149 |
| 400  | 20  | 0      | Cov | 0.9560  | 0.9406  | 0.9546 | 0.9494  | 0.9386  |
|      |     |        | Len | 16.8880 | 11.2745 | 4.1413 | 14.1885 | 13.7495 |
| 400  | 40  | 0      | Cov | 0.9536  | 0.9368  | 0.9424 | 0.9484  | 0.9352  |
|      |     |        | Len | 16.8798 | 11.2835 | 4.0347 | 16.3877 | 13.7694 |
| 400  | 100 | 0      | Cov | 0.9578  | 0.9366  | 0.8878 | 0.9534  | 0.9362  |
|      |     |        | Len | 16.8601 | 11.2398 | 3.6805 | 17.6605 | 13.7644 |
| 400  | 200 | 0      | Cov | 0.9626  | 0.9456  | 0.7152 | 0.9624  | 0.9448  |
|      |     |        | Len | 16.8816 | 11.2793 | 3.0607 | 18.0954 | 13.7507 |
| 1000 | 20  | 0      | Cov | 0.9562  | 0.9562  | 0.9562 | 0.9562  | 0.9562  |
|      |     |        | Len | 4.1766  | 4.1748  | 4.1748 | 4.1748  | 4.1748  |
| 1000 | 40  | 0      | Cov | 0.9608  | 0.9608  | 0.9590 | 0.9492  | 0.9612  |
|      |     |        | Len | 4.3715  | 4.1689  | 4.1326 | 12.7165 | 4.1742  |
| 1000 | 100 | 0      | Cov | 0.9588  | 0.9566  | 0.9312 | 0.9520  | 0.9560  |
|      |     |        | Len | 4.4814  | 4.1688  | 4.0025 | 16.0971 | 4.1773  |
| 1000 | 200 | 0      | Cov | 0.9602  | 0.9592  | 0.9124 | 0.9534  | 0.9612  |
|      |     |        | Len | 4.5501  | 4.1662  | 3.7736 | 17.2043 | 4.1772  |

Table 5.92. Etype = 1, J=50, k=19,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 100  | 20  | 0.2236 | Cov | 0.9562  | 0.9216  | 0.9612  | 0.9626  | 0.9380  |
|      |     |        | Len | 69.9410 | 16.0978 | 4.4847  | 4.5077  | 46.7607 |
| 100  | 40  | 0.1581 | Cov | 0.9618  | 0.9138  | 0.9652  | 0.9660  | 0.9400  |
|      |     |        | Len | 69.0296 | 17.3231 | 12.2787 | 12.5083 | 48.9391 |
| 100  | 100 | 0.1    | Cov | 0.9568  | 0.8828  | 0.9674  | 0.9680  | 0.9308  |
|      |     |        | Len | 67.4039 | 18.1965 | 16.0752 | 16.2952 | 50.1546 |
| 100  | 200 | 0.07   | Cov | 0.9664  | 0.8910  | 0.9662  | 0.9672  | 0.9268  |
|      |     |        | Len | 78.0264 | 25.2935 | 17.4825 | 17.6891 | 50.2308 |
| 400  | 20  | 0.2236 | Cov | 0.9608  | 0.9480  | 0.9512  | 0.9610  | 0.9282  |
|      |     |        | Len | 65.7279 | 10.5271 | 4.1424  | 4.2154  | 16.5096 |
| 400  | 40  | 0.1581 | Cov | 0.9666  | 0.9518  | 0.9398  | 0.9546  | 0.9306  |
|      |     |        | Len | 64.8389 | 11.3629 | 4.0269  | 10.5969 | 17.8209 |
| 400  | 100 | 0.1    | Cov | 0.9604  | 0.9452  | 0.8840  | 0.9528  | 0.9306  |
|      |     |        | Len | 63.4733 | 12.2446 | 3.6796  | 14.6193 | 18.9651 |
| 400  | 200 | 0.07   | Cov | 0.9606  | 0.9284  | 0.7244  | 0.9550  | 0.9232  |
|      |     |        | Len | 62.0785 | 12.8653 | 3.0995  | 16.2242 | 19.6209 |
| 1000 | 20  | 0.2236 | Cov | 0.9606  | 0.9640  | 0.9640  | 0.9640  | 0.9640  |
|      |     |        | Len | 4.2233  | 4.1775  | 4.1775  | 4.1775  | 4.1775  |
| 1000 | 40  | 0.1581 | Cov | 0.9624  | 0.9638  | 0.9590  | 0.9562  | 0.9620  |
|      |     |        | Len | 18.4591 | 4.1882  | 4.1336  | 8.4430  | 4.1777  |
| 1000 | 100 | 0.1    | Cov | 0.9592  | 0.9590  | 0.9406  | 0.9534  | 0.9606  |
|      |     |        | Len | 31.8376 | 4.3490  | 4.0047  | 13.3385 | 4.1842  |
| 1000 | 200 | 0.07   | Cov | 0.9640  | 0.9670  | 0.9068  | 0.9560  | 0.9648  |
|      |     |        | Len | 40.3358 | 4.6655  | 3.7700  | 15.3907 | 4.1840  |

Table 5.93. Etype = 1, J=50, k=19,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|----------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9640   | 0.9504 | 0.9642 | 0.9642 | 0.9406 |
|      |     |        | Len | 230.3125 | 5.0901 | 4.4468 | 4.4469 | 8.1380 |
| 100  | 40  | 0.9    | Cov | 0.9694   | 0.9326 | 0.9648 | 0.9648 | 0.9314 |
|      |     |        | Len | 329.4575 | 5.0132 | 4.6427 | 4.6427 | 8.0488 |
| 100  | 100 | 0.9    | Cov | 0.9642   | 0.9250 | 0.9638 | 0.9638 | 0.9314 |
|      |     |        | Len | 523.1495 | 4.8393 | 4.7692 | 4.7692 | 7.9676 |
| 100  | 200 | 0.9    | Cov | 0.9668   | 0.9242 | 0.9656 | 0.9656 | 0.9176 |
|      |     |        | Len | 894.3372 | 4.7773 | 4.8036 | 4.8036 | 7.8539 |
| 400  | 20  | 0.9    | Cov | 0.9604   | 0.9542 | 0.9564 | 0.9618 | 0.9472 |
|      |     |        | Len | 98.3940  | 4.5470 | 4.1391 | 4.2048 | 4.4899 |
| 400  | 40  | 0.9    | Cov | 0.9632   | 0.9548 | 0.9412 | 0.9620 | 0.9430 |
|      |     |        | Len | 65.9523  | 4.8753 | 4.0289 | 4.3607 | 4.4909 |
| 400  | 100 | 0.9    | Cov | 0.9634   | 0.9476 | 0.8848 | 0.9558 | 0.9386 |
|      |     |        | Len | 129.8894 | 4.9145 | 3.6826 | 4.4869 | 4.4795 |
| 400  | 200 | 0.9    | Cov | 0.9604   | 0.9466 | 0.7210 | 0.9574 | 0.9298 |
|      |     |        | Len | 224.2285 | 4.8506 | 3.0147 | 4.5284 | 4.4653 |
| 1000 | 20  | 0.9    | Cov | 0.9620   | 0.9578 | 0.9590 | 0.9590 | 0.9590 |
|      |     |        | Len | 9.6749   | 4.4566 | 4.1681 | 4.1681 | 4.1681 |
| 1000 | 40  | 0.9    | Cov | 0.9610   | 0.9584 | 0.9570 | 0.9618 | 0.9584 |
|      |     |        | Len | 13.5545  | 4.9061 | 4.0839 | 4.2151 | 4.1170 |
| 1000 | 100 | 0.9    | Cov | 0.9632   | 0.9546 | 0.9350 | 0.9504 | 0.9490 |
|      |     |        | Len | 21.9358  | 5.0150 | 3.9333 | 4.3319 | 4.0864 |
| 1000 | 200 | 0.9    | Cov | 0.9566   | 0.9470 | 0.8924 | 0.9550 | 0.9480 |
|      |     |        | Len | 30.2731  | 4.9927 | 3.6980 | 4.3807 | 4.0663 |

Table 5.94. Etype = 1, J=50, k=p-1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 100  | 40  | 0      | Cov | 0.9556  | 0.9060  | 0.8976  | 0.9622  | 0.9502  |
|      |     |        | Len | 25.8853 | 21.2128 | 11.6716 | 27.3372 | 26.2609 |
| 400  | 40  | 0      | Cov | 0.9566  | 0.9276  | 0.9472  | 0.9508  | 0.9284  |
|      |     |        | Len | 24.7543 | 19.5415 | 4.0305  | 23.0815 | 21.8629 |
| 1000 | 40  | 0      | Cov | 0.9612  | 0.9432  | 0.9568  | 0.9418  | 0.9374  |
|      |     |        | Len | 23.7491 | 15.2274 | 4.1325  | 17.6457 | 17.2327 |
| 100  | 100 | 0      | Cov | 0.9574  | 0.8910  | 0.8224  | 0.9646  | 0.9498  |
|      |     |        | Len | 41.2114 | 33.9555 | 19.7567 | 43.8151 | 41.9417 |
| 400  | 100 | 0      | Cov | 0.9600  | 0.9246  | 0.8824  | 0.9602  | 0.9368  |
|      |     |        | Len | 40.0437 | 34.1317 | 3.6847  | 39.3218 | 36.8697 |
| 1000 | 100 | 0      | Cov | 0.9580  | 0.9366  | 0.9386  | 0.9520  | 0.9362  |
|      |     |        | Len | 39.4544 | 32.0313 | 4.0031  | 35.7395 | 33.7181 |
| 100  | 200 | 0      | Cov | 0.9558  | 0.9172  | 0.7410  | 0.9614  | 0.9474  |
|      |     |        | Len | 60.9147 | 53.2067 | 26.5292 | 62.0366 | 59.1702 |
| 400  | 200 | 0      | Cov | 0.9516  | 0.9086  | 0.6818  | 0.9536  | 0.9194  |
|      |     |        | Len | 55.2096 | 47.9602 | 3.3959  | 55.2905 | 51.5772 |
| 1000 | 200 | 0      | Cov | 0.9594  | 0.9270  | 0.9086  | 0.9538  | 0.9336  |
|      |     |        | Len | 56.6913 | 48.7435 | 3.7709  | 54.1541 | 50.6050 |

Table 5.95. Etype = 1, J=50, k=p-1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL       | PLS    | PCR    | FS       |
|------|-----|--------|-----|----------|----------|--------|--------|----------|
| 100  | 40  | 0.1581 | Cov | 0.9566   | 0.9040   | 0.9692 | 0.9690 | 0.9364   |
|      |     |        | Len | 140.0937 | 34.4102  | 4.5917 | 4.6233 | 99.0403  |
| 100  | 100 | 0.1    | Cov | 0.9588   | 0.8700   | 0.9634 | 0.9642 | 0.9272   |
|      |     |        | Len | 344.3750 | 85.0367  | 5.0036 | 5.0470 | 253.7439 |
| 100  | 200 | 0.07   | Cov | 0.9612   | 0.8784   | 0.9600 | 0.9610 | 0.9224   |
|      |     |        | Len | 798.0916 | 236.3951 | 5.7416 | 5.7983 | 506.6544 |
| 400  | 40  | 0.1581 | Cov | 0.9656   | 0.9404   | 0.9424 | 0.9614 | 0.9196   |
|      |     |        | Len | 131.8545 | 24.5389  | 4.0276 | 4.2377 | 38.8563  |
| 400  | 100 | 0.1    | Cov | 0.9606   | 0.9200   | 0.8936 | 0.9582 | 0.9048   |
|      |     |        | Len | 324.8278 | 65.2592  | 3.6777 | 4.3359 | 105.7099 |
| 400  | 200 | 0.07   | Cov | 0.9598   | 0.9104   | 0.7196 | 0.9558 | 0.8936   |
|      |     |        | Len | 638.0791 | 130.4837 | 3.0166 | 4.5329 | 215.5089 |
| 1000 | 40  | 0.1581 | Cov | 0.9576   | 0.9548   | 0.9594 | 0.9614 | 0.9284   |
|      |     |        | Len | 128.8195 | 18.1126  | 4.1318 | 4.1817 | 20.0686  |
| 1000 | 100 | 0.1    | Cov | 0.9554   | 0.9382   | 0.9404 | 0.9580 | 0.9080   |
|      |     |        | Len | 320.4182 | 52.8284  | 3.9998 | 4.2183 | 63.1281  |
| 1000 | 200 | 0.07   | Cov | 0.9606   | 0.9410   | 0.9088 | 0.9616 | 0.9044   |
|      |     |        | Len | 625.9141 | 107.6711 | 3.7775 | 4.2988 | 133.2647 |

Table 5.96. Etype = 1, J=50, k=p-1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso     | RL      | PLS    | PCR    | FS      |
|------|-----|--------|-----|-----------|---------|--------|--------|---------|
| 100  | 40  | 0.9    | Cov | 0.9680    | 0.9338  | 0.9694 | 0.9694 | 0.9350  |
|      |     |        | Len | 676.8153  | 7.2241  | 4.4306 | 4.4306 | 14.9842 |
| 400  | 40  | 0.9    | Cov | 0.9632    | 0.9454  | 0.9400 | 0.9576 | 0.9184  |
|      |     |        | Len | 293.9916  | 6.6371  | 4.0309 | 4.2069 | 6.0584  |
| 1000 | 40  | 0.9    | Cov | 0.9628    | 0.9568  | 0.9558 | 0.9612 | 0.9424  |
|      |     |        | Len | 29.0129   | 6.8282  | 4.1355 | 4.1778 | 4.5960  |
| 100  | 100 | 0.9    | Cov | 0.9672    | 0.9212  | 0.9620 | 0.9620 | 0.9236  |
|      |     |        | Len | 2725.3180 | 14.5819 | 4.4368 | 4.4368 | 36.3098 |
| 400  | 100 | 0.9    | Cov | 0.9648    | 0.9428  | 0.8802 | 0.9596 | 0.8980  |
|      |     |        | Len | 1847.9650 | 13.7507 | 3.6735 | 4.2034 | 12.8067 |
| 1000 | 100 | 0.9    | Cov | 0.9656    | 0.9572  | 0.9392 | 0.9596 | 0.9164  |
|      |     |        | Len | 121.6069  | 14.2116 | 4.0010 | 4.1748 | 8.0172  |
| 100  | 200 | 0.9    | Cov | 0.9656    | 0.9296  | 0.9652 | 0.9652 | 0.9192  |
|      |     |        | Len | 9359.3370 | 28.7351 | 4.4382 | 4.4382 | 71.5347 |
| 400  | 200 | 0.9    | Cov | 0.9686    | 0.9402  | 0.7196 | 0.9614 | 0.8950  |
|      |     |        | Len | 6070.9600 | 25.8279 | 3.0096 | 4.1987 | 24.7068 |
| 1000 | 200 | 0.9    | Cov | 0.9680    | 0.9486  | 0.9076 | 0.9606 | 0.9056  |
|      |     |        | Len | 429.7017  | 26.0982 | 3.7736 | 4.1765 | 15.0494 |

Table 5.97. Etype = 2, J=5, k=1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0      | Cov | 0.9328 | 0.9460 | 0.9328 | 0.9328 | 0.9328 |
|      |     |        | Len | 6.5367 | 6.8943 | 6.5367 | 6.5367 | 6.5367 |
| 100  | 40  | 0      | Cov | 0.9234 | 0.9272 | 0.8390 | 0.9294 | 0.8662 |
|      |     |        | Len | 6.2763 | 6.6211 | 5.7301 | 6.9415 | 5.8941 |
| 100  | 100 | 0      | Cov | 0.9190 | 0.9042 | 0.1398 | 0.9310 | 0.7438 |
|      |     |        | Len | 6.1082 | 6.1505 | 1.5813 | 7.2358 | 4.8563 |
| 100  | 200 | 0      | Cov | 0.9216 | 0.8872 | 0.0002 | 0.9326 | 0.6350 |
|      |     |        | Len | 6.1467 | 5.9040 | 0.0041 | 7.3917 | 4.1432 |
| 400  | 20  | 0      | Cov | 0.9400 | 0.9474 | 0.9396 | 0.9396 | 0.9396 |
|      |     |        | Len | 6.3123 | 6.3881 | 6.3119 | 6.3119 | 6.3119 |
| 400  | 40  | 0      | Cov | 0.9356 | 0.9460 | 0.9356 | 0.9356 | 0.9356 |
|      |     |        | Len | 6.1444 | 6.3061 | 6.1443 | 6.1443 | 6.1443 |
| 400  | 100 | 0      | Cov | 0.9134 | 0.9276 | 0.8942 | 0.9128 | 0.8952 |
|      |     |        | Len | 5.6648 | 6.0531 | 5.6223 | 5.9019 | 5.6266 |
| 400  | 200 | 0      | Cov | 0.9116 | 0.9070 | 0.7212 | 0.9112 | 0.7856 |
|      |     |        | Len | 5.4361 | 5.7600 | 4.6689 | 6.1916 | 4.9721 |
| 1000 | 20  | 0      | Cov | 0.9464 | 0.9478 | 0.9466 | 0.9466 | 0.9466 |
|      |     |        | Len | 6.2915 | 6.3200 | 6.2913 | 6.2913 | 6.2913 |
| 1000 | 40  | 0      | Cov | 0.9438 | 0.9454 | 0.9434 | 0.9434 | 0.9434 |
|      |     |        | Len | 6.2215 | 6.2836 | 6.2209 | 6.2209 | 6.2209 |
| 1000 | 100 | 0      | Cov | 0.9326 | 0.9406 | 0.9316 | 0.9316 | 0.9316 |
|      |     |        | Len | 6.0328 | 6.1968 | 6.0327 | 6.0327 | 6.0327 |
| 1000 | 200 | 0      | Cov | 0.9114 | 0.9344 | 0.9100 | 0.9100 | 0.9100 |
|      |     |        | Len | 5.7002 | 6.0298 | 5.7006 | 5.7006 | 5.7006 |

Table 5.98. Etype = 2, J=5, k=1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0.2236 | Cov | 0.9236 | 0.9454 | 0.9230 | 0.9230 | 0.9230 |
|      |     |        | Len | 6.5479 | 7.0617 | 6.5470 | 6.5470 | 6.5470 |
| 100  | 40  | 0.1581 | Cov | 0.9286 | 0.9502 | 0.8484 | 0.9356 | 0.8702 |
|      |     |        | Len | 6.2714 | 7.0629 | 5.7123 | 6.8191 | 5.8840 |
| 100  | 100 | 0.1    | Cov | 0.9222 | 0.9480 | 0.1482 | 0.9332 | 0.7226 |
|      |     |        | Len | 6.0889 | 6.9642 | 1.6236 | 7.0970 | 4.8504 |
| 100  | 200 | 0.07   | Cov | 0.9252 | 0.9444 | 0.0006 | 0.9382 | 0.6352 |
|      |     |        | Len | 6.1285 | 6.9227 | 0.0061 | 7.2463 | 4.1295 |
| 400  | 20  | 0.2236 | Cov | 0.9402 | 0.9424 | 0.9396 | 0.9396 | 0.9396 |
|      |     |        | Len | 6.3043 | 6.4135 | 6.3040 | 6.3040 | 6.3040 |
| 400  | 40  | 0.1581 | Cov | 0.9344 | 0.9482 | 0.9342 | 0.9342 | 0.9342 |
|      |     |        | Len | 6.1334 | 6.3800 | 6.1328 | 6.1328 | 6.1328 |
| 400  | 100 | 0.1    | Cov | 0.9040 | 0.9392 | 0.8858 | 0.9078 | 0.8866 |
|      |     |        | Len | 5.6490 | 6.2932 | 5.6073 | 5.8673 | 5.6119 |
| 400  | 200 | 0.07   | Cov | 0.9006 | 0.9382 | 0.7138 | 0.9116 | 0.7846 |
|      |     |        | Len | 5.4609 | 6.3427 | 4.6765 | 6.1535 | 4.9868 |
| 1000 | 20  | 0.2236 | Cov | 0.9486 | 0.9502 | 0.9476 | 0.9476 | 0.9476 |
|      |     |        | Len | 6.2980 | 6.3428 | 6.2973 | 6.2973 | 6.2973 |
| 1000 | 40  | 0.1581 | Cov | 0.9400 | 0.9454 | 0.9402 | 0.9402 | 0.9402 |
|      |     |        | Len | 6.2319 | 6.3328 | 6.2315 | 6.2315 | 6.2315 |
| 1000 | 100 | 0.1    | Cov | 0.9392 | 0.9454 | 0.9384 | 0.9384 | 0.9384 |
|      |     |        | Len | 6.0356 | 6.2889 | 6.0356 | 6.0356 | 6.0356 |
| 1000 | 200 | 0.07   | Cov | 0.9048 | 0.9390 | 0.9040 | 0.9040 | 0.9040 |
|      |     |        | Len | 5.6972 | 6.1994 | 5.6977 | 5.6977 | 5.6977 |

Table 5.99. Etype = 2, J=5, k=1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9516 | 0.9516 | 0.9240 | 0.9240 | 0.9240 |
|      |     |        | Len | 7.2521 | 7.2363 | 6.5957 | 6.5957 | 6.5957 |
| 100  | 40  | 0.9    | Cov | 0.9550 | 0.9546 | 0.8350 | 0.9358 | 0.8650 |
|      |     |        | Len | 7.2828 | 7.2730 | 5.7666 | 6.6348 | 5.9382 |
| 100  | 100 | 0.9    | Cov | 0.9518 | 0.9500 | 0.1390 | 0.9344 | 0.7350 |
|      |     |        | Len | 7.3890 | 7.3171 | 1.6286 | 6.6899 | 4.8838 |
| 100  | 200 | 0.9    | Cov | 0.9584 | 0.9504 | 0.0008 | 0.9452 | 0.6362 |
|      |     |        | Len | 7.4891 | 7.2657 | 0.0059 | 6.7301 | 4.1683 |
| 400  | 20  | 0.9    | Cov | 0.9454 | 0.9446 | 0.9356 | 0.9356 | 0.9356 |
|      |     |        | Len | 6.4493 | 6.4470 | 6.3144 | 6.3144 | 6.3144 |
| 400  | 40  | 0.9    | Cov | 0.9456 | 0.9462 | 0.9324 | 0.9324 | 0.9324 |
|      |     |        | Len | 6.4483 | 6.4388 | 6.1431 | 6.1431 | 6.1431 |
| 400  | 100 | 0.9    | Cov | 0.9512 | 0.9502 | 0.8902 | 0.9144 | 0.8882 |
|      |     |        | Len | 6.4798 | 6.4385 | 5.6337 | 5.8129 | 5.6393 |
| 400  | 200 | 0.9    | Cov | 0.9460 | 0.9440 | 0.7116 | 0.9234 | 0.7988 |
|      |     |        | Len | 6.4580 | 6.3991 | 4.6681 | 5.7969 | 4.9719 |
| 1000 | 20  | 0.9    | Cov | 0.9466 | 0.9464 | 0.9442 | 0.9442 | 0.9442 |
|      |     |        | Len | 6.3450 | 6.3423 | 6.2909 | 6.2909 | 6.2909 |
| 1000 | 40  | 0.9    | Cov | 0.9434 | 0.9438 | 0.9388 | 0.9388 | 0.9388 |
|      |     |        | Len | 6.3467 | 6.3373 | 6.2253 | 6.2253 | 6.2253 |
| 1000 | 100 | 0.9    | Cov | 0.9490 | 0.9488 | 0.9336 | 0.9336 | 0.9336 |
|      |     |        | Len | 6.3606 | 6.3420 | 6.0293 | 6.0293 | 6.0293 |
| 1000 | 200 | 0.9    | Cov | 0.9430 | 0.9434 | 0.9016 | 0.9016 | 0.9016 |
|      |     |        | Len | 6.3684 | 6.3392 | 5.6954 | 5.6954 | 5.6954 |

Table 5.100. Etype = 2, J=5, k=19,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 100  | 20  | 0      | Cov | 0.9756  | 0.9760  | 0.9760  | 0.9760  | 0.9760  |
|      |     |        | Len | 10.0489 | 10.0443 | 10.0478 | 10.0478 | 10.0478 |
| 100  | 40  | 0      | Cov | 0.9764  | 0.9666  | 0.9338  | 0.9664  | 0.9734  |
|      |     |        | Len | 15.5734 | 10.3147 | 8.4003  | 16.9671 | 9.9483  |
| 100  | 100 | 0      | Cov | 0.9758  | 0.9548  | 0.1994  | 0.9760  | 0.9648  |
|      |     |        | Len | 17.6359 | 11.6042 | 2.2778  | 21.0897 | 9.7592  |
| 100  | 200 | 0      | Cov | 0.9726  | 0.9384  | 0.0006  | 0.9752  | 0.9476  |
|      |     |        | Len | 18.5905 | 12.6023 | 0.0092  | 22.2127 | 9.3987  |
| 400  | 20  | 0      | Cov | 0.9710  | 0.9712  | 0.9712  | 0.9712  | 0.9712  |
|      |     |        | Len | 8.4270  | 8.4250  | 8.4250  | 8.4250  | 8.4250  |
| 400  | 40  | 0      | Cov | 0.9644  | 0.9676  | 0.9642  | 0.9642  | 0.9642  |
|      |     |        | Len | 8.1467  | 8.2846  | 8.1396  | 8.1396  | 8.1396  |
| 400  | 100 | 0      | Cov | 0.9564  | 0.9650  | 0.9458  | 0.9234  | 0.9458  |
|      |     |        | Len | 7.4308  | 7.8462  | 7.2560  | 9.4788  | 7.2640  |
| 400  | 200 | 0      | Cov | 0.9558  | 0.9548  | 0.8014  | 0.9318  | 0.8886  |
|      |     |        | Len | 7.3987  | 7.4672  | 5.8517  | 13.7904 | 6.3852  |
| 1000 | 20  | 0      | Cov | 0.9586  | 0.9580  | 0.9580  | 0.9580  | 0.9580  |
|      |     |        | Len | 6.9784  | 6.9778  | 6.9778  | 6.9778  | 6.9778  |
| 1000 | 40  | 0      | Cov | 0.9586  | 0.9580  | 0.9590  | 0.9590  | 0.9590  |
|      |     |        | Len | 6.9270  | 6.9679  | 6.9250  | 6.9250  | 6.9250  |
| 1000 | 100 | 0      | Cov | 0.9534  | 0.9574  | 0.9528  | 0.9528  | 0.9528  |
|      |     |        | Len | 6.6728  | 6.8378  | 6.6687  | 6.6687  | 6.6687  |
| 1000 | 200 | 0      | Cov | 0.9358  | 0.9512  | 0.9322  | 0.9322  | 0.9322  |
|      |     |        | Len | 6.2695  | 6.6315  | 6.2658  | 6.2658  | 6.2658  |

Table 5.101. Etype = 2, J=5, k=19,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 100  | 20  | 0.2236 | Cov | 0.9778  | 0.9782  | 0.9782  | 0.9782  | 0.9782  |
|      |     |        | Len | 10.0614 | 10.0098 | 10.0214 | 10.0214 | 10.0214 |
| 100  | 40  | 0.1581 | Cov | 0.9902  | 0.9636  | 0.9360  | 0.9734  | 0.9684  |
|      |     |        | Len | 45.7718 | 11.9217 | 8.4789  | 12.9840 | 10.0205 |
| 100  | 100 | 0.1    | Cov | 0.9914  | 0.9458  | 0.2186  | 0.9778  | 0.9704  |
|      |     |        | Len | 58.0491 | 15.5082 | 2.3674  | 18.4662 | 9.9478  |
| 100  | 200 | 0.07   | Cov | 0.9910  | 0.9264  | 0.0022  | 0.9804  | 0.9666  |
|      |     |        | Len | 64.5585 | 17.8003 | 0.0136  | 21.2596 | 10.0186 |
| 400  | 20  | 0.2236 | Cov | 0.9740  | 0.9754  | 0.9754  | 0.9754  | 0.9754  |
|      |     |        | Len | 8.4459  | 8.4272  | 8.4272  | 8.4272  | 8.4272  |
| 400  | 40  | 0.1581 | Cov | 0.9708  | 0.9686  | 0.9686  | 0.9686  | 0.9686  |
|      |     |        | Len | 8.3370  | 8.3125  | 8.1346  | 8.1346  | 8.1346  |
| 400  | 100 | 0.1    | Cov | 0.9704  | 0.9688  | 0.9482  | 0.9384  | 0.9512  |
|      |     |        | Len | 8.1028  | 8.1161  | 7.2782  | 8.8427  | 7.2883  |
| 400  | 200 | 0.07   | Cov | 0.9602  | 0.9650  | 0.8074  | 0.9242  | 0.8836  |
|      |     |        | Len | 7.7605  | 7.9571  | 5.8309  | 12.6896 | 6.3658  |
| 1000 | 20  | 0.2236 | Cov | 0.9600  | 0.9606  | 0.9606  | 0.9606  | 0.9606  |
|      |     |        | Len | 7.0119  | 6.9913  | 6.9913  | 6.9913  | 6.9913  |
| 1000 | 40  | 0.1581 | Cov | 0.9578  | 0.9572  | 0.9570  | 0.9570  | 0.9570  |
|      |     |        | Len | 7.0037  | 6.9799  | 6.9299  | 6.9299  | 6.9299  |
| 1000 | 100 | 0.1    | Cov | 0.9502  | 0.9512  | 0.9440  | 0.9440  | 0.9440  |
|      |     |        | Len | 6.9322  | 6.9042  | 6.6668  | 6.6668  | 6.6668  |
| 1000 | 200 | 0.07   | Cov | 0.9586  | 0.9598  | 0.9318  | 0.9318  | 0.9318  |
|      |     |        | Len | 6.8995  | 6.8728  | 6.2666  | 6.2666  | 6.2666  |

Table 5.102. Etype = 2, J=5, k=19,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|---------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9696  | 0.9604 | 0.9538 | 0.9538 | 0.9538 |
|      |     |        | Len | 11.9344 | 8.2110 | 7.8086 | 7.8086 | 7.8086 |
| 100  | 40  | 0.9    | Cov | 0.9794  | 0.9584 | 0.8778 | 0.9520 | 0.8984 |
|      |     |        | Len | 16.4495 | 8.2539 | 6.5470 | 7.7055 | 6.7839 |
| 100  | 100 | 0.9    | Cov | 0.9770  | 0.9548 | 0.1562 | 0.9548 | 0.7750 |
|      |     |        | Len | 25.5281 | 8.0087 | 1.7863 | 7.6468 | 5.4816 |
| 100  | 200 | 0.9    | Cov | 0.9776  | 0.9458 | 0.0014 | 0.9510 | 0.6720 |
|      |     |        | Len | 37.1526 | 7.7236 | 0.0065 | 7.6230 | 4.6524 |
| 400  | 20  | 0.9    | Cov | 0.9564  | 0.9532 | 0.9560 | 0.9560 | 0.9560 |
|      |     |        | Len | 9.7518  | 7.0254 | 6.8900 | 6.8900 | 6.8900 |
| 400  | 40  | 0.9    | Cov | 0.9648  | 0.9502 | 0.9448 | 0.9448 | 0.9448 |
|      |     |        | Len | 13.8309 | 7.2221 | 6.6249 | 6.6249 | 6.6249 |
| 400  | 100 | 0.9    | Cov | 0.9588  | 0.9480 | 0.8972 | 0.9192 | 0.8946 |
|      |     |        | Len | 21.5731 | 7.2243 | 6.0202 | 6.2304 | 6.0258 |
| 400  | 200 | 0.9    | Cov | 0.9556  | 0.9454 | 0.7424 | 0.9310 | 0.8134 |
|      |     |        | Len | 31.2260 | 7.1155 | 4.9446 | 6.2583 | 5.2971 |
| 1000 | 20  | 0.9    | Cov | 0.9536  | 0.9554 | 0.9548 | 0.9548 | 0.9548 |
|      |     |        | Len | 9.2896  | 6.8012 | 6.6668 | 6.6668 | 6.6668 |
| 1000 | 40  | 0.9    | Cov | 0.9560  | 0.9510 | 0.9504 | 0.9504 | 0.9504 |
|      |     |        | Len | 13.2967 | 7.0374 | 6.5530 | 6.5530 | 6.5530 |
| 1000 | 100 | 0.9    | Cov | 0.9568  | 0.9552 | 0.9448 | 0.9448 | 0.9448 |
|      |     |        | Len | 20.7992 | 7.0688 | 6.3001 | 6.3001 | 6.3001 |
| 1000 | 200 | 0.9    | Cov | 0.9554  | 0.9494 | 0.9190 | 0.9190 | 0.9190 |
|      |     |        | Len | 30.1751 | 7.0392 | 5.9128 | 5.9128 | 5.9128 |

Table 5.103. Etype = 2, J=5, k=p-1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 40  | 0      | Cov | 0.9784  | 0.9348  | 0.9350 | 0.9670  | 0.9048  |
|      |     |        | Len | 27.5720 | 18.9041 | 8.3568 | 21.8901 | 18.1897 |
| 400  | 40  | 0      | Cov | 0.9728  | 0.9720  | 0.9720 | 0.9720  | 0.9720  |
|      |     |        | Len | 8.7191  | 8.7163  | 8.7163 | 8.7163  | 8.7163  |
| 1000 | 40  | 0      | Cov | 0.9728  | 0.9732  | 0.9732 | 0.9732  | 0.9732  |
|      |     |        | Len | 7.8926  | 7.8911  | 7.8911 | 7.8911  | 7.8911  |
| 100  | 100 | 0      | Cov | 0.9226  | 0.7958  | 0.1516 | 0.9152  | 0.6920  |
|      |     |        | Len | 35.2389 | 24.8367 | 1.8982 | 33.1127 | 22.4919 |
| 400  | 100 | 0      | Cov | 0.9804  | 0.9494  | 0.9708 | 0.9448  | 0.9288  |
|      |     |        | Len | 33.8776 | 19.3790 | 8.9249 | 18.8864 | 18.8101 |
| 1000 | 100 | 0      | Cov | 0.9738  | 0.9730  | 0.9730 | 0.9730  | 0.9730  |
|      |     |        | Len | 8.7720  | 8.7675  | 8.7675 | 8.7675  | 8.7675  |
| 100  | 200 | 0      | Cov | 0.9190  | 0.7528  | 0.0008 | 0.9270  | 0.6068  |
|      |     |        | Len | 49.9786 | 34.9318 | 0.0174 | 49.4161 | 29.1691 |
| 400  | 200 | 0      | Cov | 0.9748  | 0.9468  | 0.8880 | 0.9590  | 0.9188  |
|      |     |        | Len | 35.3882 | 21.2707 | 7.1463 | 33.9008 | 19.2347 |
| 1000 | 200 | 0      | Cov | 0.9750  | 0.9748  | 0.9748 | 0.9748  | 0.9748  |
|      |     |        | Len | 9.3472  | 9.3328  | 9.3328 | 9.3328  | 9.3328  |

Table 5.104. Etype = 2, J=5, k=p-1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL       | PLS    | PCR     | FS      |
|------|-----|--------|-----|----------|----------|--------|---------|---------|
| 100  | 40  | 0.1581 | Cov | 0.9926   | 0.9558   | 0.9340 | 0.9784  | 0.9060  |
|      |     |        | Len | 111.9761 | 24.4564  | 8.4277 | 10.0352 | 20.3508 |
| 100  | 100 | 0.1    | Cov | 0.9932   | 0.9284   | 0.2050 | 0.9776  | 0.8090  |
|      |     |        | Len | 331.0534 | 71.8418  | 2.2720 | 10.1980 | 50.5520 |
| 100  | 200 | 0.07   | Cov | 0.9944   | 0.9070   | 0.0018 | 0.9796  | 0.7172  |
|      |     |        | Len | 710.2673 | 150.8317 | 0.0087 | 10.7676 | 92.8100 |
| 400  | 40  | 0.1581 | Cov | 0.9738   | 0.9720   | 0.9720 | 0.9720  | 0.9720  |
|      |     |        | Len | 8.8442   | 8.7201   | 8.7201 | 8.7201  | 8.7201  |
| 400  | 100 | 0.1    | Cov | 0.9890   | 0.9664   | 0.9668 | 0.9730  | 0.9202  |
|      |     |        | Len | 119.0915 | 21.7437  | 8.9387 | 9.2993  | 18.8528 |
| 400  | 200 | 0.07   | Cov | 0.9900   | 0.9392   | 0.8828 | 0.9660  | 0.9062  |
|      |     |        | Len | 161.3514 | 33.3974  | 7.1552 | 23.4365 | 21.6280 |
| 1000 | 40  | 0.1581 | Cov | 0.9694   | 0.9710   | 0.9710 | 0.9710  | 0.9710  |
|      |     |        | Len | 7.9887   | 7.9020   | 7.9020 | 7.9020  | 7.9020  |
| 1000 | 100 | 0.1    | Cov | 0.9814   | 0.9722   | 0.9722 | 0.9722  | 0.9722  |
|      |     |        | Len | 16.9202  | 8.7732   | 8.7732 | 8.7732  | 8.7732  |
| 1000 | 200 | 0.07   | Cov | 0.9878   | 0.9752   | 0.9752 | 0.9752  | 0.9752  |
|      |     |        | Len | 37.0816  | 9.3373   | 9.3374 | 9.3374  | 9.3374  |

Table 5.105. Etype = 2, J=5, k=p-1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL      | PLS    | PCR    | FS      |
|------|-----|--------|-----|----------|---------|--------|--------|---------|
| 100  | 40  | 0.9    | Cov | 0.9876   | 0.9690  | 0.9046 | 0.9662 | 0.9256  |
|      |     |        | Len | 36.6262  | 10.2450 | 7.2120 | 8.5195 | 7.5336  |
| 400  | 40  | 0.9    | Cov | 0.9726   | 0.9642  | 0.9570 | 0.9570 | 0.9570  |
|      |     |        | Len | 30.5834  | 9.2919  | 7.4547 | 7.4547 | 7.4547  |
| 1000 | 40  | 0.9    | Cov | 0.9634   | 0.9558  | 0.9568 | 0.9568 | 0.9568  |
|      |     |        | Len | 28.2150  | 8.7865  | 7.0300 | 7.0300 | 7.0300  |
| 100  | 100 | 0.9    | Cov | 0.9966   | 0.9724  | 0.1930 | 0.9712 | 0.8482  |
|      |     |        | Len | 174.1334 | 19.2041 | 2.1794 | 9.5491 | 8.2058  |
| 400  | 100 | 0.9    | Cov | 0.9872   | 0.9742  | 0.9456 | 0.9600 | 0.9468  |
|      |     |        | Len | 138.9168 | 17.8661 | 7.5909 | 7.8954 | 7.6080  |
| 1000 | 100 | 0.9    | Cov | 0.9830   | 0.9778  | 0.9714 | 0.9714 | 0.9714  |
|      |     |        | Len | 132.2012 | 17.4709 | 8.1764 | 8.1764 | 8.1764  |
| 100  | 200 | 0.9    | Cov | 0.9942   | 0.9732  | 0.0016 | 0.9748 | 0.7434  |
|      |     |        | Len | 862.6756 | 32.4857 | 0.0082 | 9.9661 | 11.5052 |
| 400  | 200 | 0.9    | Cov | 0.9846   | 0.9730  | 0.8230 | 0.9632 | 0.8946  |
|      |     |        | Len | 188.5539 | 17.0783 | 6.0237 | 8.0185 | 6.7051  |
| 1000 | 200 | 0.9    | Cov | 0.9878   | 0.9812  | 0.9640 | 0.9640 | 0.9640  |
|      |     |        | Len | 389.9394 | 32.5495 | 8.1215 | 8.1215 | 8.1215  |

Table 5.106. Etype = 2, J=10, k=1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0      | Cov | 0.9496 | 0.9512 | 0.9324 | 0.9494 | 0.9408 |
|      |     |        | Len | 6.8577 | 6.9764 | 6.5832 | 7.4340 | 6.6735 |
| 100  | 40  | 0      | Cov | 0.9464 | 0.9376 | 0.8448 | 0.9476 | 0.9046 |
|      |     |        | Len | 6.8045 | 6.7801 | 5.6991 | 7.6956 | 6.3092 |
| 100  | 100 | 0      | Cov | 0.9434 | 0.9246 | 0.2848 | 0.9502 | 0.8702 |
|      |     |        | Len | 6.7510 | 6.4992 | 2.3134 | 7.8592 | 5.8149 |
| 100  | 200 | 0      | Cov | 0.9492 | 0.9264 | 0.0380 | 0.9530 | 0.8488 |
|      |     |        | Len | 6.8321 | 6.3655 | 0.2108 | 7.9407 | 5.5077 |
| 400  | 20  | 0      | Cov | 0.9456 | 0.9488 | 0.9454 | 0.9454 | 0.9454 |
|      |     |        | Len | 6.3100 | 6.3866 | 6.3094 | 6.3094 | 6.3094 |
| 400  | 40  | 0      | Cov | 0.9368 | 0.9446 | 0.9360 | 0.9360 | 0.9360 |
|      |     |        | Len | 6.1427 | 6.3078 | 6.1425 | 6.1425 | 6.1425 |
| 400  | 100 | 0      | Cov | 0.9350 | 0.9328 | 0.8932 | 0.9380 | 0.9076 |
|      |     |        | Len | 5.9791 | 6.1111 | 5.6320 | 6.6058 | 5.7563 |
| 400  | 200 | 0      | Cov | 0.9330 | 0.9188 | 0.7186 | 0.9370 | 0.8666 |
|      |     |        | Len | 5.9183 | 5.9292 | 4.6729 | 6.8079 | 5.4049 |
| 1000 | 20  | 0      | Cov | 0.9496 | 0.9488 | 0.9496 | 0.9496 | 0.9496 |
|      |     |        | Len | 6.2951 | 6.3222 | 6.2947 | 6.2947 | 6.2947 |
| 1000 | 40  | 0      | Cov | 0.9428 | 0.9438 | 0.9420 | 0.9420 | 0.9420 |
|      |     |        | Len | 6.2295 | 6.2910 | 6.2291 | 6.2291 | 6.2291 |
| 1000 | 100 | 0      | Cov | 0.9244 | 0.9342 | 0.9254 | 0.9254 | 0.9254 |
|      |     |        | Len | 6.0306 | 6.1965 | 6.0305 | 6.0305 | 6.0305 |
| 1000 | 200 | 0      | Cov | 0.9326 | 0.9374 | 0.9052 | 0.9322 | 0.9092 |
|      |     |        | Len | 5.8876 | 6.0601 | 5.6966 | 6.3854 | 5.7485 |

Table 5.107. Etype = 2, J=10, k=1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0.2236 | Cov | 0.9424 | 0.9514 | 0.9258 | 0.9438 | 0.9284 |
|      |     |        | Len | 6.7954 | 7.1022 | 6.5075 | 7.1879 | 6.5930 |
| 100  | 40  | 0.1581 | Cov | 0.9440 | 0.9502 | 0.8366 | 0.9474 | 0.9092 |
|      |     |        | Len | 6.7382 | 7.0274 | 5.6437 | 7.4226 | 6.2427 |
| 100  | 100 | 0.1    | Cov | 0.9456 | 0.9464 | 0.3058 | 0.9518 | 0.8752 |
|      |     |        | Len | 6.7895 | 7.0056 | 2.4519 | 7.7155 | 5.8596 |
| 100  | 200 | 0.07   | Cov | 0.9442 | 0.9450 | 0.0562 | 0.9500 | 0.8426 |
|      |     |        | Len | 6.8382 | 6.9267 | 0.3036 | 7.8072 | 5.5210 |
| 400  | 20  | 0.2236 | Cov | 0.9518 | 0.9554 | 0.9514 | 0.9514 | 0.9514 |
|      |     |        | Len | 6.3055 | 6.4200 | 6.3050 | 6.3050 | 6.3050 |
| 400  | 40  | 0.1581 | Cov | 0.9320 | 0.9438 | 0.9316 | 0.9316 | 0.9316 |
|      |     |        | Len | 6.1380 | 6.3801 | 6.1372 | 6.1372 | 6.1372 |
| 400  | 100 | 0.1    | Cov | 0.9302 | 0.9436 | 0.8876 | 0.9308 | 0.9022 |
|      |     |        | Len | 5.9633 | 6.3741 | 5.6062 | 6.5035 | 5.7293 |
| 400  | 200 | 0.07   | Cov | 0.9326 | 0.9462 | 0.7012 | 0.9374 | 0.8668 |
|      |     |        | Len | 5.9082 | 6.3635 | 4.6723 | 6.7190 | 5.4057 |
| 1000 | 20  | 0.2236 | Cov | 0.9478 | 0.9512 | 0.9474 | 0.9474 | 0.9474 |
|      |     |        | Len | 6.2971 | 6.3411 | 6.2969 | 6.2969 | 6.2969 |
| 1000 | 40  | 0.1581 | Cov | 0.9474 | 0.9498 | 0.9474 | 0.9474 | 0.9474 |
|      |     |        | Len | 6.2383 | 6.3370 | 6.2378 | 6.2378 | 6.2378 |
| 1000 | 100 | 0.1    | Cov | 0.9364 | 0.9510 | 0.9356 | 0.9356 | 0.9356 |
|      |     |        | Len | 6.0359 | 6.2949 | 6.0356 | 6.0356 | 6.0356 |
| 1000 | 200 | 0.07   | Cov | 0.9300 | 0.9434 | 0.9042 | 0.9326 | 0.9108 |
|      |     |        | Len | 5.8937 | 6.3021 | 5.7016 | 6.3446 | 5.7518 |

Table 5.108. Etype = 2, J=10, k=1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9552 | 0.9548 | 0.9232 | 0.9464 | 0.9272 |
|      |     |        | Len | 7.2407 | 7.2367 | 6.5651 | 6.9814 | 6.6473 |
| 100  | 40  | 0.9    | Cov | 0.9610 | 0.9612 | 0.8456 | 0.9524 | 0.9126 |
|      |     |        | Len | 7.2507 | 7.2387 | 5.7184 | 7.0028 | 6.2944 |
| 100  | 100 | 0.9    | Cov | 0.9538 | 0.9528 | 0.3068 | 0.9506 | 0.8730 |
|      |     |        | Len | 7.4077 | 7.2842 | 2.4612 | 7.1039 | 5.8732 |
| 100  | 200 | 0.9    | Cov | 0.9538 | 0.9472 | 0.0494 | 0.9492 | 0.8402 |
|      |     |        | Len | 8.9931 | 7.2323 | 0.2941 | 7.1416 | 5.5394 |
| 400  | 20  | 0.9    | Cov | 0.9478 | 0.9466 | 0.9412 | 0.9412 | 0.9412 |
|      |     |        | Len | 6.4460 | 6.4431 | 6.3097 | 6.3097 | 6.3097 |
| 400  | 40  | 0.9    | Cov | 0.9486 | 0.9476 | 0.9348 | 0.9348 | 0.9348 |
|      |     |        | Len | 6.4554 | 6.4469 | 6.1487 | 6.1487 | 6.1487 |
| 400  | 100 | 0.9    | Cov | 0.9518 | 0.9498 | 0.8942 | 0.9408 | 0.9068 |
|      |     |        | Len | 6.4600 | 6.4211 | 5.6218 | 6.1423 | 5.7481 |
| 400  | 200 | 0.9    | Cov | 0.9476 | 0.9448 | 0.7146 | 0.9422 | 0.8622 |
|      |     |        | Len | 6.4873 | 6.4239 | 4.6836 | 6.1604 | 5.4101 |
| 1000 | 20  | 0.9    | Cov | 0.9506 | 0.9492 | 0.9468 | 0.9468 | 0.9468 |
|      |     |        | Len | 6.3473 | 6.3445 | 6.2936 | 6.2936 | 6.2936 |
| 1000 | 40  | 0.9    | Cov | 0.9538 | 0.9542 | 0.9476 | 0.9476 | 0.9476 |
|      |     |        | Len | 6.3485 | 6.3398 | 6.2245 | 6.2245 | 6.2245 |
| 1000 | 100 | 0.9    | Cov | 0.9454 | 0.9468 | 0.9306 | 0.9306 | 0.9306 |
|      |     |        | Len | 6.3630 | 6.3430 | 6.0297 | 6.0297 | 6.0297 |
| 1000 | 200 | 0.9    | Cov | 0.9492 | 0.9494 | 0.9044 | 0.9356 | 0.9066 |
|      |     |        | Len | 6.3651 | 6.3363 | 5.6952 | 6.0319 | 5.7450 |

Table 5.109. Etype = 2, J=10, k=19,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 20  | 0      | Cov | 0.9808  | 0.9576  | 0.9644 | 0.9746  | 0.9504  |
|      |     |        | Len | 20.6985 | 14.7362 | 8.7340 | 16.8180 | 15.1901 |
| 100  | 40  | 0      | Cov | 0.9800  | 0.9544  | 0.9090 | 0.9730  | 0.9440  |
|      |     |        | Len | 20.6776 | 14.7442 | 7.3104 | 20.0622 | 15.1411 |
| 100  | 100 | 0      | Cov | 0.9768  | 0.9478  | 0.3532 | 0.9768  | 0.9268  |
|      |     |        | Len | 20.1815 | 14.6303 | 3.0335 | 21.4056 | 14.5779 |
| 100  | 200 | 0      | Cov | 0.9676  | 0.9266  | 0.0426 | 0.9710  | 0.8904  |
|      |     |        | Len | 19.5333 | 14.3629 | 0.3865 | 20.9442 | 13.7128 |
| 400  | 20  | 0      | Cov | 0.9728  | 0.9720  | 0.9720 | 0.9720  | 0.9720  |
|      |     |        | Len | 8.4326  | 8.4313  | 8.4313 | 8.4313  | 8.4313  |
| 400  | 40  | 0      | Cov | 0.9716  | 0.9724  | 0.9724 | 0.9724  | 0.9724  |
|      |     |        | Len | 8.1520  | 8.2946  | 8.1447 | 8.1447  | 8.1447  |
| 400  | 100 | 0      | Cov | 0.9646  | 0.9680  | 0.9424 | 0.9508  | 0.9562  |
|      |     |        | Len | 8.2135  | 8.0251  | 7.2778 | 15.3863 | 7.6129  |
| 400  | 200 | 0      | Cov | 0.9678  | 0.9592  | 0.8044 | 0.9620  | 0.9388  |
|      |     |        | Len | 8.3468  | 7.8437  | 5.8222 | 17.7929 | 7.2413  |
| 1000 | 20  | 0      | Cov | 0.9554  | 0.9550  | 0.9550 | 0.9550  | 0.9550  |
|      |     |        | Len | 6.9925  | 6.9917  | 6.9917 | 6.9917  | 6.9917  |
| 1000 | 40  | 0      | Cov | 0.9600  | 0.9596  | 0.9600 | 0.9600  | 0.9600  |
|      |     |        | Len | 6.9412  | 6.9795  | 6.9382 | 6.9382  | 6.9382  |
| 1000 | 100 | 0      | Cov | 0.9500  | 0.9550  | 0.9494 | 0.9494  | 0.9494  |
|      |     |        | Len | 6.6847  | 6.8454  | 6.6806 | 6.6806  | 6.6806  |
| 1000 | 200 | 0      | Cov | 0.9436  | 0.9446  | 0.9228 | 0.9116  | 0.9266  |
|      |     |        | Len | 6.5822  | 6.6774  | 6.2757 | 12.2616 | 6.3498  |

Table 5.110. Etype = 2, J=10, k=19,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 20  | 0.2236 | Cov | 0.9870  | 0.9640  | 0.9608 | 0.9704  | 0.9404  |
|      |     |        | Len | 73.8682 | 16.2784 | 8.6945 | 9.3741  | 15.8663 |
| 100  | 40  | 0.1581 | Cov | 0.9894  | 0.9600  | 0.9176 | 0.9810  | 0.9342  |
|      |     |        | Len | 76.8364 | 18.7473 | 7.3840 | 14.3215 | 17.2066 |
| 100  | 100 | 0.1    | Cov | 0.9882  | 0.9450  | 0.4014 | 0.9782  | 0.9040  |
|      |     |        | Len | 78.9188 | 21.0942 | 3.3246 | 19.0053 | 18.4751 |
| 100  | 200 | 0.07   | Cov | 0.9862  | 0.9362  | 0.0668 | 0.9788  | 0.8738  |
|      |     |        | Len | 83.0190 | 23.2825 | 0.5835 | 21.0602 | 18.9800 |
| 400  | 20  | 0.2236 | Cov | 0.9710  | 0.9712  | 0.9712 | 0.9712  | 0.9712  |
|      |     |        | Len | 8.4553  | 8.4316  | 8.4322 | 8.4322  | 8.4322  |
| 400  | 40  | 0.1581 | Cov | 0.9740  | 0.9730  | 0.9714 | 0.9714  | 0.9714  |
|      |     |        | Len | 8.3755  | 8.3505  | 8.1642 | 8.1642  | 8.1642  |
| 400  | 100 | 0.1    | Cov | 0.9634  | 0.9630  | 0.9418 | 0.9554  | 0.9528  |
|      |     |        | Len | 8.4131  | 8.1382  | 7.2849 | 13.2883 | 7.6229  |
| 400  | 200 | 0.07   | Cov | 0.9692  | 0.9648  | 0.8086 | 0.9644  | 0.9412  |
|      |     |        | Len | 11.4040 | 8.0171  | 5.8294 | 16.1963 | 7.2706  |
| 1000 | 20  | 0.2236 | Cov | 0.9582  | 0.9588  | 0.9588 | 0.9588  | 0.9588  |
|      |     |        | Len | 7.0052  | 6.9827  | 6.9827 | 6.9827  | 6.9827  |
| 1000 | 40  | 0.1581 | Cov | 0.9594  | 0.9600  | 0.9574 | 0.9574  | 0.9574  |
|      |     |        | Len | 6.9943  | 6.9712  | 6.9204 | 6.9204  | 6.9204  |
| 1000 | 100 | 0.1    | Cov | 0.9514  | 0.9518  | 0.9442 | 0.9442  | 0.9442  |
|      |     |        | Len | 6.9424  | 6.9166  | 6.6784 | 6.6784  | 6.6784  |
| 1000 | 200 | 0.07   | Cov | 0.9548  | 0.9572  | 0.9302 | 0.9242  | 0.9356  |
|      |     |        | Len | 6.8895  | 6.8657  | 6.2631 | 11.3002 | 6.3427  |

Table 5.111. Etype = 2, J=10, k=19,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|----------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9772   | 0.9618 | 0.9484 | 0.9638 | 0.9520 |
|      |     |        | Len | 35.6038  | 8.2307 | 7.8132 | 8.4075 | 7.9651 |
| 100  | 40  | 0.9    | Cov | 0.9826   | 0.9572 | 0.8820 | 0.9650 | 0.9356 |
|      |     |        | Len | 35.1546  | 8.2650 | 6.5688 | 8.3254 | 7.4254 |
| 100  | 100 | 0.9    | Cov | 0.9768   | 0.9484 | 0.3420 | 0.9586 | 0.9016 |
|      |     |        | Len | 103.0599 | 7.9733 | 2.6554 | 8.2223 | 6.7265 |
| 100  | 200 | 0.9    | Cov | 0.9796   | 0.9460 | 0.0588 | 0.9606 | 0.8532 |
|      |     |        | Len | 281.6071 | 7.6404 | 0.3178 | 8.1760 | 6.2701 |
| 400  | 20  | 0.9    | Cov | 0.9552   | 0.9494 | 0.9506 | 0.9506 | 0.9506 |
|      |     |        | Len | 9.7501   | 7.0309 | 6.8865 | 6.8865 | 6.8865 |
| 400  | 40  | 0.9    | Cov | 0.9604   | 0.9510 | 0.9464 | 0.9464 | 0.9464 |
|      |     |        | Len | 13.8206  | 7.2269 | 6.6317 | 6.6317 | 6.6317 |
| 400  | 100 | 0.9    | Cov | 0.9636   | 0.9590 | 0.9120 | 0.9532 | 0.9238 |
|      |     |        | Len | 21.5691  | 7.2096 | 6.0136 | 6.6713 | 6.1771 |
| 400  | 200 | 0.9    | Cov | 0.9638   | 0.9488 | 0.7336 | 0.9484 | 0.8840 |
|      |     |        | Len | 31.2092  | 7.1215 | 4.9418 | 6.6870 | 5.8102 |
| 1000 | 20  | 0.9    | Cov | 0.9556   | 0.9528 | 0.9526 | 0.9526 | 0.9526 |
|      |     |        | Len | 9.2856   | 6.8027 | 6.6672 | 6.6672 | 6.6672 |
| 1000 | 40  | 0.9    | Cov | 0.9540   | 0.9522 | 0.9498 | 0.9498 | 0.9498 |
|      |     |        | Len | 13.3109  | 7.0267 | 6.5357 | 6.5357 | 6.5357 |
| 1000 | 100 | 0.9    | Cov | 0.9540   | 0.9530 | 0.9416 | 0.9416 | 0.9416 |
|      |     |        | Len | 20.7995  | 7.0668 | 6.2931 | 6.2931 | 6.2931 |
| 1000 | 200 | 0.9    | Cov | 0.9556   | 0.9470 | 0.9114 | 0.9398 | 0.9142 |
|      |     |        | Len | 30.1532  | 7.0387 | 5.9209 | 6.3430 | 5.9846 |

Table 5.112. Etype = 2, J=10, k=p-1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 40  | 0      | Cov | 0.9762  | 0.9270  | 0.9004 | 0.9694  | 0.9134  |
|      |     |        | Len | 29.2411 | 22.0838 | 7.0977 | 26.4435 | 22.3143 |
| 400  | 40  | 0      | Cov | 0.9762  | 0.9762  | 0.9762 | 0.9762  | 0.9762  |
|      |     |        | Len | 8.7423  | 8.7409  | 8.7409 | 8.7409  | 8.7409  |
| 1000 | 40  | 0      | Cov | 0.9690  | 0.9686  | 0.9686 | 0.9686  | 0.9686  |
|      |     |        | Len | 7.8990  | 7.8983  | 7.8983 | 7.8983  | 7.8983  |
| 100  | 100 | 0      | Cov | 0.9466  | 0.8656  | 0.2946 | 0.9458  | 0.8304  |
|      |     |        | Len | 40.8300 | 31.3864 | 3.3631 | 39.7969 | 31.3018 |
| 400  | 100 | 0      | Cov | 0.9774  | 0.9432  | 0.9522 | 0.9628  | 0.9164  |
|      |     |        | Len | 41.5598 | 29.8979 | 7.8157 | 33.4445 | 29.2930 |
| 1000 | 100 | 0      | Cov | 0.9738  | 0.9742  | 0.9742 | 0.9742  | 0.9742  |
|      |     |        | Len | 8.8012  | 8.7963  | 8.7963 | 8.7963  | 8.7963  |
| 100  | 200 | 0      | Cov | 0.9468  | 0.8428  | 0.0394 | 0.9510  | 0.7970  |
|      |     |        | Len | 57.1290 | 44.0315 | 0.8811 | 56.7772 | 42.3968 |
| 400  | 200 | 0      | Cov | 0.9318  | 0.8438  | 0.7212 | 0.9152  | 0.8068  |
|      |     |        | Len | 49.6837 | 37.9770 | 4.8771 | 45.1776 | 36.7644 |
| 1000 | 200 | 0      | Cov | 0.9772  | 0.9470  | 0.9694 | 0.9608  | 0.9244  |
|      |     |        | Len | 56.3705 | 38.5022 | 8.1634 | 41.6062 | 37.5568 |

Table 5.113. Etype = 2, J=10, k=p-1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL       | PLS    | PCR     | FS       |
|------|-----|--------|-----|----------|----------|--------|---------|----------|
| 100  | 40  | 0.1581 | Cov | 0.9866   | 0.9508   | 0.9134 | 0.9766  | 0.9108   |
|      |     |        | Len | 160.3446 | 36.9186  | 7.3979 | 9.4746  | 33.9655  |
| 100  | 100 | 0.1    | Cov | 0.9878   | 0.9276   | 0.3756 | 0.9714  | 0.8632   |
|      |     |        | Len | 412.2464 | 97.2709  | 3.0133 | 9.6752  | 85.8916  |
| 100  | 200 | 0.07   | Cov | 0.9886   | 0.9170   | 0.0678 | 0.9742  | 0.8288   |
|      |     |        | Len | 867.9363 | 209.8252 | 0.3719 | 10.1468 | 166.2737 |
| 400  | 40  | 0.1581 | Cov | 0.9730   | 0.9722   | 0.9722 | 0.9722  | 0.9722   |
|      |     |        | Len | 8.8387   | 8.7227   | 8.7242 | 8.7242  | 8.7242   |
| 400  | 100 | 0.1    | Cov | 0.9802   | 0.9552   | 0.9604 | 0.9784  | 0.9030   |
|      |     |        | Len | 258.3750 | 46.0803  | 7.7999 | 8.7642  | 38.3404  |
| 400  | 200 | 0.07   | Cov | 0.9842   | 0.9356   | 0.8332 | 0.9734  | 0.8508   |
|      |     |        | Len | 568.8111 | 105.0332 | 6.2417 | 8.8534  | 80.9983  |
| 1000 | 40  | 0.1581 | Cov | 0.9662   | 0.9654   | 0.9654 | 0.9654  | 0.9654   |
|      |     |        | Len | 7.9912   | 7.9065   | 7.9065 | 7.9065  | 7.9065   |
| 1000 | 100 | 0.1    | Cov | 0.9802   | 0.9732   | 0.9732 | 0.9732  | 0.9732   |
|      |     |        | Len | 16.8888  | 8.7906   | 8.7906 | 8.7906  | 8.7906   |
| 1000 | 200 | 0.07   | Cov | 0.9854   | 0.9586   | 0.9656 | 0.9800  | 0.9050   |
|      |     |        | Len | 362.4080 | 56.6132  | 8.1670 | 8.8258  | 46.6793  |

Table 5.114. Etype = 2, J=10, k=p-1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso     | RL      | PLS    | PCR    | FS      |
|------|-----|--------|-----|-----------|---------|--------|--------|---------|
| 100  | 40  | 0.9    | Cov | 0.9894    | 0.9648  | 0.9068 | 0.9704 | 0.9430  |
|      |     |        | Len | 163.5508  | 10.1662 | 7.1915 | 9.1634 | 8.3984  |
| 400  | 40  | 0.9    | Cov | 0.9772    | 0.9602  | 0.9578 | 0.9578 | 0.9578  |
|      |     |        | Len | 30.5761   | 9.2889  | 7.4415 | 7.4415 | 7.4415  |
| 1000 | 40  | 0.9    | Cov | 0.9626    | 0.9594  | 0.9592 | 0.9592 | 0.9592  |
|      |     |        | Len | 28.2281   | 8.7765  | 7.0298 | 7.0298 | 7.0298  |
| 100  | 100 | 0.9    | Cov | 0.9908    | 0.9616  | 0.3672 | 0.9720 | 0.8766  |
|      |     |        | Len | 1622.7910 | 16.8749 | 2.9870 | 9.4344 | 11.7703 |
| 400  | 100 | 0.9    | Cov | 0.9868    | 0.9758  | 0.9488 | 0.9718 | 0.9434  |
|      |     |        | Len | 138.6274  | 17.8940 | 7.6177 | 8.5205 | 8.1880  |
| 1000 | 100 | 0.9    | Cov | 0.9800    | 0.9778  | 0.9678 | 0.9678 | 0.9678  |
|      |     |        | Len | 132.2754  | 17.5113 | 8.1753 | 8.1753 | 8.1753  |
| 100  | 200 | 0.9    | Cov | 0.9898    | 0.9612  | 0.0656 | 0.9746 | 0.8248  |
|      |     |        | Len | 7481.8090 | 29.1677 | 0.3581 | 9.4423 | 19.6532 |
| 400  | 200 | 0.9    | Cov | 0.9854    | 0.9762  | 0.8376 | 0.9742 | 0.8978  |
|      |     |        | Len | 392.6941  | 31.9719 | 6.2487 | 8.7302 | 10.8023 |
| 1000 | 200 | 0.9    | Cov | 0.9854    | 0.9814  | 0.9638 | 0.9736 | 0.9524  |
|      |     |        | Len | 387.8449  | 32.6106 | 8.1019 | 8.7189 | 8.7801  |

Table 5.115. Etype = 2, J=20, k=1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0      | Cov | 0.9538 | 0.9490 | 0.9270 | 0.9530 | 0.9444 |
|      |     |        | Len | 7.1212 | 7.0271 | 6.5550 | 7.9473 | 6.8862 |
| 100  | 40  | 0      | Cov | 0.9534 | 0.9470 | 0.8558 | 0.9544 | 0.9370 |
|      |     |        | Len | 7.1407 | 6.9454 | 5.7675 | 8.1222 | 6.7515 |
| 100  | 100 | 0      | Cov | 0.9536 | 0.9424 | 0.5274 | 0.9586 | 0.9282 |
|      |     |        | Len | 7.1023 | 6.7633 | 3.3538 | 8.1766 | 6.5226 |
| 400  | 20  | 0      | Cov | 0.9388 | 0.9394 | 0.9382 | 0.9382 | 0.9382 |
|      |     |        | Len | 6.3172 | 6.3934 | 6.3170 | 6.3170 | 6.3170 |
| 400  | 40  | 0      | Cov | 0.9438 | 0.9444 | 0.9356 | 0.9412 | 0.9360 |
|      |     |        | Len | 6.2496 | 6.3191 | 6.1466 | 6.7352 | 6.1777 |
| 400  | 100 | 0      | Cov | 0.9354 | 0.9326 | 0.8892 | 0.9380 | 0.9170 |
|      |     |        | Len | 6.2103 | 6.1923 | 5.6205 | 7.0338 | 5.9519 |
| 400  | 200 | 0      | Cov | 0.9440 | 0.9352 | 0.7118 | 0.9444 | 0.9110 |
|      |     |        | Len | 6.1918 | 6.0927 | 4.6736 | 7.1619 | 5.7816 |
| 1000 | 20  | 0      | Cov | 0.9424 | 0.9460 | 0.9422 | 0.9422 | 0.9422 |
|      |     |        | Len | 6.2999 | 6.3281 | 6.2999 | 6.2999 | 6.2999 |
| 1000 | 40  | 0      | Cov | 0.9404 | 0.9454 | 0.9408 | 0.9408 | 0.9408 |
|      |     |        | Len | 6.2388 | 6.3041 | 6.2387 | 6.2387 | 6.2387 |
| 1000 | 100 | 0      | Cov | 0.9434 | 0.9438 | 0.9354 | 0.9390 | 0.9366 |
|      |     |        | Len | 6.1358 | 6.2153 | 6.0350 | 6.5979 | 6.0603 |
| 1000 | 200 | 0      | Cov | 0.9322 | 0.9306 | 0.8944 | 0.9366 | 0.9146 |
|      |     |        | Len | 6.0959 | 6.1170 | 5.6973 | 6.8368 | 5.8879 |

