

Plots and Testing for Multivariate Linear Regression

by

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By

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## AN ABSTRACT OF THE DISSERTATION OF

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To use the multivariate linear regression model, tests of hypotheses are needed. This paper gives  $F$  approximations to the widely used Wilk, Pillai, and Hotelling Lawley test statistics, and gives plots to check goodness and lack of fit, to check for outliers and influential cases, and to check whether the error distribution is multivariate normal.

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# CHAPTER 1

## INTRODUCTION

For the multivariate linear regression model, this paper provides useful plots, and gives  $F$  approximations to the widely used Wilk, Pillai, and Hotelling Lawley test statistics. First notation is given for the multivariate linear regression model. We follow Olive (2013) closely.

The *multivariate linear regression model*  $\mathbf{y}_i = \mathbf{B}^T \mathbf{x}_i + \boldsymbol{\epsilon}_i$  for  $i = 1, \dots, n$  has  $m \geq 2$  response variables  $Y_1, \dots, Y_m$  and  $p$  predictor variables  $x_1, x_2, \dots, x_p$ , with  $x_{i1} = 1$ . The  $i$ th case is  $(\mathbf{x}_i^T, \mathbf{y}_i^T) = (x_{i1}, x_{i2}, \dots, x_{ip}, Y_{i1}, \dots, Y_{im})$ . The model is written in matrix form as  $\mathbf{Z} = \mathbf{X}\mathbf{B} + \mathbf{E}$  where the matrices are defined below. The model has  $E(\boldsymbol{\epsilon}_k) = \mathbf{0}$  and  $\text{Cov}(\boldsymbol{\epsilon}_k) = \boldsymbol{\Sigma}_\boldsymbol{\epsilon} = (\sigma_{ij})$  for  $k = 1, \dots, n$ . Also  $E(\mathbf{e}_i) = \mathbf{0}$  while  $\text{Cov}(\mathbf{e}_i, \mathbf{e}_j) = \sigma_{ij} \mathbf{I}_n$  for  $i, j = 1, \dots, m$  where  $\mathbf{I}_n$  is the  $n \times n$  identity matrix and  $\mathbf{e}_i$  is defined below. Then the  $p \times m$  coefficient matrix  $\mathbf{B} = \begin{bmatrix} \boldsymbol{\beta}_1 & \boldsymbol{\beta}_2 & \dots & \boldsymbol{\beta}_m \end{bmatrix}$  and the  $m \times m$  covariance matrix  $\boldsymbol{\Sigma}_\boldsymbol{\epsilon}$  are to be estimated, and  $E(\mathbf{Z}) = \mathbf{X}\mathbf{B}$  while  $E(Y_{ij}) = \mathbf{x}_i^T \boldsymbol{\beta}_j$ .

The  $n \times m$  matrix of response variables

$$\mathbf{Z} = \begin{bmatrix} \mathbf{Y}_1 & \mathbf{Y}_2 & \dots & \mathbf{Y}_m \end{bmatrix} = \begin{bmatrix} \mathbf{y}_1^T \\ \vdots \\ \mathbf{y}_n^T \end{bmatrix}.$$

The  $n \times p$  design matrix of predictor variables

$$\mathbf{X} = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \dots & \mathbf{v}_p \end{bmatrix} = \begin{bmatrix} \mathbf{x}_1^T \\ \vdots \\ \mathbf{x}_n^T \end{bmatrix}$$

where  $\mathbf{v}_1 = \mathbf{1}$ .

The  $n \times m$  matrix of errors

$$\mathbf{E} = \begin{bmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \dots & \mathbf{e}_m \end{bmatrix} = \begin{bmatrix} \boldsymbol{\epsilon}_1^T \\ \vdots \\ \boldsymbol{\epsilon}_n^T \end{bmatrix}.$$

Least squares is the classical method for fitting the multivariate linear model. The *least squares estimators* are  $\hat{\mathbf{B}} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{Z} = \begin{bmatrix} \hat{\boldsymbol{\beta}}_1 & \hat{\boldsymbol{\beta}}_2 & \dots & \hat{\boldsymbol{\beta}}_m \end{bmatrix}$ . The *predicted values* or *fitted values*  $\hat{\mathbf{Z}} = \mathbf{X} \hat{\mathbf{B}} = \begin{bmatrix} \hat{Y}_1 & \hat{Y}_2 & \dots & \hat{Y}_m \end{bmatrix}$ . The *residuals*

$$\hat{\mathbf{E}} = \mathbf{Z} - \hat{\mathbf{Z}} = \mathbf{Z} - \mathbf{X} \hat{\mathbf{B}} = \begin{bmatrix} \hat{\boldsymbol{\epsilon}}_1^T \\ \hat{\boldsymbol{\epsilon}}_2^T \\ \vdots \\ \hat{\boldsymbol{\epsilon}}_n^T \end{bmatrix} = \begin{bmatrix} \hat{r}_1 & \hat{r}_2 & \dots & \hat{r}_m \end{bmatrix}.$$

These quantities can be found from the  $m$  multiple linear regressions of  $Y_j$  on the predictors:

$\hat{\boldsymbol{\beta}}_j = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{Y}_j$ ,  $\hat{Y}_j = \mathbf{X} \hat{\boldsymbol{\beta}}_j$  and  $\hat{r}_j = \mathbf{Y}_j - \hat{Y}_j$  for  $j = 1, \dots, m$ . Hence  $\hat{\boldsymbol{\epsilon}}_{i,j} = Y_{i,j} - \hat{Y}_{i,j}$  where  $\hat{Y}_j = (\hat{Y}_{1,j}, \dots, \hat{Y}_{n,j})^T$ . Finally,

$$\hat{\boldsymbol{\Sigma}}_{\boldsymbol{\epsilon},d} = \frac{(\mathbf{Z} - \hat{\mathbf{Z}})^T (\mathbf{Z} - \hat{\mathbf{Z}})}{n-d} = \frac{(\mathbf{Z} - \mathbf{X} \hat{\mathbf{B}})^T (\mathbf{Z} - \mathbf{X} \hat{\mathbf{B}})}{n-d} = \frac{\hat{\mathbf{E}}^T \hat{\mathbf{E}}}{n-d} = \frac{1}{n-d} \sum_{i=1}^n \hat{\boldsymbol{\epsilon}}_i \hat{\boldsymbol{\epsilon}}_i^T.$$

The choices  $d = 0$  and  $d = p$  are common. If  $d = 1$ , then  $\hat{\boldsymbol{\Sigma}}_{\boldsymbol{\epsilon},d=1} = \mathbf{S}_r$ , the sample covariance matrix of the residual vectors  $\hat{\boldsymbol{\epsilon}}_i$  since the sample mean of the  $\hat{\boldsymbol{\epsilon}}_i$  is  $\mathbf{0}$ . Let  $\hat{\boldsymbol{\Sigma}}_{\boldsymbol{\epsilon}} = \hat{\boldsymbol{\Sigma}}_{\boldsymbol{\epsilon},p}$  be the unbiased estimator of  $\boldsymbol{\Sigma}_{\boldsymbol{\epsilon}}$ .

The  $\boldsymbol{\epsilon}_i$  are assumed to be iid. Some important joint distributions for  $\boldsymbol{\epsilon}$  are completely specified by an  $m \times 1$  population *location* vector  $\boldsymbol{\mu}$  and an  $m \times m$  symmetric positive definite population *dispersion* matrix  $\boldsymbol{\Sigma}$ . An important model is the elliptically contoured  $EC_m(\boldsymbol{\mu}, \boldsymbol{\Sigma}, g)$  distribution with probability density function

$$f(\mathbf{z}) = k_m |\boldsymbol{\Sigma}|^{-1/2} g[(\mathbf{z} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{z} - \boldsymbol{\mu})]$$

where  $k_m > 0$  is some constant and  $g$  is some known function. The multivariate normal (MVN)  $N_m(\boldsymbol{\mu}, \boldsymbol{\Sigma})$  distribution is a special case.

Some additional notation will be useful. Assume that  $\mathbf{x}_1, \dots, \mathbf{x}_n$  are iid from a multivariate distribution. The classical estimator  $(\bar{\mathbf{x}}, \mathbf{S})$  of multivariate location and dispersion is the sample mean and sample covariance matrix where

$$\bar{\mathbf{x}} = \frac{1}{n} \sum_{i=1}^n \mathbf{x}_i \quad \text{and} \quad \mathbf{S} = \frac{1}{n-1} \sum_{i=1}^n (\mathbf{x}_i - \bar{\mathbf{x}})(\mathbf{x}_i - \bar{\mathbf{x}})^T. \quad (1.1)$$

Let the  $p \times 1$  column vector  $T$  be a multivariate location estimator, and let the  $p \times p$  symmetric positive definite matrix  $\mathbf{C}$  be a dispersion estimator. Then the  $i$ th *squared sample Mahalanobis distance* is the scalar

$$D_i^2 = D_i^2(T, \mathbf{C}) = (\mathbf{x}_i - T)^T \mathbf{C}^{-1} (\mathbf{x}_i - T) \quad (1.2)$$

for each observation  $\mathbf{x}_i$ . Notice that the Euclidean distance of  $\mathbf{x}_i$  from the estimate of center  $T$  is  $D_i(T, \mathbf{I}_p)$ . The classical Mahalanobis distance uses  $(T, \mathbf{C}) = (\bar{\mathbf{x}}, \mathbf{S})$ . Following Johnson (1987, pp. 107-108), the population squared Mahalanobis distance

$$U \equiv D^2(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = (\mathbf{x} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{x} - \boldsymbol{\mu}), \quad (1.3)$$

and for elliptically contoured distributions,  $U$  has probability density function (pdf)

$$h(u) = \frac{\pi^{p/2}}{\Gamma(p/2)} k_p u^{p/2-1} g(u). \quad (1.4)$$

The next Section suggests plots for multivariate linear models, Chapter 2 shows that the Hotelling Lawley test statistic performs well asymptotically for a large class of zero mean error distributions with fourth moments, and Chapter 3 gives some examples and simulations.