Table 5.116. Etype = 2, J=20, k=1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0.2236 | Cov | 0.9524 | 0.9538 | 0.9262 | 0.9528 | 0.9420 |
|      |     |        | Len | 7.1041 | 7.1029 | 6.5198 | 7.6275 | 6.8445 |
| 100  | 40  | 0.1581 | Cov | 0.9474 | 0.9484 | 0.8590 | 0.9562 | 0.9354 |
|      |     |        | Len | 7.1582 | 7.0864 | 5.7693 | 7.8327 | 6.7390 |
| 100  | 100 | 0.1    | Cov | 0.9502 | 0.9446 | 0.6128 | 0.9522 | 0.9264 |
|      |     |        | Len | 7.1914 | 6.9734 | 3.7396 | 7.9920 | 6.5412 |
| 100  | 200 | 0.07   | Cov | 0.9476 | 0.9406 | 0.3728 | 0.9538 | 0.9188 |
|      |     |        | Len | 7.2525 | 6.9099 | 1.9756 | 8.0722 | 6.4255 |
| 400  | 20  | 0.2236 | Cov | 0.9376 | 0.9422 | 0.9374 | 0.9374 | 0.9374 |
|      |     |        | Len | 6.3036 | 6.4150 | 6.3033 | 6.3033 | 6.3033 |
| 400  | 40  | 0.1581 | Cov | 0.9384 | 0.9442 | 0.9314 | 0.9438 | 0.9332 |
|      |     |        | Len | 6.2589 | 6.4266 | 6.1583 | 6.6231 | 6.1888 |
| 400  | 100 | 0.1    | Cov | 0.9400 | 0.9466 | 0.8882 | 0.9414 | 0.9206 |
|      |     |        | Len | 6.2038 | 6.3949 | 5.6196 | 6.8871 | 5.9467 |
| 400  | 200 | 0.07   | Cov | 0.9392 | 0.9448 | 0.7072 | 0.9384 | 0.9044 |
|      |     |        | Len | 6.1853 | 6.3782 | 4.6525 | 7.0258 | 5.7709 |
| 1000 | 20  | 0.2236 | Cov | 0.9562 | 0.9568 | 0.9564 | 0.9564 | 0.9564 |
|      |     |        | Len | 6.2957 | 6.3409 | 6.2954 | 6.2954 | 6.2954 |
| 1000 | 40  | 0.1581 | Cov | 0.9432 | 0.9454 | 0.9434 | 0.9434 | 0.9434 |
|      |     |        | Len | 6.2376 | 6.3377 | 6.2371 | 6.2371 | 6.2371 |
| 1000 | 100 | 0.1    | Cov | 0.9390 | 0.9466 | 0.9324 | 0.9390 | 0.9346 |
|      |     |        | Len | 6.1258 | 6.3190 | 6.0231 | 6.5141 | 6.0481 |
| 1000 | 200 | 0.07   | Cov | 0.9426 | 0.9452 | 0.9102 | 0.9372 | 0.9264 |
|      |     |        | Len | 6.1055 | 6.3305 | 5.7035 | 6.7594 | 5.8991 |

Table 5.117. Etype = 2, J=20, k=1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|---------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9576  | 0.9568 | 0.9296 | 0.9570 | 0.9454 |
|      |     |        | Len | 7.3167  | 7.2374 | 6.5763 | 7.1968 | 6.8642 |
| 100  | 40  | 0.9    | Cov | 0.9564  | 0.9540 | 0.8698 | 0.9550 | 0.9348 |
|      |     |        | Len | 7.9689  | 7.2302 | 5.7784 | 7.1967 | 6.6949 |
| 100  | 100 | 0.9    | Cov | 0.9670  | 0.9594 | 0.6080 | 0.9576 | 0.9308 |
|      |     |        | Len | 15.1272 | 7.2773 | 3.7338 | 7.2919 | 6.5663 |
| 100  | 200 | 0.9    | Cov | 0.9666  | 0.9574 | 0.3778 | 0.9598 | 0.9220 |
|      |     |        | Len | 34.3723 | 7.2533 | 1.9070 | 7.3680 | 6.4522 |
| 400  | 20  | 0.9    | Cov | 0.9452  | 0.9448 | 0.9386 | 0.9386 | 0.9386 |
|      |     |        | Len | 6.4316  | 6.4285 | 6.2919 | 6.2919 | 6.2919 |
| 400  | 40  | 0.9    | Cov | 0.9424  | 0.9428 | 0.9266 | 0.9380 | 0.9288 |
|      |     |        | Len | 6.4440  | 6.4339 | 6.1377 | 6.3136 | 6.1681 |
| 400  | 100 | 0.9    | Cov | 0.9492  | 0.9492 | 0.8846 | 0.9428 | 0.9204 |
|      |     |        | Len | 6.4678  | 6.4281 | 5.6477 | 6.3274 | 5.9626 |
| 400  | 200 | 0.9    | Cov | 0.9508  | 0.9474 | 0.7174 | 0.9474 | 0.9124 |
|      |     |        | Len | 6.4716  | 6.4130 | 4.6796 | 6.3180 | 5.7763 |
| 1000 | 20  | 0.9    | Cov | 0.9456  | 0.9452 | 0.9448 | 0.9448 | 0.9448 |
|      |     |        | Len | 6.3514  | 6.3485 | 6.2975 | 6.2975 | 6.2975 |
| 1000 | 40  | 0.9    | Cov | 0.9460  | 0.9454 | 0.9408 | 0.9408 | 0.9408 |
|      |     |        | Len | 6.3586  | 6.3501 | 6.2317 | 6.2317 | 6.2317 |
| 1000 | 100 | 0.9    | Cov | 0.9524  | 0.9520 | 0.9372 | 0.9460 | 0.9412 |
|      |     |        | Len | 6.3536  | 6.3320 | 6.0176 | 6.1901 | 6.0443 |
| 1000 | 200 | 0.9    | Cov | 0.9444  | 0.9420 | 0.9042 | 0.9396 | 0.9208 |
|      |     |        | Len | 6.3632  | 6.3354 | 5.6974 | 6.2020 | 5.8894 |

Table 5.118. Etype = 2, J=20, k=19,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 20  | 0      | Cov | 0.9790  | 0.9526  | 0.9528 | 0.9750  | 0.9542  |
|      |     |        | Len | 21.5374 | 16.8205 | 8.1244 | 20.0649 | 18.4171 |
| 100  | 40  | 0      | Cov | 0.9768  | 0.9528  | 0.9048 | 0.9726  | 0.9526  |
|      |     |        | Len | 21.4205 | 16.7150 | 6.9849 | 21.4213 | 18.2926 |
| 100  | 100 | 0      | Cov | 0.9690  | 0.9376  | 0.5542 | 0.9696  | 0.9428  |
|      |     |        | Len | 20.7046 | 16.1859 | 4.7312 | 21.6300 | 17.6379 |
| 100  | 200 | 0      | Cov | 0.9686  | 0.9332  | 0.2496 | 0.9680  | 0.9308  |
|      |     |        | Len | 20.6402 | 16.5447 | 2.4642 | 21.3088 | 17.0586 |
| 400  | 20  | 0      | Cov | 0.9728  | 0.9732  | 0.9732 | 0.9732  | 0.9732  |
|      |     |        | Len | 8.4320  | 8.4307  | 8.4308 | 8.4308  | 8.4308  |
| 400  | 40  | 0      | Cov | 0.9728  | 0.9710  | 0.9704 | 0.9652  | 0.9720  |
|      |     |        | Len | 9.3604  | 8.3855  | 8.1220 | 15.1714 | 8.4240  |
| 400  | 100 | 0      | Cov | 0.9692  | 0.9666  | 0.9414 | 0.9644  | 0.9664  |
|      |     |        | Len | 10.0930 | 8.4027  | 7.2819 | 18.7989 | 8.4531  |
| 400  | 200 | 0      | Cov | 0.9746  | 0.9726  | 0.8154 | 0.9736  | 0.9722  |
|      |     |        | Len | 10.6519 | 8.3787  | 5.8068 | 20.0465 | 8.4291  |
| 1000 | 20  | 0      | Cov | 0.9582  | 0.9584  | 0.9584 | 0.9584  | 0.9584  |
|      |     |        | Len | 6.9982  | 6.9973  | 6.9973 | 6.9973  | 6.9973  |
| 1000 | 40  | 0      | Cov | 0.9574  | 0.9600  | 0.9566 | 0.9566  | 0.9566  |
|      |     |        | Len | 6.9186  | 6.9567  | 6.9166 | 6.9166  | 6.9166  |
| 1000 | 100 | 0      | Cov | 0.9540  | 0.9534  | 0.9462 | 0.9418  | 0.9486  |
|      |     |        | Len | 6.8880  | 6.8822  | 6.7000 | 13.1252 | 6.7487  |
| 1000 | 200 | 0      | Cov | 0.9528  | 0.9518  | 0.9292 | 0.9470  | 0.9442  |
|      |     |        | Len | 6.9051  | 6.7842  | 6.2755 | 15.7877 | 6.5771  |

Table 5.119. Etype = 2, J=20, k=19,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 20  | 0.2236 | Cov | 0.9828  | 0.9512  | 0.9502 | 0.9688  | 0.9400  |
|      |     |        | Len | 81.1712 | 19.5505 | 8.1745 | 9.1209  | 26.1005 |
| 100  | 40  | 0.1581 | Cov | 0.9834  | 0.9492  | 0.9076 | 0.9774  | 0.9410  |
|      |     |        | Len | 80.0670 | 20.9045 | 7.1599 | 15.0091 | 27.9565 |
| 100  | 100 | 0.1    | Cov | 0.9798  | 0.9272  | 0.6780 | 0.9778  | 0.9232  |
|      |     |        | Len | 78.5992 | 21.6632 | 5.4714 | 19.1129 | 29.3061 |
| 100  | 200 | 0.07   | Cov | 0.9830  | 0.9244  | 0.3960 | 0.9828  | 0.9066  |
|      |     |        | Len | 88.9480 | 27.8414 | 3.6561 | 20.8356 | 29.7058 |
| 400  | 20  | 0.2236 | Cov | 0.9764  | 0.9762  | 0.9762 | 0.9762  | 0.9762  |
|      |     |        | Len | 8.4385  | 8.4169  | 8.4169 | 8.4169  | 8.4169  |
| 400  | 40  | 0.1581 | Cov | 0.9788  | 0.9716  | 0.9654 | 0.9674  | 0.9722  |
|      |     |        | Len | 43.9822 | 8.7689  | 8.1415 | 11.2924 | 8.4295  |
| 400  | 100 | 0.1    | Cov | 0.9788  | 0.9684  | 0.9416 | 0.9698  | 0.9708  |
|      |     |        | Len | 55.7322 | 10.4395 | 7.2701 | 16.0327 | 8.4244  |
| 400  | 200 | 0.07   | Cov | 0.9808  | 0.9680  | 0.8006 | 0.9696  | 0.9700  |
|      |     |        | Len | 60.3479 | 12.3289 | 5.8287 | 18.2031 | 8.4406  |
| 1000 | 20  | 0.2236 | Cov | 0.9570  | 0.9564  | 0.9564 | 0.9564  | 0.9564  |
|      |     |        | Len | 7.0065  | 6.9848  | 6.9848 | 6.9848  | 6.9848  |
| 1000 | 40  | 0.1581 | Cov | 0.9598  | 0.9616  | 0.9594 | 0.9594  | 0.9594  |
|      |     |        | Len | 6.9917  | 6.9700  | 6.9193 | 6.9193  | 6.9193  |
| 1000 | 100 | 0.1    | Cov | 0.9534  | 0.9534  | 0.9472 | 0.9392  | 0.9466  |
|      |     |        | Len | 6.9444  | 6.9216  | 6.6832 | 11.3450 | 6.7317  |
| 1000 | 200 | 0.07   | Cov | 0.9576  | 0.9552  | 0.9268 | 0.9444  | 0.9452  |
|      |     |        | Len | 6.9703  | 6.8642  | 6.2661 | 14.3565 | 6.5712  |

Table 5.120. Etype = 2, J=20, k=19,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|----------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9822   | 0.9604 | 0.9528 | 0.9690 | 0.9570 |
|      |     |        | Len | 220.4046 | 8.4646 | 7.7830 | 8.6460 | 8.3207 |
| 100  | 40  | 0.9    | Cov | 0.9828   | 0.9572 | 0.8946 | 0.9670 | 0.9472 |
|      |     |        | Len | 283.7220 | 8.3159 | 6.6182 | 8.5811 | 8.0704 |
| 100  | 100 | 0.9    | Cov | 0.9824   | 0.9560 | 0.6554 | 0.9672 | 0.9442 |
|      |     |        | Len | 489.9401 | 7.9707 | 4.0782 | 8.4906 | 7.6958 |
| 100  | 200 | 0.9    | Cov | 0.9762   | 0.9468 | 0.4096 | 0.9672 | 0.9310 |
|      |     |        | Len | 878.6499 | 7.7549 | 2.0579 | 8.4357 | 7.4702 |
| 400  | 20  | 0.9    | Cov | 0.9622   | 0.9498 | 0.9524 | 0.9524 | 0.9524 |
|      |     |        | Len | 9.7305   | 7.0275 | 6.8905 | 6.8905 | 6.8905 |
| 400  | 40  | 0.9    | Cov | 0.9610   | 0.9520 | 0.9454 | 0.9518 | 0.9474 |
|      |     |        | Len | 13.8407  | 7.2236 | 6.6331 | 6.8716 | 6.6873 |
| 400  | 100 | 0.9    | Cov | 0.9644   | 0.9534 | 0.9096 | 0.9578 | 0.9356 |
|      |     |        | Len | 21.5739  | 7.2194 | 6.0202 | 6.9109 | 6.4614 |
| 400  | 200 | 0.9    | Cov | 0.9610   | 0.9512 | 0.7324 | 0.9524 | 0.9246 |
|      |     |        | Len | 31.2258  | 7.1287 | 4.9339 | 6.9259 | 6.2788 |
| 1000 | 20  | 0.9    | Cov | 0.9562   | 0.9550 | 0.9528 | 0.9528 | 0.9528 |
|      |     |        | Len | 9.2746   | 6.7931 | 6.6608 | 6.6608 | 6.6608 |
| 1000 | 40  | 0.9    | Cov | 0.9596   | 0.9504 | 0.9488 | 0.9488 | 0.9488 |
|      |     |        | Len | 13.3123  | 7.0373 | 6.5459 | 6.5459 | 6.5459 |
| 1000 | 100 | 0.9    | Cov | 0.9522   | 0.9520 | 0.9410 | 0.9472 | 0.9424 |
|      |     |        | Len | 20.8156  | 7.0636 | 6.2972 | 6.5392 | 6.3357 |
| 1000 | 200 | 0.9    | Cov | 0.9536   | 0.9440 | 0.9070 | 0.9408 | 0.9234 |
|      |     |        | Len | 30.1717  | 7.0332 | 5.9177 | 6.5729 | 6.1663 |

Table 5.121. Etype = 2, J=20, k=p-1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 40  | 0      | Cov | 0.9736  | 0.9338  | 0.8938 | 0.9706  | 0.9446  |
|      |     |        | Len | 29.2430 | 23.5993 | 6.8840 | 28.5153 | 25.6398 |
| 400  | 40  | 0      | Cov | 0.9748  | 0.9486  | 0.9656 | 0.9574  | 0.9378  |
|      |     |        | Len | 26.3682 | 18.2862 | 8.1510 | 20.1316 | 19.0416 |
| 1000 | 40  | 0      | Cov | 0.9718  | 0.9712  | 0.9712 | 0.9712  | 0.9712  |
|      |     |        | Len | 7.8979  | 7.8967  | 7.8967 | 7.8967  | 7.8967  |
| 100  | 100 | 0      | Cov | 0.9614  | 0.8882  | 0.5056 | 0.9636  | 0.9076  |
|      |     |        | Len | 42.4335 | 34.4251 | 6.9115 | 43.0733 | 37.6979 |
| 400  | 100 | 0      | Cov | 0.9750  | 0.9468  | 0.9426 | 0.9682  | 0.9346  |
|      |     |        | Len | 43.7023 | 34.8833 | 7.2636 | 39.6100 | 35.3324 |
| 1000 | 100 | 0      | Cov | 0.9766  | 0.9490  | 0.9634 | 0.9604  | 0.9462  |
|      |     |        | Len | 40.7200 | 27.7899 | 8.1925 | 30.1580 | 28.4460 |
| 100  | 200 | 0      | Cov | 0.9548  | 0.8926  | 0.2348 | 0.9564  | 0.8936  |
|      |     |        | Len | 60.6161 | 50.4533 | 6.0313 | 60.6210 | 52.1210 |
| 400  | 200 | 0      | Cov | 0.9460  | 0.8800  | 0.7352 | 0.9408  | 0.8692  |
|      |     |        | Len | 53.9743 | 44.5909 | 4.8447 | 51.6779 | 45.0022 |
| 1000 | 200 | 0      | Cov | 0.9726  | 0.9418  | 0.9584 | 0.9638  | 0.9248  |
|      |     |        | Len | 60.5156 | 47.2984 | 7.5970 | 52.9147 | 47.5329 |

Table 5.122. Etype = 2, J=20, k=p-1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL       | PLS    | PCR    | FS       |
|------|-----|--------|-----|----------|----------|--------|--------|----------|
| 100  | 40  | 0.1581 | Cov | 0.9844   | 0.9346   | 0.9086 | 0.9680 | 0.9274   |
|      |     |        | Len | 161.9680 | 39.9110  | 6.9853 | 9.1384 | 55.6385  |
| 100  | 100 | 0.1    | Cov | 0.9760   | 0.9206   | 0.6712 | 0.9652 | 0.9114   |
|      |     |        | Len | 399.3103 | 98.5401  | 4.3246 | 9.3489 | 142.5211 |
| 100  | 200 | 0.07   | Cov | 0.9814   | 0.9210   | 0.4160 | 0.9724 | 0.8914   |
|      |     |        | Len | 912.3747 | 258.4274 | 2.2466 | 9.8268 | 281.9183 |
| 400  | 40  | 0.1581 | Cov | 0.9804   | 0.9666   | 0.9710 | 0.9736 | 0.9314   |
|      |     |        | Len | 130.3426 | 22.3853  | 8.1462 | 8.4443 | 21.6264  |
| 400  | 100 | 0.1    | Cov | 0.9802   | 0.9526   | 0.9406 | 0.9700 | 0.9068   |
|      |     |        | Len | 354.8938 | 69.0009  | 7.2577 | 8.4563 | 63.1102  |
| 400  | 200 | 0.07   | Cov | 0.9768   | 0.9460   | 0.8100 | 0.9744 | 0.8930   |
|      |     |        | Len | 709.5221 | 142.5953 | 5.8047 | 8.5713 | 129.7019 |
| 1000 | 40  | 0.1581 | Cov | 0.9674   | 0.9648   | 0.9648 | 0.9648 | 0.9648   |
|      |     |        | Len | 7.9914   | 7.9038   | 7.9038 | 7.9038 | 7.9038   |
| 1000 | 100 | 0.1    | Cov | 0.9802   | 0.9704   | 0.9716 | 0.9744 | 0.9338   |
|      |     |        | Len | 264.6062 | 37.9248  | 8.1688 | 8.4829 | 34.5788  |
| 1000 | 200 | 0.07   | Cov | 0.9764   | 0.9620   | 0.9524 | 0.9718 | 0.9086   |
|      |     |        | Len | 585.2002 | 92.6510  | 7.5944 | 8.5256 | 79.8276  |

Table 5.123. Etype = 2, J=20, k=p-1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso      | RL      | PLS    | PCR    | FS      |
|------|-----|--------|-----|------------|---------|--------|--------|---------|
| 100  | 40  | 0.9    | Cov | 0.9854     | 0.9634  | 0.9060 | 0.9720 | 0.9426  |
|      |     |        | Len | 686.0242   | 10.1880 | 6.9451 | 9.0482 | 9.9590  |
| 400  | 40  | 0.9    | Cov | 0.9754     | 0.9626  | 0.9586 | 0.9604 | 0.9584  |
|      |     |        | Len | 30.5694    | 9.3187  | 7.4639 | 7.7195 | 7.5502  |
| 1000 | 40  | 0.9    | Cov | 0.9670     | 0.9588  | 0.9590 | 0.9590 | 0.9590  |
|      |     |        | Len | 28.2714    | 8.7817  | 7.0316 | 7.0316 | 7.0316  |
| 100  | 100 | 0.9    | Cov | 0.9854     | 0.9590  | 0.6914 | 0.9724 | 0.9176  |
|      |     |        | Len | 3024.8650  | 17.4693 | 4.3063 | 9.1080 | 18.3011 |
| 400  | 100 | 0.9    | Cov | 0.9822     | 0.9666  | 0.9486 | 0.9736 | 0.9350  |
|      |     |        | Len | 137.1596   | 16.9780 | 7.2639 | 8.4323 | 9.7609  |
| 1000 | 100 | 0.9    | Cov | 0.9798     | 0.9726  | 0.9674 | 0.9720 | 0.9602  |
|      |     |        | Len | 131.5229   | 17.3572 | 8.1286 | 8.4236 | 8.4040  |
| 100  | 200 | 0.9    | Cov | 0.9828     | 0.9516  | 0.4238 | 0.9722 | 0.8882  |
|      |     |        | Len | 10605.4900 | 32.8438 | 2.1427 | 9.0460 | 33.8312 |
| 400  | 200 | 0.9    | Cov | 0.9824     | 0.9634  | 0.8120 | 0.9734 | 0.9060  |
|      |     |        | Len | 500.0967   | 29.5161 | 5.8085 | 8.4179 | 15.5416 |
| 1000 | 200 | 0.9    | Cov | 0.9804     | 0.9712  | 0.9550 | 0.9720 | 0.9294  |
|      |     |        | Len | 364.7455   | 30.6382 | 7.5801 | 8.4798 | 10.9341 |

Table 5.124. Etype = 2, J=50, k=1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 400  | 20  | 0      | Cov | 0.9432 | 0.9428 | 0.9390 | 0.9416 | 0.9412 |
|      |     |        | Len | 6.4005 | 6.4079 | 6.3168 | 7.0021 | 6.3514 |
| 400  | 40  | 0      | Cov | 0.9522 | 0.9498 | 0.9384 | 0.9500 | 0.9462 |
|      |     |        | Len | 6.3988 | 6.3723 | 6.1548 | 7.2048 | 6.2965 |
| 400  | 100 | 0      | Cov | 0.9450 | 0.9434 | 0.8870 | 0.9448 | 0.9356 |
|      |     |        | Len | 6.3962 | 6.3208 | 5.6368 | 7.3475 | 6.2155 |
| 400  | 200 | 0      | Cov | 0.9440 | 0.9366 | 0.7290 | 0.9462 | 0.9320 |
|      |     |        | Len | 6.3747 | 6.2500 | 4.6732 | 7.3815 | 6.1354 |
| 1000 | 20  | 0      | Cov | 0.9500 | 0.9498 | 0.9506 | 0.9506 | 0.9506 |
|      |     |        | Len | 6.2950 | 6.3263 | 6.2947 | 6.2947 | 6.2947 |
| 1000 | 40  | 0      | Cov | 0.9414 | 0.9432 | 0.9404 | 0.9456 | 0.9402 |
|      |     |        | Len | 6.2755 | 6.3012 | 6.2323 | 6.7429 | 6.2445 |
| 1000 | 100 | 0      | Cov | 0.9438 | 0.9396 | 0.9292 | 0.9430 | 0.9362 |
|      |     |        | Len | 6.2600 | 6.2507 | 6.0255 | 7.0419 | 6.1524 |
| 1000 | 200 | 0      | Cov | 0.9432 | 0.9416 | 0.9024 | 0.9410 | 0.9346 |
|      |     |        | Len | 6.2530 | 6.2086 | 5.6974 | 7.1547 | 6.0877 |

Table 5.125. Etype = 2, J=50, k=1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0.2236 | Cov | 0.9506 | 0.9490 | 0.9552 | 0.9544 | 0.9504 |
|      |     |        | Len | 7.5901 | 7.1186 | 7.8306 | 7.8977 | 7.2338 |
| 100  | 40  | 0.1581 | Cov | 0.9564 | 0.9508 | 0.9552 | 0.9568 | 0.9538 |
|      |     |        | Len | 7.6260 | 7.0776 | 7.9543 | 8.0231 | 7.2221 |
| 100  | 100 | 0.1    | Cov | 0.9548 | 0.9512 | 0.9572 | 0.9568 | 0.9544 |
|      |     |        | Len | 7.6813 | 7.0319 | 8.1033 | 8.1755 | 7.2312 |
| 100  | 200 | 0.07   | Cov | 0.9492 | 0.9430 | 0.3694 | 0.9532 | 0.9200 |
|      |     |        | Len | 7.2619 | 6.9162 | 1.9806 | 8.0762 | 6.4327 |
| 400  | 20  | 0.2236 | Cov | 0.9476 | 0.9486 | 0.9428 | 0.9496 | 0.9436 |
|      |     |        | Len | 6.3872 | 6.4314 | 6.3045 | 6.7658 | 6.3370 |
| 400  | 40  | 0.1581 | Cov | 0.9496 | 0.9488 | 0.9344 | 0.9458 | 0.9440 |
|      |     |        | Len | 6.3937 | 6.4296 | 6.1435 | 6.9736 | 6.2872 |
| 400  | 100 | 0.1    | Cov | 0.9434 | 0.9448 | 0.8824 | 0.9482 | 0.9332 |
|      |     |        | Len | 6.3754 | 6.3937 | 5.6181 | 7.1498 | 6.1947 |
| 400  | 200 | 0.07   | Cov | 0.9448 | 0.9460 | 0.7136 | 0.9410 | 0.9372 |
|      |     |        | Len | 6.3898 | 6.3890 | 4.7034 | 7.2537 | 6.1477 |
| 1000 | 20  | 0.2236 | Cov | 0.9420 | 0.9442 | 0.9420 | 0.9420 | 0.9420 |
|      |     |        | Len | 6.2987 | 6.3455 | 6.2984 | 6.2984 | 6.2984 |
| 1000 | 40  | 0.1581 | Cov | 0.9486 | 0.9486 | 0.9432 | 0.9446 | 0.9458 |
|      |     |        | Len | 6.2697 | 6.3377 | 6.2250 | 6.6079 | 6.2367 |
| 1000 | 100 | 0.1    | Cov | 0.9484 | 0.9510 | 0.9388 | 0.9514 | 0.9436 |
|      |     |        | Len | 6.2586 | 6.3343 | 6.0236 | 6.9030 | 6.1512 |
| 1000 | 200 | 0.07   | Cov | 0.9452 | 0.9474 | 0.9102 | 0.9478 | 0.9368 |
|      |     |        | Len | 6.2503 | 6.3277 | 5.6923 | 7.0434 | 6.0799 |

Table 5.126. Etype = 2, J=50, k=1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|---------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9622  | 0.9554 | 0.9568 | 0.9568 | 0.9552 |
|      |     |        | Len | 11.4641 | 7.1477 | 7.2202 | 7.2203 | 7.1482 |
| 100  | 40  | 0.9    | Cov | 0.9660  | 0.9544 | 0.9556 | 0.9556 | 0.9544 |
|      |     |        | Len | 16.2700 | 7.1169 | 7.2034 | 7.2034 | 7.1184 |
| 100  | 100 | 0.9    | Cov | 0.9628  | 0.9522 | 0.9538 | 0.9538 | 0.9520 |
|      |     |        | Len | 27.5277 | 7.0810 | 7.2039 | 7.2039 | 7.1037 |
| 100  | 200 | 0.9    | Cov | 0.9660  | 0.9494 | 0.3876 | 0.9548 | 0.9184 |
|      |     |        | Len | 34.7328 | 7.2738 | 1.8950 | 7.3897 | 6.4533 |
| 400  | 20  | 0.9    | Cov | 0.9508  | 0.9530 | 0.9472 | 0.9518 | 0.9470 |
|      |     |        | Len | 6.4537  | 6.4494 | 6.3208 | 6.4251 | 6.3489 |
| 400  | 40  | 0.9    | Cov | 0.9506  | 0.9514 | 0.9360 | 0.9508 | 0.9450 |
|      |     |        | Len | 6.5303  | 6.4281 | 6.1341 | 6.4118 | 6.2712 |
| 400  | 100 | 0.9    | Cov | 0.9512  | 0.9504 | 0.8856 | 0.9534 | 0.9406 |
|      |     |        | Len | 7.9637  | 6.4211 | 5.6341 | 6.4196 | 6.1965 |
| 400  | 200 | 0.9    | Cov | 0.9486  | 0.9456 | 0.7188 | 0.9472 | 0.9308 |
|      |     |        | Len | 11.4184 | 6.4088 | 4.6984 | 6.4234 | 6.1293 |
| 1000 | 20  | 0.9    | Cov | 0.9508  | 0.9516 | 0.9504 | 0.9504 | 0.9504 |
|      |     |        | Len | 6.3514  | 6.3486 | 6.2962 | 6.2962 | 6.2962 |
| 1000 | 40  | 0.9    | Cov | 0.9458  | 0.9458 | 0.9420 | 0.9446 | 0.9418 |
|      |     |        | Len | 6.3598  | 6.3521 | 6.2363 | 6.3084 | 6.2476 |
| 1000 | 100 | 0.9    | Cov | 0.9446  | 0.9460 | 0.9296 | 0.9434 | 0.9378 |
|      |     |        | Len | 6.3642  | 6.3448 | 6.0302 | 6.3045 | 6.1579 |
| 1000 | 200 | 0.9    | Cov | 0.9434  | 0.9414 | 0.9018 | 0.9428 | 0.9330 |
|      |     |        | Len | 6.3702  | 6.3418 | 5.6905 | 6.3108 | 6.0845 |

Table 5.127. Etype = 2, J=50, k=19,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 100  | 20  | 0      | Cov | 0.9584  | 0.9252  | 0.9322  | 0.9668  | 0.9516  |
|      |     |        | Len | 18.9583 | 15.2762 | 9.0545  | 19.8850 | 19.1533 |
| 100  | 40  | 0      | Cov | 0.9544  | 0.9194  | 0.9034  | 0.9574  | 0.9488  |
|      |     |        | Len | 18.9799 | 15.2839 | 9.8864  | 20.2173 | 19.1771 |
| 100  | 100 | 0      | Cov | 0.9564  | 0.9232  | 0.8420  | 0.9612  | 0.9522  |
|      |     |        | Len | 18.9161 | 15.2528 | 10.0530 | 20.4157 | 19.1606 |
| 400  | 20  | 0      | Cov | 0.9592  | 0.9466  | 0.9496  | 0.9526  | 0.9406  |
|      |     |        | Len | 17.7479 | 12.5782 | 6.8650  | 15.2967 | 14.7167 |
| 400  | 40  | 0      | Cov | 0.9588  | 0.9398  | 0.9462  | 0.9556  | 0.9396  |
|      |     |        | Len | 17.7717 | 12.6147 | 6.6424  | 17.2981 | 14.7264 |
| 400  | 100 | 0      | Cov | 0.9584  | 0.9406  | 0.9076  | 0.9554  | 0.9404  |
|      |     |        | Len | 17.7482 | 12.5902 | 6.0566  | 18.4991 | 14.7197 |
| 400  | 200 | 0      | Cov | 0.9586  | 0.9454  | 0.7464  | 0.9576  | 0.9442  |
|      |     |        | Len | 17.7658 | 12.6397 | 5.0228  | 18.9566 | 14.7154 |
| 1000 | 20  | 0      | Cov | 0.9570  | 0.9574  | 0.9574  | 0.9574  | 0.9574  |
|      |     |        | Len | 6.9931  | 6.9919  | 6.9919  | 6.9919  | 6.9919  |
| 1000 | 40  | 0      | Cov | 0.9570  | 0.9556  | 0.9536  | 0.9464  | 0.9560  |
|      |     |        | Len | 7.2849  | 6.9812  | 6.9091  | 13.8384 | 6.9916  |
| 1000 | 100 | 0      | Cov | 0.9598  | 0.9576  | 0.9474  | 0.9504  | 0.9594  |
|      |     |        | Len | 7.4455  | 6.9730  | 6.6607  | 17.0307 | 6.9878  |
| 1000 | 200 | 0      | Cov | 0.9604  | 0.9566  | 0.9224  | 0.9528  | 0.9564  |
|      |     |        | Len | 7.5678  | 6.9857  | 6.2667  | 18.0883 | 7.0081  |

Table 5.128. Etype = 2, J=50, k=19,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 100  | 20  | 0.2236 | Cov | 0.9606  | 0.9208  | 0.9516  | 0.9522  | 0.9418  |
|      |     |        | Len | 70.1772 | 17.0137 | 7.1955  | 7.2282  | 47.1172 |
| 100  | 40  | 0.1581 | Cov | 0.9654  | 0.9098  | 0.9582  | 0.9586  | 0.9420  |
|      |     |        | Len | 69.2113 | 18.0709 | 13.4783 | 13.7013 | 49.2083 |
| 100  | 100 | 0.1    | Cov | 0.9600  | 0.8736  | 0.9566  | 0.9580  | 0.9302  |
|      |     |        | Len | 67.6777 | 18.8009 | 17.0353 | 17.2557 | 50.3659 |
| 100  | 200 | 0.07   | Cov | 0.9646  | 0.8966  | 0.9684  | 0.9698  | 0.9266  |
|      |     |        | Len | 78.1272 | 25.6772 | 18.3781 | 18.5874 | 50.4854 |
| 400  | 20  | 0.2236 | Cov | 0.9590  | 0.9528  | 0.9522  | 0.9564  | 0.9372  |
|      |     |        | Len | 66.0801 | 11.8086 | 6.8437  | 6.9664  | 17.2978 |
| 400  | 40  | 0.1581 | Cov | 0.9596  | 0.9502  | 0.9478  | 0.9562  | 0.9354  |
|      |     |        | Len | 65.0529 | 12.5213 | 6.6600  | 11.8958 | 18.5329 |
| 400  | 100 | 0.1    | Cov | 0.9634  | 0.9494  | 0.9084  | 0.9608  | 0.9378  |
|      |     |        | Len | 63.7522 | 13.4485 | 6.0602  | 15.6146 | 19.6616 |
| 400  | 200 | 0.07   | Cov | 0.9584  | 0.9360  | 0.7566  | 0.9552  | 0.9274  |
|      |     |        | Len | 62.3217 | 13.9437 | 5.0488  | 17.1377 | 20.2760 |
| 1000 | 20  | 0.2236 | Cov | 0.9584  | 0.9580  | 0.9580  | 0.9580  | 0.9580  |
|      |     |        | Len | 7.0086  | 6.9872  | 6.9872  | 6.9872  | 6.9872  |
| 1000 | 40  | 0.1581 | Cov | 0.9618  | 0.9544  | 0.9518  | 0.9532  | 0.9546  |
|      |     |        | Len | 23.6899 | 6.9935  | 6.9131  | 10.0250 | 6.9940  |
| 1000 | 100 | 0.1    | Cov | 0.9616  | 0.9572  | 0.9466  | 0.9526  | 0.9570  |
|      |     |        | Len | 36.7551 | 7.0878  | 6.6609  | 14.4274 | 6.9887  |
| 1000 | 200 | 0.07   | Cov | 0.9676  | 0.9588  | 0.9252  | 0.9548  | 0.9588  |
|      |     |        | Len | 43.8059 | 7.3659  | 6.2614  | 16.3528 | 6.9917  |

Table 5.129. Etype = 2, J=50, k=19,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|----------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9682   | 0.9358 | 0.9468 | 0.9468 | 0.9396 |
|      |     |        | Len | 230.1111 | 7.2560 | 7.1776 | 7.1776 | 9.5797 |
| 100  | 40  | 0.9    | Cov | 0.9644   | 0.9426 | 0.9572 | 0.9572 | 0.9376 |
|      |     |        | Len | 330.1891 | 7.1906 | 7.3318 | 7.3319 | 9.5258 |
| 100  | 100 | 0.9    | Cov | 0.9674   | 0.9220 | 0.9484 | 0.9484 | 0.9330 |
|      |     |        | Len | 523.0168 | 6.9405 | 7.3504 | 7.3505 | 9.3648 |
| 100  | 200 | 0.9    | Cov | 0.9644   | 0.9320 | 0.9566 | 0.9566 | 0.9278 |
|      |     |        | Len | 892.4198 | 6.8276 | 7.3623 | 7.3623 | 9.2164 |
| 400  | 20  | 0.9    | Cov | 0.9608   | 0.9496 | 0.9492 | 0.9536 | 0.9456 |
|      |     |        | Len | 86.1834  | 7.0244 | 6.8299 | 6.9510 | 6.9466 |
| 400  | 40  | 0.9    | Cov | 0.9634   | 0.9520 | 0.9442 | 0.9552 | 0.9490 |
|      |     |        | Len | 57.1676  | 7.2066 | 6.6201 | 7.0176 | 6.9122 |
| 400  | 100 | 0.9    | Cov | 0.9644   | 0.9502 | 0.9032 | 0.9532 | 0.9450 |
|      |     |        | Len | 124.6415 | 7.1835 | 6.0164 | 7.0517 | 6.8286 |
| 400  | 200 | 0.9    | Cov | 0.9590   | 0.9484 | 0.7518 | 0.9558 | 0.9410 |
|      |     |        | Len | 238.2169 | 7.0971 | 4.9563 | 7.0688 | 6.7754 |
| 1000 | 20  | 0.9    | Cov | 0.9502   | 0.9490 | 0.9520 | 0.9520 | 0.9520 |
|      |     |        | Len | 9.2947   | 6.7964 | 6.6627 | 6.6627 | 6.6627 |
| 1000 | 40  | 0.9    | Cov | 0.9556   | 0.9506 | 0.9480 | 0.9502 | 0.9474 |
|      |     |        | Len | 13.3055  | 7.0442 | 6.5553 | 6.6705 | 6.5867 |
| 1000 | 100 | 0.9    | Cov | 0.9528   | 0.9446 | 0.9296 | 0.9496 | 0.9414 |
|      |     |        | Len | 20.7900  | 7.0753 | 6.2970 | 6.7065 | 6.4979 |
| 1000 | 200 | 0.9    | Cov | 0.9548   | 0.9458 | 0.9122 | 0.9492 | 0.9390 |
|      |     |        | Len | 30.1728  | 7.0457 | 5.9273 | 6.7325 | 6.4411 |

Table 5.130. Etype = 2, J=50, k=p-1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 100  | 40  | 0      | Cov | 0.9552  | 0.9132  | 0.8972  | 0.9626  | 0.9538  |
|      |     |        | Len | 26.5480 | 21.8828 | 12.6635 | 27.9967 | 26.8942 |
| 400  | 40  | 0      | Cov | 0.9586  | 0.9312  | 0.9474  | 0.9548  | 0.9348  |
|      |     |        | Len | 25.4171 | 20.3135 | 6.6681  | 23.7507 | 22.5362 |
| 1000 | 40  | 0      | Cov | 0.9590  | 0.9416  | 0.9532  | 0.9446  | 0.9360  |
|      |     |        | Len | 24.3712 | 16.2510 | 6.9096  | 18.5311 | 18.0250 |
| 100  | 100 | 0      | Cov | 0.9522  | 0.8950  | 0.8156  | 0.9642  | 0.9520  |
|      |     |        | Len | 41.5565 | 34.3004 | 20.2161 | 44.1726 | 42.3064 |
| 400  | 100 | 0      | Cov | 0.9556  | 0.9174  | 0.9044  | 0.9536  | 0.9290  |
|      |     |        | Len | 40.4580 | 34.5753 | 6.0455  | 39.7366 | 37.2616 |
| 1000 | 100 | 0      | Cov | 0.9620  | 0.9388  | 0.9484  | 0.9530  | 0.9396  |
|      |     |        | Len | 39.8304 | 32.4597 | 6.6682  | 36.2189 | 34.1379 |
| 100  | 200 | 0      | Cov | 0.9596  | 0.9138  | 0.7456  | 0.9654  | 0.9506  |
|      |     |        | Len | 61.3696 | 53.6703 | 26.7716 | 62.5747 | 59.7265 |
| 400  | 200 | 0      | Cov | 0.9476  | 0.9070  | 0.7164  | 0.9488  | 0.9246  |
|      |     |        | Len | 55.3694 | 48.1885 | 5.1228  | 55.5002 | 51.7576 |
| 1000 | 200 | 0      | Cov | 0.9572  | 0.9238  | 0.9178  | 0.9526  | 0.9264  |
|      |     |        | Len | 57.0099 | 49.1057 | 6.2388  | 54.4820 | 50.9032 |

Table 5.131. Etype = 2, J=50, k=p-1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL       | PLS    | PCR    | FS       |
|------|-----|--------|-----|----------|----------|--------|--------|----------|
| 100  | 40  | 0.1581 | Cov | 0.9586   | 0.8942   | 0.9470 | 0.9480 | 0.9312   |
|      |     |        | Len | 140.5067 | 34.6822  | 7.2568 | 7.3007 | 99.2809  |
| 100  | 100 | 0.1    | Cov | 0.9584   | 0.8684   | 0.9536 | 0.9544 | 0.9286   |
|      |     |        | Len | 345.5624 | 84.7901  | 7.4842 | 7.5420 | 254.2178 |
| 100  | 200 | 0.07   | Cov | 0.9654   | 0.8774   | 0.9520 | 0.9536 | 0.9262   |
|      |     |        | Len | 801.0432 | 236.1797 | 7.9756 | 8.0465 | 506.5613 |
| 400  | 40  | 0.1581 | Cov | 0.9608   | 0.9406   | 0.9406 | 0.9572 | 0.9194   |
|      |     |        | Len | 132.0458 | 25.1414  | 6.6418 | 6.9883 | 39.1995  |
| 400  | 100 | 0.1    | Cov | 0.9628   | 0.9288   | 0.9122 | 0.9608 | 0.9014   |
|      |     |        | Len | 325.0597 | 65.4746  | 6.0540 | 7.0481 | 105.7312 |
| 400  | 200 | 0.07   | Cov | 0.9576   | 0.9054   | 0.7590 | 0.9584 | 0.8896   |
|      |     |        | Len | 637.1236 | 130.5701 | 4.9819 | 7.1373 | 215.6005 |
| 1000 | 40  | 0.1581 | Cov | 0.9608   | 0.9492   | 0.9492 | 0.9528 | 0.9286   |
|      |     |        | Len | 128.9721 | 18.9184  | 6.8960 | 6.9834 | 20.7333  |
| 1000 | 100 | 0.1    | Cov | 0.9612   | 0.9458   | 0.9486 | 0.9600 | 0.9158   |
|      |     |        | Len | 320.4346 | 53.2134  | 6.6627 | 7.0101 | 63.2724  |
| 1000 | 200 | 0.07   | Cov | 0.9660   | 0.9322   | 0.9186 | 0.9530 | 0.8978   |
|      |     |        | Len | 625.8531 | 107.5340 | 6.2703 | 7.0568 | 133.2922 |

Table 5.132. Etype = 2, J=50, k=p-1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso     | RL      | PLS    | PCR    | FS      |
|------|-----|--------|-----|-----------|---------|--------|--------|---------|
| 100  | 40  | 0.9    | Cov | 0.9690    | 0.9384  | 0.9552 | 0.9552 | 0.9338  |
|      |     |        | Len | 675.8627  | 8.8043  | 7.1853 | 7.1853 | 15.8398 |
| 400  | 40  | 0.9    | Cov | 0.9620    | 0.9480  | 0.9434 | 0.9546 | 0.9374  |
|      |     |        | Len | 265.5928  | 8.4272  | 6.6474 | 6.9738 | 7.8268  |
| 1000 | 40  | 0.9    | Cov | 0.9616    | 0.9522  | 0.9544 | 0.9568 | 0.9524  |
|      |     |        | Len | 27.8963   | 8.6637  | 6.9081 | 6.9882 | 7.0419  |
| 100  | 100 | 0.9    | Cov | 0.9634    | 0.9206  | 0.9516 | 0.9516 | 0.9222  |
|      |     |        | Len | 2728.1010 | 15.2971 | 7.1634 | 7.1634 | 36.5661 |
| 400  | 100 | 0.9    | Cov | 0.9642    | 0.9414  | 0.9074 | 0.9546 | 0.9140  |
|      |     |        | Len | 1817.5780 | 14.7407 | 6.0672 | 6.9756 | 13.6985 |
| 1000 | 100 | 0.9    | Cov | 0.9624    | 0.9550  | 0.9490 | 0.9570 | 0.9344  |
|      |     |        | Len | 119.0170  | 15.3255 | 6.6580 | 6.9867 | 9.3890  |
| 100  | 200 | 0.9    | Cov | 0.9654    | 0.9220  | 0.9486 | 0.9486 | 0.9244  |
|      |     |        | Len | 9352.3730 | 28.9194 | 7.2055 | 7.2055 | 71.5173 |
| 400  | 200 | 0.9    | Cov | 0.9612    | 0.9402  | 0.7578 | 0.9520 | 0.8974  |
|      |     |        | Len | 6129.4250 | 26.3750 | 5.0168 | 6.9826 | 25.1643 |
| 1000 | 200 | 0.9    | Cov | 0.9638    | 0.9534  | 0.9282 | 0.9590 | 0.9122  |
|      |     |        | Len | 417.8301  | 26.8580 | 6.2561 | 6.9922 | 15.8057 |

Table 5.133. Etype = 3, J=5, k=1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0      | Cov | 0.9226 | 0.9446 | 0.9218 | 0.9218 | 0.9218 |
|      |     |        | Len | 3.8302 | 3.9042 | 3.8351 | 3.8351 | 3.8351 |
| 100  | 40  | 0      | Cov | 0.9230 | 0.9292 | 0.8278 | 0.9152 | 0.8502 |
|      |     |        | Len | 3.5680 | 3.8366 | 3.4180 | 4.5491 | 3.5122 |
| 100  | 100 | 0      | Cov | 0.9170 | 0.9026 | 0.1430 | 0.9216 | 0.7322 |
|      |     |        | Len | 3.4555 | 3.6636 | 0.9477 | 5.0643 | 2.9244 |
| 100  | 200 | 0      | Cov | 0.9256 | 0.8870 | 0.0012 | 0.9386 | 0.6274 |
|      |     |        | Len | 3.4581 | 3.5273 | 0.0027 | 5.2838 | 2.4863 |
| 400  | 20  | 0      | Cov | 0.9468 | 0.9488 | 0.9466 | 0.9466 | 0.9466 |
|      |     |        | Len | 3.4027 | 3.3568 | 3.4103 | 3.4103 | 3.4103 |
| 400  | 40  | 0      | Cov | 0.9396 | 0.9490 | 0.9384 | 0.9384 | 0.9384 |
|      |     |        | Len | 3.4401 | 3.4134 | 3.4467 | 3.4467 | 3.4467 |
| 400  | 100 | 0      | Cov | 0.9034 | 0.9260 | 0.8814 | 0.8960 | 0.8834 |
|      |     |        | Len | 3.2795 | 3.4435 | 3.3264 | 3.6255 | 3.3291 |
| 400  | 200 | 0      | Cov | 0.8808 | 0.8974 | 0.7020 | 0.8948 | 0.7700 |
|      |     |        | Len | 3.0886 | 3.3829 | 2.8176 | 4.1603 | 2.9986 |
| 1000 | 20  | 0      | Cov | 0.9480 | 0.9498 | 0.9468 | 0.9468 | 0.9468 |
|      |     |        | Len | 3.2472 | 3.2037 | 3.2557 | 3.2557 | 3.2557 |
| 1000 | 40  | 0      | Cov | 0.9436 | 0.9472 | 0.9434 | 0.9434 | 0.9434 |
|      |     |        | Len | 3.3125 | 3.2613 | 3.3207 | 3.3207 | 3.3207 |
| 1000 | 100 | 0      | Cov | 0.9266 | 0.9370 | 0.9266 | 0.9266 | 0.9266 |
|      |     |        | Len | 3.3717 | 3.3409 | 3.3784 | 3.3784 | 3.3784 |
| 1000 | 200 | 0      | Cov | 0.9000 | 0.9308 | 0.8978 | 0.8978 | 0.8978 |
|      |     |        | Len | 3.3225 | 3.3778 | 3.3277 | 3.3277 | 3.3277 |

Table 5.134. Etype = 3, J=5, k=1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0.2236 | Cov | 0.9232 | 0.9572 | 0.9212 | 0.9212 | 0.9212 |
|      |     |        | Len | 3.8088 | 3.8805 | 3.8162 | 3.8162 | 3.8162 |
| 100  | 40  | 0.1581 | Cov | 0.9234 | 0.9548 | 0.8244 | 0.9238 | 0.8546 |
|      |     |        | Len | 3.5689 | 3.8850 | 3.4172 | 4.3544 | 3.5170 |
| 100  | 100 | 0.1    | Cov | 0.9210 | 0.9544 | 0.1426 | 0.9324 | 0.7318 |
|      |     |        | Len | 3.4673 | 3.8915 | 0.9802 | 4.8602 | 2.9334 |
| 100  | 200 | 0.07   | Cov | 0.9214 | 0.9480 | 0.0002 | 0.9322 | 0.6422 |
|      |     |        | Len | 3.4583 | 3.8885 | 0.0040 | 5.1050 | 2.4861 |
| 400  | 20  | 0.2236 | Cov | 0.9458 | 0.9530 | 0.9462 | 0.9462 | 0.9462 |
|      |     |        | Len | 3.3948 | 3.3023 | 3.4060 | 3.4060 | 3.4060 |
| 400  | 40  | 0.1581 | Cov | 0.9386 | 0.9494 | 0.9374 | 0.9374 | 0.9374 |
|      |     |        | Len | 3.4378 | 3.3543 | 3.4468 | 3.4468 | 3.4468 |
| 400  | 100 | 0.1    | Cov | 0.9016 | 0.9484 | 0.8808 | 0.8952 | 0.8796 |
|      |     |        | Len | 3.2851 | 3.4158 | 3.3340 | 3.5872 | 3.3354 |
| 400  | 200 | 0.07   | Cov | 0.8890 | 0.9452 | 0.6918 | 0.9014 | 0.7668 |
|      |     |        | Len | 3.0888 | 3.3986 | 2.8112 | 4.0741 | 2.9935 |
| 1000 | 20  | 0.2236 | Cov | 0.9452 | 0.9486 | 0.9454 | 0.9454 | 0.9454 |
|      |     |        | Len | 3.2445 | 3.1660 | 3.2570 | 3.2570 | 3.2570 |
| 1000 | 40  | 0.1581 | Cov | 0.9434 | 0.9504 | 0.9440 | 0.9440 | 0.9440 |
|      |     |        | Len | 3.3101 | 3.2029 | 3.3215 | 3.3215 | 3.3215 |
| 1000 | 100 | 0.1    | Cov | 0.9268 | 0.9432 | 0.9268 | 0.9268 | 0.9268 |
|      |     |        | Len | 3.3667 | 3.2671 | 3.3760 | 3.3760 | 3.3760 |
| 1000 | 200 | 0.07   | Cov | 0.8986 | 0.9416 | 0.8980 | 0.8980 | 0.8980 |
|      |     |        | Len | 3.3201 | 3.3404 | 3.3267 | 3.3267 | 3.3267 |

Table 5.135. Etype = 3, J=5, k=1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9586 | 0.9598 | 0.9240 | 0.9240 | 0.9240 |
|      |     |        | Len | 3.7951 | 3.8078 | 3.8169 | 3.8169 | 3.8169 |
| 100  | 40  | 0.9    | Cov | 0.9656 | 0.9650 | 0.8356 | 0.9440 | 0.8640 |
|      |     |        | Len | 3.7948 | 3.8249 | 3.4331 | 3.8363 | 3.5235 |
| 100  | 100 | 0.9    | Cov | 0.9670 | 0.9616 | 0.1402 | 0.9532 | 0.7248 |
|      |     |        | Len | 3.8069 | 3.8881 | 0.9642 | 3.8560 | 2.9246 |
| 100  | 200 | 0.9    | Cov | 0.9656 | 0.9562 | 0.0018 | 0.9540 | 0.6224 |
|      |     |        | Len | 3.9116 | 3.9213 | 0.0035 | 3.8732 | 2.4780 |
| 400  | 20  | 0.9    | Cov | 0.9516 | 0.9520 | 0.9420 | 0.9420 | 0.9420 |
|      |     |        | Len | 3.2372 | 3.2639 | 3.4003 | 3.4003 | 3.4003 |
| 400  | 40  | 0.9    | Cov | 0.9496 | 0.9500 | 0.9310 | 0.9310 | 0.9310 |
|      |     |        | Len | 3.2297 | 3.2803 | 3.4392 | 3.4392 | 3.4392 |
| 400  | 100 | 0.9    | Cov | 0.9506 | 0.9472 | 0.8756 | 0.9030 | 0.8752 |
|      |     |        | Len | 3.2509 | 3.3072 | 3.3256 | 3.3915 | 3.3280 |
| 400  | 200 | 0.9    | Cov | 0.9516 | 0.9490 | 0.6998 | 0.9248 | 0.7796 |
|      |     |        | Len | 3.2846 | 3.3155 | 2.8056 | 3.3936 | 2.9895 |
| 1000 | 20  | 0.9    | Cov | 0.9530 | 0.9528 | 0.9506 | 0.9506 | 0.9506 |
|      |     |        | Len | 3.1177 | 3.1427 | 3.2548 | 3.2548 | 3.2548 |
| 1000 | 40  | 0.9    | Cov | 0.9528 | 0.9514 | 0.9458 | 0.9458 | 0.9458 |
|      |     |        | Len | 3.1234 | 3.1567 | 3.3205 | 3.3205 | 3.3205 |
| 1000 | 100 | 0.9    | Cov | 0.9504 | 0.9488 | 0.9260 | 0.9260 | 0.9260 |
|      |     |        | Len | 3.1565 | 3.1689 | 3.3776 | 3.3776 | 3.3776 |
| 1000 | 200 | 0.9    | Cov | 0.9464 | 0.9474 | 0.8872 | 0.8872 | 0.8872 |
|      |     |        | Len | 3.2032 | 3.1796 | 3.3254 | 3.3254 | 3.3254 |

Table 5.136. Etype = 3, J=5, k=19,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS     |
|------|-----|--------|-----|---------|---------|--------|---------|--------|
| 100  | 20  | 0      | Cov | 0.9776  | 0.9778  | 0.9778 | 0.9778  | 0.9778 |
|      |     |        | Len | 5.6340  | 5.6294  | 5.6294 | 5.6294  | 5.6294 |
| 100  | 40  | 0      | Cov | 0.9818  | 0.9754  | 0.9392 | 0.9670  | 0.9782 |
|      |     |        | Len | 12.7926 | 6.2596  | 5.0102 | 15.0668 | 5.9242 |
| 100  | 100 | 0      | Cov | 0.9830  | 0.9668  | 0.2186 | 0.9828  | 0.9692 |
|      |     |        | Len | 15.9409 | 8.5272  | 1.4652 | 20.0250 | 6.6896 |
| 100  | 200 | 0      | Cov | 0.9838  | 0.9498  | 0.0008 | 0.9828  | 0.9234 |
|      |     |        | Len | 17.2683 | 10.3000 | 0.0080 | 21.3071 | 7.4545 |
| 400  | 20  | 0      | Cov | 0.9768  | 0.9756  | 0.9756 | 0.9756  | 0.9756 |
|      |     |        | Len | 4.3357  | 4.3314  | 4.3314 | 4.3314  | 4.3314 |
| 400  | 40  | 0      | Cov | 0.9660  | 0.9694  | 0.9658 | 0.9658  | 0.9658 |
|      |     |        | Len | 4.3419  | 4.3535  | 4.3547 | 4.3547  | 4.3547 |
| 400  | 100 | 0      | Cov | 0.9460  | 0.9558  | 0.9326 | 0.9012  | 0.9324 |
|      |     |        | Len | 4.1461  | 4.3258  | 4.1558 | 7.3943  | 4.1589 |
| 400  | 200 | 0      | Cov | 0.9488  | 0.9450  | 0.7938 | 0.9176  | 0.8716 |
|      |     |        | Len | 4.1011  | 4.2142  | 3.4609 | 12.4733 | 3.7503 |
| 1000 | 20  | 0      | Cov | 0.9604  | 0.9598  | 0.9598 | 0.9598  | 0.9598 |
|      |     |        | Len | 3.5669  | 3.5601  | 3.5601 | 3.5601  | 3.5601 |
| 1000 | 40  | 0      | Cov | 0.9554  | 0.9568  | 0.9546 | 0.9546  | 0.9546 |
|      |     |        | Len | 3.6120  | 3.6013  | 3.6282 | 3.6282  | 3.6282 |
| 1000 | 100 | 0      | Cov | 0.9426  | 0.9526  | 0.9428 | 0.9428  | 0.9428 |
|      |     |        | Len | 3.6562  | 3.6564  | 3.6790 | 3.6790  | 3.6790 |
| 1000 | 200 | 0      | Cov | 0.9228  | 0.9440  | 0.9200 | 0.9200  | 0.9200 |
|      |     |        | Len | 3.5991  | 3.6808  | 3.6173 | 3.6173  | 3.6173 |

Table 5.137. Etype = 3, J=5, k=19,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 20  | 0.2236 | Cov | 0.9814  | 0.9834  | 0.9834 | 0.9834  | 0.9834  |
|      |     |        | Len | 5.7611  | 5.6451  | 5.6451 | 5.6451  | 5.6451  |
| 100  | 40  | 0.1581 | Cov | 0.9920  | 0.9688  | 0.9338 | 0.9710  | 0.9674  |
|      |     |        | Len | 46.2975 | 9.3708  | 4.9680 | 10.3597 | 6.6955  |
| 100  | 100 | 0.1    | Cov | 0.9922  | 0.9486  | 0.2248 | 0.9846  | 0.9378  |
|      |     |        | Len | 58.2488 | 14.1642 | 1.4816 | 16.7227 | 8.8008  |
| 100  | 200 | 0.07   | Cov | 0.9920  | 0.9354  | 0.0024 | 0.9816  | 0.8748  |
|      |     |        | Len | 64.4681 | 16.7586 | 0.0114 | 19.6707 | 10.2374 |
| 400  | 20  | 0.2236 | Cov | 0.9740  | 0.9756  | 0.9756 | 0.9756  | 0.9756  |
|      |     |        | Len | 4.4502  | 4.3278  | 4.3278 | 4.3278  | 4.3278  |
| 400  | 40  | 0.1581 | Cov | 0.9754  | 0.9740  | 0.9708 | 0.9708  | 0.9708  |
|      |     |        | Len | 4.4544  | 4.3575  | 4.3595 | 4.3595  | 4.3595  |
| 400  | 100 | 0.1    | Cov | 0.9712  | 0.9692  | 0.9382 | 0.9148  | 0.9404  |
|      |     |        | Len | 4.4315  | 4.3639  | 4.1684 | 6.5713  | 4.1728  |
| 400  | 200 | 0.07   | Cov | 0.9664  | 0.9638  | 0.7890 | 0.9200  | 0.8678  |
|      |     |        | Len | 4.3929  | 4.3455  | 3.4689 | 11.2693 | 3.7608  |
| 1000 | 20  | 0.2236 | Cov | 0.9604  | 0.9596  | 0.9596 | 0.9596  | 0.9596  |
|      |     |        | Len | 3.7059  | 3.5606  | 3.5606 | 3.5606  | 3.5606  |
| 1000 | 40  | 0.1581 | Cov | 0.9578  | 0.9566  | 0.9562 | 0.9562  | 0.9562  |
|      |     |        | Len | 3.7129  | 3.5963  | 3.6306 | 3.6306  | 3.6306  |
| 1000 | 100 | 0.1    | Cov | 0.9540  | 0.9544  | 0.9406 | 0.9406  | 0.9406  |
|      |     |        | Len | 3.7204  | 3.6339  | 3.6874 | 3.6874  | 3.6874  |
| 1000 | 200 | 0.07   | Cov | 0.9564  | 0.9570  | 0.9198 | 0.9198  | 0.9198  |
|      |     |        | Len | 3.7178  | 3.6523  | 3.6185 | 3.6185  | 3.6185  |

Table 5.138. Etype = 3, J=5, k=19,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|---------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9860  | 0.9646 | 0.9546 | 0.9546 | 0.9546 |
|      |     |        | Len | 12.3532 | 5.2507 | 4.7728 | 4.7728 | 4.7728 |
| 100  | 40  | 0.9    | Cov | 0.9858  | 0.9728 | 0.8922 | 0.9686 | 0.9116 |
|      |     |        | Len | 17.4712 | 5.6409 | 4.1691 | 4.8085 | 4.3378 |
| 100  | 100 | 0.9    | Cov | 0.9832  | 0.9660 | 0.1634 | 0.9698 | 0.8064 |
|      |     |        | Len | 27.5324 | 5.6146 | 1.1353 | 4.9499 | 3.6492 |
| 100  | 200 | 0.9    | Cov | 0.9872  | 0.9570 | 0.0016 | 0.9670 | 0.7012 |
|      |     |        | Len | 40.8098 | 5.3460 | 0.0042 | 5.0047 | 3.1168 |
| 400  | 20  | 0.9    | Cov | 0.9720  | 0.9610 | 0.9594 | 0.9594 | 0.9594 |
|      |     |        | Len | 10.3137 | 4.5248 | 3.9536 | 3.9536 | 3.9536 |
| 400  | 40  | 0.9    | Cov | 0.9706  | 0.9598 | 0.9512 | 0.9512 | 0.9512 |
|      |     |        | Len | 14.3319 | 5.0091 | 3.8847 | 3.8847 | 3.8847 |
| 400  | 100 | 0.9    | Cov | 0.9658  | 0.9598 | 0.9028 | 0.9286 | 0.9074 |
|      |     |        | Len | 22.9714 | 5.0501 | 3.6872 | 3.8064 | 3.6912 |
| 400  | 200 | 0.9    | Cov | 0.9678  | 0.9554 | 0.7444 | 0.9432 | 0.8224 |
|      |     |        | Len | 31.8920 | 4.9775 | 3.0758 | 3.9101 | 3.3132 |
| 1000 | 20  | 0.9    | Cov | 0.9596  | 0.9606 | 0.9594 | 0.9594 | 0.9594 |
|      |     |        | Len | 9.7000  | 4.2030 | 3.5509 | 3.5509 | 3.5509 |
| 1000 | 40  | 0.9    | Cov | 0.9622  | 0.9574 | 0.9538 | 0.9538 | 0.9538 |
|      |     |        | Len | 13.5762 | 4.7975 | 3.5629 | 3.5629 | 3.5629 |
| 1000 | 100 | 0.9    | Cov | 0.9574  | 0.9542 | 0.9432 | 0.9432 | 0.9432 |
|      |     |        | Len | 21.9140 | 4.9090 | 3.6019 | 3.6019 | 3.6019 |
| 1000 | 200 | 0.9    | Cov | 0.9580  | 0.9460 | 0.9136 | 0.9136 | 0.9136 |
|      |     |        | Len | 30.2704 | 4.9036 | 3.5268 | 3.5268 | 3.5268 |

Table 5.139. Etype = 3, J=5, k=p-1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 40  | 0      | Cov | 0.9812  | 0.9398  | 0.9356 | 0.9674  | 0.9056  |
|      |     |        | Len | 27.0555 | 18.0519 | 4.9611 | 20.5946 | 17.2978 |
| 400  | 40  | 0      | Cov | 0.9762  | 0.9760  | 0.9760 | 0.9760  | 0.9760  |
|      |     |        | Len | 4.6790  | 4.6697  | 4.6697 | 4.6697  | 4.6697  |
| 1000 | 40  | 0      | Cov | 0.9704  | 0.9706  | 0.9706 | 0.9706  | 0.9706  |
|      |     |        | Len | 4.0545  | 4.0441  | 4.0441 | 4.0441  | 4.0441  |
| 100  | 100 | 0      | Cov | 0.9244  | 0.7888  | 0.1640 | 0.9140  | 0.7070  |
|      |     |        | Len | 34.9969 | 24.5169 | 1.3056 | 32.6585 | 22.2452 |
| 400  | 100 | 0      | Cov | 0.9798  | 0.9448  | 0.9664 | 0.9356  | 0.9116  |
|      |     |        | Len | 33.3676 | 18.3124 | 5.1083 | 17.3951 | 17.7940 |
| 1000 | 100 | 0      | Cov | 0.9800  | 0.9800  | 0.9800 | 0.9800  | 0.9800  |
|      |     |        | Len | 4.6811  | 4.6590  | 4.6590 | 4.6590  | 4.6590  |
| 100  | 200 | 0      | Cov | 0.9160  | 0.7600  | 0.0010 | 0.9172  | 0.5984  |
|      |     |        | Len | 49.8590 | 34.8547 | 0.0171 | 48.9535 | 28.9974 |
| 400  | 200 | 0      | Cov | 0.9760  | 0.9438  | 0.8682 | 0.9600  | 0.9062  |
|      |     |        | Len | 34.7478 | 20.2067 | 4.2485 | 33.1043 | 18.1816 |
| 1000 | 200 | 0      | Cov | 0.9784  | 0.9778  | 0.9778 | 0.9778  | 0.9778  |
|      |     |        | Len | 5.2496  | 5.2097  | 5.2097 | 5.2097  | 5.2097  |

Table 5.140. Etype = 3, J=5, k=p-1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL       | PLS    | PCR     | FS      |
|------|-----|--------|-----|----------|----------|--------|---------|---------|
| 100  | 40  | 0.1581 | Cov | 0.9950   | 0.9454   | 0.9410 | 0.9840  | 0.8876  |
|      |     |        | Len | 111.9310 | 23.6763  | 4.9821 | 5.7325  | 19.4589 |
| 100  | 100 | 0.1    | Cov | 0.9920   | 0.9200   | 0.2034 | 0.9852  | 0.7988  |
|      |     |        | Len | 332.3305 | 71.5824  | 1.3766 | 6.2761  | 50.2735 |
| 100  | 200 | 0.07   | Cov | 0.9912   | 0.9016   | 0.0016 | 0.9848  | 0.7260  |
|      |     |        | Len | 712.0865 | 150.7050 | 0.0055 | 7.1587  | 92.6378 |
| 400  | 40  | 0.1581 | Cov | 0.9782   | 0.9754   | 0.9754 | 0.9754  | 0.9754  |
|      |     |        | Len | 7.0948   | 4.6777   | 4.6777 | 4.6777  | 4.6777  |
| 400  | 100 | 0.1    | Cov | 0.9904   | 0.9642   | 0.9736 | 0.9812  | 0.9272  |
|      |     |        | Len | 119.5054 | 20.7053  | 5.1052 | 5.2454  | 17.8558 |
| 400  | 200 | 0.07   | Cov | 0.9902   | 0.9490   | 0.8674 | 0.9696  | 0.8964  |
|      |     |        | Len | 161.7559 | 32.8690  | 4.2537 | 22.2694 | 20.8019 |
| 1000 | 40  | 0.1581 | Cov | 0.9744   | 0.9684   | 0.9684 | 0.9684  | 0.9684  |
|      |     |        | Len | 6.2624   | 4.0353   | 4.0353 | 4.0353  | 4.0353  |
| 1000 | 100 | 0.1    | Cov | 0.9844   | 0.9774   | 0.9774 | 0.9774  | 0.9774  |
|      |     |        | Len | 16.3846  | 4.6526   | 4.6526 | 4.6526  | 4.6526  |
| 1000 | 200 | 0.07   | Cov | 0.9894   | 0.9766   | 0.9766 | 0.9766  | 0.9766  |
|      |     |        | Len | 36.4034  | 5.2074   | 5.2074 | 5.2074  | 5.2074  |

Table 5.141. Etype = 3, J=5, k=p-1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL      | PLS    | PCR    | FS      |
|------|-----|--------|-----|----------|---------|--------|--------|---------|
| 100  | 40  | 0.9    | Cov | 0.9902   | 0.9748  | 0.9124 | 0.9810 | 0.9320  |
|      |     |        | Len | 38.7914  | 8.2395  | 4.5324 | 5.1281 | 4.8568  |
| 400  | 40  | 0.9    | Cov | 0.9800   | 0.9758  | 0.9742 | 0.9742 | 0.9742  |
|      |     |        | Len | 33.6180  | 7.7883  | 4.4343 | 4.4343 | 4.4343  |
| 1000 | 40  | 0.9    | Cov | 0.9756   | 0.9642  | 0.9678 | 0.9678 | 0.9678  |
|      |     |        | Len | 31.4668  | 7.4662  | 4.0324 | 4.0324 | 4.0324  |
| 100  | 100 | 0.9    | Cov | 0.9966   | 0.9776  | 0.1938 | 0.9858 | 0.8116  |
|      |     |        | Len | 185.6657 | 18.4464 | 1.3474 | 5.6151 | 6.7027  |
| 400  | 100 | 0.9    | Cov | 0.9914   | 0.9840  | 0.9534 | 0.9682 | 0.9484  |
|      |     |        | Len | 150.6547 | 18.0687 | 4.7176 | 4.8233 | 4.8279  |
| 1000 | 100 | 0.9    | Cov | 0.9882   | 0.9836  | 0.9778 | 0.9778 | 0.9778  |
|      |     |        | Len | 141.6021 | 17.3735 | 4.6547 | 4.6547 | 4.6547  |
| 100  | 200 | 0.9    | Cov | 0.9970   | 0.9670  | 0.0018 | 0.9846 | 0.7288  |
|      |     |        | Len | 875.3991 | 31.6983 | 0.0049 | 5.6368 | 10.6829 |
| 400  | 200 | 0.9    | Cov | 0.9924   | 0.9780  | 0.8376 | 0.9722 | 0.9050  |
|      |     |        | Len | 200.1816 | 16.6949 | 3.7951 | 5.2076 | 4.4877  |
| 1000 | 200 | 0.9    | Cov | 0.9964   | 0.9930  | 0.9774 | 0.9774 | 0.9774  |
|      |     |        | Len | 448.1583 | 36.8166 | 5.2090 | 5.2090 | 5.2090  |

Table 5.142. Etype = 3, J=10, k=1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0      | Cov | 0.9546 | 0.9538 | 0.9258 | 0.9452 | 0.9346 |
|      |     |        | Len | 3.7684 | 3.8879 | 3.8227 | 4.9171 | 3.8475 |
| 100  | 40  | 0      | Cov | 0.9482 | 0.9390 | 0.8246 | 0.9474 | 0.9038 |
|      |     |        | Len | 3.7362 | 3.8718 | 3.4257 | 5.3687 | 3.7317 |
| 100  | 100 | 0      | Cov | 0.9432 | 0.9256 | 0.2812 | 0.9450 | 0.8548 |
|      |     |        | Len | 3.7389 | 3.8060 | 1.4069 | 5.6672 | 3.5137 |
| 100  | 200 | 0      | Cov | 0.9492 | 0.9224 | 0.0354 | 0.9510 | 0.8208 |
|      |     |        | Len | 3.7805 | 3.7466 | 0.1376 | 5.7942 | 3.3133 |
| 400  | 20  | 0      | Cov | 0.9444 | 0.9486 | 0.9432 | 0.9432 | 0.9432 |
|      |     |        | Len | 3.4068 | 3.3609 | 3.4141 | 3.4141 | 3.4141 |
| 400  | 40  | 0      | Cov | 0.9334 | 0.9414 | 0.9336 | 0.9336 | 0.9336 |
|      |     |        | Len | 3.4378 | 3.4075 | 3.4445 | 3.4445 | 3.4445 |
| 400  | 100 | 0      | Cov | 0.9326 | 0.9350 | 0.8822 | 0.9206 | 0.8968 |
|      |     |        | Len | 3.2627 | 3.4441 | 3.3286 | 4.4796 | 3.3760 |
| 400  | 200 | 0      | Cov | 0.9292 | 0.9204 | 0.7042 | 0.9248 | 0.8538 |
|      |     |        | Len | 3.2021 | 3.4247 | 2.8160 | 4.8197 | 3.2302 |
| 1000 | 20  | 0      | Cov | 0.9468 | 0.9464 | 0.9466 | 0.9466 | 0.9466 |
|      |     |        | Len | 3.2497 | 3.2061 | 3.2582 | 3.2582 | 3.2582 |
| 1000 | 40  | 0      | Cov | 0.9466 | 0.9504 | 0.9462 | 0.9462 | 0.9462 |
|      |     |        | Len | 3.3140 | 3.2628 | 3.3221 | 3.3221 | 3.3221 |
| 1000 | 100 | 0      | Cov | 0.9288 | 0.9418 | 0.9284 | 0.9284 | 0.9284 |
|      |     |        | Len | 3.3701 | 3.3397 | 3.3768 | 3.3768 | 3.3768 |
| 1000 | 200 | 0      | Cov | 0.9250 | 0.9310 | 0.8908 | 0.9212 | 0.8998 |
|      |     |        | Len | 3.2194 | 3.3777 | 3.3292 | 4.2303 | 3.3429 |

Table 5.143. Etype = 3, J=10, k=1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0.2236 | Cov | 0.9514 | 0.9582 | 0.9218 | 0.9420 | 0.9284 |
|      |     |        | Len | 3.7993 | 3.8891 | 3.8450 | 4.6052 | 3.8746 |
| 100  | 40  | 0.1581 | Cov | 0.9512 | 0.9596 | 0.8224 | 0.9514 | 0.9054 |
|      |     |        | Len | 3.7495 | 3.8850 | 3.4288 | 5.0472 | 3.7337 |
| 100  | 100 | 0.1    | Cov | 0.9468 | 0.9524 | 0.3166 | 0.9488 | 0.8630 |
|      |     |        | Len | 3.7563 | 3.8971 | 1.4899 | 5.4359 | 3.5221 |
| 100  | 200 | 0.07   | Cov | 0.9482 | 0.9472 | 0.0478 | 0.9490 | 0.8328 |
|      |     |        | Len | 3.7784 | 3.8775 | 0.1971 | 5.6099 | 3.3224 |
| 400  | 20  | 0.2236 | Cov | 0.9458 | 0.9524 | 0.9456 | 0.9456 | 0.9456 |
|      |     |        | Len | 3.3999 | 3.3105 | 3.4110 | 3.4110 | 3.4110 |
| 400  | 40  | 0.1581 | Cov | 0.9310 | 0.9504 | 0.9300 | 0.9300 | 0.9300 |
|      |     |        | Len | 3.4400 | 3.3543 | 3.4490 | 3.4490 | 3.4490 |
| 400  | 100 | 0.1    | Cov | 0.9284 | 0.9476 | 0.8682 | 0.9244 | 0.8906 |
|      |     |        | Len | 3.2676 | 3.3566 | 3.3308 | 4.3255 | 3.3748 |
| 400  | 200 | 0.07   | Cov | 0.9250 | 0.9426 | 0.7010 | 0.9246 | 0.8450 |
|      |     |        | Len | 3.2081 | 3.3608 | 2.8123 | 4.6796 | 3.2287 |
| 1000 | 20  | 0.2236 | Cov | 0.9466 | 0.9510 | 0.9468 | 0.9468 | 0.9468 |
|      |     |        | Len | 3.2458 | 3.1673 | 3.2581 | 3.2581 | 3.2581 |
| 1000 | 40  | 0.1581 | Cov | 0.9392 | 0.9474 | 0.9384 | 0.9384 | 0.9384 |
|      |     |        | Len | 3.3134 | 3.2052 | 3.3247 | 3.3247 | 3.3247 |
| 1000 | 100 | 0.1    | Cov | 0.9280 | 0.9446 | 0.9266 | 0.9266 | 0.9266 |
|      |     |        | Len | 3.3674 | 3.2702 | 3.3767 | 3.3767 | 3.3767 |
| 1000 | 200 | 0.07   | Cov | 0.9236 | 0.9492 | 0.8940 | 0.9230 | 0.9008 |
|      |     |        | Len | 3.2220 | 3.2605 | 3.3297 | 4.1395 | 3.3443 |

Table 5.144. Etype = 3, J=10, k=1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9608 | 0.9616 | 0.9244 | 0.9538 | 0.9322 |
|      |     |        | Len | 3.7860 | 3.7993 | 3.8118 | 3.9068 | 3.8413 |
| 100  | 40  | 0.9    | Cov | 0.9670 | 0.9666 | 0.8294 | 0.9626 | 0.9018 |
|      |     |        | Len | 3.8001 | 3.8264 | 3.4297 | 3.9340 | 3.7261 |
| 100  | 100 | 0.9    | Cov | 0.9628 | 0.9564 | 0.2920 | 0.9590 | 0.8622 |
|      |     |        | Len | 4.2284 | 3.8908 | 1.4485 | 3.9552 | 3.4988 |
| 100  | 200 | 0.9    | Cov | 0.9678 | 0.9610 | 0.0542 | 0.9654 | 0.8298 |
|      |     |        | Len | 5.9374 | 3.9317 | 0.1745 | 3.9763 | 3.3083 |
| 400  | 20  | 0.9    | Cov | 0.9504 | 0.9500 | 0.9436 | 0.9436 | 0.9436 |
|      |     |        | Len | 3.2407 | 3.2689 | 3.4074 | 3.4074 | 3.4074 |
| 400  | 40  | 0.9    | Cov | 0.9484 | 0.9478 | 0.9274 | 0.9274 | 0.9274 |
|      |     |        | Len | 3.2405 | 3.2920 | 3.4445 | 3.4445 | 3.4445 |
| 400  | 100 | 0.9    | Cov | 0.9504 | 0.9474 | 0.8806 | 0.9372 | 0.8958 |
|      |     |        | Len | 3.2444 | 3.3030 | 3.3211 | 3.4574 | 3.3695 |
| 400  | 200 | 0.9    | Cov | 0.9512 | 0.9450 | 0.6986 | 0.9380 | 0.8478 |
|      |     |        | Len | 3.2905 | 3.3223 | 2.8100 | 3.4666 | 3.2250 |
| 1000 | 20  | 0.9    | Cov | 0.9526 | 0.9530 | 0.9488 | 0.9488 | 0.9488 |
|      |     |        | Len | 3.1182 | 3.1426 | 3.2567 | 3.2567 | 3.2567 |
| 1000 | 40  | 0.9    | Cov | 0.9550 | 0.9554 | 0.9474 | 0.9474 | 0.9474 |
|      |     |        | Len | 3.1218 | 3.1549 | 3.3196 | 3.3196 | 3.3196 |
| 1000 | 100 | 0.9    | Cov | 0.9516 | 0.9504 | 0.9286 | 0.9286 | 0.9286 |
|      |     |        | Len | 3.1587 | 3.1709 | 3.3782 | 3.3782 | 3.3782 |
| 1000 | 200 | 0.9    | Cov | 0.9552 | 0.9528 | 0.8994 | 0.9408 | 0.9054 |
|      |     |        | Len | 3.2045 | 3.1818 | 3.3259 | 3.3907 | 3.3398 |

Table 5.145. Etype = 3, J=10, k=19,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 20  | 0      | Cov | 0.9788  | 0.9524  | 0.9646 | 0.9666  | 0.9398  |
|      |     |        | Len | 19.6001 | 13.2688 | 4.9286 | 15.0776 | 13.8277 |
| 100  | 40  | 0      | Cov | 0.9788  | 0.9534  | 0.8980 | 0.9726  | 0.9428  |
|      |     |        | Len | 19.6749 | 13.4095 | 4.3629 | 18.8059 | 13.8385 |
| 100  | 100 | 0      | Cov | 0.9768  | 0.9448  | 0.3628 | 0.9766  | 0.9318  |
|      |     |        | Len | 19.5973 | 13.5437 | 2.1058 | 20.7390 | 13.6591 |
| 100  | 200 | 0      | Cov | 0.9720  | 0.9354  | 0.0418 | 0.9700  | 0.9048  |
|      |     |        | Len | 19.1471 | 13.5717 | 0.3423 | 20.7451 | 13.0908 |
| 400  | 20  | 0      | Cov | 0.9724  | 0.9734  | 0.9734 | 0.9734  | 0.9734  |
|      |     |        | Len | 4.3367  | 4.3322  | 4.3322 | 4.3322  | 4.3322  |
| 400  | 40  | 0      | Cov | 0.9736  | 0.9758  | 0.9730 | 0.9730  | 0.9730  |
|      |     |        | Len | 4.3342  | 4.3438  | 4.3478 | 4.3478  | 4.3478  |
| 400  | 100 | 0      | Cov | 0.9724  | 0.9692  | 0.9366 | 0.9480  | 0.9560  |
|      |     |        | Len | 4.4685  | 4.3381  | 4.1506 | 14.0484 | 4.2561  |
| 400  | 200 | 0      | Cov | 0.9664  | 0.9642  | 0.7964 | 0.9576  | 0.9392  |
|      |     |        | Len | 4.6656  | 4.3183  | 3.4556 | 16.6834 | 4.1455  |
| 1000 | 20  | 0      | Cov | 0.9590  | 0.9588  | 0.9588 | 0.9588  | 0.9588  |
|      |     |        | Len | 3.5649  | 3.5583  | 3.5583 | 3.5583  | 3.5583  |
| 1000 | 40  | 0      | Cov | 0.9590  | 0.9606  | 0.9580 | 0.9580  | 0.9580  |
|      |     |        | Len | 3.6131  | 3.6018  | 3.6279 | 3.6279  | 3.6279  |
| 1000 | 100 | 0      | Cov | 0.9496  | 0.9548  | 0.9488 | 0.9488  | 0.9488  |
|      |     |        | Len | 3.6619  | 3.6631  | 3.6845 | 3.6845  | 3.6845  |
| 1000 | 200 | 0      | Cov | 0.9514  | 0.9532  | 0.9236 | 0.9146  | 0.9296  |
|      |     |        | Len | 3.5758  | 3.6846  | 3.6169 | 11.1176 | 3.6391  |

Table 5.146. Etype = 3, J=10, k=19,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 20  | 0.2236 | Cov | 0.9878  | 0.9646  | 0.9610 | 0.9772  | 0.9410  |
|      |     |        | Len | 73.8863 | 15.0499 | 4.9320 | 5.1319  | 14.7297 |
| 100  | 40  | 0.1581 | Cov | 0.9866  | 0.9532  | 0.9070 | 0.9792  | 0.9290  |
|      |     |        | Len | 76.8949 | 17.6715 | 4.3534 | 12.3721 | 16.1603 |
| 100  | 100 | 0.1    | Cov | 0.9840  | 0.9408  | 0.4032 | 0.9810  | 0.9070  |
|      |     |        | Len | 78.8115 | 20.2014 | 2.1698 | 17.5847 | 17.6214 |
| 100  | 200 | 0.07   | Cov | 0.9888  | 0.9318  | 0.0588 | 0.9792  | 0.8740  |
|      |     |        | Len | 83.0431 | 22.5835 | 0.4851 | 19.7142 | 18.1619 |
| 400  | 20  | 0.2236 | Cov | 0.9706  | 0.9714  | 0.9714 | 0.9714  | 0.9714  |
|      |     |        | Len | 4.4511  | 4.3280  | 4.3280 | 4.3280  | 4.3280  |
| 400  | 40  | 0.1581 | Cov | 0.9700  | 0.9700  | 0.9638 | 0.9638  | 0.9638  |
|      |     |        | Len | 4.4400  | 4.3425  | 4.3493 | 4.3493  | 4.3493  |
| 400  | 100 | 0.1    | Cov | 0.9710  | 0.9698  | 0.9374 | 0.9502  | 0.9550  |
|      |     |        | Len | 4.5890  | 4.3514  | 4.1561 | 11.7677 | 4.2597  |
| 400  | 200 | 0.07   | Cov | 0.9716  | 0.9676  | 0.7910 | 0.9608  | 0.9352  |
|      |     |        | Len | 6.6530  | 4.3479  | 3.4572 | 14.9650 | 4.1557  |
| 1000 | 20  | 0.2236 | Cov | 0.9604  | 0.9614  | 0.9614 | 0.9614  | 0.9614  |
|      |     |        | Len | 3.7036  | 3.5590  | 3.5590 | 3.5590  | 3.5590  |
| 1000 | 40  | 0.1581 | Cov | 0.9558  | 0.9586  | 0.9568 | 0.9568  | 0.9568  |
|      |     |        | Len | 3.7171  | 3.6013  | 3.6350 | 3.6350  | 3.6350  |
| 1000 | 100 | 0.1    | Cov | 0.9564  | 0.9528  | 0.9434 | 0.9434  | 0.9434  |
|      |     |        | Len | 3.7123  | 3.6267  | 3.6790 | 3.6790  | 3.6790  |
| 1000 | 200 | 0.07   | Cov | 0.9492  | 0.9486  | 0.9134 | 0.9052  | 0.9160  |
|      |     |        | Len | 3.7163  | 3.6512  | 3.6149 | 10.0234 | 3.6356  |

Table 5.147. Etype = 3, J=10, k=19,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|----------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9872   | 0.9696 | 0.9620 | 0.9818 | 0.9642 |
|      |     |        | Len | 46.4625  | 5.2638 | 4.7516 | 4.8899 | 4.9430 |
| 100  | 40  | 0.9    | Cov | 0.9860   | 0.9640 | 0.8820 | 0.9770 | 0.9408 |
|      |     |        | Len | 40.8380  | 5.6230 | 4.1615 | 5.0899 | 4.7882 |
| 100  | 100 | 0.9    | Cov | 0.9842   | 0.9644 | 0.3518 | 0.9746 | 0.9160 |
|      |     |        | Len | 121.7086 | 5.5387 | 1.7084 | 5.2560 | 4.5443 |
| 100  | 200 | 0.9    | Cov | 0.9880   | 0.9594 | 0.0530 | 0.9764 | 0.8856 |
|      |     |        | Len | 338.2368 | 5.2524 | 0.2093 | 5.3096 | 4.3166 |
| 400  | 20  | 0.9    | Cov | 0.9688   | 0.9652 | 0.9672 | 0.9672 | 0.9672 |
|      |     |        | Len | 10.3028  | 4.5201 | 3.9529 | 3.9529 | 3.9529 |
| 400  | 40  | 0.9    | Cov | 0.9722   | 0.9588 | 0.9508 | 0.9508 | 0.9508 |
|      |     |        | Len | 14.3214  | 5.0185 | 3.8862 | 3.8862 | 3.8862 |
| 400  | 100 | 0.9    | Cov | 0.9714   | 0.9566 | 0.9074 | 0.9514 | 0.9226 |
|      |     |        | Len | 22.9580  | 5.0681 | 3.6958 | 4.0828 | 3.7890 |
| 400  | 200 | 0.9    | Cov | 0.9676   | 0.9594 | 0.7454 | 0.9568 | 0.8904 |
|      |     |        | Len | 31.8857  | 4.9742 | 3.0828 | 4.1788 | 3.6454 |
| 1000 | 20  | 0.9    | Cov | 0.9642   | 0.9586 | 0.9582 | 0.9582 | 0.9582 |
|      |     |        | Len | 9.7005   | 4.2009 | 3.5545 | 3.5545 | 3.5545 |
| 1000 | 40  | 0.9    | Cov | 0.9664   | 0.9600 | 0.9568 | 0.9568 | 0.9568 |
|      |     |        | Len | 13.5838  | 4.8044 | 3.5640 | 3.5640 | 3.5640 |
| 1000 | 100 | 0.9    | Cov | 0.9618   | 0.9552 | 0.9348 | 0.9348 | 0.9348 |
|      |     |        | Len | 21.9143  | 4.9263 | 3.6006 | 3.6006 | 3.6006 |
| 1000 | 200 | 0.9    | Cov | 0.9608   | 0.9550 | 0.9104 | 0.9474 | 0.9178 |
|      |     |        | Len | 30.2962  | 4.9202 | 3.5235 | 3.7862 | 3.5440 |

Table 5.148. Etype = 3, J=10, k=p-1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 40  | 0      | Cov | 0.9794  | 0.9358  | 0.8934 | 0.9738  | 0.9240  |
|      |     |        | Len | 28.7206 | 21.4576 | 4.2258 | 25.6033 | 21.6939 |
| 400  | 40  | 0      | Cov | 0.9722  | 0.9718  | 0.9718 | 0.9718  | 0.9718  |
|      |     |        | Len | 4.6726  | 4.6645  | 4.6645 | 4.6645  | 4.6645  |
| 1000 | 40  | 0      | Cov | 0.9718  | 0.9710  | 0.9710 | 0.9710  | 0.9710  |
|      |     |        | Len | 4.0490  | 4.0371  | 4.0371 | 4.0371  | 4.0371  |
| 100  | 100 | 0      | Cov | 0.9540  | 0.8652  | 0.2800 | 0.9478  | 0.8392  |
|      |     |        | Len | 40.5428 | 30.9988 | 2.6673 | 39.4264 | 30.9970 |
| 400  | 100 | 0      | Cov | 0.9784  | 0.9426  | 0.9508 | 0.9652  | 0.9212  |
|      |     |        | Len | 41.1893 | 29.3608 | 4.4661 | 32.7627 | 28.7542 |
| 1000 | 100 | 0      | Cov | 0.9746  | 0.9746  | 0.9746 | 0.9746  | 0.9746  |
|      |     |        | Len | 4.6786  | 4.6569  | 4.6569 | 4.6569  | 4.6569  |
| 100  | 200 | 0      | Cov | 0.9468  | 0.8376  | 0.0350 | 0.9514  | 0.7954  |
|      |     |        | Len | 56.7085 | 43.7123 | 0.8558 | 56.2363 | 42.0791 |
| 400  | 200 | 0      | Cov | 0.9312  | 0.8432  | 0.7130 | 0.9112  | 0.8148  |
|      |     |        | Len | 49.5097 | 37.7606 | 2.9329 | 44.9334 | 36.5774 |
| 1000 | 200 | 0      | Cov | 0.9816  | 0.9448  | 0.9632 | 0.9592  | 0.9242  |
|      |     |        | Len | 56.0662 | 38.0983 | 4.5456 | 40.9701 | 37.1177 |

Table 5.149. Etype = 3, J=10, k=p-1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL       | PLS    | PCR    | FS       |
|------|-----|--------|-----|----------|----------|--------|--------|----------|
| 100  | 40  | 0.1581 | Cov | 0.9876   | 0.9516   | 0.8962 | 0.9794 | 0.9116   |
|      |     |        | Len | 160.2108 | 36.4350  | 4.3762 | 5.3188 | 33.5340  |
| 100  | 100 | 0.1    | Cov | 0.9846   | 0.9212   | 0.3634 | 0.9818 | 0.8598   |
|      |     |        | Len | 411.6806 | 97.1076  | 1.8084 | 5.9039 | 85.9486  |
| 100  | 200 | 0.07   | Cov | 0.9834   | 0.9122   | 0.0638 | 0.9804 | 0.8216   |
|      |     |        | Len | 870.5535 | 211.1051 | 0.2408 | 6.8403 | 166.3864 |
| 400  | 40  | 0.1581 | Cov | 0.9778   | 0.9778   | 0.9778 | 0.9778 | 0.9778   |
|      |     |        | Len | 7.0944   | 4.6720   | 4.6720 | 4.6720 | 4.6720   |
| 400  | 100 | 0.1    | Cov | 0.9828   | 0.9558   | 0.9486 | 0.9808 | 0.8944   |
|      |     |        | Len | 258.1246 | 45.6793  | 4.4678 | 4.8153 | 37.9353  |
| 400  | 200 | 0.07   | Cov | 0.9830   | 0.9418   | 0.8200 | 0.9786 | 0.8686   |
|      |     |        | Len | 569.7375 | 104.8685 | 3.7179 | 5.0964 | 80.7645  |
| 1000 | 40  | 0.1581 | Cov | 0.9738   | 0.9712   | 0.9712 | 0.9712 | 0.9712   |
|      |     |        | Len | 6.2691   | 4.0395   | 4.0395 | 4.0395 | 4.0395   |
| 1000 | 100 | 0.1    | Cov | 0.9854   | 0.9746   | 0.9746 | 0.9746 | 0.9746   |
|      |     |        | Len | 16.3900  | 4.6592   | 4.6592 | 4.6592 | 4.6592   |
| 1000 | 200 | 0.07   | Cov | 0.9868   | 0.9620   | 0.9638 | 0.9794 | 0.9060   |
|      |     |        | Len | 362.5307 | 56.2049  | 4.5509 | 4.7610 | 46.3504  |

Table 5.150. Etype = 3, J=10, k=p-1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso     | RL      | PLS    | PCR    | FS      |
|------|-----|--------|-----|-----------|---------|--------|--------|---------|
| 100  | 40  | 0.9    | Cov | 0.9926    | 0.9690  | 0.9084 | 0.9840 | 0.9352  |
|      |     |        | Len | 186.9498  | 7.8522  | 4.3641 | 5.0806 | 5.9160  |
| 400  | 40  | 0.9    | Cov | 0.9848    | 0.9742  | 0.9724 | 0.9724 | 0.9724  |
|      |     |        | Len | 33.5986   | 7.8173  | 4.4396 | 4.4396 | 4.4396  |
| 1000 | 40  | 0.9    | Cov | 0.9754    | 0.9722  | 0.9744 | 0.9744 | 0.9744  |
|      |     |        | Len | 31.4384   | 7.4723  | 4.0335 | 4.0335 | 4.0335  |
| 100  | 100 | 0.9    | Cov | 0.9900    | 0.9578  | 0.3676 | 0.9846 | 0.8732  |
|      |     |        | Len | 1633.5880 | 15.6240 | 1.7824 | 5.0723 | 10.5509 |
| 400  | 100 | 0.9    | Cov | 0.9890    | 0.9826  | 0.9478 | 0.9774 | 0.9286  |
|      |     |        | Len | 142.2156  | 17.0294 | 4.4574 | 4.6716 | 5.9278  |
| 1000 | 100 | 0.9    | Cov | 0.9888    | 0.9818  | 0.9772 | 0.9772 | 0.9772  |
|      |     |        | Len | 141.5850  | 17.4739 | 4.6587 | 4.6587 | 4.6587  |
| 100  | 200 | 0.9    | Cov | 0.9902    | 0.9538  | 0.0630 | 0.9800 | 0.8394  |
|      |     |        | Len | 7570.0150 | 28.5303 | 0.2115 | 5.0610 | 19.0220 |
| 400  | 200 | 0.9    | Cov | 0.9892    | 0.9754  | 0.8162 | 0.9788 | 0.8800  |
|      |     |        | Len | 393.1899  | 31.0077 | 3.7086 | 4.6724 | 9.5054  |

Table 5.151. Etype = 3, J=20, k=1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0      | Cov | 0.9594 | 0.9532 | 0.9248 | 0.9540 | 0.9450 |
|      |     |        | Len | 3.8409 | 3.8864 | 3.8254 | 5.6337 | 3.9032 |
| 100  | 40  | 0      | Cov | 0.9632 | 0.9584 | 0.8412 | 0.9582 | 0.9416 |
|      |     |        | Len | 3.8833 | 3.8989 | 3.4697 | 5.8620 | 3.8897 |
| 100  | 100 | 0      | Cov | 0.9582 | 0.9438 | 0.5258 | 0.9526 | 0.9332 |
|      |     |        | Len | 3.9306 | 3.8773 | 2.0990 | 6.0233 | 3.8390 |
| 100  | 200 | 0      | Cov | 0.9564 | 0.9362 | 0.2422 | 0.9582 | 0.9176 |
|      |     |        | Len | 3.9901 | 3.8542 | 0.9016 | 6.0676 | 3.7892 |
| 400  | 20  | 0      | Cov | 0.9422 | 0.9462 | 0.9428 | 0.9428 | 0.9428 |
|      |     |        | Len | 3.4094 | 3.3576 | 3.4169 | 3.4169 | 3.4169 |
| 400  | 40  | 0      | Cov | 0.9380 | 0.9396 | 0.9256 | 0.9310 | 0.9306 |
|      |     |        | Len | 3.3152 | 3.4026 | 3.4460 | 4.5002 | 3.4440 |
| 400  | 100 | 0      | Cov | 0.9450 | 0.9442 | 0.8790 | 0.9400 | 0.9192 |
|      |     |        | Len | 3.2690 | 3.4360 | 3.3205 | 5.0164 | 3.4274 |
| 400  | 200 | 0      | Cov | 0.9430 | 0.9318 | 0.7060 | 0.9424 | 0.8996 |
|      |     |        | Len | 3.2537 | 3.4444 | 2.8099 | 5.2109 | 3.3857 |
| 1000 | 20  | 0      | Cov | 0.9520 | 0.9556 | 0.9520 | 0.9520 | 0.9520 |
|      |     |        | Len | 3.2445 | 3.2015 | 3.2530 | 3.2530 | 3.2530 |
| 1000 | 40  | 0      | Cov | 0.9406 | 0.9454 | 0.9404 | 0.9404 | 0.9404 |
|      |     |        | Len | 3.3152 | 3.2640 | 3.3233 | 3.3233 | 3.3233 |
| 1000 | 100 | 0      | Cov | 0.9376 | 0.9386 | 0.9254 | 0.9296 | 0.9266 |
|      |     |        | Len | 3.2337 | 3.3307 | 3.3740 | 4.3942 | 3.3730 |
| 1000 | 200 | 0      | Cov | 0.9362 | 0.9326 | 0.8882 | 0.9348 | 0.9110 |
|      |     |        | Len | 3.1866 | 3.3657 | 3.3240 | 4.8206 | 3.3687 |

Table 5.152. Etype = 3, J=20, k=1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0.2236 | Cov | 0.9594 | 0.9550 | 0.9246 | 0.9534 | 0.9450 |
|      |     |        | Len | 3.8479 | 3.8475 | 3.8151 | 5.1233 | 3.8882 |
| 100  | 40  | 0.1581 | Cov | 0.9542 | 0.9552 | 0.8442 | 0.9536 | 0.9372 |
|      |     |        | Len | 3.9122 | 3.8764 | 3.4893 | 5.4462 | 3.8787 |
| 100  | 100 | 0.1    | Cov | 0.9608 | 0.9570 | 0.6168 | 0.9620 | 0.9352 |
|      |     |        | Len | 3.9827 | 3.8779 | 2.3433 | 5.7160 | 3.8349 |
| 100  | 200 | 0.07   | Cov | 0.9588 | 0.9530 | 0.3786 | 0.9548 | 0.9204 |
|      |     |        | Len | 4.0711 | 3.8829 | 1.2928 | 5.8581 | 3.7789 |
| 400  | 20  | 0.2236 | Cov | 0.9418 | 0.9488 | 0.9406 | 0.9406 | 0.9406 |
|      |     |        | Len | 3.4059 | 3.3140 | 3.4171 | 3.4171 | 3.4171 |
| 400  | 40  | 0.1581 | Cov | 0.9460 | 0.9530 | 0.9318 | 0.9448 | 0.9350 |
|      |     |        | Len | 3.3186 | 3.3155 | 3.4452 | 4.2617 | 3.4446 |
| 400  | 100 | 0.1    | Cov | 0.9410 | 0.9446 | 0.8720 | 0.9338 | 0.9148 |
|      |     |        | Len | 3.2771 | 3.3339 | 3.3322 | 4.8082 | 3.4344 |
| 400  | 200 | 0.07   | Cov | 0.9438 | 0.9526 | 0.6926 | 0.9406 | 0.8946 |
|      |     |        | Len | 3.2616 | 3.3477 | 2.8103 | 5.0411 | 3.3874 |
| 1000 | 20  | 0.2236 | Cov | 0.9482 | 0.9520 | 0.9480 | 0.9480 | 0.9480 |
|      |     |        | Len | 3.2450 | 3.1657 | 3.2576 | 3.2576 | 3.2576 |
| 1000 | 40  | 0.1581 | Cov | 0.9356 | 0.9426 | 0.9354 | 0.9354 | 0.9354 |
|      |     |        | Len | 3.3072 | 3.1991 | 3.3184 | 3.3184 | 3.3184 |
| 1000 | 100 | 0.1    | Cov | 0.9376 | 0.9494 | 0.9252 | 0.9288 | 0.9250 |
|      |     |        | Len | 3.2376 | 3.2117 | 3.3784 | 4.2447 | 3.3767 |
| 1000 | 200 | 0.07   | Cov | 0.9420 | 0.9506 | 0.8988 | 0.9318 | 0.9224 |
|      |     |        | Len | 3.1909 | 3.2164 | 3.3287 | 4.6835 | 3.3722 |

Table 5.153. Etype = 3, J=20, k=1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|---------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9644  | 0.9636 | 0.9266 | 0.9610 | 0.9474 |
|      |     |        | Len | 4.1823  | 3.8170 | 3.8274 | 3.9116 | 3.9078 |
| 100  | 40  | 0.9    | Cov | 0.9666  | 0.9622 | 0.8524 | 0.9656 | 0.9378 |
|      |     |        | Len | 6.2813  | 3.8303 | 3.4626 | 3.9182 | 3.8842 |
| 100  | 100 | 0.9    | Cov | 0.9664  | 0.9592 | 0.6014 | 0.9652 | 0.9234 |
|      |     |        | Len | 16.6351 | 3.8700 | 2.2084 | 3.9266 | 3.8250 |
| 100  | 200 | 0.9    | Cov | 0.9662  | 0.9572 | 0.3632 | 0.9644 | 0.9190 |
|      |     |        | Len | 33.1788 | 3.9145 | 1.1149 | 3.9540 | 3.7791 |
| 400  | 20  | 0.9    | Cov | 0.9478  | 0.9472 | 0.9416 | 0.9416 | 0.9416 |
|      |     |        | Len | 3.2446  | 3.2724 | 3.4105 | 3.4105 | 3.4105 |
| 400  | 40  | 0.9    | Cov | 0.9534  | 0.9526 | 0.9372 | 0.9504 | 0.9358 |
|      |     |        | Len | 3.2272  | 3.2799 | 3.4397 | 3.4220 | 3.4401 |
| 400  | 100 | 0.9    | Cov | 0.9544  | 0.9504 | 0.8766 | 0.9494 | 0.9160 |
|      |     |        | Len | 3.2492  | 3.3075 | 3.3275 | 3.4407 | 3.4287 |
| 400  | 200 | 0.9    | Cov | 0.9546  | 0.9516 | 0.7076 | 0.9518 | 0.9042 |
|      |     |        | Len | 3.2909  | 3.3225 | 2.8113 | 3.4479 | 3.3884 |
| 1000 | 20  | 0.9    | Cov | 0.9532  | 0.9530 | 0.9502 | 0.9502 | 0.9502 |
|      |     |        | Len | 3.1170  | 3.1414 | 3.2549 | 3.2549 | 3.2549 |
| 1000 | 40  | 0.9    | Cov | 0.9516  | 0.9508 | 0.9422 | 0.9422 | 0.9422 |
|      |     |        | Len | 3.1239  | 3.1561 | 3.3191 | 3.3191 | 3.3191 |
| 1000 | 100 | 0.9    | Cov | 0.9468  | 0.9466 | 0.9206 | 0.9380 | 0.9246 |
|      |     |        | Len | 3.1589  | 3.1711 | 3.3791 | 3.3594 | 3.3779 |
| 1000 | 200 | 0.9    | Cov | 0.9540  | 0.9552 | 0.8982 | 0.9520 | 0.9190 |
|      |     |        | Len | 3.2039  | 3.1784 | 3.3229 | 3.3667 | 3.3683 |

Table 5.154. Etype = 3, J=20, k=19,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 20  | 0      | Cov | 0.9798  | 0.9536  | 0.9592 | 0.9754  | 0.9540  |
|      |     |        | Len | 20.5181 | 15.6455 | 4.6449 | 18.8976 | 17.3240 |
| 100  | 40  | 0      | Cov | 0.9810  | 0.9544  | 0.8910 | 0.9796  | 0.9546  |
|      |     |        | Len | 20.5022 | 15.6291 | 4.3567 | 20.4026 | 17.2886 |
| 100  | 100 | 0      | Cov | 0.9762  | 0.9422  | 0.5488 | 0.9778  | 0.9474  |
|      |     |        | Len | 20.0419 | 15.3022 | 3.6595 | 20.9538 | 16.9013 |
| 100  | 200 | 0      | Cov | 0.9658  | 0.9386  | 0.2490 | 0.9686  | 0.9326  |
|      |     |        | Len | 20.0872 | 15.7833 | 2.1669 | 20.7276 | 16.3862 |
| 400  | 20  | 0      | Cov | 0.9744  | 0.9750  | 0.9750 | 0.9750  | 0.9750  |
|      |     |        | Len | 4.3355  | 4.3300  | 4.3300 | 4.3300  | 4.3300  |
| 400  | 40  | 0      | Cov | 0.9768  | 0.9758  | 0.9710 | 0.9700  | 0.9762  |
|      |     |        | Len | 5.3821  | 4.3399  | 4.3538 | 13.8034 | 4.3318  |
| 400  | 100 | 0      | Cov | 0.9726  | 0.9754  | 0.9320 | 0.9664  | 0.9758  |
|      |     |        | Len | 5.9271  | 4.3375  | 4.1502 | 17.6694 | 4.3281  |
| 400  | 200 | 0      | Cov | 0.9780  | 0.9784  | 0.7858 | 0.9790  | 0.9792  |
|      |     |        | Len | 6.3172  | 4.3368  | 3.4518 | 18.9838 | 4.3254  |
| 1000 | 20  | 0      | Cov | 0.9612  | 0.9610  | 0.9610 | 0.9610  | 0.9610  |
|      |     |        | Len | 3.5587  | 3.5516  | 3.5516 | 3.5516  | 3.5516  |
| 1000 | 40  | 0      | Cov | 0.9596  | 0.9600  | 0.9594 | 0.9594  | 0.9594  |
|      |     |        | Len | 3.6177  | 3.6047  | 3.6325 | 3.6325  | 3.6325  |
| 1000 | 100 | 0      | Cov | 0.9530  | 0.9494  | 0.9396 | 0.9366  | 0.9416  |
|      |     |        | Len | 3.6417  | 3.6530  | 3.6831 | 11.9768 | 3.6812  |
| 1000 | 200 | 0      | Cov | 0.9542  | 0.9498  | 0.9100 | 0.9366  | 0.9364  |
|      |     |        | Len | 3.7185  | 3.6764  | 3.6185 | 14.8414 | 3.6824  |

Table 5.155. Etype = 3, J=20, k=19,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 20  | 0.2236 | Cov | 0.9808  | 0.9588  | 0.9558 | 0.9806  | 0.9448  |
|      |     |        | Len | 80.7544 | 18.5012 | 4.5949 | 4.8365  | 25.4635 |
| 100  | 40  | 0.1581 | Cov | 0.9804  | 0.9480  | 0.9014 | 0.9768  | 0.9344  |
|      |     |        | Len | 79.7130 | 19.9509 | 4.4166 | 13.3319 | 27.3010 |
| 100  | 100 | 0.1    | Cov | 0.9832  | 0.9302  | 0.6524 | 0.9780  | 0.9254  |
|      |     |        | Len | 78.2454 | 20.9453 | 4.1682 | 17.8505 | 28.7847 |
| 100  | 200 | 0.07   | Cov | 0.9818  | 0.9390  | 0.3874 | 0.9804  | 0.9092  |
|      |     |        | Len | 88.9069 | 27.3658 | 3.1461 | 19.6499 | 29.2473 |
| 400  | 20  | 0.2236 | Cov | 0.9770  | 0.9750  | 0.9750 | 0.9750  | 0.9750  |
|      |     |        | Len | 4.4446  | 4.3219  | 4.3219 | 4.3219  | 4.3219  |
| 400  | 40  | 0.1581 | Cov | 0.9792  | 0.9716  | 0.9682 | 0.9642  | 0.9742  |
|      |     |        | Len | 41.6633 | 5.1623  | 4.3514 | 9.2795  | 4.4112  |
| 400  | 100 | 0.1    | Cov | 0.9776  | 0.9732  | 0.9300 | 0.9636  | 0.9712  |
|      |     |        | Len | 55.1140 | 7.9854  | 4.1501 | 14.6708 | 4.5877  |
| 400  | 200 | 0.07   | Cov | 0.9820  | 0.9624  | 0.7848 | 0.9698  | 0.9752  |
|      |     |        | Len | 59.8878 | 10.4482 | 3.4502 | 17.0150 | 4.8688  |
| 1000 | 20  | 0.2236 | Cov | 0.9620  | 0.9602  | 0.9602 | 0.9602  | 0.9602  |
|      |     |        | Len | 3.7048  | 3.5596  | 3.5596 | 3.5596  | 3.5596  |
| 1000 | 40  | 0.1581 | Cov | 0.9630  | 0.9624  | 0.9608 | 0.9608  | 0.9608  |
|      |     |        | Len | 3.7131  | 3.5962  | 3.6317 | 3.6317  | 3.6317  |
| 1000 | 100 | 0.1    | Cov | 0.9524  | 0.9502  | 0.9386 | 0.9352  | 0.9420  |
|      |     |        | Len | 3.7185  | 3.6324  | 3.6866 | 10.0281 | 3.6836  |
| 1000 | 200 | 0.07   | Cov | 0.9540  | 0.9516  | 0.9086 | 0.9428  | 0.9336  |
|      |     |        | Len | 3.7775  | 3.6516  | 3.6147 | 13.3205 | 3.6774  |

Table 5.156. Etype = 3, J=20, k=19,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|----------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9882   | 0.9662 | 0.9582 | 0.9814 | 0.9580 |
|      |     |        | Len | 230.5138 | 5.6653 | 4.6041 | 4.7165 | 5.6399 |
| 100  | 40  | 0.9    | Cov | 0.9862   | 0.9658 | 0.9052 | 0.9824 | 0.9500 |
|      |     |        | Len | 301.3927 | 5.6810 | 4.1164 | 5.1234 | 5.5612 |
| 100  | 100 | 0.9    | Cov | 0.9838   | 0.9614 | 0.6750 | 0.9800 | 0.9340 |
|      |     |        | Len | 518.4631 | 5.5382 | 2.5907 | 5.3000 | 5.4307 |
| 100  | 200 | 0.9    | Cov | 0.9838   | 0.9606 | 0.4094 | 0.9772 | 0.9290 |
|      |     |        | Len | 931.1808 | 5.4782 | 1.3341 | 5.3940 | 5.3057 |
| 400  | 20  | 0.9    | Cov | 0.9746   | 0.9666 | 0.9664 | 0.9664 | 0.9664 |
|      |     |        | Len | 10.3007  | 4.5350 | 3.9571 | 3.9571 | 3.9571 |
| 400  | 40  | 0.9    | Cov | 0.9658   | 0.9616 | 0.9542 | 0.9644 | 0.9572 |
|      |     |        | Len | 14.3170  | 5.0194 | 3.8902 | 4.0470 | 3.9477 |
| 400  | 100 | 0.9    | Cov | 0.9678   | 0.9614 | 0.9122 | 0.9648 | 0.9490 |
|      |     |        | Len | 22.9923  | 5.0510 | 3.6877 | 4.2360 | 3.9457 |
| 400  | 200 | 0.9    | Cov | 0.9628   | 0.9520 | 0.7498 | 0.9540 | 0.9286 |
|      |     |        | Len | 32.0266  | 4.9834 | 3.0841 | 4.3196 | 3.9267 |
| 1000 | 20  | 0.9    | Cov | 0.9630   | 0.9606 | 0.9634 | 0.9634 | 0.9634 |
|      |     |        | Len | 9.6983   | 4.2080 | 3.5504 | 3.5504 | 3.5504 |
| 1000 | 40  | 0.9    | Cov | 0.9590   | 0.9554 | 0.9522 | 0.9522 | 0.9522 |
|      |     |        | Len | 13.5718  | 4.7993 | 3.5642 | 3.5642 | 3.5642 |
| 1000 | 100 | 0.9    | Cov | 0.9614   | 0.9536 | 0.9442 | 0.9520 | 0.9472 |
|      |     |        | Len | 21.9369  | 4.9144 | 3.5977 | 3.8168 | 3.5980 |
| 1000 | 200 | 0.9    | Cov | 0.9608   | 0.9530 | 0.9128 | 0.9518 | 0.9334 |
|      |     |        | Len | 30.2929  | 4.8915 | 3.5255 | 3.9654 | 3.5915 |

Table 5.157. Etype = 3, J=20, k=p-1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 40  | 0      | Cov | 0.9748  | 0.9302  | 0.8726 | 0.9718  | 0.9430  |
|      |     |        | Len | 28.7973 | 22.9522 | 4.4790 | 28.0329 | 25.0917 |
| 400  | 40  | 0      | Cov | 0.9736  | 0.9532  | 0.9688 | 0.9592  | 0.9486  |
|      |     |        | Len | 25.6791 | 17.3084 | 4.3567 | 19.0658 | 18.1278 |
| 1000 | 40  | 0      | Cov | 0.9704  | 0.9688  | 0.9688 | 0.9688  | 0.9688  |
|      |     |        | Len | 4.0540  | 4.0434  | 4.0434 | 4.0434  | 4.0434  |
| 100  | 100 | 0      | Cov | 0.9576  | 0.8968  | 0.4878 | 0.9628  | 0.9104  |
|      |     |        | Len | 42.0968 | 34.0909 | 6.2725 | 42.6676 | 37.3366 |
| 400  | 100 | 0      | Cov | 0.9720  | 0.9404  | 0.9450 | 0.9650  | 0.9282  |
|      |     |        | Len | 43.3056 | 34.3924 | 4.1456 | 39.0735 | 34.8798 |
| 1000 | 100 | 0      | Cov | 0.9742  | 0.9484  | 0.9718 | 0.9598  | 0.9388  |
|      |     |        | Len | 40.2335 | 27.1522 | 4.3416 | 29.4349 | 27.8204 |
| 100  | 200 | 0      | Cov | 0.9582  | 0.8922  | 0.2206 | 0.9614  | 0.8920  |
|      |     |        | Len | 60.2862 | 50.1462 | 5.9030 | 60.3033 | 51.7738 |
| 400  | 200 | 0      | Cov | 0.9460  | 0.8852  | 0.7250 | 0.9420  | 0.8698  |
|      |     |        | Len | 53.8445 | 44.4312 | 2.9209 | 51.5346 | 44.8378 |
| 1000 | 200 | 0      | Cov | 0.9770  | 0.9440  | 0.9444 | 0.9622  | 0.9342  |
|      |     |        | Len | 60.1818 | 46.9544 | 4.2433 | 52.4976 | 47.1855 |

Table 5.158. Etype = 3, J=20, k=p-1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL       | PLS    | PCR    | FS       |
|------|-----|--------|-----|----------|----------|--------|--------|----------|
| 100  | 40  | 0.1581 | Cov | 0.9810   | 0.9382   | 0.9022 | 0.9814 | 0.9260   |
|      |     |        | Len | 162.1831 | 39.4737  | 4.0988 | 5.0513 | 55.2863  |
| 100  | 100 | 0.1    | Cov | 0.9834   | 0.9148   | 0.6672 | 0.9766 | 0.9096   |
|      |     |        | Len | 398.4739 | 98.2809  | 2.6254 | 5.7128 | 142.2078 |
| 100  | 200 | 0.07   | Cov | 0.9792   | 0.9284   | 0.4112 | 0.9792 | 0.8992   |
|      |     |        | Len | 912.1363 | 258.7397 | 1.4618 | 6.6378 | 282.1162 |
| 400  | 40  | 0.1581 | Cov | 0.9774   | 0.9650   | 0.9686 | 0.9742 | 0.9334   |
|      |     |        | Len | 130.5280 | 21.5731  | 4.3596 | 4.3862 | 20.8450  |
| 400  | 100 | 0.1    | Cov | 0.9756   | 0.9442   | 0.9396 | 0.9774 | 0.9144   |
|      |     |        | Len | 354.5971 | 68.7641  | 4.1539 | 4.5738 | 62.9146  |
| 400  | 200 | 0.07   | Cov | 0.9770   | 0.9436   | 0.8006 | 0.9758 | 0.8874   |
|      |     |        | Len | 709.3075 | 142.5961 | 3.4605 | 4.9155 | 129.4806 |
| 1000 | 40  | 0.1581 | Cov | 0.9740   | 0.9682   | 0.9682 | 0.9682 | 0.9682   |
|      |     |        | Len | 6.2698   | 4.0324   | 4.0324 | 4.0324 | 4.0324   |
| 1000 | 100 | 0.1    | Cov | 0.9772   | 0.9644   | 0.9678 | 0.9762 | 0.9294   |
|      |     |        | Len | 264.0273 | 37.4464  | 4.3433 | 4.3802 | 34.1463  |
| 1000 | 200 | 0.07   | Cov | 0.9756   | 0.9550   | 0.9442 | 0.9706 | 0.9190   |
|      |     |        | Len | 583.7044 | 92.5322  | 4.2420 | 4.5316 | 79.6843  |

Table 5.159. Etype = 3, J=20, k=p-1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso      | RL      | PLS    | PCR    | FS      |
|------|-----|--------|-----|------------|---------|--------|--------|---------|
| 100  | 40  | 0.9    | Cov | 0.9832     | 0.9612  | 0.8978 | 0.9808 | 0.9324  |
|      |     |        | Len | 694.0027   | 8.0494  | 4.1011 | 4.7155 | 8.1372  |
| 400  | 40  | 0.9    | Cov | 0.9802     | 0.9734  | 0.9698 | 0.9784 | 0.9626  |
|      |     |        | Len | 32.9074    | 7.6428  | 4.3445 | 4.3219 | 4.9601  |
| 1000 | 40  | 0.9    | Cov | 0.9794     | 0.9720  | 0.9716 | 0.9716 | 0.9716  |
|      |     |        | Len | 31.4758    | 7.4645  | 4.0315 | 4.0315 | 4.0315  |
| 100  | 100 | 0.9    | Cov | 0.9874     | 0.9534  | 0.6692 | 0.9814 | 0.9066  |
|      |     |        | Len | 3025.6710  | 16.3995 | 2.5480 | 4.7150 | 17.5771 |
| 400  | 100 | 0.9    | Cov | 0.9848     | 0.9644  | 0.9382 | 0.9788 | 0.9240  |
|      |     |        | Len | 137.2993   | 15.9028 | 4.1608 | 4.3296 | 8.1336  |
| 1000 | 100 | 0.9    | Cov | 0.9846     | 0.9770  | 0.9700 | 0.9780 | 0.9542  |
|      |     |        | Len | 132.0469   | 16.2782 | 4.3435 | 4.3150 | 5.7393  |
| 100  | 200 | 0.9    | Cov | 0.9860     | 0.9588  | 0.4192 | 0.9788 | 0.8912  |
|      |     |        | Len | 10591.7900 | 32.3534 | 1.2805 | 4.7283 | 33.4140 |
| 400  | 200 | 0.9    | Cov | 0.9810     | 0.9648  | 0.7982 | 0.9806 | 0.8880  |
|      |     |        | Len | 509.4511   | 28.7289 | 3.4622 | 4.3285 | 14.6730 |
| 1000 | 200 | 0.9    | Cov | 0.9790     | 0.9768  | 0.9488 | 0.9764 | 0.9172  |
|      |     |        | Len | 364.7521   | 29.9700 | 4.2375 | 4.3139 | 9.4939  |

Table 5.160. Etype = 3, J=50, k=1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0      | Cov | 0.9624 | 0.9602 | 0.9386 | 0.9602 | 0.9638 |
|      |     |        | Len | 3.9944 | 3.8048 | 4.1532 | 6.0891 | 3.7380 |
| 100  | 40  | 0      | Cov | 0.9614 | 0.9568 | 0.9120 | 0.9622 | 0.9614 |
|      |     |        | Len | 4.0635 | 3.8167 | 4.0964 | 6.1446 | 3.7435 |
| 100  | 100 | 0      | Cov | 0.9650 | 0.9578 | 0.8504 | 0.9656 | 0.9692 |
|      |     |        | Len | 4.1285 | 3.8177 | 3.7153 | 6.1969 | 3.7442 |
| 100  | 200 | 0      | Cov | 0.9592 | 0.9532 | 0.7758 | 0.9608 | 0.9612 |
|      |     |        | Len | 4.1968 | 3.8145 | 3.1906 | 6.1966 | 3.7401 |
| 400  | 20  | 0      | Cov | 0.9476 | 0.9494 | 0.9410 | 0.9412 | 0.9462 |
|      |     |        | Len | 3.2873 | 3.3461 | 3.4134 | 4.8534 | 3.3967 |
| 400  | 40  | 0      | Cov | 0.9514 | 0.9484 | 0.9314 | 0.9412 | 0.9424 |
|      |     |        | Len | 3.2865 | 3.3750 | 3.4474 | 5.1800 | 3.4227 |
| 400  | 100 | 0      | Cov | 0.9440 | 0.9368 | 0.8710 | 0.9398 | 0.9308 |
|      |     |        | Len | 3.2983 | 3.4080 | 3.3279 | 5.3826 | 3.4427 |
| 400  | 200 | 0      | Cov | 0.9484 | 0.9394 | 0.7072 | 0.9450 | 0.9316 |
|      |     |        | Len | 3.3079 | 3.4203 | 2.8149 | 5.4615 | 3.4476 |
| 1000 | 20  | 0      | Cov | 0.9442 | 0.9474 | 0.9446 | 0.9446 | 0.9446 |
|      |     |        | Len | 3.2496 | 3.2063 | 3.2584 | 3.2584 | 3.2584 |
| 1000 | 40  | 0      | Cov | 0.9466 | 0.9458 | 0.9410 | 0.9406 | 0.9424 |
|      |     |        | Len | 3.2024 | 3.2551 | 3.3201 | 4.5350 | 3.3126 |
| 1000 | 100 | 0      | Cov | 0.9512 | 0.9480 | 0.9310 | 0.9448 | 0.9404 |
|      |     |        | Len | 3.1813 | 3.3082 | 3.3811 | 5.0626 | 3.3602 |
| 1000 | 200 | 0      | Cov | 0.9488 | 0.9460 | 0.8894 | 0.9420 | 0.9346 |
|      |     |        | Len | 3.1709 | 3.3313 | 3.3249 | 5.2323 | 3.3717 |

Table 5.161. Etype = 3, J=50, k=1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0.2236 | Cov | 0.9674 | 0.9650 | 0.9622 | 0.9616 | 0.9682 |
|      |     |        | Len | 4.3200 | 3.8327 | 5.3951 | 5.4754 | 3.7457 |
| 100  | 40  | 0.1581 | Cov | 0.9640 | 0.9582 | 0.9598 | 0.9602 | 0.9630 |
|      |     |        | Len | 4.3323 | 3.8258 | 5.6169 | 5.6879 | 3.7268 |
| 100  | 100 | 0.1    | Cov | 0.9592 | 0.9586 | 0.9608 | 0.9622 | 0.9654 |
|      |     |        | Len | 4.4333 | 3.8560 | 5.8365 | 5.8983 | 3.7444 |
| 100  | 200 | 0.07   | Cov | 0.9620 | 0.9564 | 0.9554 | 0.9568 | 0.9642 |
|      |     |        | Len | 4.5215 | 3.8601 | 5.9394 | 6.0002 | 3.7434 |
| 400  | 20  | 0.2236 | Cov | 0.9496 | 0.9518 | 0.9430 | 0.9454 | 0.9426 |
|      |     |        | Len | 3.2828 | 3.2824 | 3.4065 | 4.4104 | 3.3897 |
| 400  | 40  | 0.1581 | Cov | 0.9538 | 0.9552 | 0.9360 | 0.9504 | 0.9478 |
|      |     |        | Len | 3.2816 | 3.2980 | 3.4459 | 4.8107 | 3.4179 |
| 400  | 100 | 0.1    | Cov | 0.9502 | 0.9514 | 0.8816 | 0.9480 | 0.9372 |
|      |     |        | Len | 3.3054 | 3.3314 | 3.3274 | 5.1294 | 3.4463 |
| 400  | 200 | 0.07   | Cov | 0.9460 | 0.9462 | 0.7204 | 0.9446 | 0.9296 |
|      |     |        | Len | 3.3126 | 3.3426 | 2.8288 | 5.2755 | 3.4453 |
| 1000 | 20  | 0.2236 | Cov | 0.9472 | 0.9490 | 0.9470 | 0.9470 | 0.9470 |
|      |     |        | Len | 3.2464 | 3.1684 | 3.2589 | 3.2589 | 3.2589 |
| 1000 | 40  | 0.1581 | Cov | 0.9508 | 0.9528 | 0.9460 | 0.9450 | 0.9468 |
|      |     |        | Len | 3.2020 | 3.1730 | 3.3185 | 4.2709 | 3.3108 |
| 1000 | 100 | 0.1    | Cov | 0.9464 | 0.9474 | 0.9288 | 0.9414 | 0.9392 |
|      |     |        | Len | 3.1811 | 3.1880 | 3.3781 | 4.8283 | 3.3567 |
| 1000 | 200 | 0.07   | Cov | 0.9446 | 0.9482 | 0.8904 | 0.9410 | 0.9322 |
|      |     |        | Len | 3.1721 | 3.1981 | 3.3247 | 5.0570 | 3.3711 |

Table 5.162. Etype = 3, J=50, k=1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|---------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9626  | 0.9590 | 0.9626 | 0.9626 | 0.9590 |
|      |     |        | Len | 10.9414 | 3.7808 | 3.8323 | 3.8323 | 3.7985 |
| 100  | 40  | 0.9    | Cov | 0.9634  | 0.9626 | 0.9654 | 0.9654 | 0.9638 |
|      |     |        | Len | 16.4666 | 3.7920 | 3.8418 | 3.8418 | 3.8058 |
| 100  | 100 | 0.9    | Cov | 0.9680  | 0.9608 | 0.9650 | 0.9650 | 0.9630 |
|      |     |        | Len | 27.1510 | 3.8277 | 3.8333 | 3.8333 | 3.7936 |
| 100  | 200 | 0.9    | Cov | 0.9642  | 0.9548 | 0.9638 | 0.9638 | 0.9596 |
|      |     |        | Len | 46.3657 | 3.8531 | 3.8387 | 3.8387 | 3.7939 |
| 400  | 20  | 0.9    | Cov | 0.9502  | 0.9486 | 0.9422 | 0.9492 | 0.9422 |
|      |     |        | Len | 3.2605  | 3.2743 | 3.4101 | 3.3706 | 3.3958 |
| 400  | 40  | 0.9    | Cov | 0.9512  | 0.9510 | 0.9314 | 0.9510 | 0.9418 |
|      |     |        | Len | 3.4655  | 3.2881 | 3.4455 | 3.3820 | 3.4289 |
| 400  | 100 | 0.9    | Cov | 0.9522  | 0.9512 | 0.8770 | 0.9530 | 0.9364 |
|      |     |        | Len | 4.6748  | 3.3082 | 3.3300 | 3.3912 | 3.4523 |
| 400  | 200 | 0.9    | Cov | 0.9516  | 0.9450 | 0.7190 | 0.9486 | 0.9222 |
|      |     |        | Len | 7.7790  | 3.3177 | 2.8191 | 3.3909 | 3.4567 |
| 1000 | 20  | 0.9    | Cov | 0.9538  | 0.9528 | 0.9510 | 0.9510 | 0.9510 |
|      |     |        | Len | 3.1219  | 3.1452 | 3.2583 | 3.2583 | 3.2583 |
| 1000 | 40  | 0.9    | Cov | 0.9512  | 0.9504 | 0.9420 | 0.9480 | 0.9408 |
|      |     |        | Len | 3.1219  | 3.1543 | 3.3182 | 3.2879 | 3.3102 |
| 1000 | 100 | 0.9    | Cov | 0.9494  | 0.9474 | 0.9302 | 0.9458 | 0.9370 |
|      |     |        | Len | 3.1593  | 3.1692 | 3.3769 | 3.3095 | 3.3576 |
| 1000 | 200 | 0.9    | Cov | 0.9540  | 0.9508 | 0.8954 | 0.9512 | 0.9362 |
|      |     |        | Len | 3.2081  | 3.1826 | 3.3277 | 3.3192 | 3.3751 |

Table 5.163. Etype = 3, J=50, k=19,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 20  | 0      | Cov | 0.9568  | 0.9254  | 0.9322 | 0.9612  | 0.9502  |
|      |     |        | Len | 18.0945 | 14.2843 | 7.4280 | 18.9841 | 18.3152 |
| 100  | 40  | 0      | Cov | 0.9626  | 0.9278  | 0.8946 | 0.9672  | 0.9580  |
|      |     |        | Len | 18.0581 | 14.2584 | 8.5690 | 19.3488 | 18.3198 |
| 100  | 100 | 0      | Cov | 0.9534  | 0.9136  | 0.8184 | 0.9584  | 0.9508  |
|      |     |        | Len | 18.0820 | 14.2339 | 9.1398 | 19.6271 | 18.3224 |
| 100  | 200 | 0      | Cov | 0.9678  | 0.9460  | 0.7438 | 0.9684  | 0.9606  |
|      |     |        | Len | 19.0913 | 16.1341 | 8.5696 | 19.6777 | 18.2836 |
| 400  | 20  | 0      | Cov | 0.9548  | 0.9390  | 0.9542 | 0.9422  | 0.9384  |
|      |     |        | Len | 16.9204 | 11.3395 | 3.6648 | 14.2100 | 13.7638 |
| 400  | 40  | 0      | Cov | 0.9550  | 0.9358  | 0.9446 | 0.9510  | 0.9332  |
|      |     |        | Len | 16.8844 | 11.3228 | 3.7000 | 16.3942 | 13.7577 |
| 400  | 100 | 0      | Cov | 0.9568  | 0.9416  | 0.8952 | 0.9558  | 0.9398  |
|      |     |        | Len | 16.8874 | 11.2838 | 3.5654 | 17.6544 | 13.7700 |
| 400  | 200 | 0      | Cov | 0.9542  | 0.9418  | 0.7514 | 0.9552  | 0.9368  |
|      |     |        | Len | 16.8928 | 11.2742 | 3.0564 | 18.1475 | 13.7715 |
| 1000 | 20  | 0      | Cov | 0.9618  | 0.9620  | 0.9620 | 0.9620  | 0.9620  |
|      |     |        | Len | 3.5654  | 3.5587  | 3.5587 | 3.5587  | 3.5587  |
| 1000 | 40  | 0      | Cov | 0.9684  | 0.9638  | 0.9618 | 0.9502  | 0.9654  |
|      |     |        | Len | 4.0219  | 3.5686  | 3.6227 | 12.7115 | 3.5575  |
| 1000 | 100 | 0      | Cov | 0.9590  | 0.9602  | 0.9456 | 0.9482  | 0.9624  |
|      |     |        | Len | 4.2084  | 3.5768  | 3.6801 | 16.1112 | 3.5598  |
| 1000 | 200 | 0      | Cov | 0.9598  | 0.9606  | 0.9170 | 0.9562  | 0.9620  |
|      |     |        | Len | 4.3099  | 3.5756  | 3.6105 | 17.2043 | 3.5552  |

Table 5.164. Etype = 3, J=50, k=19,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 100  | 20  | 0.2236 | Cov | 0.9648  | 0.9216  | 0.9650  | 0.9646  | 0.9364  |
|      |     |        | Len | 70.0260 | 16.0576 | 3.9333  | 3.9585  | 46.7489 |
| 100  | 40  | 0.1581 | Cov | 0.9604  | 0.9084  | 0.9662  | 0.9670  | 0.9316  |
|      |     |        | Len | 69.0525 | 17.3747 | 12.2804 | 12.5084 | 48.8759 |
| 100  | 100 | 0.1    | Cov | 0.9596  | 0.8822  | 0.9634  | 0.9648  | 0.9320  |
|      |     |        | Len | 67.5682 | 18.1072 | 16.0544 | 16.2718 | 50.1554 |
| 100  | 200 | 0.07   | Cov | 0.9626  | 0.8858  | 0.9656  | 0.9662  | 0.9288  |
|      |     |        | Len | 78.0310 | 25.2198 | 17.4941 | 17.7004 | 50.2342 |
| 400  | 20  | 0.2236 | Cov | 0.9588  | 0.9524  | 0.9616  | 0.9628  | 0.9310  |
|      |     |        | Len | 65.8737 | 10.5683 | 3.6687  | 3.6172  | 16.5396 |
| 400  | 40  | 0.1581 | Cov | 0.9600  | 0.9476  | 0.9422  | 0.9548  | 0.9262  |
|      |     |        | Len | 64.8400 | 11.3780 | 3.7012  | 10.6168 | 17.8030 |
| 400  | 100 | 0.1    | Cov | 0.9580  | 0.9474  | 0.8962  | 0.9548  | 0.9276  |
|      |     |        | Len | 63.4644 | 12.3146 | 3.5616  | 14.6268 | 18.9488 |
| 400  | 200 | 0.07   | Cov | 0.9568  | 0.9372  | 0.7402  | 0.9592  | 0.9268  |
|      |     |        | Len | 61.9664 | 12.8776 | 3.0998  | 16.2209 | 19.6442 |
| 1000 | 20  | 0.2236 | Cov | 0.9660  | 0.9630  | 0.9630  | 0.9630  | 0.9630  |
|      |     |        | Len | 3.7035  | 3.5576  | 3.5576  | 3.5576  | 3.5576  |
| 1000 | 40  | 0.1581 | Cov | 0.9626  | 0.9566  | 0.9546  | 0.9436  | 0.9568  |
|      |     |        | Len | 18.3475 | 3.5911  | 3.6191  | 8.4917  | 3.5561  |
| 1000 | 100 | 0.1    | Cov | 0.9634  | 0.9560  | 0.9372  | 0.9500  | 0.9562  |
|      |     |        | Len | 31.4030 | 3.7841  | 3.6748  | 13.3632 | 3.5626  |
| 1000 | 200 | 0.07   | Cov | 0.9666  | 0.9596  | 0.9176  | 0.9538  | 0.9630  |
|      |     |        | Len | 40.4800 | 4.2393  | 3.6119  | 15.3912 | 3.5693  |

Table 5.165. Etype = 3, J=50, k=19,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|----------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9706   | 0.9388 | 0.9650 | 0.9650 | 0.9366 |
|      |     |        | Len | 230.6223 | 5.0067 | 3.7450 | 3.7450 | 8.1529 |
| 100  | 40  | 0.9    | Cov | 0.9642   | 0.9400 | 0.9662 | 0.9662 | 0.9270 |
|      |     |        | Len | 329.9553 | 4.9791 | 4.2715 | 4.2715 | 8.0814 |
| 100  | 100 | 0.9    | Cov | 0.9694   | 0.9292 | 0.9618 | 0.9618 | 0.9308 |
|      |     |        | Len | 523.5096 | 4.7851 | 4.4544 | 4.4544 | 7.9819 |
| 100  | 200 | 0.9    | Cov | 0.9680   | 0.9298 | 0.9610 | 0.9610 | 0.9178 |
|      |     |        | Len | 893.6305 | 4.7265 | 4.5118 | 4.5118 | 7.8609 |
| 400  | 20  | 0.9    | Cov | 0.9636   | 0.9572 | 0.9576 | 0.9646 | 0.9496 |
|      |     |        | Len | 96.2901  | 4.3825 | 3.6590 | 3.5715 | 4.3506 |
| 400  | 40  | 0.9    | Cov | 0.9594   | 0.9518 | 0.9424 | 0.9584 | 0.9414 |
|      |     |        | Len | 69.2012  | 4.7417 | 3.6920 | 3.9697 | 4.3524 |
| 400  | 100 | 0.9    | Cov | 0.9588   | 0.9498 | 0.8956 | 0.9576 | 0.9414 |
|      |     |        | Len | 124.7301 | 4.8335 | 3.5631 | 4.1959 | 4.3771 |
| 400  | 200 | 0.9    | Cov | 0.9614   | 0.9508 | 0.7554 | 0.9598 | 0.9378 |
|      |     |        | Len | 215.4489 | 4.7771 | 3.0108 | 4.2744 | 4.3809 |
| 1000 | 20  | 0.9    | Cov | 0.9664   | 0.9598 | 0.9602 | 0.9602 | 0.9602 |
|      |     |        | Len | 9.6969   | 4.1974 | 3.5466 | 3.5466 | 3.5466 |
| 1000 | 40  | 0.9    | Cov | 0.9582   | 0.9586 | 0.9592 | 0.9614 | 0.9592 |
|      |     |        | Len | 13.5855  | 4.8134 | 3.5632 | 3.7576 | 3.5731 |
| 1000 | 100 | 0.9    | Cov | 0.9624   | 0.9560 | 0.9394 | 0.9592 | 0.9514 |
|      |     |        | Len | 21.9236  | 4.9075 | 3.5995 | 4.0062 | 3.6186 |
| 1000 | 200 | 0.9    | Cov | 0.9606   | 0.9528 | 0.9108 | 0.9584 | 0.9504 |
|      |     |        | Len | 30.2919  | 4.9066 | 3.5292 | 4.1001 | 3.6562 |