## 1.1 PLOTS FOR THE MULTIVARIATE LINEAR MODEL

A *response plot* for the  $j$ th response variable is a plot of the fitted values  $\widehat{Y}_{ij}$  versus the response  $Y_{ij}$  where  $i = 1, \dots, n$ . The identity line with slope one and zero intercept is added

to the plot as a visual aid. A *residual plot* corresponding to the  $j$ th response variable is a plot of  $\hat{Y}_{ij}$  versus  $r_{ij}$ .

Make the  $m$  response and residual plots for the multivariate linear model. In a response plot, the vertical deviations from the identity line are the residuals  $r_{ij} = Y_{ij} - \hat{Y}_{ij}$ . Suppose the model is good, the error distribution is not highly skewed, and  $n \geq 10p$ . Then the plotted points should cluster about the identity line in each of the  $m$  response plots. If outliers are present or if the plot is not linear, then the current model or data need to be changed or corrected. Each of the  $m$  residual plots should be ellipsoidal with no trend and should be centered about the  $r = 0$  line. There should not be any pattern in the residual plot: as a narrow vertical strip is moved from left to right, the behavior of the residuals within the strip should show little change. Outliers and patterns such as curvature or a fan shaped plot are bad. In other words, the response and residual plots should be used as for multiple linear regression where  $m = 1$ . See Olive and Hawkins (2005), Cook and Weisberg (1999, p. 432), and Example 1 in chapter 3.

The Rousseeuw and Van Driessen (1999) DD plot is a plot of classical Mahalanobis distances versus robust Mahalanobis distances. Simulations and results from Olive (2002) suggest the plotted points in the DD plot will cluster about the identity line if the  $\epsilon_i$  are iid from a multivariate normal  $N_m(\mathbf{0}, \Sigma_{\epsilon})$  distribution and about some line through the origin with slope greater than one for a large class of elliptically contoured distributions. Hence make a DD plot of the residuals  $\hat{\epsilon}_i$  to check the error distribution. Make a DD plot of the continuous predictor variables to check for  $\mathbf{x}$ -outliers. See Example 1.

**CHAPTER 2**  
**TESTING HYPOTHESES**

Consider testing a linear hypothesis  $H_0 : \mathbf{L}\mathbf{B} = \mathbf{0}$  versus  $H_1 : \mathbf{L}\mathbf{B} \neq \mathbf{0}$  where  $\mathbf{L}$  is a full rank  $r \times p$  matrix. Let  $\mathbf{H} = \hat{\mathbf{B}}^T \mathbf{L}^T [\mathbf{L}(\mathbf{X}^T \mathbf{X})^{-1} \mathbf{L}^T]^{-1} \mathbf{L} \hat{\mathbf{B}}$ . Let the error or residual sum of squares and cross products matrix be

$$\mathbf{W}_e = \hat{\mathbf{E}}^T \hat{\mathbf{E}} = (\mathbf{Z} - \hat{\mathbf{Z}})^T (\mathbf{Z} - \hat{\mathbf{Z}}) = \mathbf{Z}^T \mathbf{Z} - \mathbf{Z}^T \mathbf{X} \hat{\mathbf{B}} = \mathbf{Z}^T [\mathbf{I}_n - \mathbf{X}(\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T] \mathbf{Z}.$$

Then  $\mathbf{W}_e / (n - p) = \hat{\Sigma}_\epsilon$ . Let  $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_m$  be the ordered eigenvalues of  $\mathbf{W}_e^{-1} \mathbf{H}$ . Then there are four commonly used test statistics.

The Roy's maximum root statistic is  $\lambda_{max}(\mathbf{L}) = \lambda_1$ .

The Wilk's  $\Lambda$  statistic is  $\Lambda(\mathbf{L}) = |(\mathbf{H} + \mathbf{W}_e)^{-1} \mathbf{W}_e| = |\mathbf{W}_e^{-1} \mathbf{H} + \mathbf{I}|^{-1} = \prod_{i=1}^m (1 + \lambda_i)^{-1}$ .

The Pillai's trace statistic is  $V(\mathbf{L}) = tr[(\mathbf{H} + \mathbf{W}_e)^{-1} \mathbf{H}] = \sum_{i=1}^m \frac{\lambda_i}{1 + \lambda_i}$ .

The Hotelling-Lawley trace statistic is  $U(\mathbf{L}) = tr[\mathbf{W}_e^{-1} \mathbf{H}] = \sum_{i=1}^m \lambda_i$ .

*Theorem 1.* The Hotelling-Lawley trace statistic

$$U(\mathbf{L}) = \frac{1}{n - p} [vec(\mathbf{L}\hat{\mathbf{B}})]^T [\hat{\Sigma}_\epsilon^{-1} \otimes (\mathbf{L}(\mathbf{X}^T \mathbf{X})^{-1} \mathbf{L}^T)^{-1}] [vec(\mathbf{L}\hat{\mathbf{B}})]. \quad (2.1)$$

The proof of Theorem 1 is in the appendix. Some notation is useful to show (2.1) and to show that  $(n - p)U(\mathbf{L}) \xrightarrow{D} \chi_{rm}^2$  under mild conditions if  $H_0$  is true. Following Henderson and Searle (1979), let matrix  $\mathbf{A} = [\mathbf{a}_1 \ \mathbf{a}_2 \ \dots \ \mathbf{a}_p]$ . Then the vec operator stacks the columns of  $\mathbf{A}$  on top of one another so

$$vec(\mathbf{A}) = \begin{pmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \vdots \\ \mathbf{a}_p \end{pmatrix}.$$

Let  $\mathbf{A} = (a_{ij})$  be an  $m \times n$  matrix and  $\mathbf{B}$  a  $p \times q$  matrix. Then the Kronecker product of  $\mathbf{A}$  and  $\mathbf{B}$  is the  $mp \times nq$  matrix

$$\mathbf{A} \otimes \mathbf{B} = \begin{bmatrix} a_{11}\mathbf{B} & a_{12}\mathbf{B} & \cdots & a_{1n}\mathbf{B} \\ a_{21}\mathbf{B} & a_{22}\mathbf{B} & \cdots & a_{2n}\mathbf{B} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1}\mathbf{B} & a_{m2}\mathbf{B} & \cdots & a_{mn}\mathbf{B} \end{bmatrix}.$$

An important fact is that if  $\mathbf{A}$  and  $\mathbf{B}$  are nonsingular square matrices, then  $[\mathbf{A} \otimes \mathbf{B}]^{-1} = \mathbf{A}^{-1} \otimes \mathbf{B}^{-1}$ . The following assumption is important.

Assumption D1: Let  $h_i$  be the  $i$ th diagonal element of  $\mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T$ . Assume  $\max_{1 \leq i \leq n} h_i \rightarrow 0$  as  $n \rightarrow \infty$ , assume that the zero mean iid errors have finite fourth moments, and assume that  $\frac{1}{n}\mathbf{X}^T\mathbf{X} \xrightarrow{P} \mathbf{W}^{-1}$ .

Then for the least squares estimator, Su and Cook (2012) show that if assumption D1 holds, then  $\hat{\Sigma}_\epsilon$  is  $\sqrt{n}$  consistent and  $\sqrt{n} \text{vec}(\hat{\mathbf{B}} - \mathbf{B}) \xrightarrow{D} N_{pm}(\mathbf{0}, \Sigma_\epsilon \otimes \mathbf{W})$ .

*Theorem 2.* If assumption D1 holds and if  $H_0$  is true, then  $(n-p)U(\mathbf{L}) \xrightarrow{D} \chi_{rm}^2$ .

The proof of Theorem 2 is in the appendix. Kakizawa (2009) shows, under stronger assumptions than Theorem 2, that for a large class of iid error distributions, the following test statistics have the same  $\chi_{rm}^2$  limiting distribution when  $H_0$  is true, and the same noncentral  $\chi_{rm}^2(\omega^2)$  limiting distribution with noncentrality parameter  $\omega^2$  when  $H_0$  is false under a local alternative. Hence the three tests are robust to the assumption of normality. The limiting null distribution is well known when the zero mean errors are iid from a multivariate normal distribution. See Khattree and Naik (1999, p. 68):  $(n-p)U(\mathbf{L}) \xrightarrow{D} \chi_{rm}^2$ ,  $(n-p)V(\mathbf{L}) \xrightarrow{D} \chi_{rm}^2$ , and  $-[n-p-0.5(m-r+3)]\log(\Lambda(\mathbf{L})) \xrightarrow{D} \chi_{rm}^2$ . Results from Kshirsagar (1972, p. 301) suggest that the chi-square approximation is very good if  $n \geq 3(m^2 + p^2)$  for multivariate normal errors.

Theorems 1 and 2 are useful for relating multivariate tests with the partial  $F$  test for multiple linear regression that tests whether a reduced model that omits some of the

predictors can be used instead of the full model that uses all  $p$  predictors. The partial  $F$  test statistic is

$$F_R = \left[ \frac{SSE(R) - SSE(F)}{df_R - df_F} \right] / MSE(F)$$

where the residual sums of squares  $SSE(F)$  and  $SSE(R)$  and degrees of freedom  $df_F$  and  $df_r$  are for the full and reduced model while the mean square error  $MSE(F)$  is for the full model. Let the null hypothesis for the partial  $F$  test be  $H_0 : \mathbf{L}\boldsymbol{\beta} = \mathbf{0}$  where  $\mathbf{L}$  sets the coefficients of the predictors in the full model but not in the reduced model to 0. Seber and Lee (2003, p. 100) shows that

$$F_R = \frac{[\mathbf{L}\hat{\boldsymbol{\beta}}]^T (\mathbf{L}(\mathbf{X}^T \mathbf{X})^{-1} \mathbf{L}^T)^{-1} [\mathbf{L}\hat{\boldsymbol{\beta}}]}{r\hat{\sigma}^2}$$

is distributed as  $F_{r,n-p}$  if  $H_0$  is true and the errors are iid  $N(0, \sigma^2)$ . Note that for multiple linear regression with  $m = 1$ ,  $F_R = (n - p)U(\mathbf{L})/r$  since  $\hat{\boldsymbol{\Sigma}}_\epsilon^{-1} = 1/\hat{\sigma}^2$ . Hence the scaled Hotelling Lawley test statistic is the partial  $F$  test statistic extended to  $m > 1$  predictor variables by Theorem 1.