Table 5.166. Etype = 3, J=50, k=p-1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 100  | 40  | 0      | Cov | 0.9584  | 0.9106  | 0.8906  | 0.9646  | 0.9508  |
|      |     |        | Len | 25.9217 | 21.2087 | 11.6953 | 27.3291 | 26.2453 |
| 400  | 40  | 0      | Cov | 0.9554  | 0.9288  | 0.9424  | 0.9502  | 0.9292  |
|      |     |        | Len | 24.7895 | 19.5493 | 3.7066  | 23.0648 | 21.8866 |
| 1000 | 40  | 0      | Cov | 0.9580  | 0.9430  | 0.9576  | 0.9438  | 0.9420  |
|      |     |        | Len | 23.7430 | 15.2729 | 3.6203  | 17.6395 | 17.2337 |
| 100  | 100 | 0      | Cov | 0.9500  | 0.8800  | 0.8100  | 0.9582  | 0.9376  |
|      |     |        | Len | 41.2669 | 34.0048 | 19.7201 | 43.8646 | 41.9770 |
| 400  | 100 | 0      | Cov | 0.9542  | 0.9170  | 0.8954  | 0.9554  | 0.9360  |
|      |     |        | Len | 40.0461 | 34.1398 | 3.5688  | 39.3103 | 36.8825 |
| 1000 | 100 | 0      | Cov | 0.9576  | 0.9332  | 0.9496  | 0.9526  | 0.9314  |
|      |     |        | Len | 39.4560 | 32.0500 | 3.6792  | 35.7837 | 33.7183 |
| 400  | 200 | 0      | Cov | 0.9462  | 0.9108  | 0.6976  | 0.9472  | 0.9226  |
|      |     |        | Len | 55.1043 | 47.9555 | 3.3893  | 55.1655 | 51.5128 |
| 1000 | 200 | 0      | Cov | 0.9622  | 0.9306  | 0.9176  | 0.9552  | 0.9280  |
|      |     |        | Len | 56.7019 | 48.7998 | 3.6097  | 54.1929 | 50.5728 |

Table 5.167. Etype = 3, J=50, k=p-1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL       | PLS    | PCR    | FS       |
|------|-----|--------|-----|----------|----------|--------|--------|----------|
| 100  | 40  | 0.1581 | Cov | 0.9642   | 0.9006   | 0.9658 | 0.9662 | 0.9400   |
|      |     |        | Len | 140.3552 | 34.2855  | 4.1832 | 4.2165 | 99.0408  |
| 100  | 100 | 0.1    | Cov | 0.9584   | 0.8626   | 0.9618 | 0.9632 | 0.9218   |
|      |     |        | Len | 345.5432 | 85.3149  | 4.8078 | 4.8526 | 254.1073 |
| 100  | 200 | 0.07   | Cov | 0.9640   | 0.8790   | 0.9580 | 0.9588 | 0.9194   |
|      |     |        | Len | 799.7382 | 236.6724 | 5.6739 | 5.7304 | 506.2869 |
| 400  | 40  | 0.1581 | Cov | 0.9588   | 0.9376   | 0.9476 | 0.9606 | 0.9162   |
|      |     |        | Len | 131.8074 | 24.5767  | 3.6941 | 3.6915 | 38.8234  |
| 400  | 100 | 0.1    | Cov | 0.9554   | 0.9284   | 0.8930 | 0.9616 | 0.9094   |
|      |     |        | Len | 324.7592 | 65.1255  | 3.5620 | 3.9346 | 105.7167 |
| 400  | 200 | 0.07   | Cov | 0.9584   | 0.9152   | 0.7336 | 0.9606 | 0.8996   |
|      |     |        | Len | 636.9782 | 130.6055 | 3.0075 | 4.2697 | 215.5844 |
| 1000 | 40  | 0.1581 | Cov | 0.9640   | 0.9520   | 0.9582 | 0.9596 | 0.9346   |
|      |     |        | Len | 128.9778 | 18.0767  | 3.6194 | 3.5833 | 20.0816  |
| 1000 | 100 | 0.1    | Cov | 0.9550   | 0.9414   | 0.9488 | 0.9618 | 0.9164   |
|      |     |        | Len | 320.4386 | 52.9085  | 3.6781 | 3.7016 | 63.0990  |
| 1000 | 200 | 0.07   | Cov | 0.9582   | 0.9364   | 0.9136 | 0.9594 | 0.9086   |
|      |     |        | Len | 625.8618 | 107.7568 | 3.6102 | 3.8816 | 133.2308 |

Table 5.168. Etype = 3, J=50, k=p-1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso     | RL      | PLS    | PCR    | FS      |
|------|-----|--------|-----|-----------|---------|--------|--------|---------|
| 100  | 40  | 0.9    | Cov | 0.9660    | 0.9296  | 0.9608 | 0.9608 | 0.9328  |
|      |     |        | Len | 675.2869  | 7.2529  | 3.7490 | 3.7490 | 15.0066 |
| 400  | 40  | 0.9    | Cov | 0.9676    | 0.9480  | 0.9472 | 0.9624 | 0.9248  |
|      |     |        | Len | 295.8083  | 6.6366  | 3.6964 | 3.5787 | 6.0805  |
| 1000 | 40  | 0.9    | Cov | 0.9594    | 0.9532  | 0.9566 | 0.9580 | 0.9478  |
|      |     |        | Len | 28.9976   | 6.8519  | 3.6209 | 3.5550 | 4.4977  |
| 100  | 100 | 0.9    | Cov | 0.9684    | 0.9194  | 0.9658 | 0.9658 | 0.9214  |
|      |     |        | Len | 2723.3590 | 14.6029 | 3.7405 | 3.7405 | 36.3488 |
| 400  | 100 | 0.9    | Cov | 0.9642    | 0.9522  | 0.8908 | 0.9616 | 0.9048  |
|      |     |        | Len | 1849.4080 | 13.7998 | 3.5630 | 3.5818 | 12.8216 |
| 1000 | 100 | 0.9    | Cov | 0.9648    | 0.9504  | 0.9400 | 0.9596 | 0.9210  |
|      |     |        | Len | 122.0355  | 14.1967 | 3.6803 | 3.5590 | 8.0451  |
| 100  | 200 | 0.9    | Cov | 0.9672    | 0.9192  | 0.9640 | 0.9640 | 0.9152  |
|      |     |        | Len | 9360.4090 | 28.6047 | 3.7406 | 3.7406 | 71.3896 |
| 400  | 200 | 0.9    | Cov | 0.9626    | 0.9390  | 0.7484 | 0.9638 | 0.8986  |
|      |     |        | Len | 6157.3260 | 25.8033 | 3.0048 | 3.5753 | 24.7109 |
| 1000 | 200 | 0.9    | Cov | 0.9618    | 0.9456  | 0.9212 | 0.9656 | 0.9040  |
|      |     |        | Len | 425.1361  | 26.2287 | 3.6143 | 3.5604 | 15.0387 |

Table 5.169. Etype = 4, J=5, k=1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0      | Cov | 0.9106 | 0.9526 | 0.9082 | 0.9082 | 0.9082 |
|      |     |        | Len | 2.1562 | 2.2071 | 2.1607 | 2.1607 | 2.1607 |
| 100  | 40  | 0      | Cov | 0.9114 | 0.9290 | 0.7950 | 0.9062 | 0.8250 |
|      |     |        | Len | 2.0180 | 2.1664 | 1.9488 | 3.1987 | 1.9996 |
| 100  | 100 | 0      | Cov | 0.8974 | 0.8792 | 0.1266 | 0.9192 | 0.6812 |
|      |     |        | Len | 1.9524 | 2.0690 | 0.5571 | 3.8545 | 1.6775 |
| 100  | 200 | 0      | Cov | 0.9034 | 0.8564 | 0.0012 | 0.9270 | 0.5932 |
|      |     |        | Len | 1.9478 | 1.9952 | 0.0019 | 4.1324 | 1.4311 |
| 400  | 20  | 0      | Cov | 0.9442 | 0.9588 | 0.9424 | 0.9424 | 0.9424 |
|      |     |        | Len | 1.9691 | 1.9688 | 1.9734 | 1.9734 | 1.9734 |
| 400  | 40  | 0      | Cov | 0.9098 | 0.9366 | 0.9088 | 0.9088 | 0.9088 |
|      |     |        | Len | 1.9649 | 1.9730 | 1.9695 | 1.9695 | 1.9695 |
| 400  | 100 | 0      | Cov | 0.8546 | 0.9004 | 0.8310 | 0.8504 | 0.8326 |
|      |     |        | Len | 1.8638 | 1.9631 | 1.8889 | 2.2831 | 1.8894 |
| 400  | 200 | 0      | Cov | 0.8346 | 0.8590 | 0.6560 | 0.8676 | 0.7232 |
|      |     |        | Len | 1.7607 | 1.9194 | 1.6074 | 3.0175 | 1.7068 |
| 1000 | 20  | 0      | Cov | 0.9474 | 0.9542 | 0.9476 | 0.9476 | 0.9476 |
|      |     |        | Len | 1.9206 | 1.9206 | 1.9236 | 1.9236 | 1.9236 |
| 1000 | 40  | 0      | Cov | 0.9352 | 0.9452 | 0.9350 | 0.9350 | 0.9350 |
|      |     |        | Len | 1.9256 | 1.9240 | 1.9298 | 1.9298 | 1.9298 |
| 1000 | 100 | 0      | Cov | 0.9038 | 0.9252 | 0.9042 | 0.9042 | 0.9042 |
|      |     |        | Len | 1.9244 | 1.9319 | 1.9291 | 1.9291 | 1.9291 |
| 1000 | 200 | 0      | Cov | 0.8458 | 0.8968 | 0.8464 | 0.8464 | 0.8464 |
|      |     |        | Len | 1.8816 | 1.9287 | 1.8857 | 1.8857 | 1.8857 |

Table 5.170. Etype = 4, J=5, k=1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0.2236 | Cov | 0.9228 | 0.9782 | 0.9192 | 0.9192 | 0.9192 |
|      |     |        | Len | 2.1508 | 2.2138 | 2.1574 | 2.1574 | 2.1574 |
| 100  | 40  | 0.1581 | Cov | 0.9078 | 0.9714 | 0.7932 | 0.9016 | 0.8226 |
|      |     |        | Len | 2.0208 | 2.2145 | 1.9469 | 2.9513 | 1.9988 |
| 100  | 100 | 0.1    | Cov | 0.8944 | 0.9620 | 0.1524 | 0.9142 | 0.6824 |
|      |     |        | Len | 1.9581 | 2.2073 | 0.5784 | 3.6156 | 1.6795 |
| 100  | 200 | 0.07   | Cov | 0.9014 | 0.9544 | 0.0010 | 0.9272 | 0.5820 |
|      |     |        | Len | 1.9579 | 2.2068 | 0.0027 | 3.9464 | 1.4401 |
| 400  | 20  | 0.2236 | Cov | 0.9380 | 0.9568 | 0.9366 | 0.9366 | 0.9366 |
|      |     |        | Len | 1.9676 | 1.9655 | 1.9738 | 1.9738 | 1.9738 |
| 400  | 40  | 0.1581 | Cov | 0.9154 | 0.9566 | 0.9128 | 0.9128 | 0.9128 |
|      |     |        | Len | 1.9614 | 1.9690 | 1.9707 | 1.9707 | 1.9707 |
| 400  | 100 | 0.1    | Cov | 0.8466 | 0.9368 | 0.8254 | 0.8562 | 0.8260 |
|      |     |        | Len | 1.8638 | 1.9742 | 1.8882 | 2.2235 | 1.8892 |
| 400  | 200 | 0.07   | Cov | 0.8458 | 0.9402 | 0.6526 | 0.8758 | 0.7280 |
|      |     |        | Len | 1.7607 | 1.9736 | 1.6048 | 2.9007 | 1.7055 |
| 1000 | 20  | 0.2236 | Cov | 0.9452 | 0.9536 | 0.9470 | 0.9470 | 0.9470 |
|      |     |        | Len | 1.9195 | 1.9195 | 1.9239 | 1.9239 | 1.9239 |
| 1000 | 40  | 0.1581 | Cov | 0.9346 | 0.9538 | 0.9328 | 0.9328 | 0.9328 |
|      |     |        | Len | 1.9246 | 1.9203 | 1.9304 | 1.9304 | 1.9304 |
| 1000 | 100 | 0.1    | Cov | 0.9056 | 0.9472 | 0.9046 | 0.9046 | 0.9046 |
|      |     |        | Len | 1.9225 | 1.9251 | 1.9289 | 1.9289 | 1.9289 |
| 1000 | 200 | 0.07   | Cov | 0.8460 | 0.9250 | 0.8436 | 0.8436 | 0.8436 |
|      |     |        | Len | 1.8797 | 1.9316 | 1.8855 | 1.8855 | 1.8855 |

Table 5.171. Etype = 4, J=5, k=1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9868 | 0.9840 | 0.9148 | 0.9148 | 0.9148 |
|      |     |        | Len | 2.1991 | 2.2052 | 2.1539 | 2.1539 | 2.1539 |
| 100  | 40  | 0.9    | Cov | 0.9900 | 0.9772 | 0.7950 | 0.9318 | 0.8228 |
|      |     |        | Len | 2.2013 | 2.2089 | 1.9431 | 2.1707 | 1.9925 |
| 100  | 100 | 0.9    | Cov | 0.9916 | 0.9778 | 0.1466 | 0.9526 | 0.6826 |
|      |     |        | Len | 2.2226 | 2.2069 | 0.5597 | 2.1802 | 1.6719 |
| 100  | 200 | 0.9    | Cov | 0.9858 | 0.9676 | 0.0004 | 0.9574 | 0.5772 |
|      |     |        | Len | 2.3143 | 2.2061 | 0.0021 | 2.1870 | 1.4232 |
| 400  | 20  | 0.9    | Cov | 0.9670 | 0.9616 | 0.9368 | 0.9368 | 0.9368 |
|      |     |        | Len | 1.9642 | 1.9640 | 1.9729 | 1.9729 | 1.9729 |
| 400  | 40  | 0.9    | Cov | 0.9668 | 0.9608 | 0.9146 | 0.9146 | 0.9146 |
|      |     |        | Len | 1.9689 | 1.9647 | 1.9687 | 1.9687 | 1.9687 |
| 400  | 100 | 0.9    | Cov | 0.9686 | 0.9606 | 0.8350 | 0.8728 | 0.8366 |
|      |     |        | Len | 1.9853 | 1.9657 | 1.8878 | 1.9288 | 1.8889 |
| 400  | 200 | 0.9    | Cov | 0.9642 | 0.9588 | 0.6560 | 0.8864 | 0.7208 |
|      |     |        | Len | 2.0083 | 1.9668 | 1.6053 | 1.9407 | 1.7056 |
| 1000 | 20  | 0.9    | Cov | 0.9596 | 0.9576 | 0.9454 | 0.9454 | 0.9454 |
|      |     |        | Len | 1.9221 | 1.9193 | 1.9239 | 1.9239 | 1.9239 |
| 1000 | 40  | 0.9    | Cov | 0.9598 | 0.9560 | 0.9308 | 0.9308 | 0.9308 |
|      |     |        | Len | 1.9268 | 1.9195 | 1.9303 | 1.9303 | 1.9303 |
| 1000 | 100 | 0.9    | Cov | 0.9538 | 0.9514 | 0.9066 | 0.9066 | 0.9066 |
|      |     |        | Len | 1.9406 | 1.9199 | 1.9286 | 1.9286 | 1.9286 |
| 1000 | 200 | 0.9    | Cov | 0.9532 | 0.9428 | 0.8514 | 0.8514 | 0.8514 |
|      |     |        | Len | 1.9645 | 1.9201 | 1.8855 | 1.8855 | 1.8855 |

Table 5.172. Etype = 4, J=5, k=19,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL     | PLS    | PCR     | FS     |
|------|-----|--------|-----|---------|--------|--------|---------|--------|
| 100  | 20  | 0      | Cov | 0.9918  | 0.9912 | 0.9912 | 0.9912  | 0.9912 |
|      |     |        | Len | 2.9735  | 2.9663 | 2.9663 | 2.9663  | 2.9663 |
| 100  | 40  | 0      | Cov | 0.9804  | 0.9860 | 0.9274 | 0.9622  | 0.9888 |
|      |     |        | Len | 10.7320 | 3.6221 | 2.7125 | 14.3093 | 3.3131 |
| 100  | 100 | 0      | Cov | 0.9834  | 0.9718 | 0.2052 | 0.9768  | 0.9772 |
|      |     |        | Len | 14.8601 | 6.4804 | 0.9336 | 19.4698 | 4.3356 |
| 100  | 200 | 0      | Cov | 0.9770  | 0.9524 | 0.0016 | 0.9808  | 0.9444 |
|      |     |        | Len | 16.7004 | 9.1315 | 0.0075 | 21.1657 | 5.5755 |
| 400  | 20  | 0      | Cov | 0.9838  | 0.9834 | 0.9834 | 0.9834  | 0.9834 |
|      |     |        | Len | 2.2302  | 2.2223 | 2.2223 | 2.2223  | 2.2223 |
| 400  | 40  | 0      | Cov | 0.9754  | 0.9804 | 0.9714 | 0.9714  | 0.9714 |
|      |     |        | Len | 2.2371  | 2.2412 | 2.2493 | 2.2493  | 2.2493 |
| 400  | 100 | 0      | Cov | 0.9334  | 0.9456 | 0.9084 | 0.8890  | 0.9112 |
|      |     |        | Len | 2.1648  | 2.2460 | 2.1973 | 6.4029  | 2.1987 |
| 400  | 200 | 0      | Cov | 0.9270  | 0.9274 | 0.7442 | 0.9060  | 0.8254 |
|      |     |        | Len | 2.1390  | 2.2172 | 1.9042 | 11.9135 | 2.0356 |
| 1000 | 20  | 0      | Cov | 0.9728  | 0.9734 | 0.9734 | 0.9734  | 0.9734 |
|      |     |        | Len | 2.0126  | 2.0060 | 2.0060 | 2.0060  | 2.0060 |
| 1000 | 40  | 0      | Cov | 0.9610  | 0.9628 | 0.9570 | 0.9570  | 0.9570 |
|      |     |        | Len | 2.0132  | 2.0138 | 2.0198 | 2.0198  | 2.0198 |
| 1000 | 100 | 0      | Cov | 0.9256  | 0.9474 | 0.9212 | 0.9212  | 0.9212 |
|      |     |        | Len | 2.0123  | 2.0268 | 2.0302 | 2.0302  | 2.0302 |
| 1000 | 200 | 0      | Cov | 0.8850  | 0.9292 | 0.8788 | 0.8788  | 0.8788 |
|      |     |        | Len | 1.9778  | 2.0294 | 1.9960 | 1.9960  | 1.9960 |

Table 5.173. Etype = 4, J=5, k=19,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS     |
|------|-----|--------|-----|---------|---------|--------|---------|--------|
| 100  | 20  | 0.2236 | Cov | 0.9916  | 0.9906  | 0.9906 | 0.9906  | 0.9906 |
|      |     |        | Len | 4.1799  | 2.9670  | 2.9670 | 2.9670  | 2.9670 |
| 100  | 40  | 0.1581 | Cov | 0.9914  | 0.9714  | 0.9318 | 0.9696  | 0.9770 |
|      |     |        | Len | 46.0972 | 8.1099  | 2.7131 | 9.1500  | 4.7542 |
| 100  | 100 | 0.1    | Cov | 0.9904  | 0.9432  | 0.2308 | 0.9776  | 0.9398 |
|      |     |        | Len | 58.0956 | 13.5807 | 0.9369 | 16.0378 | 7.5798 |
| 100  | 200 | 0.07   | Cov | 0.9924  | 0.9194  | 0.0008 | 0.9812  | 0.8762 |
|      |     |        | Len | 64.6805 | 16.3557 | 0.0104 | 19.1009 | 9.4730 |
| 400  | 20  | 0.2236 | Cov | 0.9804  | 0.9848  | 0.9848 | 0.9848  | 0.9848 |
|      |     |        | Len | 3.0769  | 2.2216  | 2.2216 | 2.2216  | 2.2216 |
| 400  | 40  | 0.1581 | Cov | 0.9794  | 0.9800  | 0.9714 | 0.9714  | 0.9714 |
|      |     |        | Len | 3.0104  | 2.2395  | 2.2505 | 2.2505  | 2.2505 |
| 400  | 100 | 0.1    | Cov | 0.9744  | 0.9654  | 0.9116 | 0.8840  | 0.9098 |
|      |     |        | Len | 2.9427  | 2.2495  | 2.1990 | 5.3969  | 2.2003 |
| 400  | 200 | 0.07   | Cov | 0.9714  | 0.9558  | 0.7418 | 0.9166  | 0.8292 |
|      |     |        | Len | 2.8713  | 2.2527  | 1.9078 | 10.6707 | 2.0386 |
| 1000 | 20  | 0.2236 | Cov | 0.9594  | 0.9666  | 0.9666 | 0.9666  | 0.9666 |
|      |     |        | Len | 2.7192  | 2.0062  | 2.0062 | 2.0062  | 2.0062 |
| 1000 | 40  | 0.1581 | Cov | 0.9586  | 0.9604  | 0.9538 | 0.9538  | 0.9538 |
|      |     |        | Len | 2.6565  | 2.0122  | 2.0196 | 2.0196  | 2.0196 |
| 1000 | 100 | 0.1    | Cov | 0.9598  | 0.9588  | 0.9306 | 0.9306  | 0.9306 |
|      |     |        | Len | 2.5730  | 2.0197  | 2.0301 | 2.0301  | 2.0301 |
| 1000 | 200 | 0.07   | Cov | 0.9604  | 0.9482  | 0.8788 | 0.8788  | 0.8788 |
|      |     |        | Len | 2.4943  | 2.0239  | 1.9944 | 1.9944  | 1.9944 |

Table 5.174. Etype = 4, J=5, k=19,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|---------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9924  | 0.9802 | 0.9798 | 0.9798 | 0.9798 |
|      |     |        | Len | 13.2013 | 3.5656 | 2.7169 | 2.7169 | 2.7169 |
| 100  | 40  | 0.9    | Cov | 0.9894  | 0.9770 | 0.8850 | 0.9770 | 0.9170 |
|      |     |        | Len | 18.3335 | 4.1014 | 2.3903 | 2.7924 | 2.5406 |
| 100  | 100 | 0.9    | Cov | 0.9888  | 0.9684 | 0.1696 | 0.9770 | 0.7970 |
|      |     |        | Len | 29.0128 | 4.1275 | 0.6937 | 3.0533 | 2.3012 |
| 100  | 200 | 0.9    | Cov | 0.9896  | 0.9644 | 0.0010 | 0.9786 | 0.7056 |
|      |     |        | Len | 43.7858 | 3.8213 | 0.0027 | 3.1783 | 2.0775 |
| 400  | 20  | 0.9    | Cov | 0.9804  | 0.9736 | 0.9830 | 0.9830 | 0.9830 |
|      |     |        | Len | 11.0222 | 3.0735 | 2.2226 | 2.2226 | 2.2226 |
| 400  | 40  | 0.9    | Cov | 0.9788  | 0.9724 | 0.9694 | 0.9694 | 0.9694 |
|      |     |        | Len | 15.7686 | 3.8606 | 2.2168 | 2.2168 | 2.2168 |
| 400  | 100 | 0.9    | Cov | 0.9768  | 0.9670 | 0.8920 | 0.9238 | 0.8938 |
|      |     |        | Len | 24.6854 | 3.9706 | 2.1371 | 2.2493 | 2.1387 |
| 400  | 200 | 0.9    | Cov | 0.9808  | 0.9692 | 0.7226 | 0.9294 | 0.7956 |
|      |     |        | Len | 33.6717 | 3.8919 | 1.8333 | 2.4613 | 1.9632 |
| 1000 | 20  | 0.9    | Cov | 0.9656  | 0.9544 | 0.9678 | 0.9678 | 0.9678 |
|      |     |        | Len | 9.7087  | 2.7306 | 2.0056 | 2.0056 | 2.0056 |
| 1000 | 40  | 0.9    | Cov | 0.9620  | 0.9620 | 0.9562 | 0.9562 | 0.9562 |
|      |     |        | Len | 14.0989 | 3.5390 | 2.0190 | 2.0190 | 2.0190 |
| 1000 | 100 | 0.9    | Cov | 0.9640  | 0.9632 | 0.9276 | 0.9276 | 0.9276 |
|      |     |        | Len | 22.3844 | 3.7271 | 2.0295 | 2.0295 | 2.0295 |
| 1000 | 200 | 0.9    | Cov | 0.9704  | 0.9568 | 0.8750 | 0.8750 | 0.8750 |
|      |     |        | Len | 30.7072 | 3.7368 | 1.9946 | 1.9946 | 1.9946 |

Table 5.175. Etype = 4, J=5, k=p-1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 40  | 0      | Cov | 0.9818  | 0.9386  | 0.9266 | 0.9636  | 0.9056  |
|      |     |        | Len | 26.7237 | 17.7299 | 2.7036 | 20.1015 | 16.9753 |
| 400  | 40  | 0      | Cov | 0.9850  | 0.9854  | 0.9854 | 0.9854  | 0.9854  |
|      |     |        | Len | 2.4245  | 2.4102  | 2.4102 | 2.4102  | 2.4102  |
| 1000 | 40  | 0      | Cov | 0.9774  | 0.9772  | 0.9772 | 0.9772  | 0.9772  |
|      |     |        | Len | 2.1424  | 2.1266  | 2.1266 | 2.1266  | 2.1266  |
| 100  | 100 | 0      | Cov | 0.9264  | 0.7932  | 0.1418 | 0.9188  | 0.7040  |
|      |     |        | Len | 34.8955 | 24.4500 | 0.9745 | 32.5132 | 22.1249 |
| 400  | 100 | 0      | Cov | 0.9780  | 0.9460  | 0.9714 | 0.9366  | 0.9132  |
|      |     |        | Len | 33.1227 | 17.9061 | 2.6979 | 16.8337 | 17.4085 |
| 1000 | 100 | 0      | Cov | 0.9832  | 0.9836  | 0.9836 | 0.9836  | 0.9836  |
|      |     |        | Len | 2.4224  | 2.3854  | 2.3854 | 2.3854  | 2.3854  |
| 100  | 200 | 0      | Cov | 0.9238  | 0.7442  | 0.0010 | 0.9226  | 0.5892  |
|      |     |        | Len | 49.8576 | 34.8051 | 0.0169 | 48.9723 | 28.9765 |
| 400  | 200 | 0      | Cov | 0.8928  | 0.7846  | 0.6790 | 0.8622  | 0.7080  |
|      |     |        | Len | 40.8303 | 28.1871 | 1.6704 | 32.8659 | 25.6120 |
| 1000 | 200 | 0      | Cov | 0.9838  | 0.9836  | 0.9836 | 0.9836  | 0.9836  |
|      |     |        | Len | 2.7869  | 2.7135  | 2.7135 | 2.7135  | 2.7135  |

Table 5.176. Etype = 4, J=5, k=p-1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL       | PLS    | PCR    | FS      |
|------|-----|--------|-----|----------|----------|--------|--------|---------|
| 100  | 40  | 0.1581 | Cov | 0.9918   | 0.9482   | 0.9322 | 0.9936 | 0.8960  |
|      |     |        | Len | 112.8022 | 23.4280  | 2.7168 | 3.1507 | 19.1970 |
| 100  | 100 | 0.1    | Cov | 0.9910   | 0.9250   | 0.1976 | 0.9880 | 0.8006  |
|      |     |        | Len | 330.8742 | 71.3678  | 0.7993 | 3.9301 | 50.1268 |
| 100  | 200 | 0.07   | Cov | 0.9902   | 0.9056   | 0.0020 | 0.9866 | 0.7232  |
|      |     |        | Len | 711.0404 | 151.1371 | 0.0037 | 5.1916 | 92.7812 |
| 400  | 40  | 0.1581 | Cov | 0.9820   | 0.9848   | 0.9848 | 0.9848 | 0.9848  |
|      |     |        | Len | 6.9122   | 2.4096   | 2.4096 | 2.4096 | 2.4096  |
| 400  | 100 | 0.1    | Cov | 0.9902   | 0.9596   | 0.9710 | 0.9878 | 0.9136  |
|      |     |        | Len | 119.7442 | 20.3280  | 2.7043 | 2.7877 | 17.4846 |
| 400  | 200 | 0.07   | Cov | 0.9858   | 0.9328   | 0.8400 | 0.9840 | 0.8280  |
|      |     |        | Len | 346.6200 | 67.3994  | 2.3387 | 3.1328 | 48.0680 |
| 1000 | 40  | 0.1581 | Cov | 0.9724   | 0.9776   | 0.9776 | 0.9776 | 0.9776  |
|      |     |        | Len | 6.0444   | 2.1262   | 2.1262 | 2.1262 | 2.1262  |
| 1000 | 100 | 0.1    | Cov | 0.9818   | 0.9854   | 0.9854 | 0.9854 | 0.9854  |
|      |     |        | Len | 17.1702  | 2.3852   | 2.3852 | 2.3852 | 2.3852  |
| 1000 | 200 | 0.07   | Cov | 0.9916   | 0.9846   | 0.9846 | 0.9846 | 0.9846  |
|      |     |        | Len | 36.4373  | 2.7143   | 2.7143 | 2.7143 | 2.7143  |

Table 5.177. Etype = 4, J=5, k=p-1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL      | PLS    | PCR    | FS      |
|------|-----|--------|-----|----------|---------|--------|--------|---------|
| 100  | 40  | 0.9    | Cov | 0.9966   | 0.9844  | 0.9198 | 0.9952 | 0.9232  |
|      |     |        | Len | 41.9408  | 7.5936  | 2.6906 | 2.9343 | 3.3564  |
| 400  | 40  | 0.9    | Cov | 0.9854   | 0.9808  | 0.9846 | 0.9846 | 0.9846  |
|      |     |        | Len | 35.1383  | 6.9596  | 2.4109 | 2.4109 | 2.4109  |
| 1000 | 40  | 0.9    | Cov | 0.9758   | 0.9698  | 0.9794 | 0.9794 | 0.9794  |
|      |     |        | Len | 31.4192  | 6.3693  | 2.1260 | 2.1260 | 2.1260  |
| 100  | 100 | 0.9    | Cov | 0.9960   | 0.9824  | 0.2046 | 0.9996 | 0.8122  |
|      |     |        | Len | 182.9307 | 18.0895 | 0.8010 | 2.9605 | 6.0123  |
| 400  | 100 | 0.9    | Cov | 0.9940   | 0.9920  | 0.9708 | 0.9880 | 0.9422  |
|      |     |        | Len | 162.6525 | 18.8913 | 2.6998 | 2.7386 | 3.1723  |
| 1000 | 100 | 0.9    | Cov | 0.9902   | 0.9836  | 0.9826 | 0.9826 | 0.9826  |
|      |     |        | Len | 141.7133 | 16.8814 | 2.3844 | 2.3844 | 2.3844  |
| 100  | 200 | 0.9    | Cov | 0.9954   | 0.9714  | 0.0010 | 0.9990 | 0.7258  |
|      |     |        | Len | 866.8701 | 31.3938 | 0.0029 | 2.9631 | 10.3606 |
| 400  | 200 | 0.9    | Cov | 0.9970   | 0.9912  | 0.8494 | 0.9960 | 0.8462  |
|      |     |        | Len | 449.9084 | 35.3552 | 2.3401 | 2.7385 | 5.6165  |
| 1000 | 200 | 0.9    | Cov | 0.9962   | 0.9924  | 0.9838 | 0.9838 | 0.9838  |
|      |     |        | Len | 448.6430 | 36.4521 | 2.7150 | 2.7150 | 2.7150  |

Table 5.178. Etype = 4, J=10, k=1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0      | Cov | 0.9588 | 0.9628 | 0.9098 | 0.9352 | 0.9236 |
|      |     |        | Len | 2.1432 | 2.2080 | 2.1581 | 3.5526 | 2.1731 |
| 100  | 40  | 0      | Cov | 0.9584 | 0.9432 | 0.7990 | 0.9394 | 0.8898 |
|      |     |        | Len | 2.1255 | 2.1873 | 1.9489 | 4.1287 | 2.1071 |
| 100  | 100 | 0      | Cov | 0.9584 | 0.9198 | 0.2844 | 0.9512 | 0.8282 |
|      |     |        | Len | 2.1239 | 2.1490 | 0.8452 | 4.4931 | 1.9884 |
| 100  | 200 | 0      | Cov | 0.9570 | 0.9140 | 0.0350 | 0.9436 | 0.7902 |
|      |     |        | Len | 2.1486 | 2.1201 | 0.0959 | 4.6376 | 1.8915 |
| 400  | 20  | 0      | Cov | 0.9392 | 0.9542 | 0.9394 | 0.9394 | 0.9394 |
|      |     |        | Len | 1.9701 | 1.9696 | 1.9742 | 1.9742 | 1.9742 |
| 400  | 40  | 0      | Cov | 0.9158 | 0.9364 | 0.9170 | 0.9170 | 0.9170 |
|      |     |        | Len | 1.9657 | 1.9739 | 1.9704 | 1.9704 | 1.9704 |
| 400  | 100 | 0      | Cov | 0.9116 | 0.9084 | 0.8386 | 0.9074 | 0.8604 |
|      |     |        | Len | 1.8806 | 1.9680 | 1.8873 | 3.3261 | 1.9151 |
| 400  | 200 | 0      | Cov | 0.8978 | 0.8832 | 0.6646 | 0.9064 | 0.8076 |
|      |     |        | Len | 1.8525 | 1.9469 | 1.6057 | 3.7463 | 1.8329 |
| 1000 | 20  | 0      | Cov | 0.9478 | 0.9488 | 0.9466 | 0.9466 | 0.9466 |
|      |     |        | Len | 1.9209 | 1.9208 | 1.9240 | 1.9240 | 1.9240 |
| 1000 | 40  | 0      | Cov | 0.9402 | 0.9438 | 0.9386 | 0.9386 | 0.9386 |
|      |     |        | Len | 1.9254 | 1.9241 | 1.9295 | 1.9295 | 1.9295 |
| 1000 | 100 | 0      | Cov | 0.8982 | 0.9236 | 0.8976 | 0.8976 | 0.8976 |
|      |     |        | Len | 1.9243 | 1.9317 | 1.9292 | 1.9292 | 1.9292 |
| 1000 | 200 | 0      | Cov | 0.8980 | 0.9094 | 0.8536 | 0.8932 | 0.8610 |
|      |     |        | Len | 1.8519 | 1.9307 | 1.8856 | 3.0480 | 1.8950 |

Table 5.179. Etype = 4, J=10, k=1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0.2236 | Cov | 0.9592 | 0.9808 | 0.9122 | 0.9340 | 0.9268 |
|      |     |        | Len | 2.1460 | 2.2117 | 2.1591 | 3.1252 | 2.1738 |
| 100  | 40  | 0.1581 | Cov | 0.9562 | 0.9708 | 0.7912 | 0.9384 | 0.8814 |
|      |     |        | Len | 2.1295 | 2.2137 | 1.9459 | 3.7227 | 2.1069 |
| 100  | 100 | 0.1    | Cov | 0.9630 | 0.9694 | 0.3022 | 0.9478 | 0.8304 |
|      |     |        | Len | 2.1322 | 2.2111 | 0.8944 | 4.1811 | 1.9952 |
| 100  | 200 | 0.07   | Cov | 0.9574 | 0.9590 | 0.0512 | 0.9510 | 0.7904 |
|      |     |        | Len | 2.1545 | 2.2062 | 0.1359 | 4.4158 | 1.9020 |
| 400  | 20  | 0.2236 | Cov | 0.9432 | 0.9602 | 0.9424 | 0.9424 | 0.9424 |
|      |     |        | Len | 1.9670 | 1.9657 | 1.9732 | 1.9732 | 1.9732 |
| 400  | 40  | 0.1581 | Cov | 0.9138 | 0.9528 | 0.9102 | 0.9102 | 0.9102 |
|      |     |        | Len | 1.9634 | 1.9686 | 1.9701 | 1.9701 | 1.9701 |
| 400  | 100 | 0.1    | Cov | 0.9012 | 0.9470 | 0.8318 | 0.9052 | 0.8596 |
|      |     |        | Len | 1.8821 | 1.9688 | 1.8885 | 3.1293 | 1.9147 |
| 400  | 200 | 0.07   | Cov | 0.9050 | 0.9514 | 0.6462 | 0.9184 | 0.8086 |
|      |     |        | Len | 1.8546 | 1.9699 | 1.6063 | 3.5698 | 1.8339 |
| 1000 | 20  | 0.2236 | Cov | 0.9528 | 0.9580 | 0.9494 | 0.9494 | 0.9494 |
|      |     |        | Len | 1.9193 | 1.9192 | 1.9239 | 1.9239 | 1.9239 |
| 1000 | 40  | 0.1581 | Cov | 0.9328 | 0.9480 | 0.9306 | 0.9306 | 0.9306 |
|      |     |        | Len | 1.9244 | 1.9205 | 1.9302 | 1.9302 | 1.9302 |
| 1000 | 100 | 0.1    | Cov | 0.9098 | 0.9458 | 0.9080 | 0.9080 | 0.9080 |
|      |     |        | Len | 1.9234 | 1.9253 | 1.9299 | 1.9299 | 1.9299 |
| 1000 | 200 | 0.07   | Cov | 0.8984 | 0.9464 | 0.8548 | 0.8976 | 0.8612 |
|      |     |        | Len | 1.8523 | 1.9242 | 1.8860 | 2.9291 | 1.8952 |

Table 5.180. Etype = 4, J=10, k=1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9912 | 0.9846 | 0.9108 | 0.9660 | 0.9250 |
|      |     |        | Len | 2.1997 | 2.2049 | 2.1533 | 2.2281 | 2.1697 |
| 100  | 40  | 0.9    | Cov | 0.9900 | 0.9828 | 0.7882 | 0.9706 | 0.8864 |
|      |     |        | Len | 2.2180 | 2.2051 | 1.9405 | 2.2417 | 2.1012 |
| 100  | 100 | 0.9    | Cov | 0.9920 | 0.9746 | 0.2946 | 0.9780 | 0.8280 |
|      |     |        | Len | 2.7138 | 2.2084 | 0.8364 | 2.2534 | 1.9837 |
| 100  | 200 | 0.9    | Cov | 0.9854 | 0.9686 | 0.0488 | 0.9754 | 0.7730 |
|      |     |        | Len | 4.4309 | 2.2062 | 0.1005 | 2.2554 | 1.8761 |
| 400  | 20  | 0.9    | Cov | 0.9656 | 0.9620 | 0.9396 | 0.9396 | 0.9396 |
|      |     |        | Len | 1.9636 | 1.9632 | 1.9728 | 1.9728 | 1.9728 |
| 400  | 40  | 0.9    | Cov | 0.9720 | 0.9626 | 0.9166 | 0.9166 | 0.9166 |
|      |     |        | Len | 1.9701 | 1.9648 | 1.9700 | 1.9700 | 1.9700 |
| 400  | 100 | 0.9    | Cov | 0.9666 | 0.9606 | 0.8302 | 0.9242 | 0.8522 |
|      |     |        | Len | 1.9841 | 1.9660 | 1.8876 | 1.9921 | 1.9138 |
| 400  | 200 | 0.9    | Cov | 0.9628 | 0.9534 | 0.6566 | 0.9344 | 0.8074 |
|      |     |        | Len | 2.0087 | 1.9668 | 1.6067 | 2.0028 | 1.8346 |
| 1000 | 20  | 0.9    | Cov | 0.9546 | 0.9516 | 0.9410 | 0.9410 | 0.9410 |
|      |     |        | Len | 1.9223 | 1.9193 | 1.9239 | 1.9239 | 1.9239 |
| 1000 | 40  | 0.9    | Cov | 0.9580 | 0.9534 | 0.9344 | 0.9344 | 0.9344 |
|      |     |        | Len | 1.9269 | 1.9195 | 1.9298 | 1.9298 | 1.9298 |
| 1000 | 100 | 0.9    | Cov | 0.9562 | 0.9576 | 0.9012 | 0.9012 | 0.9012 |
|      |     |        | Len | 1.9406 | 1.9197 | 1.9291 | 1.9291 | 1.9291 |
| 1000 | 200 | 0.9    | Cov | 0.9608 | 0.9558 | 0.8450 | 0.9180 | 0.8632 |
|      |     |        | Len | 1.9645 | 1.9203 | 1.8853 | 1.9471 | 1.8948 |

Table 5.181. Etype = 4, J=10, k=19,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 20  | 0      | Cov | 0.9864  | 0.9596  | 0.9748 | 0.9734  | 0.9462  |
|      |     |        | Len | 19.1951 | 12.7161 | 2.5897 | 14.4322 | 13.2943 |
| 100  | 40  | 0      | Cov | 0.9792  | 0.9528  | 0.8790 | 0.9726  | 0.9430  |
|      |     |        | Len | 19.2609 | 12.8101 | 2.3722 | 18.2546 | 13.2913 |
| 100  | 100 | 0      | Cov | 0.9784  | 0.9498  | 0.3426 | 0.9818  | 0.9360  |
|      |     |        | Len | 19.2995 | 13.0513 | 1.5369 | 20.4599 | 13.2503 |
| 100  | 200 | 0      | Cov | 0.9748  | 0.9408  | 0.0382 | 0.9764  | 0.9182  |
|      |     |        | Len | 19.1191 | 13.2604 | 0.3203 | 20.7168 | 12.8094 |
| 400  | 20  | 0      | Cov | 0.9856  | 0.9852  | 0.9852 | 0.9852  | 0.9852  |
|      |     |        | Len | 2.2302  | 2.2218  | 2.2218 | 2.2218  | 2.2218  |
| 400  | 40  | 0      | Cov | 0.9712  | 0.9742  | 0.9680 | 0.9680  | 0.9680  |
|      |     |        | Len | 2.2375  | 2.2413  | 2.2500 | 2.2500  | 2.2500  |
| 400  | 100 | 0      | Cov | 0.9636  | 0.9604  | 0.9050 | 0.9428  | 0.9316  |
|      |     |        | Len | 2.3196  | 2.2510  | 2.1984 | 13.5052 | 2.2333  |
| 400  | 200 | 0      | Cov | 0.9636  | 0.9554  | 0.7496 | 0.9478  | 0.9162  |
|      |     |        | Len | 2.4376  | 2.2479  | 1.9030 | 16.2156 | 2.1978  |
| 1000 | 20  | 0      | Cov | 0.9672  | 0.9646  | 0.9646 | 0.9646  | 0.9646  |
|      |     |        | Len | 2.0128  | 2.0057  | 2.0057 | 2.0057  | 2.0057  |
| 1000 | 40  | 0      | Cov | 0.9554  | 0.9598  | 0.9534 | 0.9534  | 0.9534  |
|      |     |        | Len | 2.0137  | 2.0144  | 2.0202 | 2.0202  | 2.0202  |
| 1000 | 100 | 0      | Cov | 0.9364  | 0.9488  | 0.9294 | 0.9294  | 0.9294  |
|      |     |        | Len | 2.0120  | 2.0268  | 2.0299 | 2.0299  | 2.0299  |
| 1000 | 200 | 0      | Cov | 0.9176  | 0.9300  | 0.8776 | 0.9090  | 0.8882  |
|      |     |        | Len | 1.9743  | 2.0299  | 1.9944 | 10.5960 | 2.0050  |

Table 5.182. Etype = 4, J=10, k=19,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 20  | 0.2236 | Cov | 0.9912  | 0.9656  | 0.9694 | 0.9926  | 0.9378  |
|      |     |        | Len | 74.1391 | 14.4975 | 2.5877 | 2.7097  | 14.2421 |
| 100  | 40  | 0.1581 | Cov | 0.9880  | 0.9586  | 0.8794 | 0.9700  | 0.9328  |
|      |     |        | Len | 76.9352 | 17.2799 | 2.3791 | 11.5352 | 15.7920 |
| 100  | 100 | 0.1    | Cov | 0.9860  | 0.9350  | 0.3606 | 0.9830  | 0.9022  |
|      |     |        | Len | 78.4907 | 19.8367 | 1.5478 | 16.9853 | 17.2474 |
| 100  | 200 | 0.07   | Cov | 0.9880  | 0.9336  | 0.0630 | 0.9800  | 0.8784  |
|      |     |        | Len | 82.9398 | 22.2529 | 0.4431 | 19.2205 | 17.9218 |
| 400  | 20  | 0.2236 | Cov | 0.9786  | 0.9856  | 0.9856 | 0.9856  | 0.9856  |
|      |     |        | Len | 3.0761  | 2.2230  | 2.2230 | 2.2230  | 2.2230  |
| 400  | 40  | 0.1581 | Cov | 0.9782  | 0.9832  | 0.9766 | 0.9766  | 0.9766  |
|      |     |        | Len | 3.0197  | 2.2381  | 2.2498 | 2.2498  | 2.2498  |
| 400  | 100 | 0.1    | Cov | 0.9720  | 0.9670  | 0.9094 | 0.9506  | 0.9308  |
|      |     |        | Len | 2.9558  | 2.2503  | 2.1994 | 11.1114 | 2.2339  |
| 400  | 200 | 0.07   | Cov | 0.9694  | 0.9588  | 0.7520 | 0.9538  | 0.9022  |
|      |     |        | Len | 3.7717  | 2.2554  | 1.9034 | 14.4625 | 2.1987  |
| 1000 | 20  | 0.2236 | Cov | 0.9606  | 0.9654  | 0.9654 | 0.9654  | 0.9654  |
|      |     |        | Len | 2.7176  | 2.0059  | 2.0059 | 2.0059  | 2.0059  |
| 1000 | 40  | 0.1581 | Cov | 0.9598  | 0.9642  | 0.9596 | 0.9596  | 0.9596  |
|      |     |        | Len | 2.6569  | 2.0120  | 2.0197 | 2.0197  | 2.0197  |
| 1000 | 100 | 0.1    | Cov | 0.9606  | 0.9586  | 0.9292 | 0.9292  | 0.9292  |
|      |     |        | Len | 2.5729  | 2.0189  | 2.0291 | 2.0291  | 2.0291  |
| 1000 | 200 | 0.07   | Cov | 0.9586  | 0.9522  | 0.8882 | 0.9074  | 0.8958  |
|      |     |        | Len | 2.4905  | 2.0238  | 1.9946 | 9.4448  | 2.0048  |

Table 5.183. Etype = 4, J=10, k=19,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|----------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9906   | 0.9708 | 0.9674 | 0.9954 | 0.9562 |
|      |     |        | Len | 57.7824  | 3.4488 | 2.5866 | 2.6117 | 3.0849 |
| 100  | 40  | 0.9    | Cov | 0.9878   | 0.9734 | 0.8830 | 0.9908 | 0.9402 |
|      |     |        | Len | 47.9859  | 4.0199 | 2.3594 | 3.0159 | 3.0663 |
| 100  | 100 | 0.9    | Cov | 0.9916   | 0.9674 | 0.3630 | 0.9840 | 0.9138 |
|      |     |        | Len | 139.6631 | 4.0233 | 1.0473 | 3.3067 | 3.0045 |
| 100  | 200 | 0.9    | Cov | 0.9910   | 0.9618 | 0.0546 | 0.9888 | 0.8918 |
|      |     |        | Len | 368.0577 | 3.7352 | 0.1340 | 3.4092 | 2.9171 |
| 400  | 20  | 0.9    | Cov | 0.9824   | 0.9742 | 0.9870 | 0.9870 | 0.9870 |
|      |     |        | Len | 11.0208  | 3.0687 | 2.2223 | 2.2223 | 2.2223 |
| 400  | 40  | 0.9    | Cov | 0.9830   | 0.9744 | 0.9668 | 0.9668 | 0.9668 |
|      |     |        | Len | 15.7658  | 3.8666 | 2.2158 | 2.2158 | 2.2158 |
| 400  | 100 | 0.9    | Cov | 0.9802   | 0.9686 | 0.9034 | 0.9628 | 0.9228 |
|      |     |        | Len | 24.6834  | 3.9683 | 2.1371 | 2.5752 | 2.1776 |
| 400  | 200 | 0.9    | Cov | 0.9772   | 0.9686 | 0.7144 | 0.9666 | 0.8854 |
|      |     |        | Len | 33.6608  | 3.9210 | 1.8326 | 2.7503 | 2.1342 |
| 1000 | 20  | 0.9    | Cov | 0.9608   | 0.9516 | 0.9702 | 0.9702 | 0.9702 |
|      |     |        | Len | 9.7061   | 2.7323 | 2.0060 | 2.0060 | 2.0060 |
| 1000 | 40  | 0.9    | Cov | 0.9620   | 0.9562 | 0.9572 | 0.9572 | 0.9572 |
|      |     |        | Len | 14.1112  | 3.5394 | 2.0202 | 2.0202 | 2.0202 |
| 1000 | 100 | 0.9    | Cov | 0.9592   | 0.9560 | 0.9282 | 0.9282 | 0.9282 |
|      |     |        | Len | 22.3810  | 3.7134 | 2.0292 | 2.0292 | 2.0292 |
| 1000 | 200 | 0.9    | Cov | 0.9642   | 0.9548 | 0.8668 | 0.9282 | 0.8792 |
|      |     |        | Len | 30.7298  | 3.7507 | 1.9949 | 2.3261 | 2.0047 |

Table 5.184. Etype = 4, J=10, k=p-1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 40  | 0      | Cov | 0.9792  | 0.9368  | 0.8644 | 0.9740  | 0.9190  |
|      |     |        | Len | 28.4967 | 21.2534 | 2.3227 | 25.3696 | 21.4711 |
| 400  | 40  | 0      | Cov | 0.9798  | 0.9804  | 0.9804 | 0.9804  | 0.9804  |
|      |     |        | Len | 2.4255  | 2.4110  | 2.4110 | 2.4110  | 2.4110  |
| 1000 | 40  | 0      | Cov | 0.9746  | 0.9746  | 0.9746 | 0.9746  | 0.9746  |
|      |     |        | Len | 2.1419  | 2.1259  | 2.1259 | 2.1259  | 2.1259  |
| 100  | 100 | 0      | Cov | 0.9510  | 0.8656  | 0.2744 | 0.9460  | 0.8360  |
|      |     |        | Len | 40.4959 | 31.0481 | 2.3526 | 39.3580 | 30.9095 |
| 400  | 100 | 0      | Cov | 0.9732  | 0.9374  | 0.9364 | 0.9554  | 0.9168  |
|      |     |        | Len | 40.9671 | 29.1849 | 2.3589 | 32.5562 | 28.5480 |
| 1000 | 100 | 0      | Cov | 0.9810  | 0.9840  | 0.9840 | 0.9840  | 0.9840  |
|      |     |        | Len | 2.4230  | 2.3857  | 2.3857 | 2.3857  | 2.3857  |
| 100  | 200 | 0      | Cov | 0.9462  | 0.8428  | 0.0334 | 0.9494  | 0.7902  |
|      |     |        | Len | 56.8209 | 43.7028 | 0.8547 | 56.3056 | 42.0493 |
| 400  | 200 | 0      | Cov | 0.9312  | 0.8452  | 0.6852 | 0.9158  | 0.8060  |
|      |     |        | Len | 49.4425 | 37.6991 | 1.6641 | 44.8418 | 36.5214 |
| 1000 | 200 | 0      | Cov | 0.9796  | 0.9482  | 0.9566 | 0.9634  | 0.9260  |
|      |     |        | Len | 55.8950 | 37.9218 | 2.3692 | 40.8054 | 36.9422 |

Table 5.185. Etype = 4, J=10, k=p-1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL       | PLS    | PCR    | FS       |
|------|-----|--------|-----|----------|----------|--------|--------|----------|
| 100  | 40  | 0.1581 | Cov | 0.9842   | 0.9510   | 0.8870 | 0.9918 | 0.8994   |
|      |     |        | Len | 160.5267 | 36.2654  | 2.3675 | 2.9861 | 33.3126  |
| 100  | 100 | 0.1    | Cov | 0.9896   | 0.9274   | 0.3608 | 0.9808 | 0.8534   |
|      |     |        | Len | 411.1084 | 96.8765  | 1.0673 | 3.8399 | 85.9899  |
| 100  | 200 | 0.07   | Cov | 0.9864   | 0.9222   | 0.0590 | 0.9834 | 0.8324   |
|      |     |        | Len | 868.0603 | 210.0849 | 0.1624 | 5.0880 | 166.2538 |
| 400  | 40  | 0.1581 | Cov | 0.9840   | 0.9810   | 0.9810 | 0.9810 | 0.9810   |
|      |     |        | Len | 6.9062   | 2.4099   | 2.4099 | 2.4099 | 2.4099   |
| 400  | 100 | 0.1    | Cov | 0.9816   | 0.9522   | 0.9338 | 0.9844 | 0.8972   |
|      |     |        | Len | 258.4853 | 45.5554  | 2.3571 | 2.6309 | 37.7866  |
| 400  | 200 | 0.07   | Cov | 0.9828   | 0.9438   | 0.7768 | 0.9794 | 0.8578   |
|      |     |        | Len | 567.0794 | 104.7018 | 2.0378 | 3.0456 | 80.8614  |
| 1000 | 40  | 0.1581 | Cov | 0.9780   | 0.9786   | 0.9786 | 0.9786 | 0.9786   |
|      |     |        | Len | 6.0460   | 2.1260   | 2.1260 | 2.1260 | 2.1260   |
| 1000 | 100 | 0.1    | Cov | 0.9844   | 0.9842   | 0.9842 | 0.9842 | 0.9842   |
|      |     |        | Len | 17.1686  | 2.3847   | 2.3847 | 2.3847 | 2.3847   |
| 1000 | 200 | 0.07   | Cov | 0.9878   | 0.9608   | 0.9510 | 0.9846 | 0.9124   |
|      |     |        | Len | 362.7876 | 56.1351  | 2.3681 | 2.5550 | 46.2543  |

Table 5.186. Etype = 4, J=10, k=p-1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso     | RL      | PLS    | PCR    | FS      |
|------|-----|--------|-----|-----------|---------|--------|--------|---------|
| 100  | 40  | 0.9    | Cov | 0.9926    | 0.9732  | 0.8874 | 0.9976 | 0.9196  |
|      |     |        | Len | 208.1377  | 6.6131  | 2.3721 | 2.6130 | 4.6622  |
| 400  | 40  | 0.9    | Cov | 0.9878    | 0.9830  | 0.9830 | 0.9830 | 0.9830  |
|      |     |        | Len | 35.0924   | 6.9742  | 2.4111 | 2.4111 | 2.4111  |
| 1000 | 40  | 0.9    | Cov | 0.9816    | 0.9682  | 0.9778 | 0.9778 | 0.9778  |
|      |     |        | Len | 31.4019   | 6.3547  | 2.1261 | 2.1261 | 2.1261  |
| 100  | 100 | 0.9    | Cov | 0.9888    | 0.9586  | 0.3606 | 0.9994 | 0.8606  |
|      |     |        | Len | 1634.9570 | 15.1750 | 1.0341 | 2.6107 | 10.1523 |
| 400  | 100 | 0.9    | Cov | 0.9870    | 0.9844  | 0.9316 | 0.9912 | 0.9104  |
|      |     |        | Len | 142.2538  | 16.4984 | 2.3559 | 2.4098 | 4.7807  |
| 1000 | 100 | 0.9    | Cov | 0.9880    | 0.9846  | 0.9846 | 0.9846 | 0.9846  |
|      |     |        | Len | 141.6492  | 16.8474 | 2.3854 | 2.3854 | 2.3854  |
| 100  | 200 | 0.9    | Cov | 0.9892    | 0.9558  | 0.0582 | 0.9990 | 0.8284  |
|      |     |        | Len | 7549.9320 | 28.3223 | 0.1236 | 2.6128 | 18.8425 |
| 400  | 200 | 0.9    | Cov | 0.9896    | 0.9766  | 0.7754 | 0.9962 | 0.8620  |
|      |     |        | Len | 393.1390  | 30.6572 | 2.0399 | 2.4096 | 8.9997  |
| 1000 | 200 | 0.9    | Cov | 0.9870    | 0.9820  | 0.9548 | 0.9920 | 0.9204  |
|      |     |        | Len | 391.5209  | 31.7493 | 2.3684 | 2.3845 | 5.4831  |

Table 5.187. Etype = 4, J=20, k=1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0      | Cov | 0.9862 | 0.9732 | 0.9198 | 0.9522 | 0.9608 |
|      |     |        | Len | 2.2034 | 2.2123 | 2.1618 | 4.3744 | 2.2069 |
| 100  | 40  | 0      | Cov | 0.9810 | 0.9650 | 0.8180 | 0.9518 | 0.9406 |
|      |     |        | Len | 2.2164 | 2.2052 | 1.9795 | 4.6603 | 2.1890 |
| 100  | 100 | 0      | Cov | 0.9762 | 0.9534 | 0.5096 | 0.9650 | 0.9192 |
|      |     |        | Len | 2.2383 | 2.1916 | 1.3118 | 4.8428 | 2.1643 |
| 100  | 200 | 0      | Cov | 0.9782 | 0.9468 | 0.2296 | 0.9664 | 0.9084 |
|      |     |        | Len | 2.2733 | 2.1842 | 0.6340 | 4.9272 | 2.1398 |
| 400  | 20  | 0      | Cov | 0.9436 | 0.9540 | 0.9438 | 0.9438 | 0.9438 |
|      |     |        | Len | 1.9695 | 1.9694 | 1.9738 | 1.9738 | 1.9738 |
| 400  | 40  | 0      | Cov | 0.9386 | 0.9416 | 0.9144 | 0.9236 | 0.9204 |
|      |     |        | Len | 1.9359 | 1.9739 | 1.9705 | 3.3106 | 1.9722 |
| 400  | 100 | 0      | Cov | 0.9278 | 0.9218 | 0.8288 | 0.9314 | 0.8782 |
|      |     |        | Len | 1.9175 | 1.9727 | 1.8901 | 3.9374 | 1.9511 |
| 400  | 200 | 0      | Cov | 0.9418 | 0.9162 | 0.6576 | 0.9408 | 0.8700 |
|      |     |        | Len | 1.9111 | 1.9663 | 1.6078 | 4.1674 | 1.9237 |
| 1000 | 20  | 0      | Cov | 0.9384 | 0.9454 | 0.9378 | 0.9378 | 0.9378 |
|      |     |        | Len | 1.9209 | 1.9206 | 1.9239 | 1.9239 | 1.9239 |
| 1000 | 40  | 0      | Cov | 0.9312 | 0.9414 | 0.9316 | 0.9316 | 0.9316 |
|      |     |        | Len | 1.9257 | 1.9243 | 1.9298 | 1.9298 | 1.9298 |
| 1000 | 100 | 0      | Cov | 0.9266 | 0.9292 | 0.9010 | 0.9238 | 0.9084 |
|      |     |        | Len | 1.8898 | 1.9307 | 1.9284 | 3.2285 | 1.9300 |
| 1000 | 200 | 0      | Cov | 0.9216 | 0.9200 | 0.8642 | 0.9336 | 0.8858 |
|      |     |        | Len | 1.8713 | 1.9319 | 1.8855 | 3.7433 | 1.9164 |

Table 5.188. Etype = 4, J=20, k=1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0.2236 | Cov | 0.9812 | 0.9810 | 0.9182 | 0.9592 | 0.9542 |
|      |     |        | Len | 2.2105 | 2.2106 | 2.1618 | 3.7576 | 2.2036 |
| 100  | 40  | 0.1581 | Cov | 0.9836 | 0.9742 | 0.8254 | 0.9590 | 0.9412 |
|      |     |        | Len | 2.2377 | 2.2120 | 2.0127 | 4.1705 | 2.1918 |
| 100  | 100 | 0.1    | Cov | 0.9800 | 0.9726 | 0.5786 | 0.9616 | 0.9296 |
|      |     |        | Len | 2.2725 | 2.2100 | 1.4874 | 4.5137 | 2.1642 |
| 100  | 200 | 0.07   | Cov | 0.9722 | 0.9604 | 0.3418 | 0.9572 | 0.9082 |
|      |     |        | Len | 2.3180 | 2.2049 | 0.9027 | 4.6703 | 2.1383 |
| 400  | 20  | 0.2236 | Cov | 0.9428 | 0.9614 | 0.9402 | 0.9402 | 0.9402 |
|      |     |        | Len | 1.9674 | 1.9655 | 1.9736 | 1.9736 | 1.9736 |
| 400  | 40  | 0.1581 | Cov | 0.9366 | 0.9616 | 0.9128 | 0.9246 | 0.9202 |
|      |     |        | Len | 1.9358 | 1.9657 | 1.9702 | 3.0122 | 1.9718 |
| 400  | 100 | 0.1    | Cov | 0.9436 | 0.9580 | 0.8400 | 0.9322 | 0.8886 |
|      |     |        | Len | 1.9182 | 1.9673 | 1.8876 | 3.6771 | 1.9496 |
| 400  | 200 | 0.07   | Cov | 0.9354 | 0.9542 | 0.6604 | 0.9374 | 0.8730 |
|      |     |        | Len | 1.9129 | 1.9688 | 1.6076 | 3.9618 | 1.9233 |
| 1000 | 20  | 0.2236 | Cov | 0.9408 | 0.9578 | 0.9396 | 0.9396 | 0.9396 |
|      |     |        | Len | 1.9195 | 1.9192 | 1.9240 | 1.9240 | 1.9240 |
| 1000 | 40  | 0.1581 | Cov | 0.9322 | 0.9534 | 0.9316 | 0.9316 | 0.9316 |
|      |     |        | Len | 1.9243 | 1.9202 | 1.9302 | 1.9302 | 1.9302 |
| 1000 | 100 | 0.1    | Cov | 0.9294 | 0.9506 | 0.9070 | 0.9196 | 0.9124 |
|      |     |        | Len | 1.8897 | 1.9206 | 1.9284 | 3.0381 | 1.9298 |
| 1000 | 200 | 0.07   | Cov | 0.9244 | 0.9500 | 0.8456 | 0.9300 | 0.8836 |
|      |     |        | Len | 1.8715 | 1.9208 | 1.8850 | 3.5672 | 1.9156 |

Table 5.189. Etype = 4, J=20, k=1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|---------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9818  | 0.9816 | 0.9132 | 0.9780 | 0.9488 |
|      |     |        | Len | 3.2603  | 2.2062 | 2.1538 | 2.2579 | 2.2055 |
| 100  | 40  | 0.9    | Cov | 0.9796  | 0.9788 | 0.8204 | 0.9804 | 0.9360 |
|      |     |        | Len | 6.6229  | 2.2043 | 1.9597 | 2.2633 | 2.1908 |
| 100  | 100 | 0.9    | Cov | 0.9774  | 0.9730 | 0.5636 | 0.9854 | 0.9058 |
|      |     |        | Len | 15.0905 | 2.2118 | 1.2759 | 2.2741 | 2.1674 |
| 100  | 200 | 0.9    | Cov | 0.9716  | 0.9698 | 0.3122 | 0.9858 | 0.8926 |
|      |     |        | Len | 29.2181 | 2.2095 | 0.6490 | 2.2749 | 2.1347 |
| 400  | 20  | 0.9    | Cov | 0.9644  | 0.9614 | 0.9392 | 0.9392 | 0.9392 |
|      |     |        | Len | 1.9637  | 1.9639 | 1.9737 | 1.9737 | 1.9737 |
| 400  | 40  | 0.9    | Cov | 0.9714  | 0.9666 | 0.9202 | 0.9436 | 0.9218 |
|      |     |        | Len | 1.9700  | 1.9653 | 1.9702 | 1.9979 | 1.9726 |
| 400  | 100 | 0.9    | Cov | 0.9710  | 0.9568 | 0.8410 | 0.9476 | 0.8942 |
|      |     |        | Len | 1.9842  | 1.9660 | 1.8882 | 2.0146 | 1.9513 |
| 400  | 200 | 0.9    | Cov | 0.9646  | 0.9598 | 0.6624 | 0.9538 | 0.8582 |
|      |     |        | Len | 2.0074  | 1.9658 | 1.6057 | 2.0205 | 1.9215 |
| 1000 | 20  | 0.9    | Cov | 0.9574  | 0.9568 | 0.9446 | 0.9446 | 0.9446 |
|      |     |        | Len | 1.9220  | 1.9189 | 1.9235 | 1.9235 | 1.9235 |
| 1000 | 40  | 0.9    | Cov | 0.9560  | 0.9522 | 0.9302 | 0.9302 | 0.9302 |
|      |     |        | Len | 1.9267  | 1.9192 | 1.9299 | 1.9299 | 1.9299 |
| 1000 | 100 | 0.9    | Cov | 0.9524  | 0.9544 | 0.9002 | 0.9300 | 0.9084 |
|      |     |        | Len | 1.9404  | 1.9196 | 1.9289 | 1.9550 | 1.9305 |
| 1000 | 200 | 0.9    | Cov | 0.9578  | 0.9514 | 0.8522 | 0.9376 | 0.8800 |
|      |     |        | Len | 1.9648  | 1.9205 | 1.8863 | 1.9699 | 1.9164 |

Table 5.190. Etype = 4, J=20, k=19,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 20  | 0      | Cov | 0.9794  | 0.9556  | 0.9510 | 0.9770  | 0.9560  |
|      |     |        | Len | 20.1386 | 15.1662 | 2.4581 | 18.4180 | 16.9239 |
| 100  | 40  | 0      | Cov | 0.9790  | 0.9514  | 0.8602 | 0.9748  | 0.9484  |
|      |     |        | Len | 20.0977 | 15.1258 | 2.6989 | 19.9565 | 16.9030 |
| 100  | 100 | 0      | Cov | 0.9740  | 0.9362  | 0.5446 | 0.9742  | 0.9466  |
|      |     |        | Len | 19.7129 | 14.8702 | 3.1531 | 20.5945 | 16.5851 |
| 100  | 200 | 0      | Cov | 0.9706  | 0.9388  | 0.2488 | 0.9708  | 0.9404  |
|      |     |        | Len | 19.8737 | 15.4457 | 2.0458 | 20.5198 | 16.1239 |
| 400  | 20  | 0      | Cov | 0.9856  | 0.9868  | 0.9868 | 0.9868  | 0.9868  |
|      |     |        | Len | 2.2295  | 2.2221  | 2.2221 | 2.2221  | 2.2221  |
| 400  | 40  | 0      | Cov | 0.9774  | 0.9818  | 0.9670 | 0.9546  | 0.9848  |
|      |     |        | Len | 2.8631  | 2.2273  | 2.2461 | 13.1649 | 2.2231  |
| 400  | 100 | 0      | Cov | 0.9808  | 0.9828  | 0.9110 | 0.9706  | 0.9856  |
|      |     |        | Len | 3.2020  | 2.2288  | 2.1972 | 17.2159 | 2.2227  |
| 400  | 200 | 0      | Cov | 0.9780  | 0.9820  | 0.7478 | 0.9690  | 0.9842  |
|      |     |        | Len | 3.4866  | 2.2295  | 1.9037 | 18.5827 | 2.2235  |
| 1000 | 20  | 0      | Cov | 0.9688  | 0.9676  | 0.9676 | 0.9676  | 0.9676  |
|      |     |        | Len | 2.0125  | 2.0055  | 2.0055 | 2.0055  | 2.0055  |
| 1000 | 40  | 0      | Cov | 0.9596  | 0.9668  | 0.9608 | 0.9608  | 0.9608  |
|      |     |        | Len | 2.0134  | 2.0140  | 2.0196 | 2.0196  | 2.0196  |
| 1000 | 100 | 0      | Cov | 0.9564  | 0.9508  | 0.9300 | 0.9334  | 0.9376  |
|      |     |        | Len | 2.0217  | 2.0253  | 2.0299 | 11.4562 | 2.0309  |
| 1000 | 200 | 0      | Cov | 0.9538  | 0.9414  | 0.8876 | 0.9398  | 0.9202  |
|      |     |        | Len | 2.0546  | 2.0291  | 1.9941 | 14.4397 | 2.0257  |

Table 5.191. Etype = 4, J=20, k=19,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 20  | 0.2236 | Cov | 0.9820  | 0.9580  | 0.9546 | 0.9908  | 0.9414  |
|      |     |        | Len | 80.6432 | 18.1406 | 2.4096 | 2.5950  | 25.1561 |
| 100  | 40  | 0.1581 | Cov | 0.9818  | 0.9454  | 0.8818 | 0.9792  | 0.9356  |
|      |     |        | Len | 79.6964 | 19.6076 | 2.7470 | 12.5679 | 27.0543 |
| 100  | 100 | 0.1    | Cov | 0.9826  | 0.9216  | 0.6326 | 0.9786  | 0.9226  |
|      |     |        | Len | 78.1016 | 20.7663 | 3.5614 | 17.3771 | 28.5521 |
| 100  | 200 | 0.07   | Cov | 0.9830  | 0.9336  | 0.3740 | 0.9802  | 0.9136  |
|      |     |        | Len | 88.7580 | 27.0505 | 2.9412 | 19.1943 | 29.0220 |
| 400  | 20  | 0.2236 | Cov | 0.9812  | 0.9834  | 0.9834 | 0.9834  | 0.9834  |
|      |     |        | Len | 3.0810  | 2.2217  | 2.2217 | 2.2217  | 2.2217  |
| 400  | 40  | 0.1581 | Cov | 0.9786  | 0.9814  | 0.9636 | 0.9652  | 0.9842  |
|      |     |        | Len | 39.5138 | 3.2312  | 2.2449 | 8.3391  | 2.3283  |
| 400  | 100 | 0.1    | Cov | 0.9802  | 0.9750  | 0.9150 | 0.9682  | 0.9814  |
|      |     |        | Len | 55.1051 | 6.7828  | 2.1945 | 14.1190 | 2.6051  |
| 400  | 200 | 0.07   | Cov | 0.9750  | 0.9676  | 0.7384 | 0.9710  | 0.9802  |
|      |     |        | Len | 59.8448 | 9.6148  | 1.9024 | 16.5586 | 3.0102  |
| 1000 | 20  | 0.2236 | Cov | 0.9624  | 0.9682  | 0.9682 | 0.9682  | 0.9682  |
|      |     |        | Len | 2.7150  | 2.0058  | 2.0058 | 2.0058  | 2.0058  |
| 1000 | 40  | 0.1581 | Cov | 0.9610  | 0.9630  | 0.9604 | 0.9604  | 0.9604  |
|      |     |        | Len | 2.6552  | 2.0120  | 2.0196 | 2.0196  | 2.0196  |
| 1000 | 100 | 0.1    | Cov | 0.9634  | 0.9598  | 0.9290 | 0.9344  | 0.9324  |
|      |     |        | Len | 2.5721  | 2.0189  | 2.0299 | 9.4242  | 2.0305  |
| 1000 | 200 | 0.07   | Cov | 0.9632  | 0.9534  | 0.8890 | 0.9368  | 0.9246  |
|      |     |        | Len | 2.4927  | 2.0243  | 1.9954 | 12.8533 | 2.0268  |

Table 5.192. Etype = 4, J=20, k=19,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|----------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9832   | 0.9704 | 0.9552 | 0.9958 | 0.9486 |
|      |     |        | Len | 231.9873 | 4.0297 | 2.4138 | 2.4130 | 4.1499 |
| 100  | 40  | 0.9    | Cov | 0.9850   | 0.9626 | 0.8762 | 0.9870 | 0.9402 |
|      |     |        | Len | 312.5916 | 4.0661 | 2.2380 | 2.9743 | 4.1254 |
| 100  | 100 | 0.9    | Cov | 0.9840   | 0.9544 | 0.6208 | 0.9858 | 0.9262 |
|      |     |        | Len | 525.0556 | 3.9765 | 1.5272 | 3.2690 | 4.0714 |
| 100  | 200 | 0.9    | Cov | 0.9828   | 0.9560 | 0.3686 | 0.9864 | 0.9078 |
|      |     |        | Len | 943.1499 | 3.9577 | 0.8266 | 3.3666 | 4.0119 |
| 400  | 20  | 0.9    | Cov | 0.9826   | 0.9768 | 0.9852 | 0.9852 | 0.9852 |
|      |     |        | Len | 11.0236  | 3.0671 | 2.2220 | 2.2220 | 2.2220 |
| 400  | 40  | 0.9    | Cov | 0.9816   | 0.9718 | 0.9582 | 0.9724 | 0.9708 |
|      |     |        | Len | 15.7471  | 3.8533 | 2.2161 | 2.4773 | 2.2478 |
| 400  | 100 | 0.9    | Cov | 0.9728   | 0.9684 | 0.8950 | 0.9718 | 0.9618 |
|      |     |        | Len | 24.7039  | 3.9662 | 2.1384 | 2.7945 | 2.2792 |
| 400  | 200 | 0.9    | Cov | 0.9734   | 0.9678 | 0.7290 | 0.9716 | 0.9530 |
|      |     |        | Len | 33.6315  | 3.8999 | 1.8338 | 2.9154 | 2.3068 |
| 1000 | 20  | 0.9    | Cov | 0.9660   | 0.9602 | 0.9694 | 0.9694 | 0.9694 |
|      |     |        | Len | 9.7133   | 2.7356 | 2.0063 | 2.0063 | 2.0063 |
| 1000 | 40  | 0.9    | Cov | 0.9628   | 0.9618 | 0.9620 | 0.9620 | 0.9620 |
|      |     |        | Len | 14.1156  | 3.5456 | 2.0197 | 2.0197 | 2.0197 |
| 1000 | 100 | 0.9    | Cov | 0.9606   | 0.9584 | 0.9362 | 0.9486 | 0.9422 |
|      |     |        | Len | 22.3740  | 3.7415 | 2.0293 | 2.3605 | 2.0299 |
| 1000 | 200 | 0.9    | Cov | 0.9660   | 0.9594 | 0.8844 | 0.9514 | 0.9200 |
|      |     |        | Len | 30.7152  | 3.7624 | 1.9944 | 2.5649 | 2.0256 |

Table 5.193. Etype = 4, J=20, k=p-1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 40  | 0      | Cov | 0.9720  | 0.9306  | 0.8394 | 0.9732  | 0.9416  |
|      |     |        | Len | 28.5121 | 22.7825 | 3.0382 | 27.7448 | 24.8264 |
| 400  | 40  | 0      | Cov | 0.9766  | 0.9562  | 0.9648 | 0.9604  | 0.9456  |
|      |     |        | Len | 25.4145 | 16.9398 | 2.2462 | 18.6641 | 17.7505 |
| 1000 | 40  | 0      | Cov | 0.9794  | 0.9794  | 0.9794 | 0.9794  | 0.9794  |
|      |     |        | Len | 2.1420  | 2.1262  | 2.1262 | 2.1262  | 2.1262  |
| 100  | 100 | 0      | Cov | 0.9596  | 0.8954  | 0.4930 | 0.9586  | 0.9170  |
|      |     |        | Len | 41.9656 | 33.9629 | 6.0206 | 42.5988 | 37.2741 |
| 400  | 100 | 0      | Cov | 0.9780  | 0.9416  | 0.9016 | 0.9688  | 0.9388  |
|      |     |        | Len | 43.1429 | 34.2330 | 2.1945 | 38.8630 | 34.7183 |
| 1000 | 100 | 0      | Cov | 0.9746  | 0.9518  | 0.9620 | 0.9598  | 0.9434  |
|      |     |        | Len | 40.0707 | 26.8778 | 2.2235 | 29.1308 | 27.6031 |
| 100  | 200 | 0      | Cov | 0.9590  | 0.8998  | 0.2216 | 0.9656  | 0.9028  |
|      |     |        | Len | 60.2054 | 50.1366 | 5.8707 | 60.2529 | 51.7206 |
| 400  | 200 | 0      | Cov | 0.9434  | 0.8840  | 0.6760 | 0.9366  | 0.8692  |
|      |     |        | Len | 53.8774 | 44.4156 | 1.6602 | 51.4833 | 44.8413 |
| 1000 | 200 | 0      | Cov | 0.9714  | 0.9372  | 0.9324 | 0.9606  | 0.9296  |
|      |     |        | Len | 60.0953 | 46.7967 | 2.2077 | 52.4173 | 47.0331 |

Table 5.194. Etype = 4, J=20, k=p-1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL       | PLS    | PCR    | FS       |
|------|-----|--------|-----|----------|----------|--------|--------|----------|
| 100  | 40  | 0.1581 | Cov | 0.9840   | 0.9422   | 0.8832 | 0.9834 | 0.9344   |
|      |     |        | Len | 162.0408 | 39.4019  | 2.2331 | 2.9203 | 55.1993  |
| 100  | 100 | 0.1    | Cov | 0.9816   | 0.9120   | 0.6420 | 0.9814 | 0.9080   |
|      |     |        | Len | 397.7579 | 98.1376  | 1.5681 | 3.7901 | 142.4024 |
| 100  | 200 | 0.07   | Cov | 0.9830   | 0.9304   | 0.3938 | 0.9792 | 0.8956   |
|      |     |        | Len | 914.5265 | 257.8100 | 1.0006 | 5.0027 | 281.8180 |
| 400  | 40  | 0.1581 | Cov | 0.9814   | 0.9634   | 0.9686 | 0.9846 | 0.9368   |
|      |     |        | Len | 130.6620 | 21.2663  | 2.2454 | 2.2954 | 20.5967  |
| 400  | 100 | 0.1    | Cov | 0.9776   | 0.9552   | 0.9052 | 0.9766 | 0.9030   |
|      |     |        | Len | 354.1583 | 68.6721  | 2.1972 | 2.5754 | 62.8561  |
| 400  | 200 | 0.07   | Cov | 0.9770   | 0.9412   | 0.7464 | 0.9738 | 0.8816   |
|      |     |        | Len | 709.1123 | 142.5048 | 1.9010 | 3.0177 | 129.3581 |
| 1000 | 40  | 0.1581 | Cov | 0.9748   | 0.9746   | 0.9746 | 0.9746 | 0.9746   |
|      |     |        | Len | 6.0443   | 2.1264   | 2.1264 | 2.1264 | 2.1264   |
| 1000 | 100 | 0.1    | Cov | 0.9768   | 0.9658   | 0.9658 | 0.9818 | 0.9252   |
|      |     |        | Len | 263.4808 | 37.2447  | 2.2237 | 2.2918 | 33.9591  |
| 1000 | 200 | 0.07   | Cov | 0.9852   | 0.9592   | 0.9258 | 0.9774 | 0.9186   |
|      |     |        | Len | 585.4471 | 92.5241  | 2.2073 | 2.5094 | 79.5852  |

Table 5.195. Etype = 4, J=20, k=p-1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso      | RL      | PLS    | PCR    | FS      |
|------|-----|--------|-----|------------|---------|--------|--------|---------|
| 100  | 40  | 0.9    | Cov | 0.9872     | 0.9634  | 0.8788 | 0.9976 | 0.9308  |
|      |     |        | Len | 700.7226   | 7.0684  | 2.2276 | 2.4119 | 7.3447  |
| 400  | 40  | 0.9    | Cov | 0.9818     | 0.9686  | 0.9636 | 0.9884 | 0.9380  |
|      |     |        | Len | 32.7403    | 6.5050  | 2.2464 | 2.2230 | 3.3579  |
| 1000 | 40  | 0.9    | Cov | 0.9774     | 0.9700  | 0.9768 | 0.9768 | 0.9768  |
|      |     |        | Len | 31.3584    | 6.3693  | 2.1273 | 2.1273 | 2.1273  |
| 100  | 100 | 0.9    | Cov | 0.9844     | 0.9522  | 0.6072 | 0.9990 | 0.9080  |
|      |     |        | Len | 3048.0350  | 15.9980 | 1.4633 | 2.4120 | 17.3067 |
| 400  | 100 | 0.9    | Cov | 0.9810     | 0.9676  | 0.9090 | 0.9914 | 0.9114  |
|      |     |        | Len | 136.9069   | 15.3803 | 2.1974 | 2.2232 | 7.4318  |
| 1000 | 100 | 0.9    | Cov | 0.9828     | 0.9736  | 0.9630 | 0.9876 | 0.9360  |
|      |     |        | Len | 131.9889   | 15.7172 | 2.2232 | 2.1964 | 4.4342  |
| 100  | 200 | 0.9    | Cov | 0.9832     | 0.9510  | 0.3522 | 0.9986 | 0.8888  |
|      |     |        | Len | 10596.6600 | 32.0807 | 0.7421 | 2.4124 | 33.3150 |
| 400  | 200 | 0.9    | Cov | 0.9808     | 0.9620  | 0.7342 | 0.9956 | 0.8882  |
|      |     |        | Len | 490.9135   | 28.4902 | 1.9022 | 2.2230 | 14.3662 |
| 1000 | 200 | 0.9    | Cov | 0.9822     | 0.9758  | 0.9322 | 0.9894 | 0.9188  |
|      |     |        | Len | 364.9735   | 29.5261 | 2.2068 | 2.1959 | 8.8981  |

Table 5.196. Etype = 4, J=50, k=1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0      | Cov | 0.9846 | 0.9806 | 0.9384 | 0.9630 | 0.9952 |
|      |     |        | Len | 2.3015 | 2.2049 | 2.6787 | 4.9247 | 2.1995 |
| 100  | 40  | 0      | Cov | 0.9882 | 0.9808 | 0.9000 | 0.9658 | 0.9950 |
|      |     |        | Len | 2.3263 | 2.2032 | 2.7683 | 5.0067 | 2.1994 |
| 100  | 100 | 0      | Cov | 0.9846 | 0.9770 | 0.8256 | 0.9626 | 0.9926 |
|      |     |        | Len | 2.3575 | 2.2016 | 2.6776 | 5.0590 | 2.1987 |
| 100  | 200 | 0      | Cov | 0.9798 | 0.9736 | 0.7358 | 0.9634 | 0.9920 |
|      |     |        | Len | 2.3927 | 2.2013 | 2.3972 | 5.0697 | 2.1987 |
| 400  | 20  | 0      | Cov | 0.9588 | 0.9518 | 0.9374 | 0.9392 | 0.9404 |
|      |     |        | Len | 1.9546 | 1.9690 | 1.9737 | 3.7098 | 1.9729 |
| 400  | 40  | 0      | Cov | 0.9582 | 0.9504 | 0.9192 | 0.9468 | 0.9358 |
|      |     |        | Len | 1.9534 | 1.9706 | 1.9706 | 4.1085 | 1.9748 |
| 400  | 100 | 0      | Cov | 0.9558 | 0.9432 | 0.8328 | 0.9472 | 0.9266 |
|      |     |        | Len | 1.9556 | 1.9732 | 1.8884 | 4.3526 | 1.9730 |
| 400  | 200 | 0      | Cov | 0.9512 | 0.9314 | 0.6594 | 0.9484 | 0.9156 |
|      |     |        | Len | 1.9592 | 1.9733 | 1.6134 | 4.4399 | 1.9707 |
| 1000 | 20  | 0      | Cov | 0.9456 | 0.9492 | 0.9444 | 0.9444 | 0.9444 |
|      |     |        | Len | 1.9212 | 1.9205 | 1.9242 | 1.9242 | 1.9242 |
| 1000 | 40  | 0      | Cov | 0.9460 | 0.9450 | 0.9318 | 0.9344 | 0.9372 |
|      |     |        | Len | 1.9083 | 1.9240 | 1.9303 | 3.3769 | 1.9296 |
| 1000 | 100 | 0      | Cov | 0.9448 | 0.9358 | 0.8996 | 0.9372 | 0.9210 |
|      |     |        | Len | 1.9006 | 1.9283 | 1.9290 | 3.9989 | 1.9326 |
| 1000 | 200 | 0      | Cov | 0.9434 | 0.9286 | 0.8568 | 0.9392 | 0.9108 |
|      |     |        | Len | 1.8980 | 1.9305 | 1.8851 | 4.2113 | 1.9323 |

Table 5.197. Etype = 4, J=50, k=1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso  | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|--------|--------|--------|--------|--------|
| 100  | 20  | 0.2236 | Cov | 0.9834 | 0.9828 | 0.9652 | 0.9660 | 0.9940 |
|      |     |        | Len | 2.4502 | 2.2051 | 4.1036 | 4.1893 | 2.1990 |
| 100  | 40  | 0.1581 | Cov | 0.9798 | 0.9792 | 0.9602 | 0.9606 | 0.9946 |
|      |     |        | Len | 2.4776 | 2.2059 | 4.3933 | 4.4644 | 2.1988 |
| 100  | 100 | 0.1    | Cov | 0.9784 | 0.9754 | 0.9672 | 0.9680 | 0.9954 |
|      |     |        | Len | 2.5161 | 2.2062 | 4.6475 | 4.7049 | 2.1997 |
| 100  | 200 | 0.07   | Cov | 0.9776 | 0.9696 | 0.9634 | 0.9652 | 0.9934 |
|      |     |        | Len | 2.5652 | 2.2076 | 4.7694 | 4.8226 | 2.2002 |
| 400  | 20  | 0.2236 | Cov | 0.9546 | 0.9608 | 0.9442 | 0.9432 | 0.9468 |
|      |     |        | Len | 1.9542 | 1.9643 | 1.9725 | 3.1849 | 1.9715 |
| 400  | 40  | 0.1581 | Cov | 0.9560 | 0.9614 | 0.9166 | 0.9396 | 0.9378 |
|      |     |        | Len | 1.9544 | 1.9654 | 1.9697 | 3.6664 | 1.9740 |
| 400  | 100 | 0.1    | Cov | 0.9534 | 0.9572 | 0.8346 | 0.9490 | 0.9234 |
|      |     |        | Len | 1.9585 | 1.9672 | 1.8891 | 4.0451 | 1.9751 |
| 400  | 200 | 0.07   | Cov | 0.9606 | 0.9530 | 0.6652 | 0.9514 | 0.9212 |
|      |     |        | Len | 1.9632 | 1.9685 | 1.6211 | 4.2226 | 1.9721 |
| 1000 | 20  | 0.2236 | Cov | 0.9390 | 0.9522 | 0.9392 | 0.9392 | 0.9392 |
|      |     |        | Len | 1.9196 | 1.9194 | 1.9240 | 1.9240 | 1.9240 |
| 1000 | 40  | 0.1581 | Cov | 0.9428 | 0.9546 | 0.9294 | 0.9382 | 0.9314 |
|      |     |        | Len | 1.9081 | 1.9195 | 1.9300 | 3.0593 | 1.9292 |
| 1000 | 100 | 0.1    | Cov | 0.9394 | 0.9496 | 0.8946 | 0.9414 | 0.9170 |
|      |     |        | Len | 1.9011 | 1.9199 | 1.9290 | 3.7261 | 1.9327 |
| 1000 | 200 | 0.07   | Cov | 0.9354 | 0.9464 | 0.8516 | 0.9446 | 0.9110 |
|      |     |        | Len | 1.8985 | 1.9202 | 1.8853 | 3.9950 | 1.9316 |

Table 5.198. Etype = 4, J=50, k=1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|---------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9646  | 0.9854 | 0.9900 | 0.9900 | 0.9844 |
|      |     |        | Len | 10.5127 | 2.2013 | 2.2704 | 2.2705 | 2.2206 |
| 100  | 40  | 0.9    | Cov | 0.9668  | 0.9828 | 0.9892 | 0.9892 | 0.9790 |
|      |     |        | Len | 15.9635 | 2.2040 | 2.2728 | 2.2728 | 2.2225 |
| 100  | 100 | 0.9    | Cov | 0.9678  | 0.9770 | 0.9884 | 0.9884 | 0.9766 |
|      |     |        | Len | 26.4048 | 2.2060 | 2.2764 | 2.2764 | 2.2229 |
| 100  | 200 | 0.9    | Cov | 0.9712  | 0.9752 | 0.9880 | 0.9880 | 0.9746 |
|      |     |        | Len | 45.5496 | 2.2059 | 2.2743 | 2.2743 | 2.2204 |
| 400  | 20  | 0.9    | Cov | 0.9648  | 0.9630 | 0.9384 | 0.9564 | 0.9430 |
|      |     |        | Len | 1.9793  | 1.9641 | 1.9745 | 2.0050 | 1.9727 |
| 400  | 40  | 0.9    | Cov | 0.9652  | 0.9606 | 0.9122 | 0.9534 | 0.9290 |
|      |     |        | Len | 2.1170  | 1.9643 | 1.9690 | 2.0171 | 1.9735 |
| 400  | 100 | 0.9    | Cov | 0.9646  | 0.9550 | 0.8252 | 0.9548 | 0.9210 |
|      |     |        | Len | 3.1030  | 1.9660 | 1.8884 | 2.0252 | 1.9752 |
| 400  | 200 | 0.9    | Cov | 0.9578  | 0.9570 | 0.6646 | 0.9582 | 0.9148 |
|      |     |        | Len | 5.3494  | 1.9671 | 1.6108 | 2.0287 | 1.9731 |
| 1000 | 20  | 0.9    | Cov | 0.9562  | 0.9542 | 0.9448 | 0.9448 | 0.9448 |
|      |     |        | Len | 1.9225  | 1.9195 | 1.9244 | 1.9244 | 1.9244 |
| 1000 | 40  | 0.9    | Cov | 0.9554  | 0.9518 | 0.9274 | 0.9414 | 0.9282 |
|      |     |        | Len | 1.9267  | 1.9193 | 1.9300 | 1.9538 | 1.9294 |
| 1000 | 100 | 0.9    | Cov | 0.9560  | 0.9562 | 0.9070 | 0.9460 | 0.9196 |
|      |     |        | Len | 1.9406  | 1.9196 | 1.9284 | 1.9723 | 1.9325 |
| 1000 | 200 | 0.9    | Cov | 0.9578  | 0.9560 | 0.8496 | 0.9534 | 0.9134 |
|      |     |        | Len | 1.9648  | 1.9207 | 1.8859 | 1.9800 | 1.9325 |

Table 5.199. Etype = 4, J=50, k=19,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS    | PCR     | FS      |
|------|-----|--------|-----|---------|---------|--------|---------|---------|
| 100  | 20  | 0      | Cov | 0.9568  | 0.9184  | 0.9292 | 0.9580  | 0.9526  |
|      |     |        | Len | 17.7082 | 13.8079 | 6.6031 | 18.6780 | 17.9760 |
| 100  | 40  | 0      | Cov | 0.9592  | 0.9166  | 0.8928 | 0.9634  | 0.9508  |
|      |     |        | Len | 17.7027 | 13.8601 | 7.9985 | 18.9825 | 17.9331 |
| 100  | 100 | 0      | Cov | 0.9592  | 0.9230  | 0.8244 | 0.9650  | 0.9514  |
|      |     |        | Len | 17.7156 | 13.8747 | 8.7200 | 19.2575 | 17.9586 |
| 100  | 200 | 0      | Cov | 0.9616  | 0.9296  | 0.7188 | 0.9608  | 0.9466  |
|      |     |        | Len | 18.7565 | 15.7606 | 8.3039 | 19.3416 | 17.9552 |
| 400  | 20  | 0      | Cov | 0.9558  | 0.9396  | 0.9630 | 0.9460  | 0.9346  |
|      |     |        | Len | 16.5187 | 10.6778 | 2.0501 | 13.7645 | 13.3738 |
| 400  | 40  | 0      | Cov | 0.9584  | 0.9362  | 0.9318 | 0.9538  | 0.9370  |
|      |     |        | Len | 16.5180 | 10.6751 | 2.0527 | 16.0214 | 13.3648 |
| 400  | 100 | 0      | Cov | 0.9596  | 0.9388  | 0.8574 | 0.9578  | 0.9378  |
|      |     |        | Len | 16.5096 | 10.6481 | 1.9833 | 17.3029 | 13.3743 |
| 400  | 200 | 0      | Cov | 0.9632  | 0.9460  | 0.6908 | 0.9596  | 0.9444  |
|      |     |        | Len | 16.5207 | 10.6925 | 1.7906 | 17.7818 | 13.3646 |
| 1000 | 20  | 0      | Cov | 0.9688  | 0.9690  | 0.9690 | 0.9690  | 0.9690  |
|      |     |        | Len | 2.0129  | 2.0058  | 2.0058 | 2.0058  | 2.0058  |
| 1000 | 40  | 0      | Cov | 0.9630  | 0.9640  | 0.9526 | 0.9454  | 0.9680  |
|      |     |        | Len | 2.2228  | 2.0076  | 2.0183 | 12.2247 | 2.0056  |
| 1000 | 100 | 0      | Cov | 0.9564  | 0.9624  | 0.9256 | 0.9504  | 0.9650  |
|      |     |        | Len | 2.3208  | 2.0085  | 2.0277 | 15.7201 | 2.0055  |
| 1000 | 200 | 0      | Cov | 0.9602  | 0.9640  | 0.8790 | 0.9534  | 0.9650  |
|      |     |        | Len | 2.3795  | 2.0096  | 1.9934 | 16.8589 | 2.0063  |

Table 5.200. Etype = 4, J=50, k=19,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 100  | 20  | 0.2236 | Cov | 0.9632  | 0.9266  | 0.9792  | 0.9784  | 0.9402  |
|      |     |        | Len | 69.8262 | 15.8281 | 2.3813  | 2.4000  | 46.5679 |
| 100  | 40  | 0.1581 | Cov | 0.9612  | 0.9028  | 0.9650  | 0.9656  | 0.9370  |
|      |     |        | Len | 68.8910 | 17.0394 | 11.7260 | 11.9590 | 48.7826 |
| 100  | 100 | 0.1    | Cov | 0.9616  | 0.8802  | 0.9642  | 0.9666  | 0.9320  |
|      |     |        | Len | 67.3800 | 17.9376 | 15.6507 | 15.8701 | 50.0622 |
| 100  | 200 | 0.07   | Cov | 0.9648  | 0.8874  | 0.9662  | 0.9670  | 0.9276  |
|      |     |        | Len | 78.0758 | 25.1793 | 17.0820 | 17.2846 | 50.1822 |
| 400  | 20  | 0.2236 | Cov | 0.9616  | 0.9562  | 0.9580  | 0.9752  | 0.9242  |
|      |     |        | Len | 65.6704 | 9.9439  | 2.0493  | 2.0691  | 16.2255 |
| 400  | 40  | 0.1581 | Cov | 0.9616  | 0.9542  | 0.9320  | 0.9532  | 0.9366  |
|      |     |        | Len | 64.7273 | 10.8226 | 2.0535  | 10.0211 | 17.5203 |
| 400  | 100 | 0.1    | Cov | 0.9600  | 0.9436  | 0.8550  | 0.9550  | 0.9216  |
|      |     |        | Len | 63.3828 | 11.7744 | 1.9840  | 14.1939 | 18.6899 |
| 400  | 200 | 0.07   | Cov | 0.9602  | 0.9330  | 0.6944  | 0.9586  | 0.9264  |
|      |     |        | Len | 61.9345 | 12.4122 | 1.8542  | 15.8363 | 19.3521 |
| 1000 | 20  | 0.2236 | Cov | 0.9606  | 0.9674  | 0.9674  | 0.9674  | 0.9674  |
|      |     |        | Len | 2.7167  | 2.0058  | 2.0058  | 2.0058  | 2.0058  |
| 1000 | 40  | 0.1581 | Cov | 0.9634  | 0.9658  | 0.9562  | 0.9452  | 0.9672  |
|      |     |        | Len | 13.9587 | 2.0355  | 2.0180  | 7.7092  | 2.0083  |
| 1000 | 100 | 0.1    | Cov | 0.9608  | 0.9650  | 0.9262  | 0.9512  | 0.9668  |
|      |     |        | Len | 28.2512 | 2.2605  | 2.0282  | 12.9022 | 2.0134  |
| 1000 | 200 | 0.07   | Cov | 0.9592  | 0.9664  | 0.8828  | 0.9552  | 0.9684  |
|      |     |        | Len | 37.6885 | 2.7468  | 1.9933  | 15.0167 | 2.0059  |

Table 5.201. Etype = 4, J=50, k=19,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL     | PLS    | PCR    | FS     |
|------|-----|--------|-----|----------|--------|--------|--------|--------|
| 100  | 20  | 0.9    | Cov | 0.9650   | 0.9392 | 0.9930 | 0.9930 | 0.9406 |
|      |     |        | Len | 230.8572 | 3.8273 | 2.1990 | 2.1990 | 7.4740 |
| 100  | 40  | 0.9    | Cov | 0.9674   | 0.9432 | 0.9752 | 0.9752 | 0.9402 |
|      |     |        | Len | 329.5523 | 3.7704 | 2.7333 | 2.7333 | 7.4297 |
| 100  | 100 | 0.9    | Cov | 0.9704   | 0.9346 | 0.9714 | 0.9714 | 0.9240 |
|      |     |        | Len | 523.0769 | 3.6219 | 2.9645 | 2.9645 | 7.3322 |
| 100  | 200 | 0.9    | Cov | 0.9670   | 0.9264 | 0.9686 | 0.9686 | 0.9188 |
|      |     |        | Len | 896.4131 | 3.6200 | 3.0306 | 3.0306 | 7.2336 |
| 400  | 20  | 0.9    | Cov | 0.9596   | 0.9524 | 0.9626 | 0.9776 | 0.9382 |
|      |     |        | Len | 106.3606 | 3.0093 | 2.0489 | 2.0354 | 3.0134 |
| 400  | 40  | 0.9    | Cov | 0.9622   | 0.9528 | 0.9358 | 0.9626 | 0.9378 |
|      |     |        | Len | 74.8029  | 3.4451 | 2.0520 | 2.4667 | 3.0168 |
| 400  | 100 | 0.9    | Cov | 0.9636   | 0.9492 | 0.8712 | 0.9636 | 0.9336 |
|      |     |        | Len | 137.0205 | 3.5381 | 1.9805 | 2.7234 | 3.0358 |
| 400  | 200 | 0.9    | Cov | 0.9640   | 0.9486 | 0.7010 | 0.9608 | 0.9258 |
|      |     |        | Len | 215.7979 | 3.5042 | 1.7006 | 2.8123 | 3.0535 |
| 1000 | 20  | 0.9    | Cov | 0.9602   | 0.9644 | 0.9706 | 0.9706 | 0.9706 |
|      |     |        | Len | 9.7046   | 2.7306 | 2.0061 | 2.0061 | 2.0061 |
| 1000 | 40  | 0.9    | Cov | 0.9628   | 0.9602 | 0.9564 | 0.9620 | 0.9666 |
|      |     |        | Len | 14.0998  | 3.5493 | 2.0185 | 2.2873 | 2.0071 |
| 1000 | 100 | 0.9    | Cov | 0.9652   | 0.9582 | 0.9310 | 0.9574 | 0.9694 |
|      |     |        | Len | 22.3490  | 3.7253 | 2.0289 | 2.6073 | 2.0108 |
| 1000 | 200 | 0.9    | Cov | 0.9654   | 0.9592 | 0.8712 | 0.9586 | 0.9646 |
|      |     |        | Len | 30.6906  | 3.7604 | 1.9913 | 2.7294 | 2.0111 |

Table 5.202. Etype = 4, J=50, k=p-1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 100  | 40  | 0      | Cov | 0.9548  | 0.9082  | 0.8906  | 0.9604  | 0.9516  |
|      |     |        | Len | 25.6394 | 20.9416 | 11.2454 | 27.0377 | 25.9949 |
| 400  | 40  | 0      | Cov | 0.9564  | 0.9352  | 0.9326  | 0.9518  | 0.9382  |
|      |     |        | Len | 24.5502 | 19.2615 | 2.0524  | 22.8591 | 21.6276 |
| 1000 | 40  | 0      | Cov | 0.9658  | 0.9442  | 0.9558  | 0.9456  | 0.9354  |
|      |     |        | Len | 23.4846 | 14.8657 | 2.0179  | 17.3239 | 16.9444 |
| 100  | 100 | 0      | Cov | 0.9562  | 0.8946  | 0.8258  | 0.9650  | 0.9484  |
|      |     |        | Len | 41.0737 | 33.8457 | 19.5845 | 43.6193 | 41.7534 |
| 400  | 100 | 0      | Cov | 0.9598  | 0.9232  | 0.8640  | 0.9562  | 0.9310  |
|      |     |        | Len | 39.8317 | 33.9176 | 1.9872  | 39.1089 | 36.6911 |
| 1000 | 100 | 0      | Cov | 0.9576  | 0.9258  | 0.9276  | 0.9502  | 0.9258  |
|      |     |        | Len | 39.2547 | 31.8054 | 2.0285  | 35.5793 | 33.5287 |
| 100  | 200 | 0      | Cov | 0.9592  | 0.9144  | 0.7410  | 0.9642  | 0.9472  |
|      |     |        | Len | 60.8763 | 53.1727 | 26.4803 | 62.1063 | 59.2100 |
| 400  | 200 | 0      | Cov | 0.9500  | 0.9108  | 0.6534  | 0.9518  | 0.9202  |
|      |     |        | Len | 55.1246 | 47.9290 | 2.4078  | 55.1742 | 51.4684 |
| 1000 | 200 | 0      | Cov | 0.9600  | 0.9296  | 0.8748  | 0.9548  | 0.9316  |
|      |     |        | Len | 56.6031 | 48.6421 | 1.9931  | 54.0657 | 50.4780 |

Table 5.203. Etype = 4, J=50, k=p-1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL       | PLS    | PCR    | FS       |
|------|-----|--------|-----|----------|----------|--------|--------|----------|
| 100  | 40  | 0.1581 | Cov | 0.9624   | 0.8944   | 0.9736 | 0.9744 | 0.9320   |
|      |     |        | Len | 139.9899 | 34.1956  | 2.6555 | 2.6821 | 98.8902  |
| 100  | 100 | 0.1    | Cov | 0.9632   | 0.8704   | 0.9650 | 0.9660 | 0.9346   |
|      |     |        | Len | 344.5808 | 85.0248  | 3.4051 | 3.4421 | 254.0269 |
| 100  | 200 | 0.07   | Cov | 0.9666   | 0.8764   | 0.9648 | 0.9652 | 0.9204   |
|      |     |        | Len | 799.7930 | 236.3344 | 4.4549 | 4.5041 | 505.6271 |
| 400  | 40  | 0.1581 | Cov | 0.9634   | 0.9358   | 0.9310 | 0.9700 | 0.9212   |
|      |     |        | Len | 131.8341 | 24.3325  | 2.0538 | 2.1527 | 38.7537  |
| 400  | 100 | 0.1    | Cov | 0.9596   | 0.9250   | 0.8758 | 0.9674 | 0.9100   |
|      |     |        | Len | 324.8297 | 65.1857  | 1.9802 | 2.4139 | 105.6726 |
| 400  | 200 | 0.07   | Cov | 0.9608   | 0.9168   | 0.6938 | 0.9606 | 0.8990   |
|      |     |        | Len | 637.1557 | 130.5025 | 1.7008 | 2.8109 | 215.3958 |
| 1000 | 40  | 0.1581 | Cov | 0.9558   | 0.9516   | 0.9534 | 0.9678 | 0.9296   |
|      |     |        | Len | 128.7479 | 17.7718  | 2.0180 | 2.0361 | 19.8288  |
| 1000 | 100 | 0.1    | Cov | 0.9632   | 0.9430   | 0.9324 | 0.9626 | 0.9132   |
|      |     |        | Len | 237.6814 | 52.7966  | 2.0285 | 2.2960 | 64.1876  |
| 1000 | 200 | 0.07   | Cov | 0.9620   | 0.9364   | 0.8918 | 0.9612 | 0.9054   |
|      |     |        | Len | 625.8326 | 107.7041 | 1.9927 | 2.3581 | 133.2046 |

Table 5.204. Etype = 4, J=50, k=p-1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso     | RL      | PLS    | PCR    | FS      |
|------|-----|--------|-----|-----------|---------|--------|--------|---------|
| 100  | 40  | 0.9    | Cov | 0.9648    | 0.9308  | 0.9938 | 0.9938 | 0.9304  |
|      |     |        | Len | 676.7309  | 6.4794  | 2.1991 | 2.1991 | 14.6622 |
| 400  | 40  | 0.9    | Cov | 0.9634    | 0.9486  | 0.9366 | 0.9788 | 0.9240  |
|      |     |        | Len | 311.4472  | 5.7185  | 2.0521 | 2.0348 | 5.2064  |
| 1000 | 40  | 0.9    | Cov | 0.9620    | 0.9550  | 0.9524 | 0.9678 | 0.9360  |
|      |     |        | Len | 28.9205   | 5.8617  | 2.0187 | 2.0057 | 3.1947  |
| 100  | 100 | 0.9    | Cov | 0.9660    | 0.9260  | 0.9930 | 0.9930 | 0.9204  |
|      |     |        | Len | 2729.5260 | 14.3360 | 2.1994 | 2.1994 | 36.1595 |
| 400  | 100 | 0.9    | Cov | 0.9648    | 0.9488  | 0.8596 | 0.9812 | 0.9092  |
|      |     |        | Len | 1859.1730 | 13.3744 | 1.9795 | 2.0360 | 12.4624 |
| 1000 | 100 | 0.9    | Cov | 0.9630    | 0.9580  | 0.9320 | 0.9700 | 0.9142  |
|      |     |        | Len | 121.6698  | 13.8685 | 2.0282 | 2.0058 | 7.4149  |
| 100  | 200 | 0.9    | Cov | 0.9652    | 0.9200  | 0.9950 | 0.9950 | 0.9216  |
|      |     |        | Len | 9362.4050 | 28.4303 | 2.1993 | 2.1993 | 71.3569 |
| 400  | 200 | 0.9    | Cov | 0.9636    | 0.9388  | 0.6912 | 0.9836 | 0.8968  |
|      |     |        | Len | 6111.7320 | 25.6200 | 1.6979 | 2.0356 | 24.5495 |
| 1000 | 200 | 0.9    | Cov | 0.9632    | 0.9522  | 0.8752 | 0.9732 | 0.9012  |
|      |     |        | Len | 421.7521  | 25.9116 | 1.9927 | 2.0057 | 14.7530 |

Table 5.205. Etype = 5, J=5, k=1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS      | PCR     | FS      |
|------|-----|--------|-----|---------|---------|----------|---------|---------|
| 100  | 20  | 0      | Cov | 0.9258  | 0.9376  | 0.925    | 0.925   | 0.925   |
|      |     |        | Len | 12.5796 | 13.2767 | 12.5799  | 12.5799 | 12.5799 |
| 100  | 40  | 0      | Cov | 0.9282  | 0.9344  | 0.8662   | 0.9358  | 0.8846  |
|      |     |        | Len | 12.0279 | 12.7186 | 10.9417  | 12.8198 | 11.2516 |
| 100  | 100 | 0      | Cov | 0.928   | 0.9266  | 0.146    | 0.9438  | 0.7802  |
|      |     |        | Len | 11.7349 | 11.8108 | 3.01940  | 12.9826 | 9.3105  |
| 100  | 200 | 0      | Cov | 0.9222  | 0.9086  | 2.00E-04 | 0.9378  | 0.6688  |
|      |     |        | Len | 11.5869 | 11.1475 | 0.0073   | 12.8844 | 7.8792  |
| 400  | 20  | 0      | Cov | 0.9434  | 0.9446  | 0.9434   | 0.9434  | 0.9434  |
|      |     |        | Len | 12.1740 | 12.3650 | 12.1728  | 12.1728 | 12.1728 |
| 400  | 40  | 0      | Cov | 0.936   | 0.9386  | 0.936    | 0.936   | 0.936   |
|      |     |        | Len | 11.7765 | 12.1624 | 11.7746  | 11.7746 | 11.7746 |
| 400  | 100 | 0      | Cov | 0.931   | 0.9404  | 0.9206   | 0.9326  | 0.9212  |
|      |     |        | Len | 10.6993 | 11.4872 | 10.5769  | 10.9831 | 10.5878 |
| 400  | 200 | 0      | Cov | 0.9274  | 0.928   | 0.755    | 0.934   | 0.8328  |
|      |     |        | Len | 10.3280 | 10.8774 | 8.92832  | 11.1283 | 9.4445  |
| 1000 | 20  | 0      | Cov | 0.941   | 0.9424  | 0.9412   | 0.9412  | 0.9412  |
|      |     |        | Len | 12.5242 | 12.6149 | 12.5225  | 12.5225 | 12.5225 |
| 1000 | 40  | 0      | Cov | 0.9454  | 0.945   | 0.9458   | 0.9458  | 0.9458  |
|      |     |        | Len | 12.2704 | 12.4530 | 12.2676  | 12.2676 | 12.2676 |
| 1000 | 100 | 0      | Cov | 0.9402  | 0.941   | 0.94     | 0.94    | 0.94    |
|      |     |        | Len | 11.6829 | 12.1434 | 11.6794  | 11.6794 | 11.6794 |
| 1000 | 200 | 0      | Cov | 0.9358  | 0.9454  | 0.9354   | 0.9354  | 0.9354  |
|      |     |        | Len | 10.8651 | 11.7432 | 10.8620  | 10.8620 | 10.8620 |

Table 5.206. Etype = 5, J=5, k=1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 100  | 20  | 0.2236 | Cov | 0.927   | 0.941   | 0.9264  | 0.9264  | 0.9264  |
|      |     |        | Len | 12.6014 | 13.5765 | 12.6022 | 12.6022 | 12.6022 |
| 100  | 40  | 0.1581 | Cov | 0.9332  | 0.9426  | 0.8722  | 0.9386  | 0.8936  |
|      |     |        | Len | 12.1356 | 13.6362 | 10.9956 | 12.8486 | 11.3423 |
| 100  | 100 | 0.1    | Cov | 0.9324  | 0.9412  | 0.1426  | 0.943   | 0.7694  |
|      |     |        | Len | 11.5984 | 13.2617 | 3.1010  | 12.7947 | 9.2731  |
| 100  | 200 | 0.07   | Cov | 0.9322  | 0.9428  | 0.0014  | 0.9418  | 0.6756  |
|      |     |        | Len | 11.6758 | 13.2347 | 0.0111  | 12.9646 | 7.9554  |
| 400  | 20  | 0.2236 | Cov | 0.947   | 0.9456  | 0.947   | 0.947   | 0.947   |
|      |     |        | Len | 12.1779 | 12.4600 | 12.1758 | 12.1758 | 12.1758 |
| 400  | 40  | 0.1581 | Cov | 0.9354  | 0.941   | 0.9356  | 0.9356  | 0.9356  |
|      |     |        | Len | 11.7603 | 12.3512 | 11.7579 | 11.7579 | 11.7579 |
| 400  | 100 | 0.1    | Cov | 0.9276  | 0.941   | 0.9156  | 0.9322  | 0.9158  |
|      |     |        | Len | 10.7391 | 12.1297 | 10.6181 | 11.0151 | 10.6256 |
| 400  | 200 | 0.07   | Cov | 0.924   | 0.939   | 0.7548  | 0.9328  | 0.8328  |
|      |     |        | Len | 10.4139 | 12.182  | 8.96563 | 11.1900 | 9.48812 |
| 1000 | 20  | 0.2236 | Cov | 0.9456  | 0.9488  | 0.9456  | 0.9456  | 0.9456  |
|      |     |        | Len | 12.5023 | 12.6391 | 12.4999 | 12.4999 | 12.4999 |
| 1000 | 40  | 0.1581 | Cov | 0.945   | 0.9454  | 0.945   | 0.945   | 0.945   |
|      |     |        | Len | 12.2532 | 12.5521 | 12.2500 | 12.2500 | 12.2500 |
| 1000 | 100 | 0.1    | Cov | 0.9408  | 0.9436  | 0.94    | 0.94    | 0.94    |
|      |     |        | Len | 11.7167 | 12.4605 | 11.7116 | 11.7116 | 11.7116 |
| 1000 | 200 | 0.07   | Cov | 0.9298  | 0.942   | 0.9296  | 0.9296  | 0.9296  |
|      |     |        | Len | 10.8215 | 12.1867 | 10.8185 | 10.8185 | 10.8185 |

Table 5.207. Etype = 5, J=5, k=1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 100  | 20  | 0.9    | Cov | 0.9428  | 0.9432  | 0.9362  | 0.9362  | 0.9362  |
|      |     |        | Len | 14.1825 | 14.2809 | 12.8950 | 12.8945 | 12.8945 |
| 100  | 40  | 0.9    | Cov | 0.9484  | 0.9482  | 0.8726  | 0.9402  | 0.8972  |
|      |     |        | Len | 14.6475 | 14.6444 | 11.2757 | 13.1552 | 11.6357 |
| 100  | 100 | 0.9    | Cov | 0.9482  | 0.946   | 0.142   | 0.9482  | 0.7874  |
|      |     |        | Len | 14.7963 | 14.7700 | 3.15487 | 13.3202 | 9.56021 |
| 100  | 200 | 0.9    | Cov | 0.9484  | 0.9482  | 0.0018  | 0.9504  | 0.684   |
|      |     |        | Len | 15.5186 | 15.3961 | 0.0116  | 13.8322 | 8.2162  |
| 400  | 20  | 0.9    | Cov | 0.9374  | 0.9382  | 0.9338  | 0.9338  | 0.9338  |
|      |     |        | Len | 12.5808 | 12.5789 | 12.2404 | 12.2404 | 12.2404 |
| 400  | 40  | 0.9    | Cov | 0.9422  | 0.9418  | 0.9396  | 0.9396  | 0.9396  |
|      |     |        | Len | 12.5490 | 12.5415 | 11.7984 | 11.7984 | 11.7984 |
| 400  | 100 | 0.9    | Cov | 0.9466  | 0.9474  | 0.9182  | 0.9338  | 0.92    |
|      |     |        | Len | 12.6361 | 12.6031 | 10.7286 | 11.0926 | 10.7348 |
| 400  | 200 | 0.9    | Cov | 0.9474  | 0.9474  | 0.7514  | 0.942   | 0.8328  |
|      |     |        | Len | 12.6324 | 12.5248 | 8.96418 | 11.0634 | 9.49282 |
| 1000 | 20  | 0.9    | Cov | 0.943   | 0.9434  | 0.9434  | 0.9434  | 0.9434  |
|      |     |        | Len | 12.7024 | 12.7001 | 12.5302 | 12.5302 | 12.5302 |
| 1000 | 40  | 0.9    | Cov | 0.9438  | 0.9442  | 0.9436  | 0.9436  | 0.9436  |
|      |     |        | Len | 12.7051 | 12.6960 | 12.3385 | 12.3385 | 12.3385 |
| 1000 | 100 | 0.9    | Cov | 0.945   | 0.9442  | 0.9404  | 0.9404  | 0.9404  |
|      |     |        | Len | 12.6737 | 12.6341 | 11.7139 | 11.7139 | 11.7139 |
| 1000 | 200 | 0.9    | Cov | 0.9464  | 0.9474  | 0.9298  | 0.9298  | 0.9298  |
|      |     |        | Len | 12.6283 | 12.564  | 10.8085 | 10.8085 | 10.8085 |

Table 5.208. Etype = 5, J=5, k=19,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 100  | 20  | 0      | Cov | 0.9576  | 0.9574  | 0.9574  | 0.9574  | 0.9574  |
|      |     |        | Len | 21.8486 | 21.8758 | 21.8446 | 21.8446 | 21.8446 |
| 100  | 40  | 0      | Cov | 0.9524  | 0.939   | 0.9104  | 0.9494  | 0.9266  |
|      |     |        | Len | 21.1718 | 18.0553 | 15.3198 | 21.9683 | 16.9410 |
| 100  | 100 | 0      | Cov | 0.9432  | 0.918   | 0.158   | 0.9438  | 0.8382  |
|      |     |        | Len | 19.4678 | 15.5473 | 3.4440  | 21.5672 | 13.1729 |
| 100  | 200 | 0      | Cov | 0.935   | 0.8908  | 0.0014  | 0.9392  | 0.6958  |
|      |     |        | Len | 18.9057 | 14.8884 | 0.00985 | 21.1592 | 11.3972 |
| 400  | 20  | 0      | Cov | 0.9716  | 0.9718  | 0.9718  | 0.9718  | 0.9718  |
|      |     |        | Len | 21.2146 | 21.2147 | 21.2147 | 21.2147 | 21.2147 |
| 400  | 40  | 0      | Cov | 0.9694  | 0.9714  | 0.9696  | 0.9696  | 0.9696  |
|      |     |        | Len | 20.2454 | 20.7301 | 20.2227 | 20.2227 | 20.2227 |
| 400  | 100 | 0      | Cov | 0.9626  | 0.9642  | 0.9572  | 0.9614  | 0.9568  |
|      |     |        | Len | 17.9759 | 19.1412 | 17.0824 | 18.5170 | 17.1200 |
| 400  | 200 | 0      | Cov | 0.9538  | 0.956   | 0.8676  | 0.9516  | 0.9248  |
|      |     |        | Len | 17.8300 | 17.6462 | 12.3695 | 20.2032 | 13.9946 |
| 1000 | 20  | 0      | Cov | 0.9576  | 0.9566  | 0.9566  | 0.9566  | 0.9566  |
|      |     |        | Len | 15.7503 | 15.7497 | 15.7497 | 15.7497 | 15.7497 |
| 1000 | 40  | 0      | Cov | 0.952   | 0.9514  | 0.9522  | 0.9522  | 0.9522  |
|      |     |        | Len | 15.5954 | 15.7333 | 15.585  | 15.585  | 15.585  |
| 1000 | 100 | 0      | Cov | 0.9514  | 0.9524  | 0.9508  | 0.9508  | 0.9508  |
|      |     |        | Len | 14.6822 | 15.2391 | 14.6611 | 14.6611 | 14.6611 |
| 1000 | 200 | 0      | Cov | 0.947   | 0.954   | 0.9464  | 0.9464  | 0.9464  |
|      |     |        | Len | 13.3337 | 14.5436 | 13.3045 | 13.3045 | 13.3045 |

Table 5.209. Etype = 5, J=5, k=19,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 100  | 20  | 0.2236 | Cov | 0.97    | 0.9698  | 0.9702  | 0.9702  | 0.9702  |
|      |     |        | Len | 22.3154 | 22.3216 | 22.2330 | 22.2330 | 22.2330 |
| 100  | 40  | 0.1581 | Cov | 0.9854  | 0.9588  | 0.9394  | 0.9664  | 0.9528  |
|      |     |        | Len | 42.7944 | 20.4749 | 17.0877 | 22.5241 | 18.8733 |
| 100  | 100 | 0.1    | Cov | 0.982   | 0.9384  | 0.195   | 0.9644  | 0.896   |
|      |     |        | Len | 53.9787 | 20.4416 | 4.1356  | 24.8207 | 16.6221 |
| 100  | 200 | 0.07   | Cov | 0.9822  | 0.9186  | 0.0022  | 0.9652  | 0.7968  |
|      |     |        | Len | 59.4845 | 20.8285 | 0.0179  | 25.9728 | 15.1522 |
| 400  | 20  | 0.2236 | Cov | 0.97    | 0.9702  | 0.9702  | 0.9702  | 0.9702  |
|      |     |        | Len | 21.2529 | 21.2535 | 21.2535 | 21.2535 | 21.2535 |
| 400  | 40  | 0.1581 | Cov | 0.9646  | 0.9648  | 0.964   | 0.964   | 0.964   |
|      |     |        | Len | 20.7000 | 20.9104 | 20.2895 | 20.2895 | 20.2895 |
| 400  | 100 | 0.1    | Cov | 0.9624  | 0.9652  | 0.9554  | 0.9574  | 0.9544  |
|      |     |        | Len | 18.1066 | 20.0108 | 17.1180 | 18.3718 | 17.1537 |
| 400  | 200 | 0.07   | Cov | 0.9574  | 0.9616  | 0.8692  | 0.9522  | 0.9244  |
|      |     |        | Len | 18.0041 | 19.5543 | 12.4616 | 19.7989 | 14.1080 |
| 1000 | 20  | 0.2236 | Cov | 0.9586  | 0.9582  | 0.9582  | 0.9582  | 0.9582  |
|      |     |        | Len | 15.7376 | 15.7355 | 15.7355 | 15.7355 | 15.7355 |
| 1000 | 40  | 0.1581 | Cov | 0.9556  | 0.9544  | 0.9554  | 0.9554  | 0.9554  |
|      |     |        | Len | 15.7489 | 15.7450 | 15.5582 | 15.5582 | 15.5582 |
| 1000 | 100 | 0.1    | Cov | 0.9518  | 0.9512  | 0.9508  | 0.9508  | 0.9508  |
|      |     |        | Len | 15.3422 | 15.5728 | 14.7249 | 14.7249 | 14.7249 |
| 1000 | 200 | 0.07   | Cov | 0.9462  | 0.9514  | 0.9442  | 0.9442  | 0.9442  |
|      |     |        | Len | 13.6589 | 15.1367 | 13.2567 | 13.2567 | 13.2567 |

Table 5.210. Etype = 5, J=5, k=19,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 100  | 20  | 0.9    | Cov | 0.9564  | 0.952   | 0.9456  | 0.9456  | 0.9456  |
|      |     |        | Len | 16.8047 | 15.7409 | 14.6960 | 14.6960 | 14.6960 |
| 100  | 40  | 0.9    | Cov | 0.9552  | 0.9502  | 0.8872  | 0.9482  | 0.9042  |
|      |     |        | Len | 18.5014 | 15.2640 | 12.0424 | 14.3738 | 12.4881 |
| 100  | 100 | 0.9    | Cov | 0.9618  | 0.9412  | 0.1498  | 0.9438  | 0.8014  |
|      |     |        | Len | 25.3706 | 15.0006 | 3.25912 | 14.3495 | 10.0154 |
| 100  | 200 | 0.9    | Cov | 0.9732  | 0.9432  | 0.0022  | 0.948   | 0.6964  |
|      |     |        | Len | 35.3294 | 14.4798 | 0.01169 | 14.1647 | 8.3613  |
| 400  | 20  | 0.9    | Cov | 0.9462  | 0.9472  | 0.943   | 0.943   | 0.943   |
|      |     |        | Len | 13.9099 | 13.3975 | 13.2507 | 13.2507 | 13.2507 |
| 400  | 40  | 0.9    | Cov | 0.9434  | 0.9402  | 0.9376  | 0.9376  | 0.9376  |
|      |     |        | Len | 15.0170 | 13.4708 | 12.7569 | 12.7569 | 12.7569 |
| 400  | 100 | 0.9    | Cov | 0.9502  | 0.9456  | 0.9244  | 0.9352  | 0.9244  |
|      |     |        | Len | 21.3066 | 13.1788 | 11.2113 | 11.6349 | 11.2219 |
| 400  | 200 | 0.9    | Cov | 0.9536  | 0.9458  | 0.7746  | 0.941   | 0.849   |
|      |     |        | Len | 30.1652 | 13.1159 | 9.2705  | 11.6880 | 9.8688  |
| 1000 | 20  | 0.9    | Cov | 0.9458  | 0.945   | 0.9462  | 0.9462  | 0.9462  |
|      |     |        | Len | 13.6712 | 13.3215 | 13.2764 | 13.2764 | 13.2764 |
| 1000 | 40  | 0.9    | Cov | 0.9482  | 0.9446  | 0.9432  | 0.9432  | 0.9432  |
|      |     |        | Len | 14.3903 | 13.2879 | 12.9620 | 12.9620 | 12.9620 |
| 1000 | 100 | 0.9    | Cov | 0.9482  | 0.9454  | 0.9406  | 0.9406  | 0.9406  |
|      |     |        | Len | 20.690  | 13.2044 | 12.2765 | 12.2765 | 12.2765 |
| 1000 | 200 | 0.9    | Cov | 0.9554  | 0.945   | 0.9336  | 0.9336  | 0.9336  |
|      |     |        | Len | 29.5861 | 13.1402 | 11.2574 | 11.2574 | 11.2574 |

Table 5.211. Etype = 5, J=5, k=p-1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 100  | 40  | 0      | Cov | 0.9586  | 0.9208  | 0.9292  | 0.952   | 0.8948  |
|      |     |        | Len | 28.5380 | 21.1580 | 15.5517 | 25.4152 | 20.5136 |
| 400  | 40  | 0      | Cov | 0.9698  | 0.9696  | 0.9696  | 0.9696  | 0.9696  |
|      |     |        | Len | 21.7573 | 21.7556 | 21.7556 | 21.7556 | 21.7556 |
| 1000 | 40  | 0      | Cov | 0.9676  | 0.9676  | 0.9676  | 0.9676  | 0.9676  |
|      |     |        | Len | 19.8811 | 19.8813 | 19.8813 | 19.8813 | 19.8813 |
| 100  | 100 | 0      | Cov | 0.9264  | 0.8012  | 0.1494  | 0.9232  | 0.6942  |
|      |     |        | Len | 36.7954 | 26.2149 | 3.3562  | 35.0649 | 23.7848 |
| 400  | 100 | 0      | Cov | 0.9808  | 0.9638  | 0.9694  | 0.969   | 0.9502  |
|      |     |        | Len | 36.7982 | 24.5807 | 21.1077 | 26.089  | 23.7750 |
| 1000 | 100 | 0      | Cov | 0.9748  | 0.9742  | 0.9742  | 0.9742  | 0.9742  |
|      |     |        | Len | 22.7113 | 22.7117 | 22.7117 | 22.7117 | 22.7117 |
| 100  | 200 | 0      | Cov | 0.9224  | 0.7574  | 0.001   | 0.919   | 0.605   |
|      |     |        | Len | 51.1282 | 35.8587 | 0.0189  | 50.6744 | 29.8623 |
| 400  | 200 | 0      | Cov | 0.8888  | 0.7662  | 0.779   | 0.8652  | 0.7142  |
|      |     |        | Len | 41.6048 | 29.2902 | 9.4011  | 34.6423 | 26.8346 |
| 1000 | 200 | 0      | Cov | 0.9722  | 0.971   | 0.971   | 0.971   | 0.971   |
|      |     |        | Len | 23.2572 | 23.2567 | 23.2567 | 23.2567 | 23.2567 |

Table 5.212. Etype = 5, J=5, k=p-1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL       | PLS     | PCR     | FS      |
|------|-----|--------|-----|----------|----------|---------|---------|---------|
| 100  | 40  | 0.1581 | Cov | 0.9942   | 0.9578   | 0.9434  | 0.9646  | 0.9228  |
|      |     |        | Len | 110.8926 | 28.6356  | 18.2604 | 22.7531 | 24.5963 |
| 100  | 100 | 0.1    | Cov | 0.993    | 0.925    | 0.2016  | 0.9676  | 0.8048  |
|      |     |        | Len | 331.234  | 72.6509  | 4.3968  | 22.7630 | 51.3151 |
| 100  | 200 | 0.07   | Cov | 0.9938   | 0.8988   | 0.002   | 0.9702  | 0.713   |
|      |     |        | Len | 711.8182 | 151.4833 | 0.01613 | 22.9475 | 93.3569 |
| 400  | 40  | 0.1581 | Cov | 0.9678   | 0.9676   | 0.9676  | 0.9676  | 0.9676  |
|      |     |        | Len | 21.6885  | 21.6812  | 21.6811 | 21.6811 | 21.6811 |
| 400  | 100 | 0.1    | Cov | 0.993    | 0.9698   | 0.9692  | 0.9742  | 0.947   |
|      |     |        | Len | 117.9303 | 26.6882  | 21.1363 | 22.3814 | 23.7304 |
| 400  | 200 | 0.07   | Cov | 0.9884   | 0.9336   | 0.916   | 0.9692  | 0.8364  |
|      |     |        | Len | 345.5698 | 68.8206  | 15.2221 | 22.3215 | 49.6054 |
| 1000 | 40  | 0.1581 | Cov | 0.9634   | 0.963    | 0.963   | 0.963   | 0.963   |
|      |     |        | Len | 19.8178  | 19.8024  | 19.8024 | 19.8024 | 19.8024 |
| 1000 | 100 | 0.1    | Cov | 0.9724   | 0.9716   | 0.9716  | 0.9716  | 0.9716  |
|      |     |        | Len | 22.9606  | 22.6312  | 22.6312 | 22.6312 | 22.6312 |
| 1000 | 200 | 0.07   | Cov | 0.9854   | 0.9738   | 0.9738  | 0.9738  | 0.9738  |
|      |     |        | Len | 38.2751  | 23.2984  | 23.2984 | 23.2984 | 23.2984 |

Table 5.213. Etype = 5, J=5, k=p-1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|----------|---------|---------|---------|---------|
| 100  | 40  | 0.9    | Cov | 0.9738   | 0.9528  | 0.9082  | 0.9516  | 0.924   |
|      |     |        | Len | 34.9475  | 18.2568 | 13.9740 | 17.1172 | 14.5943 |
| 400  | 40  | 0.9    | Cov | 0.9556   | 0.9486  | 0.9454  | 0.9454  | 0.9454  |
|      |     |        | Len | 29.0085  | 15.1709 | 14.1482 | 14.1482 | 14.1482 |
| 1000 | 40  | 0.9    | Cov | 0.9524   | 0.9486  | 0.9484  | 0.9484  | 0.9484  |
|      |     |        | Len | 27.8509  | 14.5081 | 13.9680 | 13.9680 | 13.9680 |
| 100  | 100 | 0.9    | Cov | 0.9918   | 0.9652  | 0.1838  | 0.9656  | 0.8766  |
|      |     |        | Len | 155.072  | 24.1433 | 3.8547  | 19.7862 | 13.4246 |
| 400  | 100 | 0.9    | Cov | 0.978    | 0.9644  | 0.9546  | 0.9592  | 0.9552  |
|      |     |        | Len | 129.5812 | 22.4868 | 16.0230 | 16.9215 | 16.0497 |
| 1000 | 100 | 0.9    | Cov | 0.9666   | 0.9612  | 0.9586  | 0.9586  | 0.9586  |
|      |     |        | Len | 120.2519 | 20.4326 | 15.9724 | 15.9724 | 15.9724 |
| 100  | 200 | 0.9    | Cov | 0.9932   | 0.9662  | 0.002   | 0.9672  | 0.7948  |
|      |     |        | Len | 778.9447 | 34.9175 | 0.0151  | 21.5343 | 14.6161 |
| 400  | 200 | 0.9    | Cov | 0.988    | 0.9702  | 0.8834  | 0.9646  | 0.9306  |
|      |     |        | Len | 381.4635 | 34.4416 | 12.9886 | 18.9996 | 14.7174 |
| 1000 | 200 | 0.9    | Cov | 0.982    | 0.9752  | 0.9652  | 0.9652  | 0.9652  |
|      |     |        | Len | 363.7216 | 34.1324 | 18.6392 | 18.6392 | 18.6392 |

Table 5.214. Etype = 5, J=10, k=1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 100  | 20  | 0      | Cov | 0.9396  | 0.9432  | 0.9328  | 0.9426  | 0.9332  |
|      |     |        | Len | 13.3066 | 13.5331 | 12.7645 | 13.8106 | 12.9253 |
| 100  | 40  | 0      | Cov | 0.9362  | 0.9336  | 0.8638  | 0.9404  | 0.9182  |
|      |     |        | Len | 13.0505 | 12.9936 | 10.9648 | 13.8454 | 12.0872 |
| 100  | 100 | 0      | Cov | 0.9424  | 0.9346  | 0.2858  | 0.9478  | 0.888   |
|      |     |        | Len | 12.9172 | 12.4509 | 4.4108  | 13.8887 | 11.0668 |
| 100  | 200 | 0      | Cov | 0.9354  | 0.9252  | 0.0406  | 0.941   | 0.8626  |
|      |     |        | Len | 12.8808 | 12.1061 | 0.3827  | 13.7991 | 10.4264 |
| 400  | 20  | 0      | Cov | 0.9408  | 0.9422  | 0.9406  | 0.9406  | 0.9406  |
|      |     |        | Len | 12.1443 | 12.3398 | 12.1431 | 12.1431 | 12.1431 |
| 400  | 40  | 0      | Cov | 0.9456  | 0.9472  | 0.9456  | 0.9456  | 0.9456  |
|      |     |        | Len | 11.745  | 12.1327 | 11.7434 | 11.7434 | 11.7434 |
| 400  | 100 | 0      | Cov | 0.944   | 0.943   | 0.9222  | 0.9444  | 0.929   |
|      |     |        | Len | 11.4722 | 11.6449 | 10.6342 | 11.9554 | 10.8754 |
| 400  | 200 | 0      | Cov | 0.9392  | 0.9354  | 0.7604  | 0.9428  | 0.9046  |
|      |     |        | Len | 11.4381 | 11.3221 | 8.9663  | 12.0807 | 10.2511 |
| 1000 | 20  | 0      | Cov | 0.9388  | 0.941   | 0.9386  | 0.9386  | 0.9386  |
|      |     |        | Len | 12.5203 | 12.6094 | 12.5184 | 12.5184 | 12.5184 |
| 1000 | 40  | 0      | Cov | 0.9444  | 0.9474  | 0.9446  | 0.9446  | 0.9446  |
|      |     |        | Len | 12.2657 | 12.4602 | 12.2633 | 12.2633 | 12.2633 |
| 1000 | 100 | 0      | Cov | 0.9442  | 0.9488  | 0.9444  | 0.9444  | 0.9444  |
|      |     |        | Len | 11.7039 | 12.1776 | 11.6998 | 11.6998 | 11.6998 |
| 1000 | 200 | 0      | Cov | 0.9428  | 0.9428  | 0.9296  | 0.9426  | 0.9336  |
|      |     |        | Len | 11.5313 | 11.7644 | 10.8226 | 11.7994 | 10.9396 |