By Theorem 2, for example,  $rF_R \xrightarrow{D} \chi_r^2$  for a large class of nonnormal error distribution. If  $Z_n \sim F_{k,d_n}$ , then  $Z_n \xrightarrow{D} \chi_k^2/k$  as  $d_n \rightarrow \infty$ . Hence using the  $F_{r,n-p}$  approximation gives a large sample test with correct asymptotic level, and the partial  $F$  test is robust to nonnormality.

Similarly, using an  $F_{rm,n-pm}$  approximation for the following test statistics gives large sample tests with correct asymptotic level by Kakizawa (2009) and similar power for large  $n$ . The large sample test will have correct asymptotic level as long as the denominator degrees of freedom  $d_n \rightarrow \infty$  as  $n \rightarrow \infty$ , and  $d_n = n - pm$  reduces to the partial  $F$  test if  $m = 1$  and  $U(\mathbf{L})$  is used. Then the three test statistics are

$$\frac{-[n - p - 0.5(m - r + 3)]}{rm} \log(\Lambda(\mathbf{L})), \quad \frac{n - p}{rm} V(\mathbf{L}), \quad \text{and} \quad \frac{n - p}{rm} U(\mathbf{L}).$$

Following Khattree and Naik (1999, p. 67) for the Roy's largest root test, if  $h = \max(r, m)$ , use

$$\frac{n - p - h + r}{h} \lambda_{max}(\mathbf{L}) \approx F(h, n - p - h + r).$$

The simulations in chapter 3 suggest that this approximation is good for  $r = 1$  but poor for  $r > 1$ . Anderson (1984, p. 333) states that Roy's largest root test has the greatest power if  $r = 1$  but is an inferior test for  $r > 1$ .

Multivariate analogs of tests for multiple linear regression can be derived with appropriate choice of  $\mathbf{L}$ . Assume a constant  $x_1 = 1$  is in the model. The analog of the ANOVA  $F$  test for multiple linear regression is the MANOVA  $F$  test that uses  $\mathbf{L} = [\mathbf{0} \ \mathbf{I}_{p-1}]$  to test whether the nontrivial predictors are needed in the model. This test should reject  $H_0$  if the response and residual plots look good,  $n$  is large enough and at least one response plot does not look like the corresponding residual plot. A response plot for  $Y_j$  will look like a residual plot if the identity line appears almost horizontal, hence the range of  $\hat{Y}_j$  is small.

The  $F_j$  test of hypotheses uses  $\mathbf{L}_j = [0, \dots, 0, 1, 0, \dots, 0]$ , where the 1 is in the  $j$ th position, to test whether the  $j$ th predictor is needed in the model given that the other  $p - 1$  predictors are in the model. This test is an analog of the  $t$  tests for multiple linear regression.

The MANOVA partial F test is used to test whether a reduced model is good where the reduced model deletes  $r$  of the variables from the full model. For this test, the  $i$ th row of  $\mathbf{L}$  has a 1 in the position corresponding to the  $i$ th variable to be deleted. Omitting the  $j$ th variable corresponds to the  $F_j$  test while omitting variables  $x_2, \dots, x_p$  corresponds to the MANOVA F test. Using  $\mathbf{L} = [\mathbf{0} \ \mathbf{I}_k]$  tests whether the last  $k$  predictors are needed in the multivariate linear regression model given that the remaining predictors are in the model.



# CHAPTER 3

## EXAMPLES AND SIMULATIONS

**Example 1.** Cook and Weisberg (1999, p. 351, 433, 447) give a data set on 82 mussels sampled off the coast of New Zealand. Let  $Y_1 = \log(S)$  and  $Y_2 = \log(M)$  where  $S$  is the shell mass and  $M$  is the muscle mass. The predictors are  $X_2 = L$ ,  $X_3 = \log(W)$  and  $X_4 = H$ : the shell length,  $\log(\text{width})$  and height. Figures 3.1 and 3.2 give the response and residual plots for  $Y_1$  and  $Y_2$ . For  $Y_1$ , case 79 sticks out while for  $Y_2$ , cases 8, 25 and 48 are not fit well. Highlighted cases had Cook's distance  $> \min(0.5, 2p/n)$ . See Cook (1977). Figure 3.3 shows the DD plot of the residual vectors. Cases 8, 48 and 79 have especially large distances. The response, residual and DD plots are effective for finding influential cases, for checking linearity and whether the error distribution is multivariate normal or some other elliptically contoured distribution.

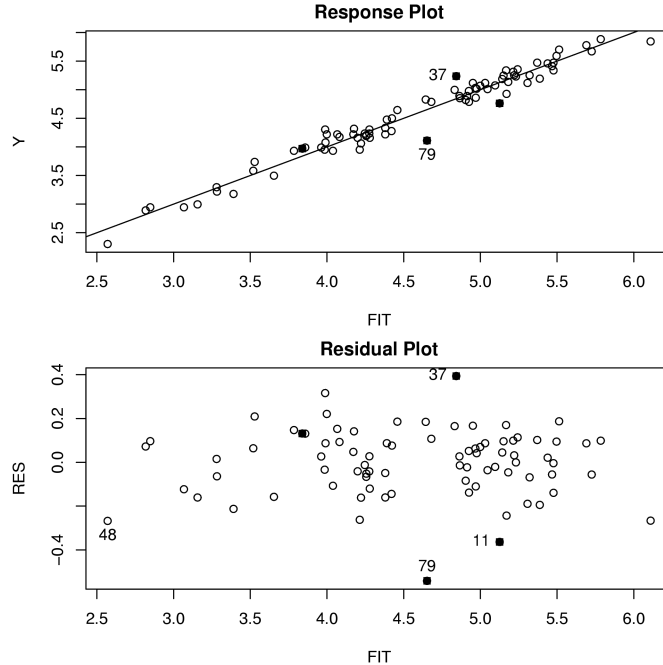


Figure 3.1. Plots for  $Y_1 = \log(W)$ .

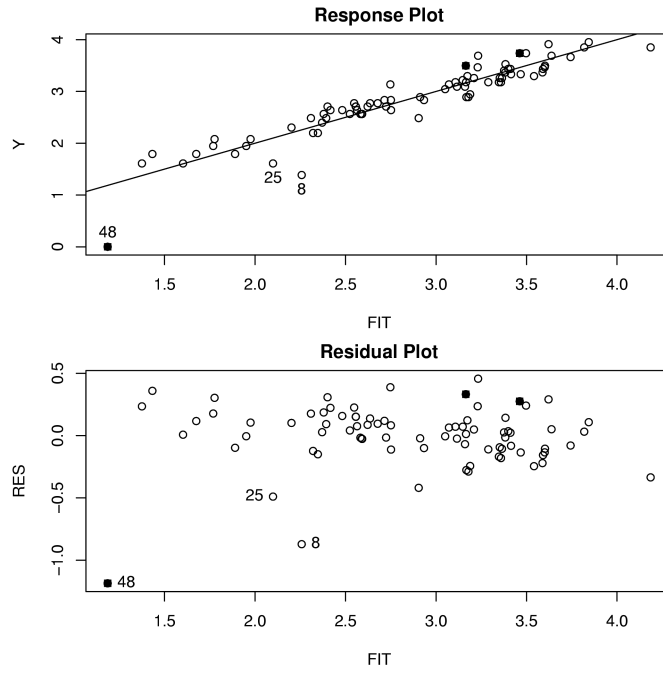


Figure 3.2. Plots for  $Y_2 = \log(M)$ .

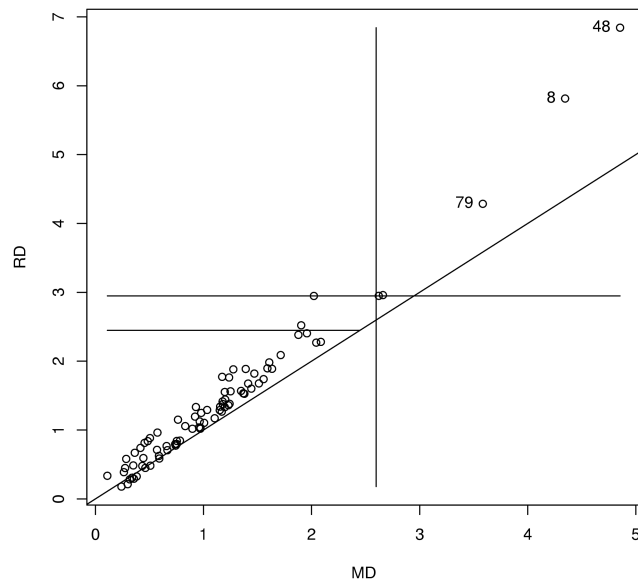


Figure 3.3. DD Plot of the Residual Vectors for the Mussel Data.

A simulation was used to study the Wilk's Lambda test, the Pillai's trace test, the Hotelling Lawley trace test, and the Roy's largest root test for the  $F_j$  tests and the MANOVA  $F$  test for multivariate linear regression. The first row of  $\mathbf{B}$  was always  $\mathbf{1}^T$  and the last row of  $\mathbf{B}$  was always  $\mathbf{0}^T$ . When the null hypothesis for the MANOVA  $F$  test is true, all but the first row corresponding to the constant are equal to  $\mathbf{0}^T$ . When  $p \geq 3$  and the null hypothesis for the MANOVA  $F$  test is false, then the second to last row of  $\mathbf{B}$  is  $(1, 0, \dots, 0)$ , the third to last row is  $(1, 1, 0, \dots, 0)$  etcetera as long as the first row is not changed from  $\mathbf{1}^T$ . First  $m$  iid errors  $\mathbf{w}_i$  are generated such that the  $m$  errors are iid with variance  $\sigma^2$ . Let the  $m \times m$  matrix  $\mathbf{A} = (a_{ij})$  with  $a_{ii} = 1$  and  $a_{ij} = \psi$  where  $0 \leq \psi < 1$  for  $i \neq j$ . Then  $\boldsymbol{\epsilon}_i = \mathbf{A}\mathbf{w}_i$  so that  $\hat{\boldsymbol{\Sigma}}_{\boldsymbol{\epsilon}} = \sigma^2 \mathbf{A}\mathbf{A}^T = (\sigma_{ij})$  where the diagonal entries  $\sigma_{ii} = \sigma^2[1 + (m-1)\psi^2]$  and the off diagonal entries  $\sigma_{ij} = \sigma^2[2\psi + (m-2)\psi^2]$  where  $\psi = 0.10$ . Hence the correlations  $(2\psi + (m-2)\psi^2)/(1 + (m-1)\psi^2)$ . As  $\psi$  gets close to 1, the data clusters about the line in the direction of  $(1, \dots, 1)^T$ . Used  $\mathbf{w}_i \sim N_m(\mathbf{0}, \mathbf{I})$ ,  $\mathbf{w}_i \sim (1 - \tau)N_m(\mathbf{0}, \mathbf{I}) + \tau N_m(\mathbf{0}, 25\mathbf{I})$  with  $0 < \tau < 1$  and  $\tau = 0.25$  in the simulation,  $\mathbf{w}_i \sim$  multivariate  $t_d$  with  $d = 7$  degrees of freedom, or  $\mathbf{w}_i \sim$  lognormal -  $E(\text{lognormal})$ : where the  $m$  components of  $\mathbf{w}_i$  were iid with distribution  $e^z - E(e^z)$  where  $z \sim N(0, 1)$ . Only the lognormal distribution is not elliptically contoured.

The simulation used 5000 runs, and  $H_0$  was rejected if the  $F$  statistic was greater than  $F_{d_1, d_2}(0.95)$  where  $P(F_{d_1, d_2} < F_{d_1, d_2}(0.95)) = 0.95$  with  $d_1 = rm$  and  $d_2 = n - mp$  for the test statistics

$$\frac{-[n - p - 0.5(m - r + 3)]}{rm} \log(\Lambda(\mathbf{L})), \quad \frac{n - p}{rm} V(\mathbf{L}), \quad \text{and} \quad \frac{n - p}{rm} U(\mathbf{L})$$

while  $d_1 = h = \max(r, m)$  and  $d_2 = n - p - h + r$  for the test statistic

$$\frac{n - p - h + r}{h} \lambda_{\max}(\mathbf{L}).$$

Denote these statistics by *Wilk's test*, *Pillai's test*, *Hotelling-Lawley* and *Roy's test*. Let the coverage be the proportion of times that  $H_0$  is rejected. Want coverage near 0.05 when

$H_0$  is true and coverage close to 1 for good power when  $H_0$  is false. With 5000 runs, coverage outside of (0.04,0.06) suggests that the true coverage is not 0.05. Coverages are tabled for the  $b_1, b_2, b_{p-1}$ , and  $b_p$  test and for the MANOVA F test denoted by MANOVA  $F$ . Error distributions are denoted as follows; *etypeI* for MVN  $N_m(0, I)$ , *etypeII* for  $(1 - \text{eps})N_m(0, I) + \text{eps}N_m(0, 25I)$   $\text{eps} = 0.1, 0.25, 0.4$ , and  $0.6$  are interesting *etypeIII* for multivariate  $t_d$  with  $d = dd$  degrees of freedom  $dd = 1, 2, 3, 5, 7$  are interesting *etypeIV* for *lognormal* –  $E(\text{lognormal})$ . The null hypothesis  $H_0$  was always true for the  $b_p$  test and always false for the  $b_1$  test. When the MANOVA  $F$  test was true,  $H_0$  was true for the  $b_j$  tests with  $j \neq 1$ . When the MANOVA  $F$  test was false,  $H_0$  was false for the  $b_j$  tests with  $j \neq p$ , but the  $b_{p-1}$  test should be hardest to reject for  $j \neq p$  by construction of  $\mathbf{B}$  and the error vectors.

Results from Berndt and Savin (1977) suggest that Pillai's test will reject  $H_0$  less often than Wilk's test which will reject less often than the Hotelling Lawley test.

## CHAPTER 4

### COVERAGES: $H_0$ IS NOT TRUE FOR ERROR TYPE I

Table 4.1.  $H_0$  false - Error type I For  $m=3, p=3, n=30$

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	0.9998	0.9674		0.0266	0.8904
Pillai's test	0.9994	0.9414		0.0128	0.6946
Hotelling-Lawley	1.0000	0.9850		0.0586	0.9616
Roy's test	1.0000	0.9826		0.0502	0.9914

Table 4.2.  $H_0$  false - Error type I For  $m=3, p=3, n=80$

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	1.0000		0.0420	1.0000
Pillai's test	1.0000	1.0000		0.0368	1.0000
Hotelling-Lawley	1.0000	1.0000		0.0520	1.0000
Roy's test	1.0000	1.0000		0.0488	1.0000

Table 4.3.  $H_0$  false - Error type I For  $m=4, p=4, n=40$

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.9996	0.9928	0.0214	1.0000
Pillai's test	1.0000	0.9988	0.9830	0.0118	0.9994
Hotelling-Lawley	1.0000	1.0000	0.9970	0.0570	1.0000
Roy's test	1.0000	1.0000	0.9960	0.0464	1.0000

Table 4.4.  $H_0$  false - Error type I For  $m=4, p=4, n=90$

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	1.0000	1.0000	0.0384	1.0000
Pillai's test	1.0000	1.0000	1.0000	0.0334	1.0000
Hotelling-Lawley	1.0000	1.0000	1.0000	0.0528	1.0000
Roy's test	1.0000	1.0000	1.0000	0.0472	1.0000

Table 4.5.  $H_0$  false - Error type I For  $m=5, p=5, n=50$

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	1.0000	0.9986	0.0266	1.0000
Pillai's test	1.0000	1.0000	0.9954	0.0154	1.0000
Hotelling-Lawley	1.0000	1.0000	0.9998	0.0540	1.0000
Roy's test	1.0000	1.0000	0.9998	0.0502	1.0000

Table 4.6.  $H_0$  false - Error type I For  $m=6, p=6, n=60$

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	1.0000	0.9990	0.0180	1.0000
Pillai's test	1.0000	1.0000	0.9982	0.0072	1.0000
Hotelling-Lawley	1.0000	1.0000	0.9994	0.0476	1.0000
Roy's test	1.0000	1.0000	0.9994	0.0464	1.0000

Table 4.7.  $H_0$  false - Error type I For  $m=7, p=7, n=70$

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	1.0000	1.0000	0.0146	1.0000
Pillai's test	1.0000	1.0000	0.9996	0.0072	1.0000
Hotelling-Lawley	1.0000	1.0000	1.0000	0.0430	1.0000
Roy's test	1.0000	1.0000	1.0000	0.0502	1.0000

Table 4.8.  $H_0$  false - Error type I For  $m=8, p=8, n=80$

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	1.0000	0.9998	0.0088	1.0000
Pillai's test	1.0000	1.0000	0.9996	0.0038	1.0000
Hotelling-Lawley	1.0000	1.0000	1.0000	0.0280	1.0000
Roy's test	1.0000	1.0000	1.0000	0.0498	1.0000

Table 4.9.  $H_0$  false - Error type I For  $m=9, p=9, n=100$

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	1.0000	1.0000	0.0102	1.0000
Pillai's test	1.0000	1.0000	1.0000	0.0046	1.0000
Hotelling-Lawley	1.0000	1.0000	1.0000	0.0308	1.0000
Roy's test	1.0000	1.0000	1.0000	0.0540	1.0000

Table 4.10.  $H_0$  false - Error type I For  $m=10, p=10, n=110$

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	1.0000	1.0000	0.0006	1.0000
Pillai's test	1.0000	1.0000	0.9998	0.0000	1.0000
Hotelling-Lawley	1.0000	1.0000	1.0000	0.0060	1.0000
Roy's test	1.0000	1.0000	1.0000	0.0558	1.0000

Table 4.11.  $H_0$  false - Error type I For  $m=11, p=11, n=140$

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	1.0000	1.0000	0.0068	1.0000
Pillai's test	1.0000	1.0000	1.0000	0.0036	1.0000
Hotelling-Lawley	1.0000	1.0000	1.0000	0.0206	1.0000
Roy's test	1.0000	1.0000	1.0000	0.0484	1.0000



Table 4.12.  $H_0$  false - Error type I For  $m=12, p=12, n=160$

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	1.0000	1.0000	0.0030	1.0000
Pillai's test	1.0000	1.0000	1.0000	0.0014	1.0000
Hotelling-Lawley	1.0000	1.0000	1.0000	0.0110	1.0000
Roy's test	1.0000	1.0000	1.0000	0.0442	1.0000

Table 4.13.  $H_0$  false - Error type I For  $m=13, p=13, n=180$

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	1.0000	1.0000	0.0006	1.0000
Pillai's test	1.0000	1.0000	1.0000	0.0000	1.0000
Hotelling-Lawley	1.0000	1.0000	1.0000	0.0034	1.0000
Roy's test	1.0000	1.0000	1.0000	0.0460	1.0000

Table 4.14.  $H_0$  false - Error type I For  $m=14, p=14, n=210$

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	1.0000	1.0000	0.0006	1.0000
Pillai's test	1.0000	1.0000	1.0000	0.0002	1.0000
Hotelling-Lawley	1.0000	1.0000	1.0000	0.0050	1.0000
Roy's test	1.0000	1.0000	1.0000	0.0514	1.0000

Table 4.15.  $H_0$  false - Error type I For  $m=15$ ,  $p=15$ ,  $n=240$

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	1.0000	1.0000	0.0026	1.0000
Pillai's test	1.0000	1.0000	1.0000	0.0020	1.0000
Hotelling-Lawley	1.0000	1.0000	1.0000	0.0054	1.0000
Roy's test	1.0000	1.0000	1.0000	0.0532	1.0000