Table 5.215. Etype = 5, J=10, k=1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 100  | 20  | 0.2236 | Cov | 0.933   | 0.939   | 0.9236  | 0.9338  | 0.9272  |
|      |     |        | Len | 12.9533 | 13.5194 | 12.4494 | 13.3624 | 12.6175 |
| 100  | 40  | 0.1581 | Cov | 0.943   | 0.9464  | 0.8682  | 0.949   | 0.9222  |
|      |     |        | Len | 12.9668 | 13.5507 | 10.9101 | 13.6110 | 12.0411 |
| 100  | 100 | 0.1    | Cov | 0.9308  | 0.933   | 0.3136  | 0.9358  | 0.8946  |
|      |     |        | Len | 12.9646 | 13.4346 | 4.6539  | 13.8222 | 11.1777 |
| 100  | 200 | 0.07   | Cov | 0.941   | 0.9408  | 0.0574  | 0.9474  | 0.874   |
|      |     |        | Len | 13.0925 | 13.3231 | 0.5578  | 13.8905 | 10.5355 |
| 400  | 20  | 0.2236 | Cov | 0.9434  | 0.9438  | 0.943   | 0.943   | 0.943   |
|      |     |        | Len | 12.1980 | 12.4705 | 12.1962 | 12.1962 | 12.1962 |
| 400  | 40  | 0.1581 | Cov | 0.9378  | 0.9404  | 0.9372  | 0.9372  | 0.9372  |
|      |     |        | Len | 11.7688 | 12.3558 | 11.7665 | 11.7665 | 11.7665 |
| 400  | 100 | 0.1    | Cov | 0.9364  | 0.9386  | 0.918   | 0.9378  | 0.9254  |
|      |     |        | Len | 11.4761 | 12.3211 | 10.6136 | 11.8998 | 10.8724 |
| 400  | 200 | 0.07   | Cov | 0.9392  | 0.942   | 0.756   | 0.9404  | 0.9056  |
|      |     |        | Len | 11.3836 | 12.3062 | 8.94166 | 11.9995 | 10.2081 |
| 1000 | 20  | 0.2236 | Cov | 0.941   | 0.9416  | 0.941   | 0.941   | 0.941   |
|      |     |        | Len | 12.4702 | 12.6068 | 12.4682 | 12.4682 | 12.4682 |
| 1000 | 40  | 0.1581 | Cov | 0.9454  | 0.9466  | 0.9458  | 0.9458  | 0.9458  |
|      |     |        | Len | 12.3311 | 12.6244 | 12.3276 | 12.3276 | 12.3276 |
| 1000 | 100 | 0.1    | Cov | 0.9452  | 0.9496  | 0.9454  | 0.9454  | 0.9454  |
|      |     |        | Len | 11.7230 | 12.4649 | 11.7182 | 11.7182 | 11.7182 |
| 1000 | 200 | 0.07   | Cov | 0.9394  | 0.9452  | 0.931   | 0.9428  | 0.9356  |
|      |     |        | Len | 11.5369 | 12.4807 | 10.8401 | 11.7956 | 10.9590 |

Table 5.216. Etype = 5, J=10, k=1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 100  | 20  | 0.9    | Cov | 0.9472  | 0.947   | 0.9288  | 0.9408  | 0.934   |
|      |     |        | Len | 14.2981 | 14.3620 | 12.9728 | 13.8087 | 13.1278 |
| 100  | 40  | 0.9    | Cov | 0.9466  | 0.9464  | 0.8736  | 0.945   | 0.9206  |
|      |     |        | Len | 14.6569 | 14.6558 | 11.3116 | 14.0733 | 12.5528 |
| 100  | 100 | 0.9    | Cov | 0.9504  | 0.951   | 0.3182  | 0.9514  | 0.9064  |
|      |     |        | Len | 15.135  | 15.1019 | 4.79503 | 14.4639 | 11.7208 |
| 100  | 200 | 0.9    | Cov | 0.9496  | 0.9492  | 0.0628  | 0.95    | 0.8858  |
|      |     |        | Len | 15.8199 | 15.2820 | 0.5748  | 14.7434 | 11.0099 |
| 400  | 20  | 0.9    | Cov | 0.939   | 0.939   | 0.9384  | 0.9384  | 0.9384  |
|      |     |        | Len | 12.5146 | 12.5121 | 12.1607 | 12.1607 | 12.1607 |
| 400  | 40  | 0.9    | Cov | 0.945   | 0.9454  | 0.9434  | 0.9434  | 0.9434  |
|      |     |        | Len | 12.6054 | 12.6011 | 11.8602 | 11.8602 | 11.8602 |
| 400  | 100 | 0.9    | Cov | 0.9402  | 0.9392  | 0.9116  | 0.9376  | 0.924   |
|      |     |        | Len | 12.5977 | 12.5689 | 10.6953 | 11.8558 | 10.9510 |
| 400  | 200 | 0.9    | Cov | 0.9418  | 0.9418  | 0.7508  | 0.9372  | 0.9032  |
|      |     |        | Len | 12.6252 | 12.5216 | 8.9820  | 11.8309 | 10.2593 |
| 1000 | 20  | 0.9    | Cov | 0.9452  | 0.945   | 0.9444  | 0.9444  | 0.9444  |
|      |     |        | Len | 12.6858 | 12.6837 | 12.5163 | 12.5163 | 12.5163 |
| 1000 | 40  | 0.9    | Cov | 0.9404  | 0.9412  | 0.9378  | 0.9378  | 0.9378  |
|      |     |        | Len | 12.6387 | 12.6324 | 12.2748 | 12.2748 | 12.2748 |
| 1000 | 100 | 0.9    | Cov | 0.9472  | 0.9468  | 0.9456  | 0.9456  | 0.9456  |
|      |     |        | Len | 12.6772 | 12.6369 | 11.7145 | 11.7145 | 11.7145 |
| 1000 | 200 | 0.9    | Cov | 0.943   | 0.9438  | 0.9318  | 0.9402  | 0.933   |
|      |     |        | Len | 12.7300 | 12.6643 | 10.869  | 11.7584 | 10.9923 |

Table 5.217. Etype = 5, J=10, k=19,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 100  | 20  | 0      | Cov | 0.9668  | 0.9558  | 0.9552  | 0.9642  | 0.9524  |
|      |     |        | Len | 26.1337 | 21.1976 | 18.9632 | 24.3208 | 21.3939 |
| 100  | 40  | 0      | Cov | 0.9618  | 0.9414  | 0.9054  | 0.962   | 0.9352  |
|      |     |        | Len | 24.7806 | 19.8996 | 14.2793 | 24.8038 | 19.8016 |
| 100  | 100 | 0      | Cov | 0.9548  | 0.9266  | 0.3318  | 0.9602  | 0.8998  |
|      |     |        | Len | 22.9028 | 18.2863 | 5.0297  | 23.9956 | 17.5996 |
| 100  | 200 | 0      | Cov | 0.949   | 0.9192  | 0.0384  | 0.9514  | 0.8614  |
|      |     |        | Len | 22.1233 | 17.7703 | 0.50863 | 23.3373 | 16.4074 |
| 400  | 20  | 0      | Cov | 0.969   | 0.969   | 0.969   | 0.969   | 0.969   |
|      |     |        | Len | 21.2833 | 21.2827 | 21.2827 | 21.2827 | 21.2827 |
| 400  | 40  | 0      | Cov | 0.9644  | 0.966   | 0.9642  | 0.9642  | 0.9642  |
|      |     |        | Len | 20.2904 | 20.7758 | 20.2695 | 20.2695 | 20.2695 |
| 400  | 100 | 0      | Cov | 0.9672  | 0.9682  | 0.9576  | 0.9664  | 0.963   |
|      |     |        | Len | 20.2051 | 19.7865 | 17.1656 | 22.4751 | 18.3265 |
| 400  | 200 | 0      | Cov | 0.969   | 0.9666  | 0.8732  | 0.9664  | 0.9606  |
|      |     |        | Len | 20.2716 | 19.1787 | 12.4487 | 23.7768 | 17.0646 |
| 1000 | 20  | 0      | Cov | 0.952   | 0.9518  | 0.9518  | 0.9518  | 0.9518  |
|      |     |        | Len | 15.7479 | 15.7467 | 15.7467 | 15.7467 | 15.7467 |
| 1000 | 40  | 0      | Cov | 0.9524  | 0.9534  | 0.9522  | 0.9522  | 0.9522  |
|      |     |        | Len | 15.6009 | 15.7432 | 15.5914 | 15.5914 | 15.5914 |
| 1000 | 100 | 0      | Cov | 0.95    | 0.9522  | 0.95    | 0.95    | 0.95    |
|      |     |        | Len | 14.6709 | 15.2194 | 14.6487 | 14.6487 | 14.6487 |
| 1000 | 200 | 0      | Cov | 0.9554  | 0.9562  | 0.947   | 0.9432  | 0.9496  |
|      |     |        | Len | 14.6322 | 14.7241 | 13.3413 | 16.9444 | 13.5913 |

Table 5.218. Etype = 5, J=10, k=19,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS       |
|------|-----|--------|-----|---------|---------|---------|---------|----------|
| 100  | 20  | 0.2236 | Cov | 0.9852  | 0.9644  | 0.9594  | 0.967   | 0.95     |
|      |     |        | Len | 73.4453 | 22.8647 | 19.8237 | 21.8029 | 22.1191  |
| 100  | 40  | 0.1581 | Cov | 0.9868  | 0.9574  | 0.9238  | 0.9676  | 0.9436   |
|      |     |        | Len | 77.4354 | 24.0287 | 15.9610 | 23.9375 | 22.51640 |
| 100  | 100 | 0.1    | Cov | 0.9846  | 0.9484  | 0.402   | 0.972   | 0.9248   |
|      |     |        | Len | 79.5408 | 25.0535 | 6.03909 | 26.4090 | 22.6700  |
| 100  | 200 | 0.07   | Cov | 0.9858  | 0.935   | 0.0604  | 0.9732  | 0.9056   |
|      |     |        | Len | 83.7529 | 26.4774 | 0.8411  | 27.6344 | 22.5119  |
| 400  | 20  | 0.2236 | Cov | 0.969   | 0.9678  | 0.9678  | 0.9678  | 0.9678   |
|      |     |        | Len | 21.2999 | 21.2949 | 21.2949 | 21.2949 | 21.2949  |
| 400  | 40  | 0.1581 | Cov | 0.9662  | 0.966   | 0.9652  | 0.9652  | 0.9652   |
|      |     |        | Len | 20.5562 | 20.7449 | 20.1328 | 20.1328 | 20.1328  |
| 400  | 100 | 0.1    | Cov | 0.9698  | 0.9686  | 0.9604  | 0.9692  | 0.9642   |
|      |     |        | Len | 20.3547 | 20.1073 | 17.1357 | 21.5153 | 18.2941  |
| 400  | 200 | 0.07   | Cov | 0.971   | 0.9648  | 0.8662  | 0.9644  | 0.9572   |
|      |     |        | Len | 23.8694 | 19.6999 | 12.4110 | 22.8896 | 17.0645  |
| 1000 | 20  | 0.2236 | Cov | 0.9508  | 0.9504  | 0.9504  | 0.9504  | 0.9504   |
|      |     |        | Len | 15.7771 | 15.7748 | 15.7748 | 15.7748 | 15.7748  |
| 1000 | 40  | 0.1581 | Cov | 0.9502  | 0.9504  | 0.951   | 0.951   | 0.951    |
|      |     |        | Len | 15.7180 | 15.7136 | 15.5389 | 15.5389 | 15.5389  |
| 1000 | 100 | 0.1    | Cov | 0.9576  | 0.9574  | 0.9568  | 0.9568  | 0.9568   |
|      |     |        | Len | 15.2865 | 15.4999 | 14.6732 | 14.6732 | 14.6732  |
| 1000 | 200 | 0.07   | Cov | 0.9554  | 0.9568  | 0.948   | 0.947   | 0.9482   |
|      |     |        | Len | 14.6737 | 15.3201 | 13.2899 | 16.3147 | 13.5331  |

Table 5.219. Etype = 5, J=10, k=19,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|----------|---------|---------|---------|---------|
| 100  | 20  | 0.9    | Cov | 0.958    | 0.9496  | 0.944   | 0.953   | 0.9472  |
|      |     |        | Len | 26.6911  | 15.7518 | 14.7045 | 15.8782 | 14.9506 |
| 100  | 40  | 0.9    | Cov | 0.959    | 0.941   | 0.892   | 0.9466  | 0.9284  |
|      |     |        | Len | 29.8533  | 15.4348 | 12.1502 | 15.6913 | 13.7295 |
| 100  | 100 | 0.9    | Cov | 0.9662   | 0.9464  | 0.3302  | 0.9506  | 0.914   |
|      |     |        | Len | 78.1624  | 14.9938 | 4.8823  | 15.4472 | 12.3045 |
| 100  | 200 | 0.9    | Cov | 0.971    | 0.9414  | 0.0724  | 0.9502  | 0.8856  |
|      |     |        | Len | 212.0299 | 14.4996 | 0.5813  | 15.3180 | 11.3432 |
| 400  | 20  | 0.9    | Cov | 0.9488   | 0.9462  | 0.9464  | 0.9464  | 0.9464  |
|      |     |        | Len | 13.9543  | 13.4436 | 13.2942 | 13.2942 | 13.2942 |
| 400  | 40  | 0.9    | Cov | 0.9496   | 0.9482  | 0.9454  | 0.9454  | 0.9454  |
|      |     |        | Len | 14.9630  | 13.4336 | 12.7112 | 12.7112 | 12.7112 |
| 400  | 100 | 0.9    | Cov | 0.9568   | 0.9452  | 0.9274  | 0.947   | 0.9348  |
|      |     |        | Len | 21.3481  | 13.2272 | 11.2437 | 12.6083 | 11.5549 |
| 400  | 200 | 0.9    | Cov | 0.9536   | 0.9464  | 0.765   | 0.9448  | 0.9094  |
|      |     |        | Len | 30.2027  | 13.0654 | 9.2316  | 12.5555 | 10.7266 |
| 1000 | 20  | 0.9    | Cov | 0.9492   | 0.9478  | 0.9468  | 0.9468  | 0.9468  |
|      |     |        | Len | 13.6881  | 13.3405 | 13.3012 | 13.3012 | 13.3012 |
| 1000 | 40  | 0.9    | Cov | 0.9496   | 0.9438  | 0.9444  | 0.9444  | 0.9444  |
|      |     |        | Len | 14.3771  | 13.2600 | 12.9519 | 12.9519 | 12.9519 |
| 1000 | 100 | 0.9    | Cov | 0.9522   | 0.9482  | 0.9426  | 0.9426  | 0.9426  |
|      |     |        | Len | 20.6842  | 13.2796 | 12.3282 | 12.3282 | 12.3282 |
| 1000 | 200 | 0.9    | Cov | 0.9522   | 0.9426  | 0.9306  | 0.9392  | 0.932   |
|      |     |        | Len | 29.5806  | 13.1923 | 11.3117 | 12.3119 | 11.4552 |

Table 5.220. Etype = 5, J=10, k=p-1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 100  | 40  | 0      | Cov | 0.9654  | 0.9274  | 0.9068  | 0.961   | 0.911   |
|      |     |        | Len | 31.3714 | 24.4421 | 13.9170 | 29.4898 | 24.6615 |
| 400  | 40  | 0      | Cov | 0.9662  | 0.9658  | 0.9658  | 0.9658  | 0.9658  |
|      |     |        | Len | 21.6148 | 21.6142 | 21.6142 | 21.6142 | 21.6142 |
| 1000 | 40  | 0      | Cov | 0.969   | 0.9684  | 0.9684  | 0.9684  | 0.9684  |
|      |     |        | Len | 19.8652 | 19.8636 | 19.8636 | 19.8636 | 19.8636 |
| 100  | 100 | 0      | Cov | 0.9496  | 0.8708  | 0.3232  | 0.953   | 0.841   |
|      |     |        | Len | 42.3279 | 32.8266 | 5.3114  | 41.5254 | 32.6744 |
| 400  | 100 | 0      | Cov | 0.9822  | 0.9436  | 0.9618  | 0.9678  | 0.9234  |
|      |     |        | Len | 44.0422 | 32.673  | 18.4536 | 36.9448 | 32.0245 |
| 1000 | 100 | 0      | Cov | 0.9708  | 0.9706  | 0.9706  | 0.9706  | 0.9706  |
|      |     |        | Len | 22.6357 | 22.6342 | 22.6342 | 22.6342 | 22.6342 |
| 100  | 200 | 0      | Cov | 0.9444  | 0.8416  | 0.0368  | 0.9508  | 0.7994  |
|      |     |        | Len | 58.3654 | 45.2207 | 0.95207 | 58.0851 | 43.4178 |
| 400  | 200 | 0      | Cov | 0.9242  | 0.8544  | 0.7644  | 0.9148  | 0.8122  |
|      |     |        | Len | 50.4237 | 38.7848 | 9.4001  | 46.2260 | 37.5964 |
| 1000 | 200 | 0      | Cov | 0.976   | 0.9474  | 0.972   | 0.9574  | 0.9286  |
|      |     |        | Len | 58.0185 | 40.7035 | 20.3115 | 44.3831 | 39.6634 |

Table 5.221. Etype = 5, J=10, k=p-1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL       | PLS     | PCR     | FS       |
|------|-----|--------|-----|----------|----------|---------|---------|----------|
| 100  | 40  | 0.1581 | Cov | 0.9878   | 0.9484   | 0.9286  | 0.9662  | 0.9114   |
|      |     |        | Len | 160.1930 | 39.2474  | 15.7768 | 21.6143 | 36.0451  |
| 100  | 100 | 0.1    | Cov | 0.9876   | 0.93     | 0.3988  | 0.9674  | 0.8592   |
|      |     |        | Len | 410.983  | 97.5192  | 5.8497  | 21.9730 | 86.5883  |
| 100  | 200 | 0.07   | Cov | 0.9888   | 0.9168   | 0.0768  | 0.9666  | 0.8412   |
|      |     |        | Len | 867.9898 | 210.8826 | 0.68871 | 22.0849 | 166.7841 |
| 400  | 40  | 0.1581 | Cov | 0.9682   | 0.9686   | 0.9686  | 0.9686  | 0.9686   |
|      |     |        | Len | 21.6487  | 21.6398  | 21.6398 | 21.6398 | 21.6398  |
| 400  | 100 | 0.1    | Cov | 0.9856   | 0.9558   | 0.9614  | 0.9706  | 0.9078   |
|      |     |        | Len | 258.1911 | 48.0096  | 18.4940 | 21.7855 | 40.1422  |
| 400  | 200 | 0.07   | Cov | 0.9812   | 0.9408   | 0.8846  | 0.9718  | 0.8642   |
|      |     |        | Len | 567.9275 | 105.7328 | 13.3486 | 21.7389 | 81.5550  |
| 1000 | 40  | 0.1581 | Cov | 0.9628   | 0.9626   | 0.9626  | 0.9626  | 0.9626   |
|      |     |        | Len | 19.7999  | 19.7917  | 19.7917 | 19.7917 | 19.7917  |
| 1000 | 100 | 0.1    | Cov | 0.975    | 0.9724   | 0.9724  | 0.9724  | 0.9724   |
|      |     |        | Len | 22.9913  | 22.6756  | 22.6756 | 22.6756 | 22.6756  |
| 1000 | 200 | 0.07   | Cov | 0.9866   | 0.9624   | 0.9696  | 0.974   | 0.9162   |
|      |     |        | Len | 362.3756 | 58.1216  | 20.3000 | 22.6633 | 48.2465  |

Table 5.222. Etype = 5, J=10, k=p-1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|----------|---------|---------|---------|---------|
| 100  | 40  | 0.9    | Cov | 0.9806   | 0.958   | 0.9168  | 0.9624  | 0.9512  |
|      |     |        | Len | 111.3717 | 18.3099 | 13.9227 | 18.6858 | 16.1720 |
| 400  | 40  | 0.9    | Cov | 0.9608   | 0.9524  | 0.9492  | 0.9492  | 0.9492  |
|      |     |        | Len | 29.0403  | 15.1962 | 14.1503 | 14.1503 | 14.1503 |
| 1000 | 40  | 0.9    | Cov | 0.9598   | 0.9532  | 0.9512  | 0.9512  | 0.9512  |
|      |     |        | Len | 27.7944  | 14.5312 | 13.9835 | 13.9835 | 13.9835 |
| 100  | 100 | 0.9    | Cov | 0.9878   | 0.9588  | 0.3814  | 0.9632  | 0.9228  |
|      |     |        | Len | 1472.005 | 22.9366 | 5.70231 | 21.3264 | 17.5129 |
| 400  | 100 | 0.9    | Cov | 0.9786   | 0.9672  | 0.959   | 0.9672  | 0.9602  |
|      |     |        | Len | 129.6064 | 22.5232 | 16.1155 | 18.8610 | 16.8217 |
| 1000 | 100 | 0.9    | Cov | 0.969    | 0.9576  | 0.9536  | 0.9536  | 0.9536  |
|      |     |        | Len | 120.2901 | 20.4005 | 16.0046 | 16.0046 | 16.0046 |
| 100  | 200 | 0.9    | Cov | 0.99     | 0.96    | 0.0762  | 0.9688  | 0.8628  |
|      |     |        | Len | 7376.066 | 32.4171 | 0.6893  | 21.8305 | 22.4140 |
| 400  | 200 | 0.9    | Cov | 0.9862   | 0.9702  | 0.8748  | 0.9662  | 0.9362  |
|      |     |        | Len | 382.2339 | 34.5566 | 12.9295 | 20.9986 | 17.0514 |
| 1000 | 200 | 0.9    | Cov | 0.9836   | 0.9736  | 0.9602  | 0.965   | 0.96    |
|      |     |        | Len | 363.0767 | 33.9823 | 18.6582 | 20.8187 | 18.9025 |

Table 5.223. Etype = 5, J=20, k=1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 400  | 20  | 0      | Cov | 0.9446  | 0.9424  | 0.9444  | 0.9444  | 0.9444  |
|      |     |        | Len | 12.1859 | 12.3768 | 12.1843 | 12.1843 | 12.1843 |
| 400  | 40  | 0      | Cov | 0.9444  | 0.9472  | 0.941   | 0.9462  | 0.943   |
|      |     |        | Len | 12.1352 | 12.2360 | 11.8156 | 12.4086 | 11.8870 |
| 400  | 100 | 0      | Cov | 0.9438  | 0.9436  | 0.9168  | 0.9466  | 0.94    |
|      |     |        | Len | 12.0327 | 11.9246 | 10.6653 | 12.5039 | 11.3728 |
| 400  | 200 | 0      | Cov | 0.938   | 0.9354  | 0.7546  | 0.938   | 0.9254  |
|      |     |        | Len | 11.9645 | 11.6604 | 8.95028 | 12.5298 | 10.9654 |
| 1000 | 20  | 0      | Cov | 0.9504  | 0.9492  | 0.9502  | 0.9502  | 0.9502  |
|      |     |        | Len | 12.4943 | 12.5861 | 12.4925 | 12.4925 | 12.4925 |
| 1000 | 40  | 0      | Cov | 0.944   | 0.9468  | 0.9442  | 0.9442  | 0.9442  |
|      |     |        | Len | 12.2908 | 12.4792 | 12.2885 | 12.2885 | 12.2885 |
| 1000 | 100 | 0      | Cov | 0.9416  | 0.9408  | 0.9388  | 0.9404  | 0.9396  |
|      |     |        | Len | 12.1052 | 12.2145 | 11.6997 | 12.2945 | 11.7683 |
| 1000 | 200 | 0      | Cov | 0.9356  | 0.9362  | 0.9254  | 0.9366  | 0.93    |
|      |     |        | Len | 12.0309 | 11.9181 | 10.8027 | 12.3389 | 11.2930 |

Table 5.224. Etype = 5, J=20, k=1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|----------|---------|---------|---------|---------|
| 100  | 20  | 0.2236 | Cov | 0.9446   | 0.944   | 0.9316  | 0.944   | 0.9402  |
|      |     |        | Len | 13.7811  | 13.8270 | 12.6646 | 14.1610 | 13.2910 |
| 100  | 40  | 0.1581 | Cov | 0.948    | 0.946   | 0.891   | 0.9512  | 0.9398  |
|      |     |        | Len | 13.5923  | 13.5082 | 10.9944 | 14.0922 | 12.8461 |
| 100  | 100 | 0.1    | Cov | 0.9446   | 0.944   | 0.668   | 0.9474  | 0.9342  |
|      |     |        | Len | 13.6559  | 13.3663 | 7.11218 | 14.1594 | 12.4785 |
| 100  | 200 | 0.07   | Cov | 0.9436   | 0.9412  | 0.4296  | 0.946   | 0.929   |
|      |     |        | Len | 13.93933 | 13.3823 | 3.64792 | 14.376  | 12.3581 |
| 400  | 20  | 0.2236 | Cov | 0.944    | 0.9468  | 0.9442  | 0.9442  | 0.9442  |
|      |     |        | Len | 12.1851  | 12.4768 | 12.1831 | 12.1831 | 12.1831 |
| 400  | 40  | 0.1581 | Cov | 0.946    | 0.9496  | 0.9432  | 0.9478  | 0.9434  |
|      |     |        | Len | 12.1061  | 12.4553 | 11.7920 | 12.3216 | 11.8695 |
| 400  | 100 | 0.1    | Cov | 0.9418   | 0.9434  | 0.918   | 0.942   | 0.9346  |
|      |     |        | Len | 12.0251  | 12.3981 | 10.6467 | 12.4267 | 11.3430 |
| 400  | 200 | 0.07   | Cov | 0.9464   | 0.9486  | 0.766   | 0.9478  | 0.933   |
|      |     |        | Len | 11.9487  | 12.3208 | 8.92650 | 12.4592 | 10.9487 |
| 1000 | 20  | 0.2236 | Cov | 0.9442   | 0.9434  | 0.9438  | 0.9438  | 0.9438  |
|      |     |        | Len | 12.4922  | 12.6356 | 12.4900 | 12.4900 | 12.4900 |
| 1000 | 40  | 0.1581 | Cov | 0.9438   | 0.9436  | 0.9434  | 0.9434  | 0.9434  |
|      |     |        | Len | 12.3343  | 12.6341 | 12.3311 | 12.3311 | 12.3311 |
| 1000 | 100 | 0.1    | Cov | 0.941    | 0.944   | 0.9398  | 0.9424  | 0.94    |
|      |     |        | Len | 12.1451  | 12.6012 | 11.7350 | 12.3088 | 11.8045 |
| 1000 | 200 | 0.07   | Cov | 0.9434   | 0.9454  | 0.9338  | 0.9434  | 0.9382  |
|      |     |        | Len | 12.0933  | 12.5909 | 10.8418 | 12.3611 | 11.3286 |

Table 5.225. Etype = 5, J=20, k=1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 100  | 20  | 0.9    | Cov | 0.9466  | 0.9472  | 0.9312  | 0.9484  | 0.941   |
|      |     |        | Len | 14.3740 | 14.3933 | 12.9794 | 14.2333 | 13.6024 |
| 100  | 40  | 0.9    | Cov | 0.9434  | 0.942   | 0.888   | 0.9424  | 0.9362  |
|      |     |        | Len | 14.6308 | 14.5202 | 11.3390 | 14.3684 | 13.3372 |
| 100  | 100 | 0.9    | Cov | 0.951   | 0.9466  | 0.6736  | 0.9474  | 0.9344  |
|      |     |        | Len | 17.4003 | 14.9332 | 7.25929 | 14.8063 | 13.1646 |
| 100  | 200 | 0.9    | Cov | 0.9628  | 0.9466  | 0.451   | 0.9492  | 0.9308  |
|      |     |        | Len | 30.6416 | 15.4294 | 3.69288 | 15.3726 | 13.0853 |
| 400  | 20  | 0.9    | Cov | 0.9448  | 0.9448  | 0.9448  | 0.9448  | 0.9448  |
|      |     |        | Len | 12.5671 | 12.5642 | 12.2188 | 12.2188 | 12.2188 |
| 400  | 40  | 0.9    | Cov | 0.9436  | 0.9428  | 0.9396  | 0.9414  | 0.9404  |
|      |     |        | Len | 12.5430 | 12.5368 | 11.7722 | 12.2049 | 11.8479 |
| 400  | 100 | 0.9    | Cov | 0.9402  | 0.9406  | 0.9152  | 0.9404  | 0.9326  |
|      |     |        | Len | 12.5274 | 12.4957 | 10.6767 | 12.1922 | 11.3347 |
| 400  | 200 | 0.9    | Cov | 0.9488  | 0.948   | 0.763   | 0.9502  | 0.9328  |
|      |     |        | Len | 12.6775 | 12.5683 | 9.01012 | 12.3050 | 11.0467 |
| 1000 | 20  | 0.9    | Cov | 0.9474  | 0.947   | 0.9464  | 0.9464  | 0.9464  |
|      |     |        | Len | 12.6094 | 12.6073 | 12.4317 | 12.4317 | 12.4317 |
| 1000 | 40  | 0.9    | Cov | 0.9482  | 0.9482  | 0.9468  | 0.9468  | 0.9468  |
|      |     |        | Len | 12.7093 | 12.7001 | 12.3349 | 12.3349 | 12.3349 |
| 1000 | 100 | 0.9    | Cov | 0.94    | 0.9408  | 0.9344  | 0.936   | 0.9364  |
|      |     |        | Len | 12.7015 | 12.6629 | 11.7309 | 12.2206 | 11.8030 |
| 1000 | 200 | 0.9    | Cov | 0.9402  | 0.939   | 0.9238  | 0.9386  | 0.9328  |
|      |     |        | Len | 12.6932 | 12.6328 | 10.8462 | 12.2145 | 11.3330 |

Table 5.226. Etype = 5, J=20, k=19,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS      | PCR      | FS       |
|------|-----|--------|-----|---------|---------|----------|----------|----------|
| 100  | 20  | 0      | Cov | 0.9692  | 0.9508  | 0.9542   | 0.9662   | 0.9552   |
|      |     |        | Len | 26.7461 | 22.2507 | 17.5727  | 25.9366  | 23.5148  |
| 100  | 40  | 0      | Cov | 0.9666  | 0.949   | 0.9058   | 0.9662   | 0.9526   |
|      |     |        | Len | 25.4490 | 21.167  | 13.4587  | 25.6807  | 22.3815  |
| 100  | 100 | 0      | Cov | 0.9562  | 0.9378  | 0.588    | 0.9568   | 0.9372   |
|      |     |        | Len | 23.9067 | 19.8895 | 7.45697  | 24.8144  | 20.9060  |
| 100  | 200 | 0      | Cov | 0.958   | 0.935   | 0.276    | 0.9588   | 0.9206   |
|      |     |        | Len | 23.6054 | 19.8687 | 3.30138  | 24.306   | 20.0789  |
| 400  | 20  | 0      | Cov | 0.9662  | 0.9658  | 0.9658   | 0.9658   | 0.9658   |
|      |     |        | Len | 21.2245 | 21.2237 | 21.2237  | 21.2237  | 21.2237  |
| 400  | 40  | 0      | Cov | 0.9674  | 0.967   | 0.963    | 0.968    | 0.9662   |
|      |     |        | Len | 21.8996 | 21.1778 | 20.2705  | 23.1278  | 21.2838  |
| 400  | 100 | 0      | Cov | 0.9716  | 0.9696  | 0.9582   | 0.9722   | 0.9696   |
|      |     |        | Len | 22.1665 | 20.9311 | 17.1206  | 24.9395  | 21.0975  |
| 400  | 200 | 0      | Cov | 0.9738  | 0.9722  | 0.8736   | 0.9734   | 0.973    |
|      |     |        | Len | 22.4569 | 20.8646 | 12.3952  | 25.7576  | 21.0855  |
| 1000 | 20  | 0      | Cov | 0.9566  | 0.957   | 0.957    | 0.957    | 0.957    |
|      |     |        | Len | 15.8181 | 15.8185 | 15.8185  | 15.8185  | 15.8185  |
| 1000 | 40  | 0      | Cov | 0.954   | 0.9554  | 0.9536   | 0.9536   | 0.9536   |
|      |     |        | Len | 15.5479 | 15.6726 | 15.5372  | 15.5372  | 15.5372  |
| 1000 | 100 | 0      | Cov | 0.951   | 0.9522  | 0.9504   | 0.9514   | 0.951    |
|      |     |        | Len | 15.4609 | 15.4207 | 14.75876 | 17.84504 | 14.9404  |
| 1000 | 200 | 0      | Cov | 0.957   | 0.9536  | 0.9454   | 0.9546   | 0.9512   |
|      |     |        | Len | 15.3416 | 15.0241 | 13.2839  | 19.7442  | 14.30685 |

Table 5.227. Etype = 5, J=20, k=19,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 100  | 20  | 0.2236 | Cov | 0.979   | 0.9582  | 0.9558  | 0.9668  | 0.9452  |
|      |     |        | Len | 82.1223 | 24.5141 | 18.4304 | 21.1269 | 29.4323 |
| 100  | 40  | 0.1581 | Cov | 0.981   | 0.9474  | 0.9278  | 0.9668  | 0.9388  |
|      |     |        | Len | 81.0446 | 25.2136 | 15.2272 | 23.7515 | 30.8728 |
| 100  | 100 | 0.1    | Cov | 0.981   | 0.929   | 0.7426  | 0.9684  | 0.9248  |
|      |     |        | Len | 79.5267 | 25.4441 | 9.27791 | 26.1787 | 31.9061 |
| 100  | 200 | 0.07   | Cov | 0.9816  | 0.9282  | 0.4576  | 0.972   | 0.9178  |
|      |     |        | Len | 90.1470 | 30.6129 | 5.2551  | 27.3502 | 32.0937 |
| 400  | 20  | 0.2236 | Cov | 0.969   | 0.9696  | 0.9696  | 0.9696  | 0.9696  |
|      |     |        | Len | 21.2368 | 21.2336 | 21.2336 | 21.2336 | 21.2336 |
| 400  | 40  | 0.1581 | Cov | 0.9788  | 0.9708  | 0.9702  | 0.9732  | 0.9706  |
|      |     |        | Len | 47.5936 | 20.9240 | 20.1480 | 21.7361 | 21.0751 |
| 400  | 100 | 0.1    | Cov | 0.9814  | 0.9666  | 0.954   | 0.9696  | 0.9688  |
|      |     |        | Len | 57.6661 | 20.7397 | 17.0732 | 23.3598 | 20.9401 |
| 400  | 200 | 0.07   | Cov | 0.976   | 0.9656  | 0.875   | 0.9702  | 0.9672  |
|      |     |        | Len | 61.9393 | 21.0080 | 12.4123 | 24.6312 | 20.9237 |
| 1000 | 20  | 0.2236 | Cov | 0.9546  | 0.9556  | 0.9556  | 0.9556  | 0.9556  |
|      |     |        | Len | 15.8135 | 15.8114 | 15.8114 | 15.8114 | 15.8114 |
| 1000 | 40  | 0.1581 | Cov | 0.949   | 0.9494  | 0.9494  | 0.9494  | 0.9494  |
|      |     |        | Len | 15.7326 | 15.7287 | 15.5534 | 15.5534 | 15.5534 |
| 1000 | 100 | 0.1    | Cov | 0.952   | 0.9528  | 0.9508  | 0.9476  | 0.9522  |
|      |     |        | Len | 15.5047 | 15.5524 | 14.7171 | 16.7889 | 14.8867 |
| 1000 | 200 | 0.07   | Cov | 0.9558  | 0.9552  | 0.9444  | 0.9524  | 0.9522  |
|      |     |        | Len | 15.4855 | 15.3984 | 13.3471 | 18.6902 | 14.3713 |

Table 5.228. Etype = 5, J=20, k=19,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|----------|---------|---------|---------|---------|
| 100  | 20  | 0.9    | Cov | 0.9746   | 0.9472  | 0.9364  | 0.9522  | 0.9462  |
|      |     |        | Len | 197.046  | 15.9120 | 14.6949 | 16.4080 | 15.5760 |
| 100  | 40  | 0.9    | Cov | 0.9688   | 0.9484  | 0.9024  | 0.9518  | 0.9424  |
|      |     |        | Len | 260.2371 | 15.5578 | 12.3149 | 16.2330 | 14.9866 |
| 100  | 100 | 0.9    | Cov | 0.9706   | 0.9478  | 0.687   | 0.9518  | 0.9408  |
|      |     |        | Len | 457.498  | 14.9798 | 7.49291 | 15.9687 | 14.1531 |
| 100  | 200 | 0.9    | Cov | 0.9696   | 0.944   | 0.459   | 0.9534  | 0.9364  |
|      |     |        | Len | 828.9359 | 14.5764 | 3.7482  | 15.7947 | 13.5763 |
| 400  | 20  | 0.9    | Cov | 0.9448   | 0.945   | 0.944   | 0.944   | 0.944   |
|      |     |        | Len | 13.9510  | 13.4556 | 13.3164 | 13.3164 | 13.3164 |
| 400  | 40  | 0.9    | Cov | 0.95     | 0.9476  | 0.9452  | 0.949   | 0.9444  |
|      |     |        | Len | 14.9919  | 13.4505 | 12.7477 | 13.2532 | 12.8385 |
| 400  | 100 | 0.9    | Cov | 0.954    | 0.9454  | 0.9236  | 0.9444  | 0.9406  |
|      |     |        | Len | 21.3577  | 13.2326 | 11.2628 | 13.1178 | 12.1053 |
| 400  | 200 | 0.9    | Cov | 0.9528   | 0.94    | 0.7676  | 0.944   | 0.9294  |
|      |     |        | Len | 30.1970  | 13.0433 | 9.23156 | 13.0590 | 11.5972 |
| 1000 | 20  | 0.9    | Cov | 0.9496   | 0.9486  | 0.9476  | 0.9476  | 0.9476  |
|      |     |        | Len | 13.6659  | 13.3181 | 13.2759 | 13.2759 | 13.2759 |
| 1000 | 40  | 0.9    | Cov | 0.9512   | 0.9482  | 0.948   | 0.948   | 0.948   |
|      |     |        | Len | 14.4213  | 13.3168 | 13.0021 | 13.0021 | 13.0021 |
| 1000 | 100 | 0.9    | Cov | 0.9554   | 0.955   | 0.9502  | 0.9534  | 0.9504  |
|      |     |        | Len | 20.6671  | 13.2421 | 12.3075 | 12.8542 | 12.3900 |
| 1000 | 200 | 0.9    | Cov | 0.9522   | 0.9508  | 0.9372  | 0.9484  | 0.9448  |
|      |     |        | Len | 29.5727  | 13.1584 | 11.2857 | 12.8217 | 11.8478 |

Table 5.229. Etype = 5, J=20, k=p-1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 100  | 40  | 0      | Cov | 0.965   | 0.9352  | 0.9088  | 0.9674  | 0.9398  |
|      |     |        | Len | 32.0211 | 26.3943 | 13.3617 | 31.6606 | 28.2460 |
| 400  | 40  | 0      | Cov | 0.9716  | 0.9594  | 0.965   | 0.9644  | 0.954   |
|      |     |        | Len | 30.2674 | 23.4527 | 20.2547 | 25.9436 | 23.8107 |
| 1000 | 40  | 0      | Cov | 0.9634  | 0.963   | 0.963   | 0.963   | 0.963   |
|      |     |        | Len | 19.8776 | 19.8771 | 19.8771 | 19.8771 | 19.8771 |
| 100  | 100 | 0      | Cov | 0.9602  | 0.8972  | 0.5454  | 0.962   | 0.908   |
|      |     |        | Len | 44.0753 | 35.934  | 9.02817 | 44.9065 | 39.2053 |
| 400  | 100 | 0      | Cov | 0.9722  | 0.944   | 0.9578  | 0.9704  | 0.9358  |
|      |     |        | Len | 45.6965 | 36.9923 | 16.8775 | 42.1005 | 37.4258 |
| 1000 | 100 | 0      | Cov | 0.9758  | 0.9538  | 0.9688  | 0.9632  | 0.9482  |
|      |     |        | Len | 43.0666 | 31.0414 | 21.1421 | 33.8939 | 31.5194 |
| 100  | 200 | 0      | Cov | 0.9558  | 0.8916  | 0.2242  | 0.9568  | 0.8966  |
|      |     |        | Len | 61.8802 | 51.7923 | 6.48181 | 61.9668 | 53.2870 |
| 400  | 200 | 0      | Cov | 0.9454  | 0.8908  | 0.7692  | 0.9398  | 0.8782  |
|      |     |        | Len | 54.9273 | 45.5485 | 9.40041 | 52.7585 | 45.9309 |
| 1000 | 200 | 0      | Cov | 0.9756  | 0.9496  | 0.9682  | 0.967   | 0.942   |
|      |     |        | Len | 62.0198 | 48.9369 | 18.8883 | 54.899  | 49.1582 |

Table 5.230. Etype = 5, J=20, k=p-1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL       | PLS     | PCR     | FS       |
|------|-----|--------|-----|----------|----------|---------|---------|----------|
| 100  | 40  | 0.1581 | Cov | 0.9806   | 0.9352   | 0.929   | 0.9638  | 0.9306   |
|      |     |        | Len | 162.9997 | 41.9764  | 15.0715 | 21.3063 | 56.9789  |
| 100  | 100 | 0.1    | Cov | 0.9794   | 0.9134   | 0.7442  | 0.9656  | 0.9064   |
|      |     |        | Len | 398.9939 | 98.9748  | 8.4506  | 21.255  | 142.818  |
| 100  | 200 | 0.07   | Cov | 0.9788   | 0.9154   | 0.4836  | 0.967   | 0.895    |
|      |     |        | Len | 915.1741 | 258.1096 | 4.18962 | 21.5494 | 282.4824 |
| 400  | 40  | 0.1581 | Cov | 0.9822   | 0.9676   | 0.9688  | 0.9728  | 0.9534   |
|      |     |        | Len | 130.5839 | 26.6883  | 20.1500 | 21.1953 | 25.4847  |
| 400  | 100 | 0.1    | Cov | 0.9812   | 0.9538   | 0.9518  | 0.9654  | 0.914    |
|      |     |        | Len | 354.1904 | 69.9699  | 17.2258 | 21.3045 | 64.2063  |
| 400  | 200 | 0.07   | Cov | 0.9798   | 0.9478   | 0.8634  | 0.966   | 0.8974   |
|      |     |        | Len | 708.8138 | 143.1781 | 12.4314 | 21.2511 | 129.9768 |
| 1000 | 40  | 0.1581 | Cov | 0.969    | 0.9688   | 0.9688  | 0.9688  | 0.9688   |
|      |     |        | Len | 19.8931  | 19.8837  | 19.8837 | 19.8837 | 19.8837  |
| 1000 | 100 | 0.1    | Cov | 0.9798   | 0.9682   | 0.97    | 0.9708  | 0.9412   |
|      |     |        | Len | 264.089  | 40.4291  | 21.1711 | 22.2720 | 36.9199  |
| 1000 | 200 | 0.07   | Cov | 0.9796   | 0.9588   | 0.9576  | 0.965   | 0.9106   |
|      |     |        | Len | 585.2276 | 93.4376  | 18.9027 | 22.2196 | 80.5964  |

Table 5.231. Etype = 5, J=20, k=p-1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|----------|---------|---------|---------|---------|
| 100  | 40  | 0.9    | Cov | 0.9822   | 0.9568  | 0.9258  | 0.9588  | 0.9492  |
|      |     |        | Len | 640.7085 | 18.2809 | 13.9845 | 19.201  | 17.6483 |
| 400  | 40  | 0.9    | Cov | 0.96     | 0.9504  | 0.946   | 0.949   | 0.947   |
|      |     |        | Len | 28.9787  | 15.2364 | 14.1942 | 14.8014 | 14.3008 |
| 1000 | 40  | 0.9    | Cov | 0.9608   | 0.954   | 0.953   | 0.953   | 0.953   |
|      |     |        | Len | 27.8166  | 14.5247 | 13.9962 | 13.9962 | 13.9962 |
| 100  | 100 | 0.9    | Cov | 0.9844   | 0.9562  | 0.7376  | 0.9654  | 0.9316  |
|      |     |        | Len | 3011.602 | 22.8198 | 8.4283  | 21.1857 | 22.3331 |
| 400  | 100 | 0.9    | Cov | 0.9794   | 0.9678  | 0.9472  | 0.962   | 0.9508  |
|      |     |        | Len | 131.5416 | 22.4727 | 16.0763 | 19.7704 | 17.6007 |
| 1000 | 100 | 0.9    | Cov | 0.9688   | 0.9552  | 0.951   | 0.956   | 0.953   |
|      |     |        | Len | 120.3505 | 20.4397 | 15.9403 | 16.7487 | 16.0088 |
| 100  | 200 | 0.9    | Cov | 0.987    | 0.9566  | 0.4982  | 0.9596  | 0.8932  |
|      |     |        | Len | 10602.13 | 35.2473 | 4.1109  | 21.0770 | 35.5272 |
| 400  | 200 | 0.9    | Cov | 0.981    | 0.971   | 0.8686  | 0.9696  | 0.9306  |
|      |     |        | Len | 476.574  | 33.1214 | 12.4462 | 21.2670 | 20.0139 |
| 1000 | 200 | 0.9    | Cov | 0.9802   | 0.9726  | 0.9626  | 0.9718  | 0.9622  |
|      |     |        | Len | 362.6699 | 34.0796 | 18.6284 | 21.8626 | 19.1634 |

Table 5.232. Etype = 5, J=50, k=1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 400  | 20  | 0      | Cov | 0.9454  | 0.946   | 0.9426  | 0.9466  | 0.9452  |
|      |     |        | Len | 12.3915 | 12.4019 | 12.1823 | 12.6629 | 12.2560 |
| 400  | 40  | 0      | Cov | 0.9408  | 0.9372  | 0.9346  | 0.9422  | 0.9386  |
|      |     |        | Len | 12.4268 | 12.3414 | 11.7999 | 12.7908 | 12.1509 |
| 400  | 100 | 0      | Cov | 0.948   | 0.9444  | 0.921   | 0.9486  | 0.9446  |
|      |     |        | Len | 12.3182 | 12.1183 | 10.6064 | 12.7753 | 11.8644 |
| 400  | 200 | 0      | Cov | 0.9418  | 0.9436  | 0.7672  | 0.945   | 0.941   |
|      |     |        | Len | 12.3559 | 12.0430 | 8.96905 | 12.8145 | 11.7499 |
| 1000 | 20  | 0      | Cov | 0.9464  | 0.9464  | 0.9466  | 0.9466  | 0.9466  |
|      |     |        | Len | 12.4872 | 12.5728 | 12.4853 | 12.4853 | 12.4853 |
| 1000 | 40  | 0      | Cov | 0.9496  | 0.9502  | 0.9492  | 0.9506  | 0.9492  |
|      |     |        | Len | 12.5241 | 12.5585 | 12.3495 | 12.6520 | 12.3843 |
| 1000 | 100 | 0      | Cov | 0.9456  | 0.9446  | 0.941   | 0.9472  | 0.9426  |
|      |     |        | Len | 12.4088 | 12.3211 | 11.6823 | 12.671  | 12.0374 |
| 1000 | 200 | 0      | Cov | 0.9424  | 0.941   | 0.927   | 0.9438  | 0.9408  |
|      |     |        | Len | 12.3956 | 12.1810 | 10.8006 | 12.6923 | 11.8234 |

Table 5.233. Etype = 5, J=50, k=1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 100  | 20  | 0.2236 | Cov | 0.945   | 0.946   | 0.9474  | 0.9472  | 0.9456  |
|      |     |        | Len | 14.0291 | 13.4693 | 14.0331 | 14.1264 | 13.5741 |
| 400  | 20  | 0.2236 | Cov | 0.9466  | 0.9458  | 0.9446  | 0.9468  | 0.9442  |
|      |     |        | Len | 12.4202 | 12.5115 | 12.2069 | 12.5933 | 12.2794 |
| 400  | 40  | 0.1581 | Cov | 0.9468  | 0.9474  | 0.9436  | 0.95    | 0.9468  |
|      |     |        | Len | 12.4045 | 12.4702 | 11.7851 | 12.6644 | 12.1322 |
| 400  | 100 | 0.1    | Cov | 0.9426  | 0.943   | 0.9124  | 0.9424  | 0.938   |
|      |     |        | Len | 12.4298 | 12.4560 | 10.6809 | 12.7641 | 11.9783 |
| 400  | 200 | 0.07   | Cov | 0.9436  | 0.9442  | 0.7678  | 0.945   | 0.942   |
|      |     |        | Len | 12.3618 | 12.3536 | 8.9913  | 12.7622 | 11.7718 |
| 1000 | 20  | 0.2236 | Cov | 0.946   | 0.9468  | 0.9462  | 0.9462  | 0.9462  |
|      |     |        | Len | 12.4881 | 12.6235 | 12.4858 | 12.4858 | 12.4858 |
| 1000 | 40  | 0.1581 | Cov | 0.9498  | 0.95    | 0.949   | 0.9494  | 0.9496  |
|      |     |        | Len | 12.4717 | 12.6282 | 12.3010 | 12.5792 | 12.3340 |
| 1000 | 100 | 0.1    | Cov | 0.9484  | 0.9482  | 0.9464  | 0.9516  | 0.9472  |
|      |     |        | Len | 12.4390 | 12.5986 | 11.7010 | 12.6384 | 12.0683 |
| 1000 | 200 | 0.07   | Cov | 0.9444  | 0.9442  | 0.9326  | 0.943   | 0.9412  |
|      |     |        | Len | 12.4438 | 12.5960 | 10.8358 | 12.6945 | 11.8880 |

Table 5.234. Etype = 5, J=50, k=1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 100  | 20  | 0.9    | Cov | 0.939   | 0.9346  | 0.9362  | 0.9362  | 0.9348  |
|      |     |        | Len | 15.8461 | 13.7948 | 13.8648 | 13.8649 | 13.7950 |
| 100  | 40  | 0.9    | Cov | 0.9568  | 0.9494  | 0.9502  | 0.9502  | 0.9494  |
|      |     |        | Len | 18.8093 | 13.6284 | 13.7169 | 13.7170 | 13.6289 |
| 100  | 100 | 0.9    | Cov | 0.9608  | 0.9434  | 0.9448  | 0.9448  | 0.945   |
|      |     |        | Len | 28.3265 | 13.6466 | 13.7506 | 13.7507 | 13.6519 |
| 100  | 200 | 0.9    | Cov | 0.9622  | 0.94    | 0.9428  | 0.9428  | 0.9406  |
|      |     |        | Len | 47.7296 | 13.5356 | 13.6894 | 13.6894 | 13.5790 |
| 400  | 20  | 0.9    | Cov | 0.9452  | 0.9452  | 0.9426  | 0.9442  | 0.9438  |
|      |     |        | Len | 12.5598 | 12.557  | 12.2106 | 12.4620 | 12.2784 |
| 400  | 40  | 0.9    | Cov | 0.9444  | 0.9446  | 0.9384  | 0.9444  | 0.9414  |
|      |     |        | Len | 12.5347 | 12.5268 | 11.7658 | 12.4213 | 12.1058 |
| 400  | 100 | 0.9    | Cov | 0.9414  | 0.9404  | 0.9156  | 0.9414  | 0.9402  |
|      |     |        | Len | 13.0874 | 12.5568 | 10.6855 | 12.4887 | 11.9203 |
| 400  | 200 | 0.9    | Cov | 0.947   | 0.9458  | 0.7718  | 0.9468  | 0.9412  |
|      |     |        | Len | 16.5664 | 12.6087 | 9.04213 | 12.6047 | 11.8836 |
| 1000 | 20  | 0.9    | Cov | 0.942   | 0.9422  | 0.9422  | 0.9422  | 0.9422  |
|      |     |        | Len | 12.6744 | 12.6716 | 12.5019 | 12.5019 | 12.5019 |
| 1000 | 40  | 0.9    | Cov | 0.9478  | 0.9484  | 0.9472  | 0.9478  | 0.9474  |
|      |     |        | Len | 12.5747 | 12.5647 | 12.2058 | 12.4113 | 12.2380 |
| 1000 | 100 | 0.9    | Cov | 0.9436  | 0.9438  | 0.9406  | 0.9438  | 0.945   |
|      |     |        | Len | 12.7085 | 12.6726 | 11.7345 | 12.5314 | 12.1018 |
| 1000 | 200 | 0.9    | Cov | 0.9468  | 0.9466  | 0.9322  | 0.946   | 0.9428  |
|      |     |        | Len | 12.6708 | 12.6002 | 10.8292 | 12.4865 | 11.8459 |

Table 5.235. Etype = 5, J=50, k=19,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 100  | 20  | 0      | Cov | 0.9514  | 0.9302  | 0.9314  | 0.9546  | 0.9486  |
|      |     |        | Len | 22.4408 | 19.1005 | 14.1953 | 23.3755 | 22.557  |
| 100  | 40  | 0      | Cov | 0.945   | 0.925   | 0.9114  | 0.949   | 0.9448  |
|      |     |        | Len | 22.3771 | 19.0338 | 14.1348 | 23.6294 | 22.5347 |
| 400  | 20  | 0      | Cov | 0.9582  | 0.9548  | 0.9564  | 0.9586  | 0.9528  |
|      |     |        | Len | 21.4165 | 17.3859 | 14.6661 | 19.4826 | 18.6547 |
| 400  | 40  | 0      | Cov | 0.955   | 0.9472  | 0.9466  | 0.9552  | 0.9448  |
|      |     |        | Len | 21.3784 | 17.3824 | 14.0195 | 21.0119 | 18.6282 |
| 400  | 100 | 0      | Cov | 0.9612  | 0.9504  | 0.9354  | 0.9582  | 0.9484  |
|      |     |        | Len | 21.3859 | 17.377  | 12.3024 | 22.0721 | 18.6338 |
| 400  | 200 | 0      | Cov | 0.9572  | 0.947   | 0.8004  | 0.9548  | 0.946   |
|      |     |        | Len | 21.3747 | 17.3574 | 9.82682 | 22.4474 | 18.6098 |
| 1000 | 20  | 0      | Cov | 0.9568  | 0.9572  | 0.9572  | 0.9572  | 0.9572  |
|      |     |        | Len | 15.7293 | 15.7290 | 15.7290 | 15.7290 | 15.7290 |
| 1000 | 40  | 0      | Cov | 0.9578  | 0.9574  | 0.9544  | 0.955   | 0.9584  |
|      |     |        | Len | 15.9594 | 15.7450 | 15.4898 | 18.4663 | 15.7845 |
| 1000 | 100 | 0      | Cov | 0.96    | 0.9598  | 0.957   | 0.9592  | 0.96    |
|      |     |        | Len | 16.0443 | 15.6778 | 14.5908 | 20.8430 | 15.7416 |
| 1000 | 200 | 0      | Cov | 0.96    | 0.9586  | 0.9478  | 0.9588  | 0.9586  |
|      |     |        | Len | 16.2156 | 15.7517 | 13.2843 | 21.7749 | 15.8218 |

Table 5.236. Etype = 5, J=50, k=19,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 100  | 20  | 0.2236 | Cov | 0.967   | 0.9264  | 0.9456  | 0.9456  | 0.9376  |
|      |     |        | Len | 71.3109 | 20.1685 | 13.6123 | 13.6736 | 48.4832 |
| 100  | 40  | 0.1581 | Cov | 0.9672  | 0.915   | 0.9538  | 0.9544  | 0.9382  |
|      |     |        | Len | 70.3299 | 20.9620 | 17.9783 | 18.1946 | 50.4730 |
| 100  | 100 | 0.1    | Cov | 0.9622  | 0.9     | 0.96    | 0.9606  | 0.9316  |
|      |     |        | Len | 68.7341 | 21.3156 | 20.8167 | 21.0495 | 51.6932 |
| 100  | 200 | 0.07   | Cov | 0.9616  | 0.8892  | 0.9528  | 0.9542  | 0.9312  |
|      |     |        | Len | 79.2503 | 27.7775 | 22.0954 | 22.3297 | 51.7552 |
| 400  | 20  | 0.2236 | Cov | 0.962   | 0.9508  | 0.9498  | 0.9528  | 0.9396  |
|      |     |        | Len | 67.2388 | 16.8425 | 14.6483 | 15.0123 | 20.5442 |
| 400  | 40  | 0.1581 | Cov | 0.9646  | 0.9464  | 0.9442  | 0.9508  | 0.9382  |
|      |     |        | Len | 66.2339 | 17.3183 | 14.0534 | 17.0587 | 21.6504 |
| 400  | 100 | 0.1    | Cov | 0.9582  | 0.947   | 0.9296  | 0.9536  | 0.935   |
|      |     |        | Len | 64.8898 | 17.7505 | 12.3283 | 19.6662 | 22.6098 |
| 400  | 200 | 0.07   | Cov | 0.96    | 0.937   | 0.8162  | 0.9522  | 0.9376  |
|      |     |        | Len | 63.4143 | 18.0138 | 9.84953 | 20.8815 | 23.1717 |
| 1000 | 20  | 0.2236 | Cov | 0.9572  | 0.9576  | 0.9576  | 0.9576  | 0.9576  |
|      |     |        | Len | 15.7741 | 15.7720 | 15.7720 | 15.7720 | 15.7720 |
| 1000 | 40  | 0.1581 | Cov | 0.9572  | 0.9496  | 0.9498  | 0.9512  | 0.9502  |
|      |     |        | Len | 35.2048 | 15.757  | 15.5120 | 16.4885 | 15.7958 |
| 1000 | 100 | 0.1    | Cov | 0.9538  | 0.9516  | 0.9492  | 0.9572  | 0.9512  |
|      |     |        | Len | 45.4432 | 15.7392 | 14.6335 | 18.8251 | 15.7818 |
| 1000 | 200 | 0.07   | Cov | 0.9624  | 0.9508  | 0.9382  | 0.9508  | 0.9514  |
|      |     |        | Len | 50.1096 | 15.7174 | 13.2299 | 20.2985 | 15.7466 |

Table 5.237. Etype = 5, J=50, k=19,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|----------|---------|---------|---------|---------|
| 100  | 20  | 0.9    | Cov | 0.9638   | 0.937   | 0.9384  | 0.9384  | 0.9364  |
|      |     |        | Len | 230.7094 | 13.3178 | 13.6365 | 13.6366 | 14.6810 |
| 100  | 40  | 0.9    | Cov | 0.9656   | 0.941   | 0.9466  | 0.9466  | 0.9422  |
|      |     |        | Len | 330.025  | 13.1917 | 13.7774 | 13.7774 | 14.5887 |
| 100  | 100 | 0.9    | Cov | 0.965    | 0.939   | 0.9476  | 0.9476  | 0.9418  |
|      |     |        | Len | 524.6121 | 13.0188 | 13.9197 | 13.9197 | 14.4896 |
| 100  | 200 | 0.9    | Cov | 0.9706   | 0.9418  | 0.9488  | 0.9488  | 0.941   |
|      |     |        | Len | 895.1404 | 12.8258 | 13.9057 | 13.9057 | 14.3035 |
| 400  | 20  | 0.9    | Cov | 0.954    | 0.9454  | 0.9438  | 0.946   | 0.9446  |
|      |     |        | Len | 73.5905  | 13.4923 | 13.3234 | 13.6279 | 13.4219 |
| 400  | 40  | 0.9    | Cov | 0.9476   | 0.9452  | 0.9408  | 0.9472  | 0.9416  |
|      |     |        | Len | 49.1354  | 13.3770 | 12.6882 | 13.5078 | 13.1087 |
| 400  | 100 | 0.9    | Cov | 0.9566   | 0.952   | 0.9274  | 0.9522  | 0.9536  |
|      |     |        | Len | 128.3472 | 13.1759 | 11.2127 | 13.3890 | 12.7796 |
| 400  | 200 | 0.9    | Cov | 0.9524   | 0.948   | 0.7934  | 0.9476  | 0.9452  |
|      |     |        | Len | 258.5456 | 13.052  | 9.25729 | 13.3346 | 12.5553 |
| 1000 | 20  | 0.9    | Cov | 0.9484   | 0.9486  | 0.9484  | 0.9484  | 0.9484  |
|      |     |        | Len | 13.6374  | 13.2911 | 13.2469 | 13.2469 | 13.2469 |
| 1000 | 40  | 0.9    | Cov | 0.9476   | 0.9474  | 0.9446  | 0.9468  | 0.9448  |
|      |     |        | Len | 14.4181  | 13.3034 | 12.9865 | 13.2167 | 13.0234 |
| 1000 | 100 | 0.9    | Cov | 0.9484   | 0.9458  | 0.939   | 0.9464  | 0.9442  |
|      |     |        | Len | 20.6800  | 13.2587 | 12.3127 | 13.2019 | 12.7261 |
| 1000 | 200 | 0.9    | Cov | 0.9536   | 0.9552  | 0.939   | 0.9532  | 0.9502  |
|      |     |        | Len | 29.585   | 13.2454 | 11.3435 | 13.2430 | 12.5434 |

Table 5.238. Etype = 5, J=50, k=p-1,  $\psi = 0$ 

| $n$  | $p$ | $\psi$ |     | Lasso   | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|---------|---------|---------|---------|---------|
| 100  | 40  | 0      | Cov | 0.9564  | 0.922   | 0.9142  | 0.9602  | 0.946   |
|      |     |        | Len | 29.1906 | 24.6615 | 16.3614 | 30.7036 | 29.5173 |
| 400  | 40  | 0      | Cov | 0.9536  | 0.9322  | 0.9442  | 0.9488  | 0.9348  |
|      |     |        | Len | 28.1708 | 23.4581 | 14.0222 | 26.735  | 25.3438 |
| 1000 | 40  | 0      | Cov | 0.9618  | 0.9518  | 0.953   | 0.9516  | 0.9456  |
|      |     |        | Len | 27.1642 | 20.1002 | 15.5121 | 22.1381 | 21.3802 |
| 100  | 100 | 0      | Cov | 0.9584  | 0.8932  | 0.836   | 0.9638  | 0.9492  |
|      |     |        | Len | 43.3244 | 36.0097 | 22.1871 | 46.0216 | 44.0343 |
| 400  | 100 | 0      | Cov | 0.955   | 0.9168  | 0.938   | 0.9534  | 0.93    |
|      |     |        | Len | 42.1864 | 36.3845 | 12.2831 | 41.5593 | 38.9758 |
| 1000 | 100 | 0      | Cov | 0.955   | 0.9416  | 0.9552  | 0.9486  | 0.9354  |
|      |     |        | Len | 41.6729 | 34.545  | 14.6152 | 38.2776 | 36.0727 |
| 100  | 200 | 0      | Cov | 0.959   | 0.9136  | 0.7464  | 0.9604  | 0.942   |
|      |     |        | Len | 62.5657 | 54.8214 | 27.8363 | 63.7463 | 60.8964 |
| 400  | 200 | 0      | Cov | 0.951   | 0.9032  | 0.764   | 0.9536  | 0.9228  |
|      |     |        | Len | 56.4717 | 49.3369 | 9.4291  | 56.6268 | 52.8048 |
| 1000 | 200 | 0      | Cov | 0.9538  | 0.9292  | 0.9482  | 0.9516  | 0.9272  |
|      |     |        | Len | 58.2664 | 50.3872 | 13.2449 | 55.8671 | 52.1573 |

Table 5.239. Etype = 5, J=50, k=p-1,  $\psi = 1/\sqrt{p}$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL       | PLS     | PCR     | FS       |
|------|-----|--------|-----|----------|----------|---------|---------|----------|
| 100  | 40  | 0.1581 | Cov | 0.9594   | 0.8948   | 0.9504  | 0.951   | 0.9328   |
|      |     |        | Len | 140.8249 | 36.2570  | 13.7963 | 13.8747 | 99.8619  |
| 100  | 100 | 0.1    | Cov | 0.9586   | 0.8668   | 0.9438  | 0.9442  | 0.925    |
|      |     |        | Len | 344.8387 | 85.7651  | 13.9712 | 14.0728 | 254.09   |
| 100  | 200 | 0.07   | Cov | 0.961    | 0.8798   | 0.9448  | 0.9452  | 0.9138   |
|      |     |        | Len | 800.815  | 236.8835 | 14.2564 | 14.3753 | 506.5833 |
| 400  | 40  | 0.1581 | Cov | 0.9606   | 0.9426   | 0.9492  | 0.952   | 0.9196   |
|      |     |        | Len | 132.6393 | 27.6932  | 14.0303 | 14.9901 | 40.7307  |
| 400  | 100 | 0.1    | Cov | 0.963    | 0.9242   | 0.933   | 0.9508  | 0.9102   |
|      |     |        | Len | 325.3014 | 66.4774  | 12.2716 | 14.9970 | 106.3686 |
| 400  | 200 | 0.07   | Cov | 0.9568   | 0.911    | 0.8082  | 0.9526  | 0.898    |
|      |     |        | Len | 637.8657 | 131.4207 | 9.8556  | 15.0841 | 215.8089 |
| 1000 | 40  | 0.1581 | Cov | 0.9588   | 0.9522   | 0.9546  | 0.9558  | 0.9382   |
|      |     |        | Len | 129.4285 | 22.2859  | 15.4511 | 15.7379 | 23.6157  |
| 1000 | 100 | 0.1    | Cov | 0.9606   | 0.9418   | 0.9488  | 0.952   | 0.9148   |
|      |     |        | Len | 320.4241 | 54.5261  | 14.6099 | 15.7492 | 64.2118  |
| 1000 | 200 | 0.07   | Cov | 0.9608   | 0.9316   | 0.9448  | 0.956   | 0.903    |
|      |     |        | Len | 625.3806 | 108.2742 | 13.2463 | 15.7776 | 133.7199 |

Table 5.240. Etype = 5, J=50, k=p-1,  $\psi = 0.9$ 

| $n$  | $p$ | $\psi$ |     | Lasso    | RL      | PLS     | PCR     | FS      |
|------|-----|--------|-----|----------|---------|---------|---------|---------|
| 100  | 40  | 0.9    | Cov | 0.9636   | 0.9364  | 0.9418  | 0.9418  | 0.9334  |
|      |     |        | Len | 673.4568 | 14.0123 | 13.7604 | 13.7605 | 19.2659 |
| 400  | 40  | 0.9    | Cov | 0.9642   | 0.9508  | 0.9494  | 0.9512  | 0.951   |
|      |     |        | Len | 216.3722 | 15.0526 | 14.0481 | 15.0189 | 14.5183 |
| 1000 | 40  | 0.9    | Cov | 0.9544   | 0.9484  | 0.9482  | 0.9498  | 0.9484  |
|      |     |        | Len | 27.8141  | 14.5430 | 14.0043 | 14.2553 | 14.0299 |
| 100  | 100 | 0.9    | Cov | 0.9676   | 0.9234  | 0.9414  | 0.9414  | 0.9198  |
|      |     |        | Len | 2732.104 | 18.6628 | 13.7253 | 13.7253 | 38.2052 |
| 400  | 100 | 0.9    | Cov | 0.966    | 0.9482  | 0.9234  | 0.9458  | 0.9262  |
|      |     |        | Len | 1746.093 | 18.5674 | 12.2691 | 14.9185 | 17.2458 |
| 1000 | 100 | 0.9    | Cov | 0.963    | 0.958   | 0.952   | 0.9576  | 0.9518  |
|      |     |        | Len | 120.0336 | 19.4086 | 14.6282 | 15.7809 | 15.1087 |
| 100  | 200 | 0.9    | Cov | 0.9678   | 0.9246  | 0.9458  | 0.9458  | 0.9092  |
|      |     |        | Len | 9353.668 | 30.9236 | 13.7974 | 13.7974 | 72.3018 |
| 400  | 200 | 0.9    | Cov | 0.9618   | 0.9452  | 0.8094  | 0.9516  | 0.9036  |
|      |     |        | Len | 6110.024 | 28.7512 | 9.85178 | 15.0412 | 27.2562 |
| 1000 | 200 | 0.9    | Cov | 0.9654   | 0.954   | 0.9436  | 0.9542  | 0.9274  |
|      |     |        | Len | 416.8401 | 29.4639 | 13.2352 | 15.7431 | 19.0295 |