## CHAPTER 5

### COVERAGES: $H_0$ IS TRUE FOR ERROR TYPES I, II, III AND IV

#### 5.1 ERROR TYPE I

Table 5.1. *Error type I For  $m=2, p=2, n=20$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	0.9944			0.0278	0.0278
Pillai's test	0.9824			0.0132	0.0132
Hotelling -Lawley test	0.9986			0.0624	0.0624
Roy's test	0.9986			0.0552	0.0552

Table 5.2. *Error type I For  $m=2, p=2, n=70$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000			0.0464	0.0464
Pillai's test	1.0000			0.0418	0.0418
Hotelling -Lawley test	1.0000			0.0544	0.0544
Roy's test	1.0000			0.0516	0.0516

Table 5.3. *Error type I For  $m=2, p=2, n=120$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000			0.0474	0.0474
Pillai's test	1.0000			0.0450	0.0450
Hotelling -Lawley test	1.0000			0.0514	0.0514
Roy's test	1.0000			0.0506	0.0506

Table 5.4. *Error type I For  $m=2, p=2, n=170$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000			0.0456	0.0456
Pillai's test	1.0000			0.0434	0.0434
Hotelling -Lawley test	1.0000			0.0492	0.0492
Roy's test	1.0000			0.0480	0.0480

Table 5.5. *Error type I For  $m=2, p=2, n=220$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000			0.0428	0.0428
Pillai's test	1.0000			0.0412	0.0412
Hotelling -Lawley test	1.0000			0.0466	0.0466
Roy's test	1.0000			0.0450	0.0450

Table 5.6. *Error type I For  $m=2, p=2, n=270$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000			0.0476	0.0476
Pillai's test	1.0000			0.0468	0.0468
Hotelling -Lawley test	1.0000			0.0496	0.0496
Roy's test	1.0000			0.0494	0.0494

Table 5.7. *Error type I For  $m=2, p=2, n=320$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000			0.0490	0.0490
Pillai's test	1.0000			0.0482	0.0482
Hotelling -Lawley test	1.0000			0.0512	0.0512
Roy's test	1.0000			0.0504	0.0504

Table 5.8. *Error type I For  $m=2, p=2, n=370$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000			0.0476	0.0476
Pillai's test	1.0000			0.0468	0.0468
Hotelling -Lawley test	1.0000			0.0488	0.0488
Roy's test	1.0000			0.0486	0.0486

Table 5.9. *Error type I For  $m=3, p=3, n=30$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0258		0.0298	0.0192
Pillai's test	1.0000	0.0142		0.0138	0.0050
Hotelling -Lawley test	1.0000	0.0560		0.0620	0.0552
Roy's test	1.0000	0.0488		0.0534	0.1704

Table 5.10. *Error type I For  $m=3, p=3, n=80$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0388		0.0410	0.0392
Pillai's test	1.0000	0.0340		0.0362	0.0296
Hotelling -Lawley test	1.0000	0.0478		0.0532	0.0542
Roy's test	1.0000	0.0440		0.0494	0.1482

Table 5.11. *Error type I For  $m=3, p=3, n=130$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0468		0.0474	0.0428
Pillai's test	1.0000	0.0438		0.0448	0.0386
Hotelling -Lawley test	1.0000	0.0552		0.0550	0.0542
Roy's test	1.0000	0.0530		0.0530	0.1630

Table 5.12. *Error type I For  $m=3, p=3, n=180$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0516		0.0398	0.0420
Pillai's test	1.0000	0.0490		0.0382	0.0378
Hotelling -Lawley test	1.0000	0.0580		0.0440	0.0494
Roy's test	1.0000	0.0558		0.0426	0.1560

Table 5.13. *Error type I For  $m=3, p=3, n=230$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0522		0.0468	0.0464
Pillai's test	1.0000	0.0500		0.0438	0.0430
Hotelling -Lawley test	1.0000	0.0558		0.0502	0.0536
Roy's test	1.0000	0.0546		0.0488	0.1626

Table 5.14. *Error type I For  $m=3, p=3, n=280$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0486		0.0494	0.0464
Pillai's test	1.0000	0.0478		0.0478	0.0434
Hotelling -Lawley test	1.0000	0.0514		0.0512	0.0520
Roy's test	1.0000	0.0506		0.0506	0.1600

Table 5.15. *Error type I For  $m=3, p=3, n=330$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0476		0.0500	0.0496
Pillai's test	1.0000	0.0458		0.0484	0.0474
Hotelling -Lawley test	1.0000	0.0528		0.0534	0.0526
Roy's test	1.0000	0.0506		0.0520	0.1596

Table 5.16. *Error type I For  $m=3, p=3, n=380$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0434		0.0482	0.0450
Pillai's test	1.0000	0.0422		0.0472	0.0422
Hotelling -Lawley test	1.0000	0.0456		0.0512	0.0488
Roy's test	1.0000	0.0444		0.0498	0.1564

Table 5.17. *Error type I For  $m=3, p=3, n=430$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0456		0.0456	0.0452
Pillai's test	1.0000	0.0446		0.0444	0.0424
Hotelling -Lawley test	1.0000	0.0486		0.0468	0.0486
Roy's test	1.0000	0.0482		0.0468	0.1538



Table 5.18. *Error type I For  $m=3, p=3, n=480$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0486		0.0468	0.0498
Pillai's test	1.0000	0.0480		0.0460	0.0476
Hotelling -Lawley test	1.0000	0.0502		0.0496	0.0522
Roy's test	1.0000	0.0496		0.0488	0.1604

Table 5.19. *Error type I For  $m=3, p=3, n=530$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0456		0.0522	0.0498
Pillai's test	1.0000	0.0448		0.0514	0.0488
Hotelling -Lawley test	1.0000	0.0476		0.0540	0.0522
Roy's test	1.0000	0.0468		0.0538	0.1580

Table 5.20. *Error type I For  $m=3, p=3, n=580$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0476		0.0502	0.0484
Pillai's test	1.0000	0.0468		0.0488	0.0476
Hotelling -Lawley test	1.0000	0.0490		0.0526	0.0506
Roy's test	1.0000	0.0486		0.0512	0.1570

Table 5.21. *Error type I For  $m=4, p=4, n=40$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0256	0.0238	0.0240	0.0120
Pillai's test	1.0000	0.0134	0.0118	0.0128	0.0014
Hotelling -Lawley test	1.0000	0.0564	0.0562	0.0574	0.0496
Roy's test	1.0000	0.0494	0.0492	0.0482	0.3400

Table 5.22. *Error type I For  $m=4, p=4, n=90$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0422	0.0372	0.0424	0.0336
Pillai's test	1.0000	0.0370	0.0316	0.0358	0.0212
Hotelling -Lawley test	1.0000	0.0608	0.0530	0.0574	0.0538
Roy's test	1.0000	0.0534	0.0478	0.0520	0.3244

Table 5.23. *Error type I For  $m=4, p=4, n=140$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0432	0.0468	0.0442	0.0376
Pillai's test	1.0000	0.0392	0.0436	0.0384	0.0306
Hotelling -Lawley test	1.0000	0.0502	0.0578	0.0530	0.0534
Roy's test	1.0000	0.0470	0.0540	0.0498	0.3266

Table 5.24. *Error type I For  $m=4, p=4, n=190$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0458	0.0486	0.0480	0.0458
Pillai's test	1.0000	0.0442	0.0444	0.0448	0.0382
Hotelling -Lawley test	1.0000	0.0526	0.0554	0.0558	0.0556
Roy's test	1.0000	0.0500	0.0518	0.0524	0.3248

Table 5.25. *Error type I For  $m=4, p=4, n=240$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0464	0.0426	0.0462	0.0464
Pillai's test	1.0000	0.0436	0.0404	0.0442	0.0404
Hotelling -Lawley test	1.0000	0.0540	0.0476	0.0518	0.0540
Roy's test	1.0000	0.0518	0.0452	0.0494	0.3236

Table 5.26. *Error type I For  $m=4, p=4, n=290$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0466	0.0462	0.0454	0.0452
Pillai's test	1.0000	0.0456	0.0444	0.0444	0.0410
Hotelling -Lawley test	1.0000	0.0512	0.0508	0.0496	0.0534
Roy's test	1.0000	0.0486	0.0490	0.0474	0.3262

Table 5.27. *Error type I For  $m=4, p=4, n=340$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0486	0.0470	0.0478	0.0408
Pillai's test	1.0000	0.0466	0.0454	0.0470	0.0386
Hotelling -Lawley test	1.0000	0.0526	0.0494	0.0528	0.0484
Roy's test	1.0000	0.0514	0.0492	0.0500	0.3218

Table 5.28. *Error type I For  $m=4, p=4, n=390$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0524	0.0496	0.0480	0.0498
Pillai's test	1.0000	0.0520	0.0480	0.0464	0.0458
Hotelling -Lawley test	1.0000	0.0550	0.0538	0.0524	0.0580
Roy's test	1.0000	0.0534	0.0522	0.0502	0.3300

Table 5.29. *Error type I For  $m=4, p=4, n=440$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0474	0.0524	0.0560	0.0498
Pillai's test	1.0000	0.0462	0.0508	0.0550	0.0464
Hotelling -Lawley test	1.0000	0.0506	0.0558	0.0588	0.0550
Roy's test	1.0000	0.0490	0.0538	0.0574	0.3308

Table 5.30. *Error type I For  $m=4, p=4, n=490$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0518	0.0474	0.0498	0.0476
Pillai's test	1.0000	0.0514	0.0460	0.0496	0.0436
Hotelling -Lawley test	1.0000	0.0538	0.0496	0.0526	0.0508
Roy's test	1.0000	0.0526	0.0484	0.0518	0.3192

Table 5.31. *Error type I For  $m=4, p=4, n=540$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0494	0.0406	0.0476	0.0470
Pillai's test	1.0000	0.0478	0.0398	0.0470	0.0436
Hotelling -Lawley test	1.0000	0.0506	0.0434	0.0492	0.0500
Roy's test	1.0000	0.0502	0.0422	0.0482	0.3080

Table 5.32. *Error type I For  $m=4, p=4, n=590$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0530	0.0468	0.0468	0.0512
Pillai's test	1.0000	0.0520	0.0460	0.0464	0.0482
Hotelling -Lawley test	1.0000	0.0540	0.0482	0.0490	0.0534
Roy's test	1.0000	0.0536	0.0478	0.0480	0.3278

Table 5.33. *Error type I For  $m=5, p=5, n=50$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0238	0.0222	0.0266	0.0052
Pillai's test	1.0000	0.0128	0.0120	0.0140	0.0004
Hotelling -Lawley test	1.0000	0.0522	0.0532	0.0554	0.0344
Roy's test	1.0000	0.0474	0.0490	0.0486	0.5506

Table 5.34. *Error type I For  $m=5, p=5, n=100$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0390	0.0358	0.0430	0.0222
Pillai's test	1.0000	0.0322	0.0300	0.0358	0.0114
Hotelling -Lawley test	1.0000	0.0564	0.0544	0.0626	0.0476
Roy's test	1.0000	0.0494	0.0458	0.0552	0.5348

Table 5.35. *Error type I For  $m=5, p=5, n=150$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0412	0.0426	0.0430	0.0300
Pillai's test	1.0000	0.0366	0.0392	0.0362	0.0202
Hotelling -Lawley test	1.0000	0.0520	0.0554	0.0530	0.0478
Roy's test	1.0000	0.0472	0.0508	0.0498	0.5324

Table 5.36. *Error type I For  $m=5, p=5, n=200$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0450	0.0456	0.0444	0.0338
Pillai's test	1.0000	0.0428	0.0420	0.0416	0.0248
Hotelling -Lawley test	1.0000	0.0536	0.0548	0.0536	0.0500
Roy's test	1.0000	0.0500	0.0504	0.0492	0.5252

Table 5.37. *Error type I For  $m=5, p=5, n=250$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0470	0.0484	0.0440	0.0390
Pillai's test	1.0000	0.0452	0.0454	0.0414	0.0314
Hotelling -Lawley test	1.0000	0.0518	0.0564	0.0494	0.0502
Roy's test	1.0000	0.0494	0.0526	0.0474	0.5210

Table 5.38. *Error type I For  $m=5, p=5, n=300$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0448	0.0420	0.0454	0.0422
Pillai's test	1.0000	0.0436	0.0396	0.0430	0.0348
Hotelling -Lawley test	1.0000	0.0496	0.0486	0.0498	0.0520
Roy's test	1.0000	0.0472	0.0454	0.0482	0.5202

Table 5.39. *Error type I For  $m=5, p=5, n=350$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0438	0.0484	0.0466	0.0434
Pillai's test	1.0000	0.0414	0.0476	0.0446	0.0374
Hotelling -Lawley test	1.0000	0.0490	0.0528	0.0502	0.0478
Roy's test	1.0000	0.0472	0.0512	0.0484	0.5142



Table 5.40. *Error type I For  $m=5, p=5, n=400$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0500	0.0488	0.0448	0.0448
Pillai's test	1.0000	0.0484	0.0470	0.0440	0.0390
Hotelling -Lawley test	1.0000	0.0556	0.0532	0.0512	0.0546
Roy's test	1.0000	0.0526	0.0510	0.0478	0.5310

Table 5.41. *Error type I For  $m=5, p=5, n=450$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0522	0.0464	0.0462	0.0490
Pillai's test	1.0000	0.0504	0.0458	0.0450	0.0436
Hotelling -Lawley test	1.0000	0.0548	0.0488	0.0506	0.0548
Roy's test	1.0000	0.0538	0.0474	0.0486	0.5120

Table 5.42. *Error type I For  $m=5, p=5, n=500$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0498	0.0472	0.0496	0.0472
Pillai's test	1.0000	0.0484	0.0458	0.0484	0.0424
Hotelling -Lawley test	1.0000	0.0554	0.0516	0.0538	0.0540
Roy's test	1.0000	0.0530	0.0502	0.0522	0.5348

Table 5.43. *Error type I For  $m=5, p=5, n=550$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0462	0.0502	0.0502	0.0412
Pillai's test	1.0000	0.0458	0.0494	0.0492	0.0364
Hotelling -Lawley test	1.0000	0.0490	0.0522	0.0532	0.0464
Roy's test	1.0000	0.0478	0.0510	0.0518	0.5170

Table 5.44. *Error type I For  $m=5, p=5, n=600$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0500	0.0478	0.0510	0.0530
Pillai's test	1.0000	0.0482	0.0470	0.0488	0.0488
Hotelling -Lawley test	1.0000	0.0518	0.0492	0.0536	0.0582
Roy's test	1.0000	0.0514	0.0484	0.0520	0.5238

## 5.2 ERROR TYPE II

Table 5.45. *Error typeII For  $m=2, p=2, n=20$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	0.4342	0.0264			0.0264
Pillai's test	0.3360	0.0114			0.0114
Hotelling -Lawley test	0.5418	0.0558			0.0558
Roy's test	0.5262	0.0498			0.0498

Table 5.46. *Error typeII For  $m=2, p=2, n=70$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	0.9402	0.0414			0.0414
Pillai's test	0.9346	0.0366			0.0366
Hotelling -Lawley test	0.9488	0.0486			0.0486
Roy's test	0.9466	0.0468			0.0468

Table 5.47. *Error typeII For m=2, p=2, n=120*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	0.9950	0.0500			0.0500
Pillai's test	0.9944	0.0486			0.0486
Hotelling -Lawley test	0.9956	0.0548			0.0548
Roy's test	0.9956	0.0542			0.0542

Table 5.48. *Error typeII For m=2, p=2, n=170*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	0.9998	0.0488			0.0488
Pillai's test	0.9998	0.0472			0.0472
Hotelling -Lawley test	0.9998	0.0518			0.0518
Roy's test	0.9998	0.0504			0.0504

Table 5.49. *Error typeII For m=2, p=2, n=220*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0464			0.0464
Pillai's test	1.0000	0.0452			0.0452
Hotelling -Lawley test	1.0000	0.0496			0.0496
Roy's test	1.0000	0.0494			0.0494

Table 5.50. *Error typeII For  $m=3, p=3, n=30$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	0.6218	0.0268		0.0244	0.0178
Pillai's test	0.5378	0.0148		0.0120	0.0034
Hotelling -Lawley test	0.7288	0.0586		0.0574	0.0506
Roy's test	0.7084	0.0520		0.0474	0.1628

Table 5.51. *Error typeII For  $m=3, p=3, n=80$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	0.9770	0.0448		0.0456	0.0380
Pillai's test	0.9746	0.0392		0.0410	0.0282
Hotelling -Lawley test	0.9820	0.0586		0.0558	0.0542
Roy's test	0.9796	0.0538		0.0524	0.1650

Table 5.52. *Error typeII For  $m=3, p=3, n=130$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	0.9994	0.0444		0.0422	0.0424
Pillai's test	0.9994	0.0404		0.0386	0.0358
Hotelling -Lawley test	0.9994	0.0492		0.0498	0.0536
Roy's test	0.9994	0.0476		0.0474	0.1586

Table 5.53. *Error typeII For  $m=3, p=3, n=180$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0462		0.0464	0.0422
Pillai's test	1.0000	0.0436		0.0446	0.0376
Hotelling -Lawley test	1.0000	0.0508		0.0508	0.0484
Roy's test	1.0000	0.0488		0.0488	0.1574

Table 5.54. *Error typeII For m=3, p=3, n=230*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0460		0.0488	0.0450
Pillai's test	1.0000	0.0436		0.0466	0.0430
Hotelling -Lawley test	1.0000	0.0502		0.0524	0.0516
Roy's test	1.0000	0.0492		0.0510	0.1544

Table 5.55. *Error typeII For m=3, p=3, n=280*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0482		0.0428	0.0482
Pillai's test	1.0000	0.0470		0.0416	0.0444
Hotelling -Lawley test	1.0000	0.0516		0.0462	0.0534
Roy's test	1.0000	0.0502		0.0446	0.1522

Table 5.56. *Error typeII For m=4, p=4, n=40*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	0.7606	0.0280	0.0258	0.0290	0.0108
Pillai's test	0.6924	0.0180	0.0136	0.0162	0.0014
Hotelling -Lawley test	0.8438	0.0602	0.0580	0.0608	0.0540
Roy's test	0.8302	0.0540	0.0490	0.0530	0.3506

Table 5.57. *Error typeII For  $m=4, p=4, n=90$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	0.9894	0.0416	0.0414	0.0414	0.0304
Pillai's test	0.9874	0.0344	0.0352	0.0366	0.0146
Hotelling -Lawley test	0.9924	0.0556	0.0538	0.0550	0.0542
Roy's test	0.9914	0.0510	0.0492	0.0502	0.3356

Table 5.58. *Error typeII For  $m=4, p=4, n=140$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0438	0.0438	0.0476	0.0362
Pillai's test	1.0000	0.0386	0.0406	0.0440	0.0282
Hotelling -Lawley test	1.0000	0.0536	0.0528	0.0584	0.0550
Roy's test	1.0000	0.0496	0.0490	0.0546	0.3350

Table 5.59. *Error typeII For  $m=4, p=4, n=190$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0462	0.0500	0.0392	0.0414
Pillai's test	1.0000	0.0424	0.0478	0.0374	0.0362
Hotelling -Lawley test	1.0000	0.0530	0.0582	0.0476	0.0518
Roy's test	1.0000	0.0498	0.0542	0.0444	0.3296



Table 5.60. *Error typeII For m=4, p=4, n=240*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0450	0.0492	0.0446	0.0422
Pillai's test	1.0000	0.0428	0.0474	0.0426	0.0380
Hotelling -Lawley test	1.0000	0.0534	0.0554	0.0516	0.0508
Roy's test	1.0000	0.0506	0.0530	0.0478	0.3236

Table 5.61. *Error typeII For m=4, p=4, n=290*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0470	0.0452	0.0444	0.0436
Pillai's test	1.0000	0.0456	0.0436	0.0432	0.0384
Hotelling -Lawley test	1.0000	0.0524	0.0494	0.0474	0.0492
Roy's test	1.0000	0.0500	0.0478	0.0460	0.3208