### 5.3 EBIC WITH FORWARD SELECTION

Table 5.241. Etype = 1, J=5, k=1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9634     | 0.9650                      | 0.9620       |
|      |     | Len | 4.4384     | 4.4430                      | 4.3939       |
| 100  | 40  | Cov | 0.9600     | 0.9618                      | 0.9600       |
|      |     | Len | 4.4467     | 4.4393                      | 4.3791       |
| 100  | 100 | Cov | 0.9658     | 0.9630                      | 0.9580       |
|      |     | Len | 4.4396     | 4.4280                      | 4.3682       |
| 100  | 200 | Cov | 0.9606     | 0.9652                      | 0.9598       |
|      |     | Len | 4.4219     | 4.4256                      | 4.3427       |
| 400  | 20  | Cov | 0.9538     | 0.9484                      | 0.9548       |
|      |     | Len | 4.0071     | 4.0058                      | 3.9979       |
| 400  | 40  | Cov | 0.9446     | 0.9538                      | 0.9524       |
|      |     | Len | 4.0055     | 4.0064                      | 3.9944       |
| 400  | 100 | Cov | 0.9496     | 0.9490                      | 0.9484       |
|      |     | Len | 4.0080     | 4.0094                      | 3.9929       |
| 400  | 200 | Cov | 0.9522     | 0.9534                      | 0.9518       |
|      |     | Len | 4.0033     | 4.0061                      | 3.9870       |
| 1000 | 20  | Cov | 0.9470     | 0.9414                      | 0.9496       |
|      |     | Len | 3.9379     | 3.9382                      | 3.9394       |
| 1000 | 40  | Cov | 0.9446     | 0.9488                      | 0.9494       |
|      |     | Len | 3.9367     | 3.9368                      | 3.9372       |
| 1000 | 100 | Cov | 0.9464     | 0.9512                      | 0.9494       |
|      |     | Len | 3.9349     | 3.9352                      | 3.9336       |
| 1000 | 200 | Cov | 0.9506     | 0.9528                      | 0.9530       |
|      |     | Len | 3.9344     | 3.9393                      | 3.9352       |

Table 5.242. Etype = 1, J=5, k=19

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9832     | 0.9862                      | 0.9616       |
|      |     | Len | 5.7094     | 5.7056                      | 5.1729       |
| 100  | 40  | Cov | 0.9796     | 0.9706                      | 0.9516       |
|      |     | Len | 5.9440     | 6.7200                      | 5.1444       |
| 100  | 100 | Cov | 0.9680     | 0.9306                      | 0.9334       |
|      |     | Len | 6.6805     | 8.7804                      | 5.0738       |
| 100  | 200 | Cov | 0.9392     | 0.8760                      | 0.9110       |
|      |     | Len | 8.3810     | 10.4374                     | 4.9965       |
| 400  | 20  | Cov | 0.9758     | 0.9792                      | 0.9634       |
|      |     | Len | 4.6929     | 4.6887                      | 4.4468       |
| 400  | 40  | Cov | 0.9756     | 0.9702                      | 0.9502       |
|      |     | Len | 4.6857     | 4.6855                      | 4.4108       |
| 400  | 100 | Cov | 0.9746     | 0.9736                      | 0.9388       |
|      |     | Len | 4.6774     | 4.6791                      | 4.3971       |
| 400  | 200 | Cov | 0.9756     | 0.9708                      | 0.9414       |
|      |     | Len | 4.6730     | 4.6753                      | 4.3848       |
| 1000 | 20  | Cov | 0.9604     | 0.9662                      | 0.9610       |
|      |     | Len | 4.1742     | 4.1791                      | 4.1663       |
| 1000 | 40  | Cov | 0.9602     | 0.9578                      | 0.9554       |
|      |     | Len | 4.1781     | 4.1782                      | 4.1481       |
| 1000 | 100 | Cov | 0.9592     | 0.9580                      | 0.9562       |
|      |     | Len | 4.1775     | 4.1778                      | 4.1539       |
| 1000 | 200 | Cov | 0.9598     | 0.9576                      | 0.9504       |
|      |     | Len | 4.1739     | 4.1783                      | 4.1495       |

Table 5.243. Etype = 1, J=5, k=p-1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 40  | Cov | 0.9040     | 0.8858                      | 0.9278       |
|      |     | Len | 17.4837    | 19.4947                     | 5.4377       |
| 100  | 100 | Cov | 0.9394     | 0.8000                      | 0.8230       |
|      |     | Len | 40.6694    | 50.2127                     | 6.8075       |
| 100  | 200 | Cov | 0.9452     | 0.7186                      | 0.7248       |
|      |     | Len | 58.9667    | 92.8953                     | 10.7159      |
| 400  | 40  | Cov | 0.9792     | 0.9824                      | 0.9614       |
|      |     | Len | 4.8971     | 4.9038                      | 4.8126       |
| 400  | 100 | Cov | 0.9252     | 0.9212                      | 0.9366       |
|      |     | Len | 17.7642    | 17.8330                     | 5.2853       |
| 400  | 200 | Cov | 0.9384     | 0.8320                      | 0.8642       |
|      |     | Len | 53.8183    | 48.1354                     | 6.4799       |
| 1000 | 40  | Cov | 0.9696     | 0.9710                      | 0.9702       |
|      |     | Len | 4.4919     | 4.4908                      | 4.4896       |
| 1000 | 100 | Cov | 0.9764     | 0.9748                      | 0.9790       |
|      |     | Len | 4.8752     | 4.8825                      | 4.8765       |
| 1000 | 200 | Cov | 0.9826     | 0.9800                      | 0.9826       |
|      |     | Len | 5.2696     | 5.2731                      | 5.2713       |

Table 5.244. Etype = 1, J=10, k=1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9604     | 0.9624                      | 0.9624       |
|      |     | Len | 4.4339     | 4.4318                      | 4.3948       |
| 100  | 40  | Cov | 0.9698     | 0.9662                      | 0.9604       |
|      |     | Len | 4.4355     | 4.4405                      | 4.3845       |
| 100  | 100 | Cov | 0.9578     | 0.9660                      | 0.9570       |
|      |     | Len | 4.4308     | 4.4335                      | 4.3642       |
| 100  | 200 | Cov | 0.9656     | 0.9658                      | 0.9598       |
|      |     | Len | 4.4277     | 4.4156                      | 4.3535       |
| 400  | 20  | Cov | 0.9472     | 0.9524                      | 0.9522       |
|      |     | Len | 4.0059     | 4.0037                      | 4.0035       |
| 400  | 40  | Cov | 0.9548     | 0.9524                      | 0.9510       |
|      |     | Len | 4.0080     | 4.0076                      | 3.9983       |
| 400  | 100 | Cov | 0.9534     | 0.9542                      | 0.9488       |
|      |     | Len | 4.0051     | 4.0045                      | 3.9888       |
| 400  | 200 | Cov | 0.9538     | 0.9464                      | 0.9470       |
|      |     | Len | 4.0063     | 4.0012                      | 3.9887       |
| 1000 | 20  | Cov | 0.9544     | 0.9430                      | 0.9474       |
|      |     | Len | 3.9383     | 3.9376                      | 3.9375       |
| 1000 | 40  | Cov | 0.9546     | 0.9526                      | 0.9470       |
|      |     | Len | 3.9342     | 3.9372                      | 3.9358       |
| 1000 | 100 | Cov | 0.9526     | 0.9514                      | 0.9554       |
|      |     | Len | 3.9366     | 3.9363                      | 3.9382       |
| 1000 | 200 | Cov | 0.9522     | 0.9442                      | 0.9482       |
|      |     | Len | 3.9381     | 3.9394                      | 3.9323       |

Table 5.245. Etype = 1, J=10, k=19

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9430     | 0.9378                      | 0.9614       |
|      |     | Len | 13.8313    | 14.6745                     | 5.1656       |
| 100  | 40  | Cov | 0.9398     | 0.9310                      | 0.9564       |
|      |     | Len | 13.8426    | 16.1301                     | 5.1360       |
| 100  | 100 | Cov | 0.9412     | 0.9008                      | 0.9270       |
|      |     | Len | 14.1143    | 17.6116                     | 5.0754       |
| 100  | 200 | Cov | 0.9356     | 0.8792                      | 0.9178       |
|      |     | Len | 15.1546    | 18.1522                     | 4.9902       |
| 400  | 20  | Cov | 0.9772     | 0.9750                      | 0.9592       |
|      |     | Len | 4.6990     | 4.6969                      | 4.4438       |
| 400  | 40  | Cov | 0.9750     | 0.9752                      | 0.9534       |
|      |     | Len | 4.6780     | 4.6868                      | 4.4024       |
| 400  | 100 | Cov | 0.9712     | 0.9740                      | 0.9448       |
|      |     | Len | 4.6775     | 4.6791                      | 4.3971       |
| 400  | 200 | Cov | 0.9718     | 0.9768                      | 0.9398       |
|      |     | Len | 4.6717     | 4.6784                      | 4.3855       |
| 1000 | 20  | Cov | 0.9554     | 0.9598                      | 0.9600       |
|      |     | Len | 4.1766     | 4.1744                      | 4.1701       |
| 1000 | 40  | Cov | 0.9612     | 0.9542                      | 0.9498       |
|      |     | Len | 4.1784     | 4.1763                      | 4.1534       |
| 1000 | 100 | Cov | 0.9580     | 0.9584                      | 0.9536       |
|      |     | Len | 4.1768     | 4.1751                      | 4.1520       |
| 1000 | 200 | Cov | 0.9642     | 0.9564                      | 0.9482       |
|      |     | Len | 4.1754     | 4.1760                      | 4.1601       |

Table 5.246. Etype = 1, J=10, k=p-1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 40  | Cov | 0.9318     | 0.9044                      | 0.9302       |
|      |     | Len | 22.7023    | 33.5587                     | 5.8339       |
| 100  | 100 | Cov | 0.9378     | 0.8650                      | 0.8748       |
|      |     | Len | 41.0337    | 85.7560                     | 10.5333      |
| 100  | 200 | Cov | 0.9484     | 0.8184                      | 0.8394       |
|      |     | Len | 58.8774    | 166.1195                    | 19.0490      |
| 400  | 40  | Cov | 0.9786     | 0.9774                      | 0.9644       |
|      |     | Len | 4.9071     | 4.9084                      | 4.8114       |
| 400  | 100 | Cov | 0.9118     | 0.8966                      | 0.9274       |
|      |     | Len | 28.7049    | 37.9356                     | 5.8302       |
| 400  | 200 | Cov | 0.9318     | 0.8570                      | 0.8648       |
|      |     | Len | 54.0818    | 80.8306                     | 9.4875       |
| 1000 | 40  | Cov | 0.9756     | 0.9694                      | 0.9680       |
|      |     | Len | 4.4868     | 4.4878                      | 4.4828       |
| 1000 | 100 | Cov | 0.9816     | 0.9772                      | 0.9760       |
|      |     | Len | 4.8849     | 4.8815                      | 4.8837       |
| 1000 | 200 | Cov | 0.9296     | 0.9028                      | 0.9222       |
|      |     | Len | 37.1150    | 46.3527                     | 6.4369       |

Table 5.247. Etype = 1, J=20, k=1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9682     | 0.9620                      | 0.9636       |
|      |     | Len | 4.4329     | 4.4406                      | 4.3942       |
| 100  | 40  | Cov | 0.9652     | 0.9658                      | 0.9606       |
|      |     | Len | 4.4356     | 4.4303                      | 4.3804       |
| 100  | 100 | Cov | 0.9720     | 0.9614                      | 0.9646       |
|      |     | Len | 4.4291     | 4.4402                      | 4.3561       |
| 100  | 200 | Cov | 0.9626     | 0.9662                      | 0.9532       |
|      |     | Len | 4.4333     | 4.4293                      | 4.3477       |
| 400  | 20  | Cov | 0.9530     | 0.9482                      | 0.9494       |
|      |     | Len | 4.0002     | 4.0033                      | 3.9947       |
| 400  | 40  | Cov | 0.9486     | 0.9434                      | 0.9570       |
|      |     | Len | 4.0051     | 4.0048                      | 3.9959       |
| 400  | 100 | Cov | 0.9496     | 0.9502                      | 0.9482       |
|      |     | Len | 4.0051     | 4.0055                      | 3.9916       |
| 400  | 200 | Cov | 0.9490     | 0.9502                      | 0.9490       |
|      |     | Len | 4.0055     | 4.0100                      | 3.9884       |
| 1000 | 20  | Cov | 0.9494     | 0.9500                      | 0.9490       |
|      |     | Len | 3.9386     | 3.9355                      | 3.9365       |
| 1000 | 40  | Cov | 0.9502     | 0.9472                      | 0.9494       |
|      |     | Len | 3.9375     | 3.9387                      | 3.9357       |
| 1000 | 100 | Cov | 0.9522     | 0.9488                      | 0.9464       |
|      |     | Len | 3.9368     | 3.9363                      | 3.9340       |
| 1000 | 200 | Cov | 0.9484     | 0.9490                      | 0.9516       |
|      |     | Len | 3.9348     | 3.9339                      | 3.9342       |

Table 5.248. Etype = 1, J=20, k=19

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9590     | 0.9462                      | 0.9582       |
|      |     | Len | 17.3245    | 25.3737                     | 5.5360       |
| 100  | 40  | Cov | 0.9576     | 0.9318                      | 0.9468       |
|      |     | Len | 17.2694    | 27.2703                     | 5.4740       |
| 100  | 100 | Cov | 0.9424     | 0.9186                      | 0.9344       |
|      |     | Len | 17.2460    | 28.7491                     | 5.3451       |
| 100  | 200 | Cov | 0.9490     | 0.9112                      | 0.9198       |
|      |     | Len | 17.4097    | 29.2283                     | 5.2282       |
| 400  | 20  | Cov | 0.9804     | 0.9756                      | 0.9628       |
|      |     | Len | 4.6963     | 4.6967                      | 4.4438       |
| 400  | 40  | Cov | 0.9758     | 0.9760                      | 0.9568       |
|      |     | Len | 4.6932     | 4.7545                      | 4.4133       |
| 400  | 100 | Cov | 0.9808     | 0.9698                      | 0.9470       |
|      |     | Len | 4.6941     | 4.8947                      | 4.3922       |
| 400  | 200 | Cov | 0.9770     | 0.9752                      | 0.9380       |
|      |     | Len | 4.6946     | 5.1311                      | 4.3772       |
| 1000 | 20  | Cov | 0.9584     | 0.9604                      | 0.9612       |
|      |     | Len | 4.1761     | 4.1755                      | 4.1741       |
| 1000 | 40  | Cov | 0.9628     | 0.9542                      | 0.9576       |
|      |     | Len | 4.1765     | 4.1773                      | 4.1553       |
| 1000 | 100 | Cov | 0.9640     | 0.9602                      | 0.9552       |
|      |     | Len | 4.1771     | 4.1751                      | 4.1516       |
| 1000 | 200 | Cov | 0.9630     | 0.9594                      | 0.9462       |
|      |     | Len | 4.1784     | 4.1763                      | 4.1563       |

Table 5.249. Etype = 1, J=20, k=p-1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 40  | Cov | 0.9508     | 0.9298                      | 0.9284       |
|      |     | Len | 25.6545    | 55.3481                     | 8.0655       |
| 400  | 40  | Cov | 0.9476     | 0.9348                      | 0.9636       |
|      |     | Len | 18.0978    | 20.8497                     | 4.9484       |
| 1000 | 40  | Cov | 0.9648     | 0.9702                      | 0.9704       |
|      |     | Len | 4.4897     | 4.4889                      | 4.4867       |
| 100  | 100 | Cov | 0.9430     | 0.9128                      | 0.9162       |
|      |     | Len | 41.5219    | 142.7038                    | 17.5271      |
| 400  | 100 | Cov | 0.9320     | 0.9142                      | 0.9190       |
|      |     | Len | 34.8639    | 62.8941                     | 8.0764       |
| 1000 | 100 | Cov | 0.9760     | 0.9324                      | 0.9430       |
|      |     | Len | 4.8837     | 34.1218                     | 5.6318       |
| 100  | 200 | Cov | 0.9486     | 0.8976                      | 0.8886       |
|      |     | Len | 59.1332    | 282.1866                    | 33.3948      |
| 400  | 200 | Cov | 0.9346     | 0.8826                      | 0.8976       |
|      |     | Len | 54.1671    | 129.3459                    | 14.6696      |
| 1000 | 200 | Cov | 0.9290     | 0.9088                      | 0.9136       |
|      |     | Len | 47.1483    | 79.6519                     | 9.4535       |

Table 5.250. Etype = 1, J=50, k=1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9670     | 0.9654                      | 0.9640       |
|      |     | Len | 4.4394     | 4.4388                      | 4.3859       |
| 100  | 40  | Cov | 0.9624     | 0.9638                      | 0.9568       |
|      |     | Len | 4.4328     | 4.4311                      | 4.3819       |
| 100  | 100 | Cov | 0.9706     | 0.9596                      | 0.9602       |
|      |     | Len | 4.4459     | 4.4376                      | 4.3695       |
| 100  | 200 | Cov | 0.9656     | 0.9658                      | 0.9600       |
|      |     | Len | 4.4495     | 4.4356                      | 4.3609       |
| 400  | 20  | Cov | 0.9558     | 0.9502                      | 0.9456       |
|      |     | Len | 4.0082     | 4.0044                      | 4.0015       |
| 400  | 40  | Cov | 0.9528     | 0.9518                      | 0.9430       |
|      |     | Len | 4.0060     | 4.0042                      | 3.9947       |
| 400  | 100 | Cov | 0.9492     | 0.9504                      | 0.9452       |
|      |     | Len | 4.0042     | 4.0051                      | 3.9912       |
| 400  | 200 | Cov | 0.9554     | 0.9494                      | 0.9438       |
|      |     | Len | 4.0020     | 4.0052                      | 3.9906       |
| 1000 | 20  | Cov | 0.9504     | 0.9506                      | 0.9500       |
|      |     | Len | 3.9387     | 3.9371                      | 3.9347       |
| 1000 | 40  | Cov | 0.9462     | 0.9526                      | 0.9478       |
|      |     | Len | 3.9363     | 3.9380                      | 3.9332       |
| 1000 | 100 | Cov | 0.9492     | 0.9440                      | 0.9508       |
|      |     | Len | 3.9376     | 3.9370                      | 3.9348       |
| 1000 | 200 | Cov | 0.9486     | 0.9554                      | 0.9508       |
|      |     | Len | 3.9362     | 3.9382                      | 3.9323       |

Table 5.251. Etype = 1, J=50, k=19

| $n$  | $p$ | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov        | 0.9502                      | 0.9380       |
|      |     | Len        | 18.3207                     | 46.7607      |
| 100  | 40  | Cov        | 0.9494                      | 0.9400       |
|      |     | Len        | 18.3136                     | 48.9391      |
| 100  | 100 | Cov        | 0.9494                      | 0.9308       |
|      |     | Len        | 18.2962                     | 50.1546      |
| 100  | 200 | Cov        | 0.9542                      | 0.9268       |
|      |     | Len        | 18.3149                     | 50.2308      |
| 400  | 20  | Cov        | 0.9386                      | 0.9282       |
|      |     | Len        | 13.7495                     | 16.5096      |
| 400  | 40  | Cov        | 0.9352                      | 0.9306       |
|      |     | Len        | 13.7694                     | 17.8209      |
| 400  | 100 | Cov        | 0.9362                      | 0.9306       |
|      |     | Len        | 13.7644                     | 18.9651      |
| 400  | 200 | Cov        | 0.9448                      | 0.9232       |
|      |     | Len        | 13.7507                     | 19.6209      |
| 1000 | 20  | Cov        | 0.9562                      | 0.9640       |
|      |     | Len        | 4.1748                      | 4.1775       |
| 1000 | 40  | Cov        | 0.9612                      | 0.9620       |
|      |     | Len        | 4.1742                      | 4.1777       |
| 1000 | 100 | Cov        | 0.9560                      | 0.9606       |
|      |     | Len        | 4.1773                      | 4.1842       |
| 1000 | 200 | Cov        | 0.9612                      | 0.9648       |
|      |     | Len        | 4.1772                      | 4.1840       |

Table 5.252. Etype = 1, J=50, k=p-1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 40  | Cov | 0.9502     | 0.9364                      | 0.9350       |
|      |     | Len | 26.2609    | 99.0403                     | 14.9842      |
| 400  | 40  | Cov | 0.9284     | 0.9196                      | 0.9184       |
|      |     | Len | 21.8629    | 38.8563                     | 6.0584       |
| 1000 | 40  | Cov | 0.9374     | 0.9284                      | 0.9424       |
|      |     | Len | 17.2327    | 20.0686                     | 4.5960       |
| 100  | 100 | Cov | 0.9498     | 0.9272                      | 0.9236       |
|      |     | Len | 41.9417    | 253.7439                    | 36.3098      |
| 400  | 100 | Cov | 0.9384     | 0.9048                      | 0.8980       |
|      |     | Len | 36.9146    | 105.7099                    | 12.8067      |
| 1000 | 100 | Cov | 0.9362     | 0.9080                      | 0.9164       |
|      |     | Len | 33.7181    | 63.1281                     | 8.0172       |
| 100  | 200 | Cov | 0.9474     | 0.9224                      | 0.9192       |
|      |     | Len | 59.1702    | 506.6544                    | 71.5347      |
| 400  | 200 | Cov | 0.9386     | 0.8936                      | 0.8950       |
|      |     | Len | 54.5270    | 215.5089                    | 24.7068      |
| 1000 | 200 | Cov | 0.9336     | 0.9044                      | 0.9056       |
|      |     | Len | 50.6054    | 133.2647                    | 15.0494      |

Table 5.253. Etype = 2, J=5, k=1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9564     | 0.9538                      | 0.9496       |
|      |     | Len | 7.2198     | 7.2275                      | 7.1927       |
| 100  | 40  | Cov | 0.9510     | 0.9578                      | 0.9504       |
|      |     | Len | 7.2597     | 7.2496                      | 7.1912       |
| 100  | 100 | Cov | 0.9492     | 0.9534                      | 0.9462       |
|      |     | Len | 7.2354     | 7.2028                      | 7.1998       |
| 100  | 200 | Cov | 0.9490     | 0.9496                      | 0.9500       |
|      |     | Len | 7.2521     | 7.2308                      | 7.1480       |
| 400  | 20  | Cov | 0.9470     | 0.9446                      | 0.9454       |
|      |     | Len | 6.4632     | 6.4483                      | 6.4468       |
| 400  | 40  | Cov | 0.9516     | 0.9520                      | 0.9458       |
|      |     | Len | 6.4629     | 6.4492                      | 6.4404       |
| 400  | 100 | Cov | 0.9528     | 0.9460                      | 0.9496       |
|      |     | Len | 6.4616     | 6.4504                      | 6.4514       |
| 400  | 200 | Cov | 0.9494     | 0.9454                      | 0.9440       |
|      |     | Len | 6.4509     | 6.4768                      | 6.4113       |
| 1000 | 20  | Cov | 0.9482     | 0.9500                      | 0.9466       |
|      |     | Len | 6.3494     | 6.3564                      | 6.3449       |
| 1000 | 40  | Cov | 0.9454     | 0.9438                      | 0.9430       |
|      |     | Len | 6.3465     | 6.3578                      | 6.3420       |
| 1000 | 100 | Cov | 0.9480     | 0.9498                      | 0.9480       |
|      |     | Len | 6.3603     | 6.3612                      | 6.3478       |
| 1000 | 200 | Cov | 0.9550     | 0.9482                      | 0.9438       |
|      |     | Len | 6.3571     | 6.3606                      | 6.3464       |

Table 5.254. Etype = 2, J=5, k=19

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9760     | 0.9780                      | 0.9620       |
|      |     | Len | 10.0696    | 10.0334                     | 8.2869       |
| 100  | 40  | Cov | 0.9726     | 0.9676                      | 0.9520       |
|      |     | Len | 10.0310    | 10.0748                     | 8.0663       |
| 100  | 100 | Cov | 0.9700     | 0.9712                      | 0.9420       |
|      |     | Len | 10.3542    | 10.0511                     | 7.8225       |
| 100  | 200 | Cov | 0.9612     | 0.9692                      | 0.9316       |
|      |     | Len | 11.2014    | 10.2027                     | 7.6814       |
| 400  | 20  | Cov | 0.9712     | 0.9754                      | 0.9532       |
|      |     | Len | 8.4250     | 8.4276                      | 6.9967       |
| 400  | 40  | Cov | 0.9688     | 0.9714                      | 0.9488       |
|      |     | Len | 8.3983     | 8.3948                      | 6.9465       |
| 400  | 100 | Cov | 0.9698     | 0.9738                      | 0.9398       |
|      |     | Len | 8.3662     | 8.3874                      | 6.9018       |
| 400  | 200 | Cov | 0.9738     | 0.9710                      | 0.9414       |
|      |     | Len | 8.3995     | 8.4170                      | 6.8531       |
| 1000 | 20  | Cov | 0.9580     | 0.9606                      | 0.9560       |
|      |     | Len | 6.9778     | 6.9918                      | 6.7105       |
| 1000 | 40  | Cov | 0.9598     | 0.9572                      | 0.9518       |
|      |     | Len | 7.0023     | 7.0053                      | 6.6845       |
| 1000 | 100 | Cov | 0.9606     | 0.9530                      | 0.9506       |
|      |     | Len | 6.9960     | 6.9858                      | 6.6595       |
| 1000 | 200 | Cov | 0.9590     | 0.9622                      | 0.9470       |
|      |     | Len | 6.9924     | 6.9981                      | 6.6346       |

Table 5.255. Etype = 2, J=5, k=p-1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 40  | Cov | 0.9098     | 0.9060                      | 0.9490       |
|      |     | Len | 18.8294    | 20.3581                     | 8.7305       |
| 400  | 40  | Cov | 0.9720     | 0.9720                      | 0.9536       |
|      |     | Len | 8.7203     | 8.7201                      | 7.6968       |
| 1000 | 40  | Cov | 0.9732     | 0.9710                      | 0.9538       |
|      |     | Len | 7.8911     | 7.9020                      | 7.1201       |
| 100  | 100 | Cov | 0.9300     | 0.8096                      | 0.8702       |
|      |     | Len | 41.0859    | 50.5781                     | 9.1799       |
| 400  | 100 | Cov | 0.9288     | 0.9202                      | 0.9394       |
|      |     | Len | 18.8219    | 18.8528                     | 8.5608       |
| 1000 | 100 | Cov | 0.9730     | 0.9722                      | 0.9668       |
|      |     | Len | 8.7675     | 8.7732                      | 8.3899       |
| 100  | 200 | Cov | 0.9490     | 0.7192                      | 0.7474       |
|      |     | Len | 59.2288    | 93.0165                     | 11.7748      |
| 400  | 200 | Cov | 0.9190     | 0.9062                      | 0.9354       |
|      |     | Len | 19.2643    | 21.6358                     | 8.6117       |
| 1000 | 200 | Cov | 0.9748     | 0.9752                      | 0.9502       |
|      |     | Len | 9.3328     | 9.3402                      | 8.8941       |

Table 5.256. Etype = 2, J=10, k=1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9610     | 0.9530                      | 0.9532       |
|      |     | Len | 7.2671     | 7.2015                      | 7.1787       |
| 100  | 40  | Cov | 0.9544     | 0.9566                      | 0.9592       |
|      |     | Len | 7.2525     | 7.1782                      | 7.1502       |
| 100  | 100 | Cov | 0.9548     | 0.9560                      | 0.9482       |
|      |     | Len | 7.2069     | 7.2400                      | 7.1713       |
| 100  | 200 | Cov | 0.9568     | 0.9522                      | 0.9448       |
|      |     | Len | 7.2377     | 7.2345                      | 7.1369       |
| 400  | 20  | Cov | 0.9512     | 0.9546                      | 0.9466       |
|      |     | Len | 6.4611     | 6.4520                      | 6.4439       |
| 400  | 40  | Cov | 0.9528     | 0.9460                      | 0.9482       |
|      |     | Len | 6.4646     | 6.4508                      | 6.4469       |
| 400  | 100 | Cov | 0.9512     | 0.9458                      | 0.9512       |
|      |     | Len | 6.4594     | 6.4451                      | 6.4296       |
| 400  | 200 | Cov | 0.9478     | 0.9480                      | 0.9490       |
|      |     | Len | 6.4549     | 6.4418                      | 6.4386       |
| 1000 | 20  | Cov | 0.9506     | 0.9520                      | 0.9502       |
|      |     | Len | 6.3530     | 6.3557                      | 6.3470       |
| 1000 | 40  | Cov | 0.9462     | 0.9510                      | 0.9530       |
|      |     | Len | 6.3529     | 6.3640                      | 6.3433       |
| 1000 | 100 | Cov | 0.9398     | 0.9526                      | 0.9448       |
|      |     | Len | 6.3599     | 6.3640                      | 6.3486       |
| 1000 | 200 | Cov | 0.9514     | 0.9456                      | 0.9492       |
|      |     | Len | 6.3588     | 6.3639                      | 6.3435       |

Table 5.257. Etype = 2, J=10, k=19

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9506     | 0.9404                      | 0.9552       |
|      |     | Len | 15.2142    | 15.8700                     | 8.3033       |
| 100  | 40  | Cov | 0.9454     | 0.9340                      | 0.9542       |
|      |     | Len | 15.2739    | 17.2104                     | 8.1063       |
| 100  | 100 | Cov | 0.9380     | 0.9040                      | 0.9362       |
|      |     | Len | 15.9292    | 18.4827                     | 7.8094       |
| 100  | 200 | Cov | 0.9400     | 0.8744                      | 0.9316       |
|      |     | Len | 17.0700    | 18.9969                     | 7.6465       |
| 400  | 20  | Cov | 0.9720     | 0.9712                      | 0.9482       |
|      |     | Len | 8.4313     | 8.4339                      | 6.9988       |
| 400  | 40  | Cov | 0.9746     | 0.9744                      | 0.9492       |
|      |     | Len | 8.3977     | 8.4271                      | 6.9512       |
| 400  | 100 | Cov | 0.9732     | 0.9662                      | 0.9516       |
|      |     | Len | 8.3930     | 8.4114                      | 6.9030       |
| 400  | 200 | Cov | 0.9706     | 0.9726                      | 0.9422       |
|      |     | Len | 8.3749     | 8.4094                      | 6.8542       |
| 1000 | 20  | Cov | 0.9550     | 0.9588                      | 0.9524       |
|      |     | Len | 6.9917     | 6.9827                      | 6.7091       |
| 1000 | 40  | Cov | 0.9594     | 0.9616                      | 0.9522       |
|      |     | Len | 7.0148     | 6.9977                      | 6.6673       |
| 1000 | 100 | Cov | 0.9596     | 0.9550                      | 0.9482       |
|      |     | Len | 7.0045     | 6.9978                      | 6.6514       |
| 1000 | 200 | Cov | 0.9558     | 0.9618                      | 0.9408       |
|      |     | Len | 6.9954     | 6.9947                      | 6.6377       |

Table 5.258. Etype = 2, J=10, k=p-1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 40  | Cov | 0.9264     | 0.9108                      | 0.9422       |
|      |     | Len | 23.7779    | 33.9659                     | 8.7436       |
| 400  | 40  | Cov | 0.9762     | 0.9722                      | 0.9526       |
|      |     | Len | 8.7409     | 8.7269                      | 7.6865       |
| 1000 | 40  | Cov | 0.9686     | 0.9654                      | 0.9574       |
|      |     | Len | 7.8983     | 7.9072                      | 7.1144       |
| 100  | 100 | Cov | 0.9424     | 0.8632                      | 0.8776       |
|      |     | Len | 41.4014    | 85.8916                     | 11.7993      |
| 400  | 100 | Cov | 0.9166     | 0.9030                      | 0.9422       |
|      |     | Len | 29.3077    | 38.3416                     | 8.5978       |
| 1000 | 100 | Cov | 0.9742     | 0.9732                      | 0.9636       |
|      |     | Len | 8.8007     | 8.7906                      | 8.3915       |
| 100  | 200 | Cov | 0.9420     | 0.8288                      | 0.8248       |
|      |     | Len | 59.2734    | 166.2737                    | 19.6584      |
| 400  | 200 | Cov | 0.9390     | 0.8508                      | 0.8978       |
|      |     | Len | 54.3612    | 80.9983                     | 10.8268      |
| 1000 | 200 | Cov | 0.9246     | 0.9050                      | 0.9484       |
|      |     | Len | 37.5619    | 46.6793                     | 8.9576       |

Table 5.259. Etype = 2, J=20, k=1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9562     | 0.9522                      | 0.9562       |
|      |     | Len | 7.2348     | 7.2008                      | 7.1770       |
| 100  | 40  | Cov | 0.9556     | 0.9506                      | 0.9520       |
|      |     | Len | 7.2390     | 7.2243                      | 7.1580       |
| 100  | 100 | Cov | 0.9526     | 0.9526                      | 0.9528       |
|      |     | Len | 7.1813     | 7.1877                      | 7.1502       |
| 100  | 200 | Cov | NA         | 0.9510                      | 0.9562       |
|      |     | Len | NA         | 7.1938                      | 7.1569       |
| 400  | 20  | Cov | 0.9460     | 0.9456                      | 0.9442       |
|      |     | Len | 6.4636     | 6.4492                      | 6.4270       |
| 400  | 40  | Cov | 0.9502     | 0.9468                      | 0.9428       |
|      |     | Len | 6.4620     | 6.4675                      | 6.4359       |
| 400  | 100 | Cov | 0.9432     | 0.9466                      | 0.9494       |
|      |     | Len | 6.4593     | 6.4515                      | 6.4400       |
| 400  | 200 | Cov | 0.9504     | 0.9452                      | 0.9480       |
|      |     | Len | 6.4558     | 6.4509                      | 6.4281       |
| 1000 | 20  | Cov | 0.9482     | 0.9566                      | 0.9444       |
|      |     | Len | 6.3591     | 6.3552                      | 6.3503       |
| 1000 | 40  | Cov | 0.9492     | 0.9478                      | 0.9462       |
|      |     | Len | 6.3643     | 6.3635                      | 6.3531       |
| 1000 | 100 | Cov | 0.9478     | 0.9456                      | 0.9486       |
|      |     | Len | 6.3639     | 6.3515                      | 6.3381       |
| 1000 | 200 | Cov | 0.9414     | 0.9478                      | 0.9420       |
|      |     | Len | 6.3574     | 6.3628                      | 6.3423       |

Table 5.260. Etype = 2, J=20, k=19

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9544     | 0.9400                      | 0.9574       |
|      |     | Len | 18.4259    | 26.1005                     | 8.3027       |
| 100  | 40  | Cov | 0.9538     | 0.9410                      | 0.9490       |
|      |     | Len | 18.4030    | 27.9565                     | 8.0949       |
| 100  | 100 | Cov | 0.9486     | 0.9232                      | 0.9464       |
|      |     | Len | 18.3614    | 29.3061                     | 7.8209       |
| 100  | 200 | Cov | 0.9470     | 0.9066                      | 0.9388       |
|      |     | Len | 18.4335    | 29.7058                     | 7.6764       |
| 400  | 20  | Cov | 0.9732     | 0.9762                      | 0.9494       |
|      |     | Len | 8.4337     | 8.4175                      | 7.0005       |
| 400  | 40  | Cov | 0.9720     | 0.9724                      | 0.9498       |
|      |     | Len | 8.4209     | 8.4301                      | 6.9515       |
| 400  | 100 | Cov | 0.9664     | 0.9708                      | 0.9490       |
|      |     | Len | 8.4559     | 8.4262                      | 6.9046       |
| 400  | 200 | Cov | 0.9722     | 0.9700                      | 0.9434       |
|      |     | Len | 8.4352     | 8.4433                      | 6.8591       |
| 1000 | 20  | Cov | 0.9584     | 0.9564                      | 0.9540       |
|      |     | Len | 6.9992     | 6.9848                      | 6.7019       |
| 1000 | 40  | Cov | 0.9596     | 0.9608                      | 0.9524       |
|      |     | Len | 6.9914     | 6.9957                      | 6.6789       |
| 1000 | 100 | Cov | 0.9576     | 0.9546                      | 0.9500       |
|      |     | Len | 7.0107     | 7.0003                      | 6.6597       |
| 1000 | 200 | Cov | 0.9578     | 0.9588                      | 0.9422       |
|      |     | Len | 6.9970     | 6.9960                      | 6.6366       |

Table 5.261. Etype = 2, J=20, k=p-1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 40  | Cov | 0.9522     | 0.9274                      | 0.9424       |
|      |     | Len | 26.3543    | 55.6385                     | 9.9646       |
| 400  | 40  | Cov | 0.9378     | 0.9314                      | 0.9564       |
|      |     | Len | 19.0416    | 21.6264                     | 7.7040       |
| 1000 | 40  | Cov | 0.9712     | 0.9648                      | 0.9558       |
|      |     | Len | 7.8967     | 7.9038                      | 7.1171       |
| 100  | 100 | Cov | 0.9458     | 0.9114                      | 0.9176       |
|      |     | Len | 41.8971    | 142.5211                    | 18.3060      |
| 400  | 100 | Cov | 0.9340     | 0.9068                      | 0.9352       |
|      |     | Len | 35.3779    | 63.1102                     | 9.7697       |
| 1000 | 100 | Cov | 0.9464     | 0.9338                      | 0.9602       |
|      |     | Len | 28.4479    | 34.5788                     | 8.4358       |
| 100  | 200 | Cov | 0.9426     | 0.8914                      | 0.8882       |
|      |     | Len | 59.3888    | 281.9183                    | 33.8312      |
| 400  | 200 | Cov | 0.9406     | 0.8930                      | 0.9060       |
|      |     | Len | 54.4805    | 129.7019                    | 15.5431      |
| 1000 | 200 | Cov | 0.9248     | 0.9086                      | 0.9292       |
|      |     | Len | 47.5347    | 79.8276                     | 10.9373      |

Table 5.262. Etype = 2, J=50, k=1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | NA         | 0.9504                      | 0.9552       |
|      |     | Len | NA         | 7.2338                      | 7.1482       |
| 100  | 40  | Cov | NA         | 0.9538                      | 0.9544       |
|      |     | Len | NA         | 7.2221                      | 7.1184       |
| 100  | 100 | Cov | NA         | 0.9544                      | 0.9520       |
|      |     | Len | NA         | 7.2312                      | 7.1037       |
| 100  | 200 | Cov | NA         | 0.9504                      | 0.9490       |
|      |     | Len | NA         | 7.1983                      | 7.1735       |
| 400  | 20  | Cov | 0.9462     | 0.9486                      | 0.9522       |
|      |     | Len | 6.4725     | 6.4550                      | 6.4490       |
| 400  | 40  | Cov | 0.9552     | 0.9512                      | 0.9518       |
|      |     | Len | 6.4736     | 6.4649                      | 6.4301       |
| 400  | 100 | Cov | 0.9486     | 0.9450                      | 0.9500       |
|      |     | Len | 6.4714     | 6.4455                      | 6.4285       |
| 400  | 200 | Cov | 0.9486     | 0.9500                      | 0.9486       |
|      |     | Len | 6.4481     | 6.4582                      | 6.4257       |
| 1000 | 20  | Cov | 0.9502     | 0.9466                      | 0.9500       |
|      |     | Len | 6.3548     | 6.3587                      | 6.3505       |
| 1000 | 40  | Cov | 0.9468     | 0.9494                      | 0.9444       |
|      |     | Len | 6.3588     | 6.3555                      | 6.3567       |
| 1000 | 100 | Cov | 0.9474     | 0.9500                      | 0.9446       |
|      |     | Len | 6.3593     | 6.3575                      | 6.3504       |
| 1000 | 200 | Cov | 0.9456     | 0.9490                      | 0.9436       |
|      |     | Len | 6.3614     | 6.3531                      | 6.3485       |

Table 5.263. Etype = 2, J=50, k=19

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9516     | 0.9418                      | 0.9396       |
|      |     | Len | 19.1533    | 47.1172                     | 9.5797       |
| 100  | 40  | Cov | 0.9488     | 0.9420                      | 0.9376       |
|      |     | Len | 19.1771    | 49.2083                     | 9.5258       |
| 100  | 100 | Cov | 0.9522     | 0.9302                      | 0.9330       |
|      |     | Len | 19.1606    | 50.3659                     | 9.3648       |
| 100  | 200 | Cov | NA         | 0.9266                      | 0.9278       |
|      |     | Len | NA         | 50.4854                     | 9.2164       |
| 400  | 20  | Cov | 0.9406     | 0.9372                      | 0.9454       |
|      |     | Len | 14.7175    | 17.2978                     | 6.9715       |
| 400  | 40  | Cov | 0.9396     | 0.9354                      | 0.9480       |
|      |     | Len | 14.7266    | 18.5329                     | 6.9556       |
| 400  | 100 | Cov | 0.9404     | 0.9378                      | 0.9450       |
|      |     | Len | 14.7204    | 19.6616                     | 6.8989       |
| 400  | 200 | Cov | 0.9440     | 0.9274                      | 0.9420       |
|      |     | Len | 14.7158    | 20.2760                     | 6.8734       |
| 1000 | 20  | Cov | 0.9574     | 0.9580                      | 0.9502       |
|      |     | Len | 6.9919     | 6.9872                      | 6.7053       |
| 1000 | 40  | Cov | 0.9560     | 0.9548                      | 0.9478       |
|      |     | Len | 6.9927     | 6.9955                      | 6.6888       |
| 1000 | 100 | Cov | 0.9594     | 0.9570                      | 0.9424       |
|      |     | Len | 6.9878     | 6.9887                      | 6.6603       |
| 1000 | 200 | Cov | 0.9564     | 0.9588                      | 0.9406       |
|      |     | Len | 7.0092     | 6.9924                      | 6.6436       |

Table 5.264. Etype = 2, J=50, k=p-1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 40  | Cov | 0.9538     | 0.9312                      | 0.9338       |
|      |     | Len | 26.8942    | 99.2809                     | 15.8398      |
| 400  | 40  | Cov | 0.9348     | 0.9194                      | 0.9374       |
|      |     | Len | 22.5364    | 39.1995                     | 7.8289       |
| 1000 | 40  | Cov | 0.9360     | 0.9286                      | 0.9528       |
|      |     | Len | 18.0250    | 20.7333                     | 7.0529       |
| 100  | 100 | Cov | 0.9520     | 0.9286                      | 0.9222       |
|      |     | Len | 42.3064    | 254.2178                    | 36.5661      |
| 400  | 100 | Cov | 0.9294     | 0.9014                      | 0.9140       |
|      |     | Len | 37.3130    | 105.7312                    | 13.6985      |
| 1000 | 100 | Cov | 0.9396     | 0.9158                      | 0.9344       |
|      |     | Len | 34.1379    | 63.2724                     | 9.3894       |
| 100  | 200 | Cov | 0.9506     | 0.9262                      | 0.9244       |
|      |     | Len | 59.7265    | 506.5613                    | 71.5173      |
| 400  | 200 | Cov | 0.9370     | 0.8896                      | 0.8974       |
|      |     | Len | 54.7432    | 215.6005                    | 25.1643      |
| 1000 | 200 | Cov | 0.9264     | 0.8978                      | 0.9122       |
|      |     | Len | 50.9035    | 133.2922                    | 15.8057      |

Table 5.265. Etype = 3, J=5, k=1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9612     | 0.9646                      | 0.9578       |
|      |     | Len | 3.7945     | 3.7679                      | 3.8340       |
| 100  | 40  | Cov | 0.9624     | 0.9634                      | 0.9648       |
|      |     | Len | 3.7741     | 3.7681                      | 3.8388       |
| 100  | 100 | Cov | 0.9618     | 0.9662                      | 0.9608       |
|      |     | Len | 3.7837     | 3.7770                      | 3.8520       |
| 100  | 200 | Cov | 0.9646     | 0.9610                      | 0.9566       |
|      |     | Len | 3.7816     | 3.7656                      | 3.8651       |
| 400  | 20  | Cov | 0.9562     | 0.9556                      | 0.9510       |
|      |     | Len | 3.2180     | 3.2105                      | 3.2729       |
| 400  | 40  | Cov | 0.9578     | 0.9576                      | 0.9496       |
|      |     | Len | 3.2167     | 3.2142                      | 3.2913       |
| 400  | 100 | Cov | 0.9564     | 0.9596                      | 0.9468       |
|      |     | Len | 3.2142     | 3.2166                      | 3.3162       |
| 400  | 200 | Cov | 0.9498     | 0.9522                      | 0.9494       |
|      |     | Len | 3.2130     | 3.2136                      | 3.3201       |
| 1000 | 20  | Cov | 0.9528     | 0.9488                      | 0.9524       |
|      |     | Len | 3.0971     | 3.0988                      | 3.1190       |
| 1000 | 40  | Cov | 0.9506     | 0.9534                      | 0.9522       |
|      |     | Len | 3.0988     | 3.0992                      | 3.1343       |
| 1000 | 100 | Cov | 0.9518     | 0.9486                      | 0.9502       |
|      |     | Len | 3.0978     | 3.0953                      | 3.1499       |
| 1000 | 200 | Cov | 0.9530     | 0.9498                      | 0.9484       |
|      |     | Len | 3.0976     | 3.1002                      | 3.1649       |

Table 5.266. Etype = 3, J=5, k=19

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9778     | 0.9834                      | 0.9576       |
|      |     | Len | 5.6294     | 5.6451                      | 5.2388       |
| 100  | 40  | Cov | 0.9782     | 0.9674                      | 0.9498       |
|      |     | Len | 5.9242     | 6.6955                      | 5.2174       |
| 100  | 100 | Cov | 0.9690     | 0.9378                      | 0.9392       |
|      |     | Len | 6.7217     | 8.8035                      | 5.1545       |
| 100  | 200 | Cov | 0.9412     | 0.8788                      | 0.9190       |
|      |     | Len | 8.4551     | 10.4310                     | 5.0499       |
| 400  | 20  | Cov | 0.9756     | 0.9756                      | 0.9570       |
|      |     | Len | 4.3314     | 4.3278                      | 4.1679       |
| 400  | 40  | Cov | 0.9728     | 0.9744                      | 0.9542       |
|      |     | Len | 4.3384     | 4.3435                      | 4.2074       |
| 400  | 100 | Cov | 0.9708     | 0.9752                      | 0.9482       |
|      |     | Len | 4.3399     | 4.3523                      | 4.2441       |
| 400  | 200 | Cov | 0.9752     | 0.9744                      | 0.9452       |
|      |     | Len | 4.3357     | 4.3480                      | 4.2610       |
| 1000 | 20  | Cov | 0.9598     | 0.9596                      | 0.9582       |
|      |     | Len | 3.5601     | 3.5606                      | 3.5657       |
| 1000 | 40  | Cov | 0.9578     | 0.9604                      | 0.9526       |
|      |     | Len | 3.5707     | 3.5722                      | 3.6925       |
| 1000 | 100 | Cov | 0.9592     | 0.9574                      | 0.9554       |
|      |     | Len | 3.5653     | 3.5739                      | 3.7609       |
| 1000 | 200 | Cov | 0.9600     | 0.9608                      | 0.9460       |
|      |     | Len | 3.5675     | 3.5706                      | 3.7959       |

Table 5.267. Etype = 3, J=5, k=p-1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 40  | Cov | 0.9068     | 0.8876                      | 0.9374       |
|      |     | Len | 17.4717    | 19.4589                     | 5.5039       |
| 400  | 40  | Cov | 0.9760     | 0.9754                      | 0.9660       |
|      |     | Len | 4.6697     | 4.6777                      | 4.7592       |
| 1000 | 40  | Cov | 0.9706     | 0.9684                      | 0.9674       |
|      |     | Len | 4.0441     | 4.0353                      | 4.0387       |
| 100  | 100 | Cov | 0.9366     | 0.7986                      | 0.8118       |
|      |     | Len | 40.6862    | 50.2978                     | 6.8339       |
| 400  | 100 | Cov | 0.9116     | 0.9272                      | 0.9408       |
|      |     | Len | 17.7940    | 17.8558                     | 5.3550       |
| 1000 | 100 | Cov | 0.9800     | 0.9774                      | 0.9776       |
|      |     | Len | 4.6590     | 4.6526                      | 4.6559       |
| 100  | 200 | Cov | 0.9422     | 0.7270                      | 0.7302       |
|      |     | Len | 58.9611    | 92.9103                     | 10.7338      |
| 400  | 200 | Cov | 0.9062     | 0.8964                      | 0.9176       |
|      |     | Len | 18.1816    | 20.8019                     | 5.5971       |
| 1000 | 200 | Cov | 0.9778     | 0.9766                      | 0.9774       |
|      |     | Len | 5.2097     | 5.2074                      | 5.2125       |

Table 5.268. Etype = 3, J=10, k=1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9660     | 0.9650                      | 0.9596       |
|      |     | Len | 3.7766     | 3.7956                      | 3.8254       |
| 100  | 40  | Cov | 0.9606     | 0.9644                      | 0.9642       |
|      |     | Len | 3.7798     | 3.7824                      | 3.8398       |
| 100  | 100 | Cov | 0.9578     | 0.9618                      | 0.9596       |
|      |     | Len | 3.7810     | 3.7811                      | 3.8588       |
| 100  | 200 | Cov | 0.9626     | 0.9596                      | 0.9626       |
|      |     | Len | 3.7755     | 3.7595                      | 3.8713       |
| 400  | 20  | Cov | 0.9520     | 0.9564                      | 0.9506       |
|      |     | Len | 3.2191     | 3.2168                      | 3.2789       |
| 400  | 40  | Cov | 0.9522     | 0.9542                      | 0.9472       |
|      |     | Len | 3.2089     | 3.2135                      | 3.3014       |
| 400  | 100 | Cov | 0.9560     | 0.9512                      | 0.9472       |
|      |     | Len | 3.2138     | 3.2172                      | 3.3121       |
| 400  | 200 | Cov | 0.9554     | 0.9506                      | 0.9438       |
|      |     | Len | 3.2155     | 3.2144                      | 3.3270       |
| 1000 | 20  | Cov | 0.9484     | 0.9524                      | 0.9524       |
|      |     | Len | 3.0986     | 3.1004                      | 3.1209       |
| 1000 | 40  | Cov | 0.9548     | 0.9498                      | 0.9554       |
|      |     | Len | 3.1024     | 3.1019                      | 3.1316       |
| 1000 | 100 | Cov | 0.9502     | 0.9482                      | 0.9504       |
|      |     | Len | 3.0960     | 3.0969                      | 3.1515       |
| 1000 | 200 | Cov | 0.9530     | 0.9530                      | 0.9528       |
|      |     | Len | 3.1002     | 3.1016                      | 3.1663       |

Table 5.269. Etype = 3, J=10, k=19

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9398     | 0.9410                      | 0.9606       |
|      |     | Len | 13.8277    | 14.7297                     | 5.2228       |
| 100  | 40  | Cov | 0.9434     | 0.9290                      | 0.9492       |
|      |     | Len | 13.8467    | 16.1603                     | 5.2057       |
| 100  | 100 | Cov | 0.9334     | 0.9070                      | 0.9344       |
|      |     | Len | 14.1469    | 17.6214                     | 5.1447       |
| 100  | 200 | Cov | 0.9328     | 0.8740                      | 0.9264       |
|      |     | Len | 15.1591    | 18.1632                     | 5.0548       |
| 400  | 20  | Cov | 0.9734     | 0.9714                      | 0.9654       |
|      |     | Len | 4.3322     | 4.3280                      | 4.1705       |
| 400  | 40  | Cov | 0.9774     | 0.9708                      | 0.9538       |
|      |     | Len | 4.3299     | 4.3319                      | 4.2099       |
| 400  | 100 | Cov | 0.9760     | 0.9752                      | 0.9462       |
|      |     | Len | 4.3323     | 4.3319                      | 4.2545       |
| 400  | 200 | Cov | 0.9784     | 0.9766                      | 0.9438       |
|      |     | Len | 4.3416     | 4.3419                      | 4.2715       |
| 1000 | 20  | Cov | 0.9588     | 0.9614                      | 0.9572       |
|      |     | Len | 3.5583     | 3.5590                      | 3.5694       |
| 1000 | 40  | Cov | 0.9600     | 0.9584                      | 0.9568       |
|      |     | Len | 3.5701     | 3.5774                      | 3.6937       |
| 1000 | 100 | Cov | 0.9594     | 0.9596                      | 0.9494       |
|      |     | Len | 3.5725     | 3.5667                      | 3.7557       |
| 1000 | 200 | Cov | 0.9652     | 0.9558                      | 0.9554       |
|      |     | Len | 3.5727     | 3.5718                      | 3.7946       |

Table 5.270. Etype = 3, J=10, k=p-1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 40  | Cov | 0.9316     | 0.9116                      | 0.9358       |
|      |     | Len | 22.7435    | 33.5340                     | 5.9343       |
| 400  | 40  | Cov | 0.9718     | 0.9778                      | 0.9666       |
|      |     | Len | 4.6645     | 4.6720                      | 4.7706       |
| 1000 | 40  | Cov | 0.9710     | 0.9712                      | 0.9746       |
|      |     | Len | 4.0371     | 4.0395                      | 4.0393       |
| 100  | 100 | Cov | 0.9418     | 0.8598                      | 0.8732       |
|      |     | Len | 41.0691    | 85.9486                     | 10.5509      |
| 400  | 100 | Cov | 0.9212     | 0.8944                      | 0.9286       |
|      |     | Len | 28.7542    | 37.9353                     | 5.9282       |
| 1000 | 100 | Cov | 0.9746     | 0.9746                      | 0.9774       |
|      |     | Len | 4.6569     | 4.6592                      | 4.6608       |
| 100  | 200 | Cov | 0.9430     | 0.8216                      | 0.8394       |
|      |     | Len | 58.8166    | 166.3864                    | 19.0220      |
| 400  | 200 | Cov | 0.9382     | 0.8686                      | 0.8800       |
|      |     | Len | 53.9478    | 80.7645                     | 9.5054       |
| 1000 | 200 | Cov | 0.9242     | 0.9060                      | 0.9330       |
|      |     | Len | 37.1177    | 46.3504                     | 6.5286       |

Table 5.271. Etype = 3, J=20, k=1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9626     | 0.9620                      | 0.9626       |
|      |     | Len | 3.7786     | 3.7689                      | 3.8432       |
| 100  | 40  | Cov | 0.9708     | 0.9590                      | 0.9620       |
|      |     | Len | 3.7851     | 3.7715                      | 3.8448       |
| 100  | 100 | Cov | 0.9624     | 0.9638                      | 0.9620       |
|      |     | Len | 3.7838     | 3.7663                      | 3.8452       |
| 100  | 200 | Cov | 0.9590     | 0.9622                      | 0.9580       |
|      |     | Len | 3.7769     | 3.7587                      | 3.8608       |
| 400  | 20  | Cov | 0.9500     | 0.9530                      | 0.9472       |
|      |     | Len | 3.2196     | 3.2192                      | 3.2816       |
| 400  | 40  | Cov | 0.9508     | 0.9564                      | 0.9540       |
|      |     | Len | 3.2139     | 3.2137                      | 3.2897       |
| 400  | 100 | Cov | 0.9562     | 0.9516                      | 0.9516       |
|      |     | Len | 3.2111     | 3.2156                      | 3.3180       |
| 400  | 200 | Cov | 0.9560     | 0.9574                      | 0.9506       |
|      |     | Len | 3.2120     | 3.2140                      | 3.3266       |
| 1000 | 20  | Cov | 0.9576     | 0.9536                      | 0.9546       |
|      |     | Len | 3.0945     | 3.0985                      | 3.1194       |
| 1000 | 40  | Cov | 0.9494     | 0.9426                      | 0.9510       |
|      |     | Len | 3.1016     | 3.0967                      | 3.1331       |
| 1000 | 100 | Cov | 0.9500     | 0.9510                      | 0.9456       |
|      |     | Len | 3.0945     | 3.0973                      | 3.1532       |
| 1000 | 200 | Cov | 0.9484     | 0.9526                      | 0.9550       |
|      |     | Len | 3.0960     | 3.0988                      | 3.1622       |

Table 5.272. Etype = 3, J=20, k=19

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9538     | 0.9448                      | 0.9580       |
|      |     | Len | 17.3250    | 25.4635                     | 5.6396       |
| 100  | 40  | Cov | 0.9554     | 0.9344                      | 0.9498       |
|      |     | Len | 17.3287    | 27.3010                     | 5.5623       |
| 100  | 100 | Cov | 0.9514     | 0.9254                      | 0.9334       |
|      |     | Len | 17.3118    | 28.7847                     | 5.4326       |
| 100  | 200 | Cov | 0.9418     | 0.9092                      | 0.9284       |
|      |     | Len | 17.3920    | 29.2473                     | 5.3111       |
| 400  | 20  | Cov | 0.9750     | 0.9750                      | 0.9616       |
|      |     | Len | 4.3300     | 4.3219                      | 4.1763       |
| 400  | 40  | Cov | 0.9762     | 0.9742                      | 0.9546       |
|      |     | Len | 4.3318     | 4.4112                      | 4.2084       |
| 400  | 100 | Cov | 0.9758     | 0.9712                      | 0.9520       |
|      |     | Len | 4.3281     | 4.5877                      | 4.2437       |
| 400  | 200 | Cov | 0.9792     | 0.9752                      | 0.9380       |
|      |     | Len | 4.3254     | 4.8688                      | 4.2666       |
| 1000 | 20  | Cov | 0.9610     | 0.9602                      | 0.9640       |
|      |     | Len | 3.5516     | 3.5596                      | 3.5646       |
| 1000 | 40  | Cov | 0.9598     | 0.9642                      | 0.9522       |
|      |     | Len | 3.5734     | 3.5727                      | 3.6952       |
| 1000 | 100 | Cov | 0.9570     | 0.9556                      | 0.9588       |
|      |     | Len | 3.5713     | 3.5718                      | 3.7548       |
| 1000 | 200 | Cov | 0.9598     | 0.9560                      | 0.9520       |
|      |     | Len | 3.5753     | 3.5719                      | 3.7946       |

Table 5.273. Etype = 3, J=20, k=p-1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 40  | Cov | 0.9472     | 0.9260                      | 0.9324       |
|      |     | Len | 25.6599    | 55.2863                     | 8.1372       |
| 400  | 40  | Cov | 0.9486     | 0.9334                      | 0.9624       |
|      |     | Len | 18.1278    | 20.8450                     | 4.9604       |
| 1000 | 40  | Cov | 0.9688     | 0.9682                      | 0.9718       |
|      |     | Len | 4.0434     | 4.0324                      | 4.0368       |
| 100  | 100 | Cov | 0.9456     | 0.9096                      | 0.9066       |
|      |     | Len | 41.5008    | 142.2078                    | 17.5771      |
| 400  | 100 | Cov | 0.9282     | 0.9144                      | 0.9240       |
|      |     | Len | 34.9094    | 62.9146                     | 8.1336       |
| 1000 | 100 | Cov | 0.9388     | 0.9294                      | 0.9542       |
|      |     | Len | 27.8204    | 34.1463                     | 5.7393       |
| 100  | 200 | Cov | 0.9446     | 0.8992                      | 0.8912       |
|      |     | Len | 59.1021    | 282.1162                    | 33.4140      |
| 400  | 200 | Cov | 0.9376     | 0.8874                      | 0.8880       |
|      |     | Len | 54.1021    | 129.4806                    | 14.6730      |
| 1000 | 200 | Cov | 0.9342     | 0.9190                      | 0.9172       |
|      |     | Len | 47.1855    | 79.6843                     | 9.4939       |

Table 5.274. Etype = 3, J=50, k=1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9638     | 0.9682                      | 0.9590       |
|      |     | Len | 3.7380     | 3.7457                      | 3.7985       |
| 100  | 40  | Cov | 0.9614     | 0.9630                      | 0.9638       |
|      |     | Len | 3.7435     | 3.7268                      | 3.8058       |
| 100  | 100 | Cov | 0.9692     | 0.9654                      | 0.9630       |
|      |     | Len | 3.7442     | 3.7444                      | 3.7936       |
| 100  | 200 | Cov | 0.9612     | 0.9642                      | 0.9596       |
|      |     | Len | 3.7401     | 3.7434                      | 3.7939       |
| 400  | 20  | Cov | 0.9518     | 0.9538                      | 0.9474       |
|      |     | Len | 3.2165     | 3.2121                      | 3.2843       |
| 400  | 40  | Cov | 0.9532     | 0.9574                      | 0.9496       |
|      |     | Len | 3.2155     | 3.2109                      | 3.2996       |
| 400  | 100 | Cov | 0.9478     | 0.9552                      | 0.9538       |
|      |     | Len | 3.2173     | 3.2165                      | 3.3160       |
| 400  | 200 | Cov | 0.9536     | 0.9508                      | 0.9462       |
|      |     | Len | 3.2157     | 3.2126                      | 3.3255       |
| 1000 | 20  | Cov | 0.9492     | 0.9506                      | 0.9532       |
|      |     | Len | 3.0987     | 3.1022                      | 3.1240       |
| 1000 | 40  | Cov | 0.9492     | 0.9530                      | 0.9518       |
|      |     | Len | 3.0996     | 3.0982                      | 3.1306       |
| 1000 | 100 | Cov | 0.9548     | 0.9506                      | 0.9488       |
|      |     | Len | 3.1021     | 3.0984                      | 3.1482       |
| 1000 | 200 | Cov | 0.9508     | 0.9486                      | 0.9526       |
|      |     | Len | 3.0989     | 3.0970                      | 3.1670       |

Table 5.275. Etype = 3, J=50, k=19

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9502     | 0.9364                      | 0.9366       |
|      |     | Len | 18.3152    | 46.7489                     | 8.1529       |
| 100  | 40  | Cov | 0.9580     | 0.9316                      | 0.9270       |
|      |     | Len | 18.3198    | 48.8759                     | 8.0814       |
| 100  | 100 | Cov | 0.9508     | 0.9320                      | 0.9308       |
|      |     | Len | 18.3224    | 50.1554                     | 7.9819       |
| 100  | 200 | Cov | 0.9606     | 0.9288                      | 0.9178       |
|      |     | Len | 18.2836    | 50.2342                     | 7.8609       |
| 400  | 20  | Cov | 0.9384     | 0.9310                      | 0.9496       |
|      |     | Len | 13.7638    | 16.5396                     | 4.3506       |
| 400  | 40  | Cov | 0.9332     | 0.9262                      | 0.9414       |
|      |     | Len | 13.7577    | 17.8030                     | 4.3524       |
| 400  | 100 | Cov | 0.9398     | 0.9276                      | 0.9414       |
|      |     | Len | 13.7700    | 18.9488                     | 4.3771       |
| 400  | 200 | Cov | 0.9368     | 0.9268                      | 0.9378       |
|      |     | Len | 13.7715    | 19.6442                     | 4.3809       |
| 1000 | 20  | Cov | 0.9620     | 0.9630                      | 0.9602       |
|      |     | Len | 3.5587     | 3.5576                      | 3.5610       |
| 1000 | 40  | Cov | 0.9654     | 0.9568                      | 0.9590       |
|      |     | Len | 3.5575     | 3.5561                      | 3.6924       |
| 1000 | 100 | Cov | 0.9624     | 0.9562                      | 0.9526       |
|      |     | Len | 3.5598     | 3.5626                      | 3.7582       |
| 1000 | 200 | Cov | 0.9620     | 0.9630                      | 0.9498       |
|      |     | Len | 3.5552     | 3.5693                      | 3.7997       |

Table 5.276. Etype = 3, J=50, k=p-1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 40  | Cov | 0.9508     | 0.9400                      | 0.9328       |
|      |     | Len | 26.2453    | 99.0408                     | 15.0066      |
| 400  | 40  | Cov | 0.9292     | 0.9162                      | 0.9248       |
|      |     | Len | 21.8866    | 38.8234                     | 6.0805       |
| 1000 | 40  | Cov | 0.9420     | 0.9346                      | 0.9478       |
|      |     | Len | 17.2337    | 20.0816                     | 4.4977       |
| 100  | 100 | Cov | 0.9376     | 0.9218                      | 0.9214       |
|      |     | Len | 41.9770    | 254.1073                    | 36.3488      |
| 400  | 100 | Cov | 0.9366     | 0.9094                      | 0.9048       |
|      |     | Len | 36.9149    | 105.7167                    | 12.8216      |
| 1000 | 100 | Cov | 0.9314     | 0.9164                      | 0.9210       |
|      |     | Len | 33.7183    | 63.0990                     | 8.0451       |
| 100  | 200 | Cov | NA         | 0.9194                      | 0.9152       |
|      |     | Len | NA         | 506.2869                    | 71.3896      |
| 400  | 200 | Cov | 0.9360     | 0.8996                      | 0.8986       |
|      |     | Len | 54.4719    | 215.5844                    | 24.7109      |
| 1000 | 200 | Cov | 0.9280     | 0.9086                      | 0.9040       |
|      |     | Len | 50.5728    | 133.2308                    | 15.0387      |

Table 5.277. Etype = 4, J=5, k=1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9896     | 0.9910                      | 0.9792       |
|      |     | Len | 2.2083     | 2.2065                      | 2.2245       |
| 100  | 40  | Cov | 0.9914     | 0.9922                      | 0.9752       |
|      |     | Len | 2.2081     | 2.2071                      | 2.2274       |
| 100  | 100 | Cov | 0.9900     | 0.9914                      | 0.9794       |
|      |     | Len | 2.2070     | 2.2052                      | 2.2238       |
| 100  | 200 | Cov | 0.9910     | 0.9924                      | 0.9706       |
|      |     | Len | 2.2049     | 2.2071                      | 2.2210       |
| 400  | 20  | Cov | 0.9702     | 0.9702                      | 0.9652       |
|      |     | Len | 1.9623     | 1.9624                      | 1.9673       |
| 400  | 40  | Cov | 0.9690     | 0.9674                      | 0.9680       |
|      |     | Len | 1.9621     | 1.9626                      | 1.9699       |
| 400  | 100 | Cov | 0.9714     | 0.9704                      | 0.9694       |
|      |     | Len | 1.9627     | 1.9624                      | 1.9736       |
| 400  | 200 | Cov | 0.9682     | 0.9678                      | 0.9648       |
|      |     | Len | 1.9626     | 1.9619                      | 1.9770       |
| 1000 | 20  | Cov | 0.9574     | 0.9562                      | 0.9570       |
|      |     | Len | 1.9188     | 1.9189                      | 1.9189       |
| 1000 | 40  | Cov | 0.9564     | 0.9548                      | 0.9576       |
|      |     | Len | 1.9187     | 1.9188                      | 1.9190       |
| 1000 | 100 | Cov | 0.9530     | 0.9584                      | 0.9550       |
|      |     | Len | 1.9192     | 1.9191                      | 1.9191       |
| 1000 | 200 | Cov | 0.9532     | 0.9562                      | 0.9506       |
|      |     | Len | 1.9192     | 1.9187                      | 1.9190       |

Table 5.278. Etype = 4, J=5, k=19

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9912     | 0.9906                      | 0.9714       |
|      |     | Len | 2.9663     | 2.9670                      | 2.9300       |
| 100  | 40  | Cov | 0.9888     | 0.9770                      | 0.9428       |
|      |     | Len | 3.3131     | 4.7542                      | 3.0338       |
| 100  | 100 | Cov | 0.9772     | 0.9398                      | 0.9108       |
|      |     | Len | 4.3395     | 7.5802                      | 3.0910       |
| 100  | 200 | Cov | 0.9550     | 0.8766                      | 0.8896       |
|      |     | Len | 5.9387     | 9.5670                      | 3.1067       |
| 400  | 20  | Cov | 0.9834     | 0.9848                      | 0.9828       |
|      |     | Len | 2.2223     | 2.2216                      | 2.2229       |
| 400  | 40  | Cov | 0.9870     | 0.9796                      | 0.9738       |
|      |     | Len | 2.2303     | 2.2313                      | 2.3126       |
| 400  | 100 | Cov | 0.9810     | 0.9810                      | 0.9652       |
|      |     | Len | 2.2299     | 2.2308                      | 2.3874       |
| 400  | 200 | Cov | 0.9816     | 0.9804                      | 0.9558       |
|      |     | Len | 2.2304     | 2.2335                      | 2.4401       |
| 1000 | 20  | Cov | 0.9734     | 0.9666                      | 0.9678       |
|      |     | Len | 2.0060     | 2.0062                      | 2.0056       |
| 1000 | 40  | Cov | 0.9682     | 0.9620                      | 0.9644       |
|      |     | Len | 2.0081     | 2.0083                      | 2.0089       |
| 1000 | 100 | Cov | 0.9662     | 0.9660                      | 0.9694       |
|      |     | Len | 2.0083     | 2.0086                      | 2.0132       |
| 1000 | 200 | Cov | 0.9656     | 0.9680                      | 0.9670       |
|      |     | Len | 2.0088     | 2.0083                      | 2.0181       |