Table 5.62. *Error typeII For m=4, p=4, n=340*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0488	0.0578	0.0518	0.0508
Pillai's test	1.0000	0.0472	0.0564	0.0502	0.0456
Hotelling -Lawley test	1.0000	0.0524	0.0628	0.0546	0.0576
Roy's test	1.0000	0.0512	0.0618	0.0538	0.3326

Table 5.63. *Error typeII For m=4, p=4, n=390*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0456	0.0516	0.0478	0.0468
Pillai's test	1.0000	0.0442	0.0508	0.0472	0.0430
Hotelling -Lawley test	1.0000	0.0490	0.0544	0.0518	0.0520
Roy's test	1.0000	0.0478	0.0534	0.0504	0.3264

Table 5.64. *Error typeII For m=4, p=4, n=440*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0500	0.0484	0.0490	0.0448
Pillai's test	1.0000	0.0494	0.0478	0.0480	0.0414
Hotelling -Lawley test	1.0000	0.0530	0.0522	0.0524	0.0498
Roy's test	1.0000	0.0516	0.0506	0.0514	0.3250

Table 5.65. *Error typeII For m=5, p=5, n=50*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	0.8400	0.0218	0.0258	0.0270	0.0056
Pillai's test	0.7828	0.0128	0.0154	0.0154	0.0000
Hotelling -Lawley test	0.9070	0.0540	0.0574	0.0622	0.0378
Roy's test	0.8970	0.0478	0.0544	0.0548	0.5718

Table 5.66. *Error typeII For m=5, p=5, n=100*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	0.9942	0.0352	0.0370	0.0364	0.0248
Pillai's test	0.9932	0.0304	0.0310	0.0300	0.0122
Hotelling -Lawley test	0.9968	0.0548	0.0512	0.0544	0.0514
Roy's test	0.9956	0.0480	0.0468	0.0472	0.5268

Table 5.67. *Error typeII For m=5, p=5, n=150*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0406	0.0408	0.0474	0.0344
Pillai's test	1.0000	0.0376	0.0368	0.0424	0.0232
Hotelling -Lawley test	1.0000	0.0538	0.0534	0.0594	0.0568
Roy's test	1.0000	0.0480	0.0484	0.0542	0.5274

Table 5.68. *Error typeII For m=5, p=5, n=200*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0470	0.0490	0.0460	0.0410
Pillai's test	1.0000	0.0424	0.0444	0.0430	0.0328
Hotelling -Lawley test	1.0000	0.0574	0.0572	0.0548	0.0548
Roy's test	1.0000	0.0532	0.0538	0.0514	0.5444

Table 5.69. *Error typeII For m=5, p=5, n=250*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0420	0.0478	0.0426	0.0372
Pillai's test	1.0000	0.0400	0.0442	0.0410	0.0324
Hotelling -Lawley test	1.0000	0.0502	0.0560	0.0500	0.0478
Roy's test	1.0000	0.0470	0.0526	0.0466	0.5118

Table 5.70. *Error typeII For m=5, p=5, n=300*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0468	0.0484	0.0544	0.0386
Pillai's test	1.0000	0.0440	0.0458	0.0520	0.0386
Hotelling -Lawley test	1.0000	0.0532	0.0512	0.0598	0.0386
Roy's test	1.0000	0.0500	0.0500	0.0568	0.0386

Table 5.71. *Error typeII For m=5, p=5, n=350*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0480	0.0484	0.0432	0.0400
Pillai's test	1.0000	0.0454	0.0472	0.0414	0.0336
Hotelling -Lawley test	1.0000	0.0524	0.0526	0.0470	0.0458
Roy's test	1.0000	0.0498	0.0506	0.0454	0.5202

Table 5.72. *Error typeII For m=5, p=5, n=400*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0490	0.0492	0.0450	0.0410
Pillai's test	1.0000	0.0474	0.0474	0.0436	0.0364
Hotelling -Lawley test	1.0000	0.0526	0.0536	0.0490	0.0498
Roy's test	1.0000	0.0510	0.0520	0.0464	0.5262

Table 5.73. *Error typeII For m=5, p=5, n=450*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0462	0.0450	0.0456	0.0460
Pillai's test	1.0000	0.0448	0.0438	0.0440	0.0414
Hotelling -Lawley test	1.0000	0.0496	0.0488	0.0504	0.0518
Roy's test	1.0000	0.0478	0.0470	0.0482	0.5238

Table 5.74. *Error typeII For m=5, p=5, n=500*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0502	0.0450	0.0468	0.0442
Pillai's test	1.0000	0.0484	0.0442	0.0458	0.0398
Hotelling -Lawley test	1.0000	0.0524	0.0488	0.0498	0.0510
Roy's test	1.0000	0.0510	0.0474	0.0486	0.5272

Table 5.75. *Error typeII For m=5, p=5, n=550*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0482	0.0446	0.0446	0.0426
Pillai's test	1.0000	0.0470	0.0432	0.0438	0.0382
Hotelling -Lawley test	1.0000	0.0514	0.0466	0.0474	0.0492
Roy's test	1.0000	0.0500	0.0452	0.0458	0.5230

Table 5.76. *Error typeII For m=5, p=5, n=600*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0470	0.0496	0.0522	0.0508
Pillai's test	1.0000	0.0462	0.0486	0.0520	0.0488
Hotelling -Lawley test	1.0000	0.0502	0.0530	0.0546	0.0552
Roy's test	1.0000	0.0486	0.0510	0.0532	0.5236

Table 5.77. *Error typeII For  $m=5, p=5, n=650$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0520	0.0444	0.0480	0.0454
Pillai's test	1.0000	0.0514	0.0430	0.0466	0.0428
Hotelling -Lawley test	1.0000	0.0548	0.0468	0.0502	0.0498
Roy's test	1.0000	0.0534	0.0450	0.0486	0.5326

### 5.3 ERROR TYPE III

Table 5.78. *Error typeIII For  $m=2, p=2, n=20$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	0.9492	0.0260			0.0260
Pillai's test	0.9070	0.0124			0.0124
Hotelling -Lawley test	0.9752	0.0524			0.0524
Roy's test	0.9720	0.0466			0.0466

Table 5.79. *Error typeIII For  $m=2, p=2, n=70$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0466			0.0466
Pillai's test	1.0000	0.0402			0.0402
Hotelling -Lawley test	1.0000	0.0550			0.0550
Roy's test	1.0000	0.0538			0.0538



Table 5.80. *Error typeIII For  $m=2, p=2, n=120$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0448			0.0448
Pillai's test	1.0000	0.0428			0.0428
Hotelling -Lawley test	1.0000	0.0492			0.0492
Roy's test	1.0000	0.0482			0.0482

Table 5.81. *Error typeIII For  $m=3, p=3, n=30$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0426		0.0408	0.0378
Pillai's test	1.0000	0.0366		0.0344	0.0274
Hotelling -Lawley test	1.0000	0.0504		0.0536	0.0550
Roy's test	1.0000	0.0482		0.0498	0.1610

Table 5.82. *Error typeIII For  $m=3, p=3, n=80$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0426		0.0408	0.0378
Pillai's test	1.0000	0.0366		0.0344	0.0274
Hotelling -Lawley test	1.0000	0.0504		0.0536	0.0550
Roy's test	1.0000	0.0482		0.0498	0.1610

Table 5.83. *Error typeIII For  $m=3, p=3, n=130$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0428		0.0426	0.0404
Pillai's test	1.0000	0.0394		0.0404	0.0346
Hotelling -Lawley test	1.0000	0.0506		0.0506	0.0508
Roy's test	1.0000	0.0472		0.0466	0.1538

Table 5.84. *Error typeIII For  $m=3, p=3, n=180$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0426		0.0488	0.0460
Pillai's test	1.0000	0.0398		0.0460	0.0398
Hotelling -Lawley test	1.0000	0.0468		0.0524	0.0518
Roy's test	1.0000	0.0456		0.0514	0.1548

Table 5.85. *Error typeIII For  $m=3, p=3, n=230$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0484		0.0478	0.0476
Pillai's test	1.0000	0.0460		0.0458	0.0440
Hotelling -Lawley test	1.0000	0.0514		0.0520	0.0524
Roy's test	1.0000	0.0502		0.0500	0.1636

Table 5.86. *Error typeIII For  $m=3, p=3, n=280$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0502		0.0476	0.0452
Pillai's test	1.0000	0.0488		0.0462	0.0416
Hotelling -Lawley test	1.0000	0.0526		0.0512	0.0498
Roy's test	1.0000	0.0518		0.0490	0.1634

Table 5.87. *Error typeIII For  $m=3, p=3, n=330$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0500		0.0476	0.0520
Pillai's test	1.0000	0.0490		0.0460	0.0492
Hotelling -Lawley test	1.0000	0.0538		0.0500	0.0560
Roy's test	1.0000	0.0526		0.0490	0.1578

Table 5.88. *Error typeIII For  $m=4, p=4, n=40$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	0.9998	0.0252	0.0266	0.0318	0.0126
Pillai's test	0.9998	0.0132	0.0140	0.0144	0.0008
Hotelling -Lawley test	1.0000	0.0604	0.0620	0.0596	0.0532
Roy's test	1.0000	0.0524	0.0542	0.0534	0.3536

Table 5.89. *Error typeIII For  $m=4, p=4, n=90$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0422	0.0410	0.0406	0.0314
Pillai's test	1.0000	0.0378	0.0350	0.0358	0.0214
Hotelling -Lawley test	1.0000	0.0586	0.0534	0.0532	0.0534
Roy's test	1.0000	0.0524	0.0482	0.0484	0.3318

Table 5.90. *Error typeIII For  $m=4, p=4, n=140$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0394	0.0422	0.0462	0.0356
Pillai's test	1.0000	0.0368	0.0368	0.0434	0.0260
Hotelling -Lawley test	1.0000	0.0518	0.0522	0.0548	0.0514
Roy's test	1.0000	0.0460	0.0478	0.0522	0.3290