Table 5.279. Etype = 4, J=5, k=p-1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 40  | Cov | 0.9060     | 0.8960                      | 0.9210       |
|      |     | Len | 17.0568    | 19.1970                     | 3.4131       |
| 400  | 40  | Cov | 0.9854     | 0.9848                      | 0.9846       |
|      |     | Len | 2.4102     | 2.4096                      | 2.4109       |
| 1000 | 40  | Cov | 0.9772     | 0.9776                      | 0.9794       |
|      |     | Len | 2.1266     | 2.1262                      | 2.1260       |
| 100  | 100 | Cov | 0.9376     | 0.8008                      | 0.8120       |
|      |     | Len | 40.5312    | 50.1514                     | 6.0249       |
| 400  | 100 | Cov | 0.9132     | 0.9136                      | 0.9422       |
|      |     | Len | 17.4085    | 17.4846                     | 3.1723       |
| 1000 | 100 | Cov | 0.9836     | 0.9854                      | 0.9826       |
|      |     | Len | 2.3854     | 2.3852                      | 2.3844       |
| 100  | 200 | Cov | 0.9448     | 0.7238                      | 0.7258       |
|      |     | Len | 58.8565    | 93.0229                     | 10.3869      |
| 400  | 200 | Cov | 0.9334     | 0.8280                      | 0.8462       |
|      |     | Len | 53.6547    | 48.0680                     | 5.6165       |
| 1000 | 200 | Cov | 0.9836     | 0.9846                      | 0.9838       |
|      |     | Len | 2.7135     | 2.7143                      | 2.7150       |

Table 5.280. Etype = 4, J=10, k=1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9918     | 0.9930                      | 0.9820       |
|      |     | Len | 2.2072     | 2.2069                      | 2.2243       |
| 100  | 40  | Cov | 0.9898     | 0.9924                      | 0.9786       |
|      |     | Len | 2.2045     | 2.2070                      | 2.2240       |
| 100  | 100 | Cov | 0.9898     | 0.9916                      | 0.9762       |
|      |     | Len | 2.2055     | 2.2067                      | 2.2252       |
| 100  | 200 | Cov | 0.9898     | 0.9902                      | 0.9688       |
|      |     | Len | 2.2065     | 2.2045                      | 2.2204       |
| 400  | 20  | Cov | 0.9716     | 0.9686                      | 0.9682       |
|      |     | Len | 1.9625     | 1.9620                      | 1.9664       |
| 400  | 40  | Cov | 0.9688     | 0.9702                      | 0.9676       |
|      |     | Len | 1.9626     | 1.9621                      | 1.9698       |
| 400  | 100 | Cov | 0.9694     | 0.9678                      | 0.9656       |
|      |     | Len | 1.9621     | 1.9624                      | 1.9740       |
| 400  | 200 | Cov | 0.9680     | 0.9700                      | 0.9632       |
|      |     | Len | 1.9625     | 1.9622                      | 1.9783       |
| 1000 | 20  | Cov | 0.9556     | 0.9582                      | 0.9558       |
|      |     | Len | 1.9188     | 1.9187                      | 1.9189       |
| 1000 | 40  | Cov | 0.9550     | 0.9596                      | 0.9586       |
|      |     | Len | 1.9188     | 1.9190                      | 1.9192       |
| 1000 | 100 | Cov | 0.9536     | 0.9548                      | 0.9558       |
|      |     | Len | 1.9191     | 1.9191                      | 1.9188       |
| 1000 | 200 | Cov | 0.9578     | 0.9546                      | 0.9598       |
|      |     | Len | 1.9190     | 1.9193                      | 1.9192       |

Table 5.281. Etype = 4, J=10, k=19

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9462     | 0.9378                      | 0.9564       |
|      |     | Len | 13.2943    | 14.2421                     | 3.0900       |
| 100  | 40  | Cov | 0.9430     | 0.9328                      | 0.9388       |
|      |     | Len | 13.2918    | 15.7920                     | 3.1069       |
| 100  | 100 | Cov | 0.9392     | 0.9022                      | 0.9226       |
|      |     | Len | 13.5050    | 17.2474                     | 3.1368       |
| 100  | 200 | Cov | 0.9354     | 0.8784                      | 0.8968       |
|      |     | Len | 14.2810    | 17.9221                     | 3.1253       |
| 400  | 20  | Cov | 0.9852     | 0.9856                      | 0.9874       |
|      |     | Len | 2.2218     | 2.2230                      | 2.2225       |
| 400  | 40  | Cov | 0.9810     | 0.9836                      | 0.9732       |
|      |     | Len | 2.2302     | 2.2302                      | 2.3127       |
| 400  | 100 | Cov | 0.9822     | 0.9760                      | 0.9630       |
|      |     | Len | 2.2308     | 2.2316                      | 2.3853       |
| 400  | 200 | Cov | 0.9814     | 0.9806                      | 0.9544       |
|      |     | Len | 2.2313     | 2.2335                      | 2.4404       |
| 1000 | 20  | Cov | 0.9646     | 0.9654                      | 0.9702       |
|      |     | Len | 2.0057     | 2.0059                      | 2.0060       |
| 1000 | 40  | Cov | 0.9656     | 0.9682                      | 0.9686       |
|      |     | Len | 2.0086     | 2.0082                      | 2.0095       |
| 1000 | 100 | Cov | 0.9708     | 0.9678                      | 0.9720       |
|      |     | Len | 2.0082     | 2.0080                      | 2.0137       |
| 1000 | 200 | Cov | 0.9628     | 0.9674                      | 0.9636       |
|      |     | Len | 2.0085     | 2.0084                      | 2.0198       |

Table 5.282. Etype = 4, J=10, k=p-1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 40  | Cov | 0.9302     | 0.8994                      | 0.9196       |
|      |     | Len | 22.4118    | 33.3126                     | 4.6622       |
| 400  | 40  | Cov | 0.9804     | 0.9810                      | 0.9830       |
|      |     | Len | 2.4110     | 2.4099                      | 2.4111       |
| 1000 | 40  | Cov | 0.9746     | 0.9786                      | 0.9778       |
|      |     | Len | 2.1259     | 2.1260                      | 2.1261       |
| 100  | 100 | Cov | 0.9374     | 0.8534                      | 0.8606       |
|      |     | Len | 40.8695    | 85.9899                     | 10.1523      |
| 400  | 100 | Cov | 0.9168     | 0.8972                      | 0.9104       |
|      |     | Len | 28.5480    | 37.7866                     | 4.7807       |
| 1000 | 100 | Cov | 0.9840     | 0.9842                      | 0.9846       |
|      |     | Len | 2.3857     | 2.3847                      | 2.3854       |
| 100  | 200 | Cov | 0.9424     | 0.8324                      | 0.8284       |
|      |     | Len | 58.8656    | 166.2538                    | 18.8425      |
| 400  | 200 | Cov | 0.9346     | 0.8578                      | 0.8620       |
|      |     | Len | 53.8147    | 80.8614                     | 8.9997       |
| 1000 | 200 | Cov | 0.9260     | 0.9124                      | 0.9204       |
|      |     | Len | 36.9422    | 46.2543                     | 5.4831       |

Table 5.283. Etype = 4, J=20, k=1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9928     | 0.9928                      | 0.9792       |
|      |     | Len | 2.2081     | 2.2055                      | 2.2265       |
| 100  | 40  | Cov | 0.9916     | 0.9942                      | 0.9764       |
|      |     | Len | 2.2067     | 2.2074                      | 2.2236       |
| 100  | 100 | Cov | 0.9912     | 0.9924                      | 0.9724       |
|      |     | Len | 2.2053     | 2.2060                      | 2.2261       |
| 100  | 200 | Cov | 0.9918     | 0.9902                      | 0.9758       |
|      |     | Len | 2.2065     | 2.2036                      | 2.2220       |
| 400  | 20  | Cov | 0.9678     | 0.9710                      | 0.9676       |
|      |     | Len | 1.9627     | 1.9626                      | 1.9667       |
| 400  | 40  | Cov | 0.9696     | 0.9682                      | 0.9684       |
|      |     | Len | 1.9629     | 1.9618                      | 1.9704       |
| 400  | 100 | Cov | 0.9644     | 0.9722                      | 0.9682       |
|      |     | Len | 1.9628     | 1.9624                      | 1.9740       |
| 400  | 200 | Cov | 0.9686     | 0.9656                      | 0.9630       |
|      |     | Len | 1.9621     | 1.9624                      | 1.9770       |
| 1000 | 20  | Cov | 0.9536     | 0.9540                      | 0.9580       |
|      |     | Len | 1.9188     | 1.9187                      | 1.9186       |
| 1000 | 40  | Cov | 0.9522     | 0.9602                      | 0.9526       |
|      |     | Len | 1.9190     | 1.9188                      | 1.9187       |
| 1000 | 100 | Cov | 0.9552     | 0.9576                      | 0.9552       |
|      |     | Len | 1.9189     | 1.9187                      | 1.9189       |
| 1000 | 200 | Cov | 0.9578     | 0.9622                      | 0.9582       |
|      |     | Len | 1.9190     | 1.9187                      | 1.9197       |

Table 5.284. Etype = 4, J=20, k=19

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9558     | 0.9414                      | 0.9486       |
|      |     | Len | 16.9240    | 25.1561                     | 4.1499       |
| 100  | 40  | Cov | 0.9480     | 0.9356                      | 0.9402       |
|      |     | Len | 16.9267    | 27.0543                     | 4.1254       |
| 100  | 100 | Cov | 0.9480     | 0.9226                      | 0.9262       |
|      |     | Len | 16.8886    | 28.5521                     | 4.0714       |
| 100  | 200 | Cov | 0.9498     | 0.9136                      | 0.9078       |
|      |     | Len | 16.9499    | 29.0220                     | 4.0119       |
| 400  | 20  | Cov | 0.9868     | 0.9834                      | 0.9852       |
|      |     | Len | 2.2221     | 2.2217                      | 2.2221       |
| 400  | 40  | Cov | 0.9848     | 0.9842                      | 0.9712       |
|      |     | Len | 2.2231     | 2.3283                      | 2.3143       |
| 400  | 100 | Cov | 0.9856     | 0.9814                      | 0.9578       |
|      |     | Len | 2.2227     | 2.6051                      | 2.3888       |
| 400  | 200 | Cov | 0.9842     | 0.9802                      | 0.9550       |
|      |     | Len | 2.2235     | 3.0102                      | 2.4418       |
| 1000 | 20  | Cov | 0.9676     | 0.9682                      | 0.9694       |
|      |     | Len | 2.0055     | 2.0058                      | 2.0063       |
| 1000 | 40  | Cov | 0.9696     | 0.9674                      | 0.9686       |
|      |     | Len | 2.0084     | 2.0080                      | 2.0091       |
| 1000 | 100 | Cov | 0.9692     | 0.9684                      | 0.9684       |
|      |     | Len | 2.0084     | 2.0080                      | 2.0134       |
| 1000 | 200 | Cov | 0.9698     | 0.9674                      | 0.9660       |
|      |     | Len | 2.0083     | 2.0090                      | 2.0192       |

Table 5.285. Etype = 4, J=20, k=p-1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 40  | Cov | 0.9440     | 0.9344                      | 0.9308       |
|      |     | Len | 25.3658    | 55.1993                     | 7.3447       |
| 400  | 40  | Cov | 0.9456     | 0.9368                      | 0.9380       |
|      |     | Len | 17.7505    | 20.5967                     | 3.3579       |
| 1000 | 40  | Cov | 0.9794     | 0.9746                      | 0.9768       |
|      |     | Len | 2.1262     | 2.1264                      | 2.1273       |
| 100  | 100 | Cov | 0.9478     | 0.9080                      | 0.9080       |
|      |     | Len | 41.3006    | 142.4024                    | 17.3067      |
| 400  | 100 | Cov | 0.9386     | 0.9030                      | 0.9114       |
|      |     | Len | 34.7406    | 62.8561                     | 7.4318       |
| 1000 | 100 | Cov | 0.9434     | 0.9252                      | 0.9360       |
|      |     | Len | 27.6031    | 33.9591                     | 4.4342       |
| 100  | 200 | Cov | 0.9488     | 0.8956                      | 0.8888       |
|      |     | Len | 58.9871    | 281.8180                    | 33.3150      |
| 400  | 200 | Cov | 0.9304     | 0.8816                      | 0.8882       |
|      |     | Len | 53.9918    | 129.3581                    | 14.3662      |
| 1000 | 200 | Cov | 0.9296     | 0.9186                      | 0.9188       |
|      |     | Len | 47.0331    | 79.5852                     | 8.8981       |

Table 5.286. Etype = 4, J=50, k=1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9952     | 0.9940                      | 0.9844       |
|      |     | Len | 2.1995     | 2.1990                      | 2.2206       |
| 100  | 40  | Cov | 0.9950     | 0.9946                      | 0.9790       |
|      |     | Len | 2.1994     | 2.1988                      | 2.2225       |
| 100  | 100 | Cov | 0.9926     | 0.9954                      | 0.9766       |
|      |     | Len | 2.1987     | 2.1997                      | 2.2229       |
| 100  | 200 | Cov | 0.9920     | 0.9934                      | 0.9746       |
|      |     | Len | 2.1987     | 2.2002                      | 2.2204       |
| 400  | 20  | Cov | 0.9690     | 0.9694                      | 0.9662       |
|      |     | Len | 1.9628     | 1.9620                      | 1.9675       |
| 400  | 40  | Cov | 0.9712     | 0.9702                      | 0.9604       |
|      |     | Len | 1.9628     | 1.9622                      | 1.9697       |
| 400  | 100 | Cov | 0.9686     | 0.9698                      | 0.9600       |
|      |     | Len | 1.9628     | 1.9625                      | 1.9748       |
| 400  | 200 | Cov | 0.9680     | 0.9704                      | 0.9654       |
|      |     | Len | 1.9628     | 1.9623                      | 1.9779       |
| 1000 | 20  | Cov | 0.9546     | 0.9592                      | 0.9540       |
|      |     | Len | 1.9186     | 1.9189                      | 1.9191       |
| 1000 | 40  | Cov | 0.9578     | 0.9582                      | 0.9528       |
|      |     | Len | 1.9191     | 1.9187                      | 1.9188       |
| 1000 | 100 | Cov | 0.9592     | 0.9538                      | 0.9588       |
|      |     | Len | 1.9191     | 1.9191                      | 1.9189       |
| 1000 | 200 | Cov | 0.9578     | 0.9528                      | 0.9566       |
|      |     | Len | 1.9192     | 1.9189                      | 1.9197       |

Table 5.287. Etype = 4, J=50, k=19

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9526     | 0.9402                      | 0.9406       |
|      |     | Len | 17.9760    | 46.5679                     | 7.4740       |
| 100  | 40  | Cov | 0.9508     | 0.9370                      | 0.9402       |
|      |     | Len | 17.9331    | 48.7826                     | 7.4297       |
| 100  | 100 | Cov | 0.9514     | 0.9320                      | 0.9240       |
|      |     | Len | 17.9586    | 50.0622                     | 7.3322       |
| 100  | 200 | Cov | 0.9466     | 0.9276                      | 0.9188       |
|      |     | Len | 17.9552    | 50.1822                     | 7.2336       |
| 400  | 20  | Cov | 0.9346     | 0.9242                      | 0.9382       |
|      |     | Len | 13.3738    | 16.2255                     | 3.0134       |
| 400  | 40  | Cov | 0.9370     | 0.9366                      | 0.9378       |
|      |     | Len | 13.3648    | 17.5203                     | 3.0168       |
| 400  | 100 | Cov | 0.9378     | 0.9216                      | 0.9336       |
|      |     | Len | 13.3743    | 18.6899                     | 3.0358       |
| 400  | 200 | Cov | 0.9444     | 0.9264                      | 0.9258       |
|      |     | Len | 13.3646    | 19.3521                     | 3.0535       |
| 1000 | 20  | Cov | 0.9690     | 0.9674                      | 0.9706       |
|      |     | Len | 2.0058     | 2.0058                      | 2.0061       |
| 1000 | 40  | Cov | 0.9680     | 0.9672                      | 0.9662       |
|      |     | Len | 2.0056     | 2.0083                      | 2.0081       |
| 1000 | 100 | Cov | 0.9650     | 0.9668                      | 0.9698       |
|      |     | Len | 2.0055     | 2.0134                      | 2.0140       |
| 1000 | 200 | Cov | 0.9650     | 0.9684                      | 0.9652       |
|      |     | Len | 2.0063     | 2.0059                      | 2.0178       |

Table 5.288. Etype = 4, J=50, k=p-1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 40  | Cov | 0.9516     | 0.9320                      | 0.9304       |
|      |     | Len | 25.9949    | 98.8902                     | 14.6622      |
| 400  | 40  | Cov | 0.9382     | 0.9212                      | 0.9240       |
|      |     | Len | 21.6276    | 38.7537                     | 5.2064       |
| 1000 | 40  | Cov | 0.9354     | 0.9296                      | 0.9360       |
|      |     | Len | 16.9444    | 19.8288                     | 3.1947       |
| 100  | 100 | Cov | 0.9484     | 0.9346                      | 0.9204       |
|      |     | Len | 41.7534    | 254.0269                    | 36.1595      |
| 400  | 100 | Cov | 0.9304     | 0.9100                      | 0.9092       |
|      |     | Len | 36.7289    | 105.6726                    | 12.4624      |
| 1000 | 100 | Cov | 0.9258     | 0.9132                      | 0.9142       |
|      |     | Len | 33.5287    | 64.1876                     | 7.4149       |
| 100  | 200 | Cov | 0.9472     | 0.9204                      | 0.9216       |
|      |     | Len | 59.2100    | 505.6271                    | 71.3569      |
| 400  | 200 | Cov | 0.9386     | 0.8990                      | 0.8968       |
|      |     | Len | 54.3540    | 215.3958                    | 24.5495      |
| 1000 | 200 | Cov | 0.9316     | 0.9054                      | 0.9012       |
|      |     | Len | 50.4785    | 133.2046                    | 14.7530      |

Table 5.289. Etype = 5, J=5, k=1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9416     | 0.9418                      | 0.9430       |
|      |     | Len | 13.7744    | 13.7533                     | 14.0450      |
| 100  | 40  | Cov | 0.9412     | 0.9424                      | 0.9498       |
|      |     | Len | 13.7591    | 13.8346                     | 14.2835      |
| 100  | 100 | Cov | 0.9440     | 0.9450                      | 0.9466       |
|      |     | Len | 13.6856    | 13.5354                     | 14.1634      |
| 100  | 200 | Cov | 0.9368     | 0.9426                      | 0.9456       |
|      |     | Len | 13.4667    | 13.5818                     | 14.4944      |
| 400  | 20  | Cov | 0.9444     | 0.9462                      | 0.9366       |
|      |     | Len | 12.5336    | 12.5414                     | 12.5615      |
| 400  | 40  | Cov | 0.9398     | 0.9400                      | 0.9420       |
|      |     | Len | 12.5302    | 12.5153                     | 12.5147      |
| 400  | 100 | Cov | 0.9446     | 0.9424                      | 0.9470       |
|      |     | Len | 12.4439    | 12.4857                     | 12.5657      |
| 400  | 200 | Cov | 0.9396     | 0.9400                      | 0.9478       |
|      |     | Len | 12.4129    | 12.4970                     | 12.5170      |
| 1000 | 20  | Cov | 0.9422     | 0.9480                      | 0.9426       |
|      |     | Len | 12.7085    | 12.6851                     | 12.6970      |
| 1000 | 40  | Cov | 0.9464     | 0.9460                      | 0.9444       |
|      |     | Len | 12.6467    | 12.6349                     | 12.6916      |
| 1000 | 100 | Cov | 0.9422     | 0.9432                      | 0.9450       |
|      |     | Len | 12.6234    | 12.6757                     | 12.6424      |
| 1000 | 200 | Cov | 0.9504     | 0.9424                      | 0.9474       |
|      |     | Len | 12.7262    | 12.6543                     | 12.5888      |

Table 5.290. Etype = 5, J=5, k=19

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9568     | 0.9706                      | 0.9534       |
|      |     | Len | 21.8404    | 22.0011                     | 15.9242      |
| 100  | 40  | Cov | 0.9476     | 0.9560                      | 0.9510       |
|      |     | Len | 20.2484    | 19.9555                     | 15.3632      |
| 100  | 100 | Cov | 0.9440     | 0.9222                      | 0.9414       |
|      |     | Len | 21.1782    | 19.8807                     | 15.0603      |
| 100  | 200 | Cov | 0.9420     | 0.8844                      | 0.9438       |
|      |     | Len | 21.8344    | 20.3214                     | 14.6771      |
| 400  | 20  | Cov | 0.9718     | 0.9702                      | 0.9456       |
|      |     | Len | 21.2147    | 21.2535                     | 13.4103      |
| 400  | 40  | Cov | 0.9724     | 0.9638                      | 0.9394       |
|      |     | Len | 21.0893    | 21.1343                     | 13.3680      |
| 400  | 100 | Cov | 0.9700     | 0.9690                      | 0.9450       |
|      |     | Len | 20.9233    | 20.8773                     | 13.0638      |
| 400  | 200 | Cov | 0.9630     | 0.9644                      | 0.9422       |
|      |     | Len | 20.8480    | 20.9003                     | 13.0574      |
| 1000 | 20  | Cov | 0.9566     | 0.9582                      | 0.9458       |
|      |     | Len | 15.7497    | 15.7355                     | 13.2877      |
| 1000 | 40  | Cov | 0.9518     | 0.9538                      | 0.9436       |
|      |     | Len | 15.8588    | 15.8285                     | 13.1573      |
| 1000 | 100 | Cov | 0.9534     | 0.9524                      | 0.9442       |
|      |     | Len | 15.7830    | 15.8592                     | 13.0396      |
| 1000 | 200 | Cov | 0.9574     | 0.9548                      | 0.9454       |
|      |     | Len | 15.7961    | 15.7520                     | 12.9725      |

Table 5.291. Etype = 5, J=5, k=p-1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 40  | Cov | 0.9318     | 0.9224                      | 0.9504       |
|      |     | Len | 25.2335    | 24.6307                     | 17.4242      |
| 400  | 40  | Cov | 0.9696     | 0.9676                      | 0.9474       |
|      |     | Len | 21.7556    | 21.6811                     | 14.5232      |
| 1000 | 40  | Cov | 0.9676     | 0.9630                      | 0.9484       |
|      |     | Len | 19.8813    | 19.8024                     | 13.9840      |
| 100  | 100 | Cov | 0.9438     | 0.8054                      | 0.9228       |
|      |     | Len | 43.2638    | 51.3765                     | 17.6832      |
| 400  | 100 | Cov | 0.9502     | 0.9470                      | 0.9546       |
|      |     | Len | 23.7750    | 23.7304                     | 17.5792      |
| 1000 | 100 | Cov | 0.9742     | 0.9716                      | 0.9584       |
|      |     | Len | 22.7117    | 22.6312                     | 16.0372      |
| 100  | 200 | Cov | 0.9486     | 0.7130                      | 0.8302       |
|      |     | Len | 60.5080    | 93.6231                     | 17.2067      |
| 400  | 200 | Cov | 0.9366     | 0.8364                      | 0.9404       |
|      |     | Len | 55.8214    | 49.6055                     | 17.9916      |
| 1000 | 200 | Cov | 0.9710     | 0.9738                      | 0.9656       |
|      |     | Len | 23.2567    | 23.2984                     | 19.1344      |

Table 5.292. Etype = 5, J=10, k=1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9456     | 0.9382                      | 0.9460       |
|      |     | Len | 13.9704    | 13.6198                     | 14.1512      |
| 100  | 40  | Cov | 0.9408     | 0.9466                      | 0.9456       |
|      |     | Len | 13.7736    | 13.6817                     | 14.2589      |
| 100  | 100 | Cov | 0.9422     | 0.9346                      | 0.9478       |
|      |     | Len | 13.6277    | 13.6479                     | 14.4238      |
| 100  | 200 | Cov | 0.9376     | 0.9432                      | 0.9460       |
|      |     | Len | 13.4761    | 13.6706                     | 14.4099      |
| 400  | 20  | Cov | 0.9456     | 0.9444                      | 0.9396       |
|      |     | Len | 12.5092    | 12.5584                     | 12.4909      |
| 400  | 40  | Cov | 0.9492     | 0.9428                      | 0.9444       |
|      |     | Len | 12.5053    | 12.5274                     | 12.5737      |
| 400  | 100 | Cov | 0.9480     | 0.9398                      | 0.9398       |
|      |     | Len | 12.4854    | 12.4890                     | 12.5329      |
| 400  | 200 | Cov | 0.9434     | 0.9450                      | 0.9426       |
|      |     | Len | 12.5495    | 12.4826                     | 12.5064      |
| 1000 | 20  | Cov | 0.9410     | 0.9416                      | 0.9452       |
|      |     | Len | 12.6959    | 12.6473                     | 12.6793      |
| 1000 | 40  | Cov | 0.9462     | 0.9456                      | 0.9404       |
|      |     | Len | 12.6464    | 12.7034                     | 12.6281      |
| 1000 | 100 | Cov | 0.9494     | 0.9494                      | 0.9470       |
|      |     | Len | 12.6679    | 12.6779                     | 12.6471      |
| 1000 | 200 | Cov | 0.9450     | 0.9436                      | 0.9428       |
|      |     | Len | 12.6439    | 12.6477                     | 12.6863      |

Table 5.293. Etype = 5, J=10, k=19

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9540     | 0.9504                      | 0.9490       |
|      |     | Len | 21.7386    | 22.1184                     | 15.9085      |
| 100  | 40  | Cov | 0.9466     | 0.9438                      | 0.9436       |
|      |     | Len | 21.6670    | 22.5236                     | 15.4813      |
| 100  | 100 | Cov | 0.9450     | 0.9244                      | 0.9442       |
|      |     | Len | 21.8859    | 22.7787                     | 15.0444      |
| 100  | 200 | Cov | 0.9474     | 0.9062                      | 0.9398       |
|      |     | Len | 22.1617    | 22.8439                     | 14.7146      |
| 400  | 20  | Cov | 0.9690     | 0.9678                      | 0.9454       |
|      |     | Len | 21.2827    | 21.2949                     | 13.4655      |
| 400  | 40  | Cov | 0.9660     | 0.9670                      | 0.9488       |
|      |     | Len | 21.1397    | 20.9619                     | 13.3353      |
| 400  | 100 | Cov | 0.9680     | 0.9698                      | 0.9464       |
|      |     | Len | 21.0313    | 20.9087                     | 13.1122      |
| 400  | 200 | Cov | 0.9698     | 0.9686                      | 0.9460       |
|      |     | Len | 20.9540    | 20.8433                     | 12.9878      |
| 1000 | 20  | Cov | 0.9518     | 0.9504                      | 0.9472       |
|      |     | Len | 15.7467    | 15.7748                     | 13.3152      |
| 1000 | 40  | Cov | 0.9526     | 0.9512                      | 0.9444       |
|      |     | Len | 15.8600    | 15.8020                     | 13.1378      |
| 1000 | 100 | Cov | 0.9540     | 0.9582                      | 0.9470       |
|      |     | Len | 15.7592    | 15.7849                     | 13.1188      |
| 1000 | 200 | Cov | 0.9604     | 0.9576                      | 0.9432       |
|      |     | Len | 15.8588    | 15.7830                     | 13.0255      |

Table 5.294. Etype = 5, J=10, k=p-1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 40  | Cov | 0.9390     | 0.9114                      | 0.9600       |
|      |     | Len | 27.8852    | 36.0451                     | 17.4160      |
| 400  | 40  | Cov | 0.9658     | 0.9686                      | 0.9506       |
|      |     | Len | 21.6142    | 21.6398                     | 14.5525      |
| 1000 | 40  | Cov | 0.9684     | 0.9626                      | 0.9530       |
|      |     | Len | 19.8636    | 19.7917                     | 14.0001      |
| 100  | 100 | Cov | 0.9462     | 0.8592                      | 0.9248       |
|      |     | Len | 43.4317    | 86.5883                     | 18.0714      |
| 400  | 100 | Cov | 0.9242     | 0.9078                      | 0.9612       |
|      |     | Len | 32.0824    | 40.1422                     | 17.6497      |
| 1000 | 100 | Cov | 0.9706     | 0.9724                      | 0.9510       |
|      |     | Len | 22.6342    | 22.6756                     | 16.0697      |
| 100  | 200 | Cov | 0.9450     | 0.8412                      | 0.8630       |
|      |     | Len | 60.5503    | 166.7841                    | 22.4312      |
| 400  | 200 | Cov | 0.9364     | 0.8642                      | 0.9374       |
|      |     | Len | 55.7946    | 81.5550                     | 17.8955      |
| 1000 | 200 | Cov | 0.9286     | 0.9162                      | 0.9598       |
|      |     | Len | 39.6634    | 48.2465                     | 19.1529      |

Table 5.295. Etype = 5, J=20, k=1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | NA         | 0.9448                      | 0.9466       |
|      |     | Len | NA         | 13.8836                     | 14.1494      |
| 100  | 40  | Cov | NA         | 0.9472                      | 0.9432       |
|      |     | Len | NA         | 13.6475                     | 14.1290      |
| 100  | 100 | Cov | NA         | 0.9440                      | 0.9448       |
|      |     | Len | NA         | 13.5954                     | 14.2865      |
| 100  | 200 | Cov | NA         | 0.9426                      | 0.9438       |
|      |     | Len | NA         | 13.6824                     | 14.4997      |
| 400  | 20  | Cov | 0.9446     | 0.9482                      | 0.9446       |
|      |     | Len | 12.5538    | 12.5561                     | 12.5513      |
| 400  | 40  | Cov | 0.9474     | 0.9504                      | 0.9438       |
|      |     | Len | 12.5944    | 12.5514                     | 12.5165      |
| 400  | 100 | Cov | 0.9436     | 0.9452                      | 0.9400       |
|      |     | Len | 12.5469    | 12.5254                     | 12.4589      |
| 400  | 200 | Cov | 0.9416     | 0.9490                      | 0.9486       |
|      |     | Len | 12.5115    | 12.4802                     | 12.5541      |
| 1000 | 20  | Cov | 0.9504     | 0.9440                      | 0.9476       |
|      |     | Len | 12.6732    | 12.6755                     | 12.6030      |
| 1000 | 40  | Cov | 0.9454     | 0.9448                      | 0.9482       |
|      |     | Len | 12.6697    | 12.7178                     | 12.6997      |
| 1000 | 100 | Cov | 0.9416     | 0.9462                      | 0.9398       |
|      |     | Len | 12.6516    | 12.6982                     | 12.6713      |
| 1000 | 200 | Cov | 0.9384     | 0.9460                      | 0.9402       |
|      |     | Len | 12.6251    | 12.6949                     | 12.6529      |

Table 5.296. Etype = 5, J=20, k=19

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9546     | 0.9452                      | 0.9472       |
|      |     | Len | 23.5938    | 29.4323                     | 15.9353      |
| 100  | 40  | Cov | 0.9562     | 0.9388                      | 0.9504       |
|      |     | Len | 22.9139    | 30.8728                     | 15.5382      |
| 100  | 100 | Cov | 0.9500     | 0.9248                      | 0.9452       |
|      |     | Len | 22.4660    | 31.9061                     | 15.0362      |
| 100  | 200 | Cov | 0.9478     | 0.9178                      | 0.9436       |
|      |     | Len | 22.3463    | 32.0937                     | 14.7010      |
| 400  | 20  | Cov | 0.9658     | 0.9696                      | 0.9450       |
|      |     | Len | 21.2237    | 21.2336                     | 13.4960      |
| 400  | 40  | Cov | 0.9662     | 0.9704                      | 0.9484       |
|      |     | Len | 21.2814    | 21.0677                     | 13.3628      |
| 400  | 100 | Cov | 0.9694     | 0.9686                      | 0.9438       |
|      |     | Len | 21.0911    | 20.9339                     | 13.1313      |
| 400  | 200 | Cov | 0.9722     | 0.9676                      | 0.9400       |
|      |     | Len | 21.0768    | 20.9259                     | 12.9866      |
| 1000 | 20  | Cov | 0.9570     | 0.9556                      | 0.9474       |
|      |     | Len | 15.8185    | 15.8114                     | 13.2855      |
| 1000 | 40  | Cov | 0.9562     | 0.9492                      | 0.9466       |
|      |     | Len | 15.7953    | 15.8180                     | 13.2025      |
| 1000 | 100 | Cov | 0.9524     | 0.9542                      | 0.9524       |
|      |     | Len | 15.8802    | 15.8333                     | 13.0893      |
| 1000 | 200 | Cov | 0.9554     | 0.9558                      | 0.9498       |
|      |     | Len | 15.7618    | 15.8330                     | 13.0066      |

Table 5.297. Etype = 5, J=20, k=p-1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 40  | Cov | 0.9496     | 0.9306                      | 0.9502       |
|      |     | Len | 29.4655    | 56.9789                     | 17.4281      |
| 400  | 40  | Cov | 0.9540     | 0.9534                      | 0.9488       |
|      |     | Len | 23.8107    | 25.4847                     | 14.5761      |
| 1000 | 40  | Cov | 0.9630     | 0.9688                      | 0.9526       |
|      |     | Len | 19.8771    | 19.8837                     | 14.0218      |
| 100  | 100 | Cov | 0.9420     | 0.9064                      | 0.9316       |
|      |     | Len | 43.7164    | 142.8182                    | 22.3331      |
| 400  | 100 | Cov | 0.9392     | 0.9140                      | 0.9504       |
|      |     | Len | 37.7221    | 64.2063                     | 17.6274      |
| 1000 | 100 | Cov | 0.9482     | 0.9412                      | 0.9514       |
|      |     | Len | 31.5194    | 36.9199                     | 16.0170      |
| 100  | 200 | Cov | 0.9494     | 0.8950                      | 0.8932       |
|      |     | Len | 60.6563    | 282.4824                    | 35.5272      |
| 400  | 200 | Cov | 0.9412     | 0.8974                      | 0.9306       |
|      |     | Len | 55.8279    | 129.9768                    | 20.0139      |
| 1000 | 200 | Cov | 0.9420     | 0.9106                      | 0.9604       |
|      |     | Len | 49.1582    | 80.5964                     | 19.1688      |

Table 5.298. Etype = 5, J=50, k=1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | NA         | 0.9456                      | 0.9348       |
|      |     | Len | NA         | 13.5741                     | 13.7950      |
| 100  | 40  | Cov | NA         | NA                          | 0.9494       |
|      |     | Len | NA         | NA                          | 13.6289      |
| 100  | 100 | Cov | NA         | NA                          | 0.9450       |
|      |     | Len | NA         | NA                          | 13.6519      |
| 100  | 200 | Cov | NA         | NA                          | 0.9406       |
|      |     | Len | NA         | NA                          | 13.5790      |
| 400  | 20  | Cov | 0.9438     | 0.9474                      | 0.9458       |
|      |     | Len | 12.5302    | 12.5731                     | 12.5434      |
| 400  | 40  | Cov | 0.9388     | 0.9478                      | 0.9446       |
|      |     | Len | 12.5783    | 12.5485                     | 12.5027      |
| 400  | 100 | Cov | 0.9470     | 0.9432                      | 0.9422       |
|      |     | Len | 12.4726    | 12.5845                     | 12.5171      |
| 400  | 200 | Cov | 0.9428     | 0.9436                      | 0.9452       |
|      |     | Len | 12.5182    | 12.5088                     | 12.5982      |
| 1000 | 20  | Cov | 0.9484     | 0.9464                      | 0.9426       |
|      |     | Len | 12.6590    | 12.6682                     | 12.6691      |
| 1000 | 40  | Cov | 0.9504     | 0.9504                      | 0.9486       |
|      |     | Len | 12.7274    | 12.6785                     | 12.5637      |
| 1000 | 100 | Cov | 0.9470     | 0.9490                      | 0.9440       |
|      |     | Len | 12.6366    | 12.6601                     | 12.6814      |
| 1000 | 200 | Cov | 0.9424     | 0.9450                      | 0.9464       |
|      |     | Len | 12.6364    | 12.6831                     | 12.6267      |

Table 5.299. Etype = 5, J=50, k=19

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 20  | Cov | 0.9486     | 0.9376                      | 0.9364       |
|      |     | Len | 22.5573    | 48.4832                     | 14.6810      |
| 100  | 40  | Cov | 0.9448     | 0.9382                      | 0.9422       |
|      |     | Len | 22.5347    | 50.4730                     | 14.5887      |
| 100  | 100 | Cov | NA         | 0.9316                      | 0.9418       |
|      |     | Len | NA         | 51.6932                     | 14.4896      |
| 100  | 200 | Cov | NA         | 0.9312                      | 0.9410       |
|      |     | Len | NA         | 51.7552                     | 14.3035      |
| 400  | 20  | Cov | 0.9528     | 0.9396                      | 0.9442       |
|      |     | Len | 18.6547    | 20.5442                     | 13.5044      |
| 400  | 40  | Cov | 0.9448     | 0.9382                      | 0.9426       |
|      |     | Len | 18.6282    | 21.6504                     | 13.2947      |
| 400  | 100 | Cov | 0.9484     | 0.9350                      | 0.9520       |
|      |     | Len | 18.6338    | 22.6098                     | 13.1027      |
| 400  | 200 | Cov | 0.9460     | 0.9376                      | 0.9464       |
|      |     | Len | 18.6098    | 23.1717                     | 12.9855      |
| 1000 | 20  | Cov | 0.9572     | 0.9576                      | 0.9484       |
|      |     | Len | 15.7290    | 15.7720                     | 13.2574      |
| 1000 | 40  | Cov | 0.9584     | 0.9502                      | 0.9476       |
|      |     | Len | 15.7845    | 15.7958                     | 13.1902      |
| 1000 | 100 | Cov | 0.9600     | 0.9512                      | 0.9446       |
|      |     | Len | 15.7416    | 15.7818                     | 13.0950      |
| 1000 | 200 | Cov | 0.9586     | 0.9514                      | 0.9538       |
|      |     | Len | 15.8218    | 15.7466                     | 13.0831      |

Table 5.300. Etype = 5, J=50, k=p-1

| $n$  | $p$ |     | $\psi = 0$ | $\psi = \frac{1}{\sqrt{p}}$ | $\psi = 0.9$ |
|------|-----|-----|------------|-----------------------------|--------------|
| 100  | 40  | Cov | 0.9460     | 0.9328                      | 0.9334       |
|      |     | Len | 29.5173    | 99.8619                     | 19.2659      |
| 400  | 40  | Cov | 0.9348     | 0.9196                      | 0.9502       |
|      |     | Len | 25.3438    | 40.7307                     | 14.5198      |
| 1000 | 40  | Cov | 0.9456     | 0.9382                      | 0.9476       |
|      |     | Len | 21.3802    | 23.6157                     | 14.0232      |
| 100  | 100 | Cov | 0.9492     | 0.9250                      | 0.9198       |
|      |     | Len | 44.0343    | 254.0900                    | 38.2052      |
| 400  | 100 | Cov | 0.9304     | 0.9102                      | 0.9262       |
|      |     | Len | 39.1433    | 106.3686                    | 17.2458      |
| 1000 | 100 | Cov | 0.9354     | 0.9148                      | 0.9518       |
|      |     | Len | 36.0727    | 64.2118                     | 15.1087      |
| 100  | 200 | Cov | 0.9420     | 0.9138                      | 0.9092       |
|      |     | Len | 60.8964    | 506.5833                    | 72.3018      |
| 400  | 200 | Cov | 0.9400     | 0.8980                      | 0.9036       |
|      |     | Len | 56.0239    | 215.8089                    | 27.2562      |
| 1000 | 200 | Cov | 0.9272     | 0.9030                      | 0.9274       |
|      |     | Len | 52.1613    | 133.7199                    | 19.0295      |

## 5.4 SIMULATIONS FOR THE NEW PI

For the model  $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{e}$ , methods such as forward selection, PCR, PLS, ridge regression, relaxed lasso, and lasso each generate  $M$  fitted models  $I_1, \dots, I_M$ , where  $M$  depends on the method, and we considered several methods for selecting the final submodel  $I_d$ . Only method i) needs  $n/p$  large.

- i) Let  $I_d = I_{min}$  be the model that minimizes  $C_p$  for forward selection, relaxed lasso, or lasso.
- ii) Let  $I_d$  use  $d = \min(\lceil n/J \rceil, p)$  variables where  $J$  is a positive integer. We used  $J = 5, 10, 20$ , and  $50$ . Forward selection used  $M = d$ . (For ridge regression, we used the model  $I_c$  with the “degrees of freedom” closest to  $d$ .) For PCR and PLS, the “variables” were the  $v_j = \gamma_j^T \mathbf{x}$ . This method uses the full OLS model if  $n/p \geq J$  for forward selection, PCR, PLS, ridge regression, relaxed lasso, and lasso. Hence large sample inference is simple for these six model selection estimators if  $p$  is fixed. For lasso, several values of  $\lambda$  may have the same degrees of freedom: we chose the model with the smallest  $\lambda$  value. In the simulation for lasso with  $d = p$ , we used the lasso model with  $\lambda_0$  instead of OLS. See Section 5.2.
- iii) Let  $I_d = I_{min}$  be the model that minimizes  $EBIC$  for forward selection or relaxed lasso. For forward selection, we used  $M = \min(\lceil n/5 \rceil, p)$ . See Section 5.3.
- iv) Choose  $I_d$  using  $k$ -fold cross validation (CV). We used 10-fold CV.

The following method is currently slow to simulate, but is a useful diagnostic. When the model underfits, PI (3.7) tends to have coverage near or greater than the nominal 0.95 coverage, but the PI length is long. When the model severely overfits, the PI length is short, but the coverage is less than 0.95. See Section 5.1.

v) Modify  $k$ -fold cross validation to compute the PI coverage and average PI length on all  $M$  models. Then  $n$  PIs are made for  $Y_i$  using  $\mathbf{x}_f = \mathbf{x}_i$  for  $i = 1, \dots, n$ . The coverage is the proportion of times the  $n$  PIs contained  $Y_i$ . For example, choose the model  $I_d$  with the shortest average PI length given that the nominal large sample  $100(1 - \delta)\%$  PI had coverage

$$\geq c_n = \max\left(1 - \delta - \frac{1}{\sqrt{n}}, 1 - \delta - 0.01\right).$$

If no model  $I_i$  had coverage  $\geq c_n$ , pick the model with the largest coverage.

The simulation for the new PIs (3.11) and (3.12) was similar to that in section 5.1. Let  $\mathbf{x} = (1 \ \mathbf{u}^T)^T$  where  $\mathbf{u}$  is the  $p - 1 \times 1$  vector of nontrivial predictors. In the simulations, for  $i = 1, \dots, n$ , we generated  $\mathbf{w}_i \sim N_{p-1}(\mathbf{0}, \mathbf{I})$  where the  $m = p - 1$  elements of the vector  $\mathbf{w}_i$  are iid  $N(0, 1)$ . Let the  $m \times m$  matrix  $\mathbf{A} = (a_{ij})$  with  $a_{ii} = 1$  and  $a_{ij} = \psi$  where  $0 \leq \psi < 1$  for  $i \neq j$ . Then the vector  $\mathbf{u}_i = \mathbf{A}\mathbf{w}_i$  so that  $Cov(\mathbf{u}_i) = \Sigma_{\mathbf{u}} = \mathbf{A}\mathbf{A}^T = (\sigma_{ij})$  where the diagonal entries  $\sigma_{ii} = [1 + (m - 1)\psi^2]$  and the off diagonal entries  $\sigma_{ij} = [2\psi + (m - 2)\psi^2]$ . Hence the correlations are  $cor(x_i, x_j) = \rho = (2\psi + (m - 2)\psi^2)/(1 + (m - 1)\psi^2)$  for  $i \neq j$  where  $x_i$  and  $x_j$  are nontrivial predictors. If  $\psi = 1/\sqrt{cp}$ , then  $\rho \rightarrow 1/(c + 1)$  as  $p \rightarrow \infty$  where  $c > 0$ . As  $\psi$  gets close to 1, the predictor vectors cluster about the line in the direction of  $(1, \dots, 1)^T$ . Then  $Y_i = 1 + 1x_{i,2} + \dots + 1x_{i,k} + e_i$  for  $i = 1, \dots, n$ . Hence  $\beta = (1, \dots, 1, 0, \dots, 0)^T$  with  $k + 1$  ones and  $p - k - 1$  zeros. The zero mean errors  $e_i$  were iid of five types: i)  $N(0, 1)$  errors, ii)  $t_3$  errors, iii)  $EXP(1) - 1$  errors, iv) uniform( $-1, 1$ ) errors, and v)  $0.9 N(0, 1) + 0.1 N(0, 100)$  errors.

The lengths of the asymptotically optimal 95% PIs are i)  $3.92 = 2(1.96)$ , ii)  $6.365$ , iii)  $2.996$ , iv)  $1.90 = 2(0.95)$ , and v)  $13.490$ . The simulation used 5000 runs, so an observed

coverage in [0.94, 0.96] gives no reason to doubt that the PI has the nominal coverage of 0.95. The simulation used  $p = 20, 40, n$ , and  $2n$ . The simulation used  $\psi = 0, 1/\sqrt{p}$ , and 0.9, and  $k = 1, 19$ , and  $p - 1$ .

Table 5.301 shows some simulation results. For lasso, often more than one  $\lambda$  value had  $d - 1$  active predictors, and we used the value of  $\lambda$  closest to 0. If  $d = p$ , lasso and relaxed lasso used the selected value of  $\lambda$  rather than the OLS full model. For  $N(0, 1)$  errors,  $\psi = 0$ , and  $d < k$ , the asymptotically optimal PI length is  $3.92\sqrt{k - d + 1}$ .

Table 5.301. Simulated Large Sample 95% PI Coverages and Lengths,  $e_i \sim N(0, 1)$ 

| n   | p   | $\psi$ | k  |     | FS     | lasso  | RL     | RR      | PLS    | PCR     |
|-----|-----|--------|----|-----|--------|--------|--------|---------|--------|---------|
| 100 | 20  | 0      | 1  | Cov | 0.9644 | 0.9570 | 0.9534 | 0.9354  | 0.9438 | 0.9772  |
|     |     |        |    | Len | 4.4490 | 4.3849 | 4.3648 | 4.1441  | 4.4149 | 5.5647  |
| 100 | 40  | 0      | 1  | Cov | 0.9654 | 0.9522 | 0.9482 | 0.8932  | 0.8810 | 0.9882  |
|     |     |        |    | Len | 4.4294 | 4.3113 | 4.2734 | 3.8982  | 4.0202 | 7.3393  |
| 100 | 100 | 0      | 1  | Cov | 0.9686 | 0.9494 | 0.9414 | 0.9554  | 0.8000 | 0.9932  |
|     |     |        |    | Len | 4.4274 | 4.2427 | 4.1600 | 5.4422  | 3.5035 | 9.5767  |
| 100 | 200 | 0      | 1  | Cov | 0.9648 | 0.9332 | 0.9222 | 0.9254  | 0.6616 | 0.9922  |
|     |     |        |    | Len | 4.4268 | 4.1546 | 4.0340 | 4.9843  | 2.7695 | 12.4116 |
| 200 | 20  | 0      | 19 | Cov | 0.9788 | 0.9766 | 0.9788 | 0.9792  | 0.9550 | 0.9786  |
|     |     |        |    | Len | 4.9613 | 4.9636 | 4.9613 | 5.0458  | 4.3211 | 4.9610  |
| 200 | 40  | 0      | 19 | Cov | 0.9742 | 0.9650 | 0.9732 | 0.9606  | 0.9324 | 0.9792  |
|     |     |        |    | Len | 4.9285 | 4.8146 | 4.8567 | 4.8044  | 4.2152 | 5.3616  |
| 200 | 100 | 0      | 19 | Cov | 0.9746 | 0.9456 | 0.9472 | 0.8416  | 0.7834 | 1.0000  |
|     |     |        |    | Len | 4.9057 | 4.5640 | 4.5551 | 3.9090  | 3.4810 | 23.3839 |
| 200 | 200 | 0      | 19 | Cov | 0.9728 | 0.9124 | 0.9136 | 0.9696  | 0.3500 | 1.0000  |
|     |     |        |    | Len | 4.8835 | 4.3197 | 4.2244 | 16.5887 | 2.1451 | 51.8962 |
| 400 | 20  | 0      | 19 | Cov | 0.9756 | 0.9756 | 0.9756 | 0.9760  | 0.9516 | 0.9756  |
|     |     |        |    | Len | 4.6934 | 4.6959 | 4.6934 | 4.7504  | 4.0704 | 4.6934  |
| 400 | 40  | 0      | 19 | Cov | 0.9738 | 0.9748 | 0.9760 | 0.9714  | 0.9412 | 0.9790  |
|     |     |        |    | Len | 4.6733 | 4.6638 | 4.6813 | 4.6776  | 4.0165 | 4.9001  |
| 400 | 100 | 0      | 19 | Cov | 0.9686 | 0.9554 | 0.9588 | 0.9250  | 0.8928 | 1.0000  |
|     |     |        |    | Len | 4.6777 | 4.5262 | 4.4992 | 4.2544  | 3.7749 | 9.6077  |
| 400 | 200 | 0      | 19 | Cov | 0.9718 | 0.9528 | 0.9430 | 0.7956  | 0.7306 | 1.0000  |
|     |     |        |    | Len | 4.6784 | 4.4430 | 4.3454 | 3.5541  | 3.1304 | 22.9925 |

## 5.5 SIMULATIONS FOR BOOTSTRAPPING

Assume  $n \geq 20p$  and that the error distribution is unimodal and not highly skewed.

The response plot and residual plot are plots with  $\hat{Y} = \mathbf{x}^T \hat{\boldsymbol{\beta}}$  on the horizontal axis and  $Y$  or  $r$  on the vertical axis, respectively. Then the plotted points in these plots should scatter in roughly even bands about the identity line with unit slope and zero intercept and  $r = 0$  lines, respectively. See Figure 5.1. If the plots for the OLS full model suggest that the error distribution is skewed or multimodal, then much larger sample sizes may be needed.

If the error distribution is unknown, then large sample theory tests are straightforward if the estimator is asymptotically equivalent to the OLS full model, e.g.  $\hat{\lambda}_{1,n} = o_P(\sqrt{n})$ , or choose the OLS full model if  $n \geq 50p$ . The latter technique may be reasonable if the large sample theory of the method is not better than that of the OLS full model (lasso and ridge regression), if it is not known how to do inference unless the model is asymptotically equivalent to the OLS full model (PCR), or if it is not known how to do inference for the model (PLS, forward selection).

The residual bootstrap with the residuals from the OLS full model can provide a lot of information. Olive (2017a: p. 128, 2017b) showed that the prediction region method can simulate well for the  $p \times 1$  vector  $\hat{\boldsymbol{\beta}}_{I_{min},0}$  obtained by adding zeroes to  $\hat{\boldsymbol{\beta}}_{I_{min}}$  where  $I_{min}$  is the model that minimizes  $C_p$  for forward selection. Asymptotically,  $\hat{\boldsymbol{\beta}}_{I_{min},0}$  is a mixture  $\sum_j \pi_j \hat{\boldsymbol{\beta}}_{I_j,0}$  where  $0 \leq \pi_j \leq 1$  and  $\sum_j \pi_j = 1$  where the sum is over all  $2^{p-as}$  submodels  $I_j$  that contain  $S$ . Results from Knight and Fu (2000) show that each component  $\hat{\boldsymbol{\beta}}_{I_j,0}$  has the correct asymptotically multivariate normal distribution, but we may need at least  $50p$  bootstrap samples per component with nonnegligible  $\pi_j$ . The number of nonnegligible  $\pi_j$

can be small if  $p - a_S$  is small or if a criterion that picks  $S$  with high probability, such as

BIC, is used. Here  $Y = \mathbf{x}^T \boldsymbol{\beta} + e = \mathbf{x}_S^T \boldsymbol{\beta}_S + e$  where  $\boldsymbol{\beta}_S$  is  $a_s \times 1$ .

Examining  $\hat{\boldsymbol{\beta}}_S$  and  $\hat{\boldsymbol{\beta}}_E$  is informative for  $I_{min}$ . First assume that the nontrivial predictors are uncorrelated or orthogonal so  $\mathbf{X}^T \mathbf{X}/n \rightarrow diag(d_1, \dots, d_p)$  as  $n \rightarrow \infty$  where each  $d_i > 0$ . Then  $\hat{\boldsymbol{\beta}}_S$  has the same limiting distribution for  $I_{min}$  and for the OLS full model. The bootstrap distribution for  $\hat{\boldsymbol{\beta}}_E$  is a mixture of zeroes and a distribution that would produce a confidence region for  $\mathbf{A}\boldsymbol{\beta}_E = \mathbf{0}$  that has asymptotic coverage of  $\mathbf{0}$  equal to  $100(1 - \delta)\%$ . Hence the asymptotic coverage is greater than the nominal coverage provided that  $\mathbf{S}_T$  is nonsingular with probability going to one (e.g.,  $p - a_S$  is small), where  $T = \mathbf{A}\hat{\boldsymbol{\beta}}_{E,I_{min}}$ . With uncorrelated predictors, the number of bootstrap samples  $B \geq 50p$  may work well for the short confidence intervals and for testing  $\mathbf{A}\boldsymbol{\beta}_S = \mathbf{0}$ .

We do not yet have a proof that the prediction region method works when the estimator is not asymptotically multivariate normal, but in the simulations for forward selection, coverages were similar regardless of the correlation of the predictors. Let  $\boldsymbol{\beta}_O$  be a vector component of  $\boldsymbol{\beta}_E$ , and consider testing  $H_0 : \mathbf{A}\boldsymbol{\beta}_O = \mathbf{0}$ . If  $\mathbf{A}\hat{\boldsymbol{\beta}}_{O,i}^* = \mathbf{0}$  for greater than  $B\delta$  of the bootstrap samples  $i = 1, \dots, B$ , then the  $100(1 - \delta)\%$  prediction region method confidence region will contain  $\mathbf{0}$ , and the test will fail to reject  $H_0$ .

Suppose we want to bootstrap  $T = \hat{\boldsymbol{\beta}}_O$ , where  $\boldsymbol{\beta} = (\boldsymbol{\beta}_I^T, \boldsymbol{\beta}_O^T)^T$ , and all  $\hat{\boldsymbol{\beta}}_{O,i}^* = \mathbf{0}$  for  $i = 1, \dots, B$ . Then  $\mathbf{S}_T$  is singular, but the singleton set  $\{\mathbf{0}\}$  is the large sample prediction region method  $100(1 - \delta)\%$  confidence region for  $\boldsymbol{\beta}_O$  and  $\delta \in (0, 1)$ , and the pvalue for  $H_0 : \boldsymbol{\beta}_O = \mathbf{0}$  is one. For large sample theory tests, the pvalue estimates the population pvalue. For the  $I_{min}$  model from forward selection, there may be strong evidence that  $\mathbf{x}_O$

is not needed in the model given  $\mathbf{x}_I$  is in the model if the confidence region is  $\{\mathbf{0}\}$ ,  $n \geq 20p$ ,  $B \geq 50p$ , and the error distribution is unimodal and not highly skewed. (Since the pvalue is one, this technique may be useful for data snooping: applying OLS theory to submodel  $I$  may have negligible selection bias.)

A small simulation was done, using the same type of data as for the prediction interval simulation, using  $B = \max(1000, n, 20p)$  and 5000 runs. The regression model used  $\boldsymbol{\beta} = (1, 1, 0, 0)^T$  with  $n = 100$  and  $p = 4$ . When  $\psi = 0$ , the design matrix  $\mathbf{X}$  consisted of iid  $N(0,1)$  random variables, and the full model least squares confidence intervals for  $\beta_i$  should have length near  $2t_{96,0.975}\sigma/\sqrt{n} \approx 2(1.96)\sigma/10 = 0.392\sigma$  when the iid zero mean errors have variance  $\sigma^2$ . The simulation computed the Frey shorth( $c$ ) interval for each  $\beta_i$  and used the prediction region method to test  $H_0 : \beta_3 = \beta_4 = 0$ . The nominal coverage was 0.95 with  $\delta = 0.05$ . Observed coverage between 0.94 and 0.96 would suggest coverage is close to the nominal value. Models with the first  $k + 1$   $\beta_i = 1$  and the last  $p - k - 1$   $\beta_i = 0$  were also considered.

The regression models used the residual bootstrap on the full model least squares estimator and on the forward selection estimator  $\hat{\boldsymbol{\beta}}_{I_{min},0}$ . Results are shown for when the iid errors  $e_i \sim N(0, 1)$ . Table 5.302 shows two rows for each model giving the observed confidence interval coverages and average lengths of the confidence intervals. The term “reg” is for the full model regression, and the term “vs” is for forward selection. The column for the “test” gives the length and coverage =  $P(\text{fail to reject } H_0)$  for the interval  $[0, D_{(U_B)}]$  where  $D_{(U_B)}$  is the cutoff for the confidence region. These lengths do not give information about the volume of the confidence region, which will decrease to 0 as  $n \rightarrow \infty$ .

The cutoff will often be near  $\sqrt{\chi^2_{r,0.95}}$  if the statistic  $T$  is asymptotically normal. Note that  $\sqrt{\chi^2_{2,0.95}} = 2.448$  is close to 2.45 for the full model regression bootstrap test. The coverages were near 0.95 for the regression bootstrap on the full model.

Suppose  $\psi = 0$ . Then  $\hat{\beta}_S$  has the same limiting distribution for  $I_{min}$  and the full model. Note that the average lengths and coverages were similar for the full model and forward selection  $I_{min}$  for  $\beta_1$  and  $\beta_2$  and  $\beta_S = (\beta_1, \beta_2)^T$ .

Table 5.302. Bootstrapping OLS Regression and Forward Selection

| model | $\psi$ | cov/len | $\beta_1$ | $\beta_2$ | $\beta_3$ | $\beta_4$ | test   |
|-------|--------|---------|-----------|-----------|-----------|-----------|--------|
| reg   | 0      | cov     | 0.9456    | 0.9474    | 0.9496    | 0.9474    | 0.9442 |
|       |        | len     | 0.3961    | 0.3997    | 0.3988    | 0.3992    | 2.4503 |
| vs    | 0      | cov     | 0.9472    | 0.9470    | 0.9980    | 0.9980    | 0.9936 |
|       |        | len     | 0.3964    | 0.3991    | 0.3246    | 0.3233    | 2.6936 |
| reg   | 0.5    | cov     | 0.9432    | 0.9452    | 0.9498    | 0.9506    | 0.9436 |
|       |        | len     | 0.3976    | 0.6642    | 0.6645    | 0.6637    | 2.4507 |
| vs    | 0.5    | cov     | 0.9458    | 0.9728    | 0.9976    | 0.9974    | 0.9926 |
|       |        | len     | 0.3966    | 0.6598    | 0.5383    | 0.5383    | 2.7055 |
| reg   | 0.9    | cov     | 0.9432    | 0.9512    | 0.9500    | 0.9498    | 0.9442 |
|       |        | len     | 0.3963    | 3.2621    | 3.2613    | 3.2611    | 2.4505 |
| vs    | 0.9    | cov     | 0.9422    | 0.9678    | 0.9944    | 0.9970    | 0.9914 |
|       |        | len     | 0.3957    | 2.7640    | 2.7356    | 2.7430    | 2.7121 |

Table 5.303. Bootstrap LASSO,  $\psi = 0$ 

| n   | eps | type |        | $\beta_1$ | $\beta_2$ | $\beta_3$ | $\beta_4$ | test   |
|-----|-----|------|--------|-----------|-----------|-----------|-----------|--------|
| 100 | 1   | 1    | cicov  | 0.9440    | 0.9376    | 0.9910    | 0.9946    | 0.9790 |
|     |     |      | avelen | 0.4143    | 0.4470    | 0.3759    | 0.3763    | 2.6444 |
|     |     | 2    | cicov  | 0.9468    | 0.9428    | 0.9946    | 0.9944    | 0.9816 |
|     |     |      | avelen | 0.6870    | 0.7565    | 0.6238    | 0.6226    | 2.6832 |
|     |     | 3    | cicov  | 0.9418    | 0.9408    | 0.9930    | 0.9948    | 0.9840 |
|     |     |      | avelen | 0.4110    | 0.4506    | 0.3743    | 0.3746    | 2.6684 |
|     | 0.5 | 5    | avelen | 0.2392    | 0.2578    | 0.2151    | 0.2153    | 2.6454 |
|     |     |      | cicov  | 0.9438    | 0.9344    | 0.9988    | 0.9970    | 0.9924 |
|     | 0.9 | 5    | avelen | 2.9380    | 2.5042    | 2.4912    | 2.4715    | 2.8536 |
|     |     |      | cicov  | 0.9506    | 0.9290    | 0.9974    | 0.9976    | 0.9956 |
|     |     |      | avelen | 3.9180    | 3.2760    | 3.2739    | 3.2702    | 2.8836 |
| 200 | 5   | 1    | cicov  | 0.9494    | 0.9390    | 0.9942    | 0.9924    | 0.9802 |
|     |     |      | avelen | 0.4132    | 0.4460    | 0.3754    | 0.3760    | 2.6455 |
|     |     | 2    | cicov  | 0.9474    | 0.9502    | 0.9966    | 0.9948    | 0.9860 |
|     |     |      | avelen | 0.4902    | 0.5365    | 0.4445    | 0.4448    | 2.6726 |
|     |     | 3    | cicov  | 0.9432    | 0.9440    | 0.9958    | 0.9966    | 0.9852 |
|     |     |      | avelen | 0.2924    | 0.3167    | 0.2641    | 0.2647    | 2.6617 |
|     |     | 4    | cicov  | 0.9504    | 0.9354    | 0.9952    | 0.9948    | 0.9858 |
|     |     |      | avelen | 0.1699    | 0.1810    | 0.1506    | 0.1510    | 2.6429 |
|     | 0.9 | 5    | cicov  | 0.9486    | 0.9352    | 0.9978    | 0.9972    | 0.9936 |
|     |     |      | avelen | 2.6679    | 2.2152    | 2.1943    | 2.2014    | 2.7952 |

## CHAPTER 6

### CONCLUSIONS

Let  $p$  be fixed and  $n \rightarrow \infty$ . For forward selection, PCR, PLS, ridge regression, relaxed lasso, and lasso, if  $P(d \rightarrow p) \rightarrow 1$  as  $n \rightarrow \infty$  then the six methods are asymptotically equivalent to the OLS full model, and the PIs (3.11) and (3.12) are asymptotically optimal on a large class of iid unimodal error distributions. For PCR and some constants  $\theta_i$ ,  $\sum_{i=1}^j \theta_i \gamma_i^T \mathbf{x}_i = \sum_{i=1}^p \beta_i x_i$  if  $j = p$ , but not if  $j < p$  in general. Hence PCR tends to give inconsistent estimators unless  $P(j = p) = P(\text{PCR uses the full OLS model})$  goes to one. Forward selection with  $C_p$  produces a  $\sqrt{n}$  consistent estimator  $\hat{\boldsymbol{\beta}}_{I_{min},0}$  of  $\boldsymbol{\beta}$ . Using  $d = \min(\lceil n/J \rceil, p)$  with forward selection, PCR, PLS, ridge regression, lasso, and relaxed lasso makes large sample inference easy since the selected model is the full OLS model if  $n/p \geq J$ .

There is massive literature on variable selection and a fairly large literature for inference after variable selection. See, for example, Bertsimas, King, and Mazumder (2016), Fan and Lv (2010), Ferrari and Yang (2015), Fithian, Sun, and Taylor (2014), Hjort and Claeskens (2003), Knight and Fu (2000), Lee, Sun, Sun, and Taylor (2016), Leeb and Pötscher (2006), Lockhart, Taylor, Tibshirani, and Tibshirani (2014), Qi, Luo, Carroll, and Zhao (2015), and Taylor, Lockhart, Tibshirani, and Tibshirani (2014).

If  $n/p$  is large, the residual bootstrap with OLS residuals should work for lasso, relaxed lasso, and ridge regression if  $\hat{\lambda}_{1,n} = o_P(\sqrt{n})$ . Also see Knight and Fu (2000). Camponovo (2015) suggests that the nonparametric bootstrap does not work for lasso. Chatterjee and

Lahiri (2011) state that the residual bootstrap with lasso does not work. Hall, Lee, and Park (2009) show that the residual bootstrap with full model OLS residuals does not work, but the  $m$  out of  $n$  residual bootstrap with OLS full model residuals does work. Rejchel (2016) is a good review of lasso theory. Fan and Lv (2010) review large sample theory for some alternative methods. See Lockhart, Taylor, Tibshirani, and Tibshirani (2014) for a partial remedy for hypothesis testing with lasso. Xu, Caramanis, and Mannor (2011) suggest that sparse algorithms are not stable.

Lei, G'Sell, Rinaldo, Tibshirani, and Wasserman (2016) and Wasserman (2014) suggest prediction intervals for estimators such as lasso. Also see Butler and Rothman (1980). Steinberger and Leeb (2016) use leave-one-out residuals to make a PI. Chao, Ning, and Liu (2014), assume that the  $e_i$  are iid  $N(0, \sigma^2)$ . Denham (1997) gave a PI for PLS when the number of PLS components  $V_j$  is selected in advance. Zhang (1992) has some  $k$ -fold CV theory.

With  $n/p$  large,  $C_p$  produced good PIs for forward selection and 10-fold CV produced good PIs for PCR and PLS. For lasso and ridge regression, 10-fold CV produced good PIs if  $\psi = 0$  or if  $k$  was small. If  $k$  was larger than about 18 and the predictors were highly correlated, 10-fold CV tended to underfit and the PI lengths were too long.

When  $n/p$  is not large, inference is currently much more difficult. Zheng and Loh (1995) show that  $BIC_S$  can work if  $p = o(\log(n))$  and there is a consistent estimator of  $\sigma^2$ . Chun and Keles (2010) show that PLS does not give a consistent estimator of  $\beta$  unless  $p/n \rightarrow 0$ . Also see Cook, Helland, and Su (2013). Fan and Lv (2010) give large sample theory for some methods if  $p = o(n^{1/5})$ . Leeb, Pötscher, and Ewald (2015) suggests that

the method Berk et al. (2013) method does not really work. Also see Dezeure et al. (2015), Javanmard and Montanari (2014), Taylor et al. (2014), and van de Geer et al. (2014).

Response plots of the fitted values  $\hat{Y}$  versus the response  $Y$  are useful for checking linearity of the MLR model and for detecting outliers. Residual plots should also be made. When  $n$  is large, the points within the pointwise PI bands can be omitted, eliminating a black band about the identity line.

The simulations were done in *R*. See R Core Team (2016). A much larger simulation study is in Pelawa Watagoda (2017). We used several *R* functions including forward selection as computed with `regsubsets` function from the `leaps` library, principal components regression with the `pcr` function and partial least squares with the `plsr` function from the `pls` library, and ridge regression and lasso with the `glmnet` and `cv.glmnet` functions from the `glmnet` library.

The collection of Olive (2017d) *R* functions `slpack`, available from (<http://lagrange.math.siu.edu/Olive/slpack.txt>), has some useful functions for the inference. Table 5.301 was made with `mspisim` while Table 5.302 was made with `regbootsim` for the OLS full model and `vsbootsim3` for forward selection. The function `lassobotsim3` uses the prediction region method for lasso. For PI (3.12), the function `valvspisim` is for forward selection using the minimum  $C_p$  model, and the function `valrelpisim` simulates the relaxed lasso model corresponding to the lasso model chosen with 10-fold CV.

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