Table 5.91. *Error typeIII For  $m=4, p=4, n=190$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0438	0.0466	0.0492	0.0428
Pillai's test	1.0000	0.0416	0.0436	0.0454	0.0346
Hotelling -Lawley test	1.0000	0.0500	0.0528	0.0550	0.0512
Roy's test	1.0000	0.0474	0.0500	0.0530	0.3320

Table 5.92. *Error typeIII For  $m=4, p=4, n=240$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0478	0.0416	0.0486	0.0440
Pillai's test	1.0000	0.0456	0.0402	0.0466	0.0370
Hotelling -Lawley test	1.0000	0.0514	0.0466	0.0554	0.0514
Roy's test	1.0000	0.0506	0.0446	0.0530	0.3258

Table 5.93. *Error typeIII For  $m=4, p=4, n=290$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0478	0.0526	0.0492	0.0472
Pillai's test	1.0000	0.0462	0.0494	0.0474	0.0432
Hotelling -Lawley test	1.0000	0.0522	0.0578	0.0518	0.0552
Roy's test	1.0000	0.0498	0.0560	0.0506	0.3282

Table 5.94. *Error typeIII For  $m=4, p=4, n=340$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0442	0.0498	0.0468	0.0508
Pillai's test	1.0000	0.0426	0.0490	0.0456	0.0468
Hotelling -Lawley test	1.0000	0.0494	0.0558	0.0506	0.0566
Roy's test	1.0000	0.0472	0.0530	0.0486	0.3354

Table 5.95. *Error typeIII For  $m=4, p=4, n=390$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0484	0.0562	0.0418	0.0478
Pillai's test	1.0000	0.0466	0.0544	0.0402	0.0442
Hotelling -Lawley test	1.0000	0.0536	0.0590	0.0448	0.0516
Roy's test	1.0000	0.0516	0.0574	0.0436	0.3356

Table 5.96. *Error typeIII For  $m=5, p=5, n=50$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0262	0.0230	0.0244	0.0030
Pillai's test	1.0000	0.0150	0.0112	0.0120	0.0000
Hotelling -Lawley test	1.0000	0.0608	0.0554	0.0548	0.0346
Roy's test	1.0000	0.0528	0.0498	0.0490	0.5600

Table 5.97. *Error typeIII For m=5, p=5, n=100*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0376	0.0426	0.0420	0.0244
Pillai's test	1.0000	0.0316	0.0350	0.0340	0.0120
Hotelling -Lawley test	1.0000	0.0550	0.0636	0.0616	0.0540
Roy's test	1.0000	0.0494	0.0550	0.0536	0.5400

Table 5.98. *Error typeIII For m=5, p=5, n=150*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0434	0.0426	0.0410	0.0294
Pillai's test	1.0000	0.0386	0.0378	0.0358	0.0184
Hotelling -Lawley test	1.0000	0.0548	0.0562	0.0496	0.0486
Roy's test	1.0000	0.0498	0.0514	0.0460	0.5312

Table 5.99. *Error typeIII For m=5, p=5, n=200*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0484	0.0442	0.0416	0.0378
Pillai's test	1.0000	0.0436	0.0410	0.0382	0.0308
Hotelling -Lawley test	1.0000	0.0586	0.0538	0.0502	0.0534
Roy's test	1.0000	0.0534	0.0476	0.0474	0.5264

Table 5.100. *Error typeIII For m=5, p=5, n=250*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0510	0.0432	0.0490	0.0412
Pillai's test	1.0000	0.0474	0.0392	0.0466	0.0338
Hotelling -Lawley test	1.0000	0.0576	0.0498	0.0546	0.0510
Roy's test	1.0000	0.0558	0.0462	0.0516	0.5168

Table 5.101. *Error typeIII For m=5, p=5, n=300*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0492	0.0476	0.0434	0.0416
Pillai's test	1.0000	0.0480	0.0452	0.0414	0.0360
Hotelling -Lawley test	1.0000	0.0552	0.0536	0.0492	0.0494
Roy's test	1.0000	0.0514	0.0510	0.0460	0.5176

Table 5.102. *Error typeIII For m=5, p=5, n=350*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0454	0.0456	0.0450	0.0454
Pillai's test	1.0000	0.0434	0.0442	0.0442	0.0388
Hotelling -Lawley test	1.0000	0.0498	0.0526	0.0500	0.0536
Roy's test	1.0000	0.0480	0.0492	0.0484	0.5106



Table 5.103. *Error typeIII For m=5, p=5, n=400*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0502	0.0426	0.0422	0.0404
Pillai's test	1.0000	0.0492	0.0404	0.0410	0.0350
Hotelling -Lawley test	1.0000	0.0554	0.0472	0.0466	0.0488
Roy's test	1.0000	0.0524	0.0444	0.0446	0.5212

Table 5.104. *Error typeIII For m=5, p=5, n=450*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0452	0.0466	0.0458	0.0412
Pillai's test	1.0000	0.0426	0.0450	0.0452	0.0362
Hotelling -Lawley test	1.0000	0.0494	0.0518	0.0494	0.0472
Roy's test	1.0000	0.0474	0.0496	0.0474	0.5224

Table 5.105. *Error typeIII For m=5, p=5, n=500*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0486	0.0462	0.0448	0.0454
Pillai's test	1.0000	0.0474	0.0450	0.0432	0.0414
Hotelling -Lawley test	1.0000	0.0528	0.0494	0.0486	0.0532
Roy's test	1.0000	0.0506	0.0478	0.0470	0.5152

Table 5.106. *Error typeIII For m=6, p=6, n=60*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0194	0.0244	0.0198	0.0014
Pillai's test	1.0000	0.0126	0.0132	0.0122	0.0000
Hotelling -Lawley test	1.0000	0.0498	0.0566	0.0512	0.0212
Roy's test	1.0000	0.0482	0.0556	0.0500	0.7532

Table 5.107. *Error typeIII For m=6, p=6, n=110*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0384	0.0374	0.0348	0.0192
Pillai's test	1.0000	0.0314	0.0300	0.0276	0.0054
Hotelling -Lawley test	1.0000	0.0570	0.0574	0.0554	0.0462
Roy's test	1.0000	0.0502	0.0490	0.0462	0.7178

Table 5.108. *Error typeIII For m=6, p=6, n=160*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0434	0.0452	0.0414	0.0244
Pillai's test	1.0000	0.0378	0.0406	0.0368	0.0136
Hotelling -Lawley test	1.0000	0.0572	0.0590	0.0566	0.0476
Roy's test	1.0000	0.0514	0.0536	0.0510	0.7158

Table 5.109. *Error typeIII For m=6, p=6, n=210*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0472	0.0514	0.0480	0.0376
Pillai's test	1.0000	0.0430	0.0468	0.0446	0.0250
Hotelling -Lawley test	1.0000	0.0572	0.0614	0.0566	0.0584
Roy's test	1.0000	0.0522	0.0564	0.0530	0.7142

Table 5.110. *Error typeIII For m=6, p=6, n=260*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0464	0.0456	0.0426	0.0354
Pillai's test	1.0000	0.0442	0.0412	0.0404	0.0272
Hotelling -Lawley test	1.0000	0.0544	0.0518	0.0522	0.0502
Roy's test	1.0000	0.0506	0.0488	0.0486	0.7122

Table 5.111. *Error typeIII For m=6, p=6, n=310*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0432	0.0422	0.0470	0.0390
Pillai's test	1.0000	0.0406	0.0398	0.0452	0.0318
Hotelling -Lawley test	1.0000	0.0486	0.0498	0.0554	0.0500
Roy's test	1.0000	0.0460	0.0464	0.0506	0.7106

Table 5.112. *Error typeIII For  $m=6, p=6, n=360$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0452	0.0468	0.0432	0.0372
Pillai's test	1.0000	0.0428	0.0446	0.0414	0.0288
Hotelling -Lawley test	1.0000	0.0500	0.0528	0.0494	0.0510
Roy's test	1.0000	0.0474	0.0500	0.0456	0.7048

Table 5.113. *Error typeIII For  $m=6, p=6, n=410$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0462	0.0486	0.0488	0.0416
Pillai's test	1.0000	0.0436	0.0468	0.0466	0.0342
Hotelling -Lawley test	1.0000	0.0510	0.0526	0.0546	0.0520
Roy's test	1.0000	0.0490	0.0496	0.0516	0.6994

Table 5.114. *Error typeIII For m=6, p=6, n=460*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0482	0.0414	0.0524	0.0404
Pillai's test	1.0000	0.0468	0.0382	0.0514	0.0350
Hotelling -Lawley test	1.0000	0.0540	0.0464	0.0560	0.0474
Roy's test	1.0000	0.0508	0.0444	0.0536	0.7008

Table 5.115. *Error typeIII For m=6, p=6, n=510*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0488	0.0484	0.0484	0.0426
Pillai's test	1.0000	0.0460	0.0476	0.0474	0.0366
Hotelling -Lawley test	1.0000	0.0520	0.0538	0.0528	0.0512
Roy's test	1.0000	0.0514	0.0508	0.0512	0.7012

Table 5.116. *Error typeIII For m=6, p=6, n=560*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0444	0.0502	0.0528	0.0448
Pillai's test	1.0000	0.0438	0.0476	0.0522	0.0380
Hotelling -Lawley test	1.0000	0.0488	0.0542	0.0558	0.0496
Roy's test	1.0000	0.0462	0.0534	0.0534	0.6994

Table 5.117. *Error typeIII For m=6, p=6, n=610*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0526	0.0498	0.0486	0.0452
Pillai's test	1.0000	0.0506	0.0492	0.0478	0.0384
Hotelling -Lawley test	1.0000	0.0556	0.0532	0.0520	0.0516
Roy's test	1.0000	0.0540	0.0508	0.0500	0.7052

Table 5.118. *Error typeIII For m=6, p=6, n=660*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0466	0.0494	0.0506	0.0478
Pillai's test	1.0000	0.0462	0.0488	0.0490	0.0434
Hotelling -Lawley test	1.0000	0.0488	0.0528	0.0530	0.0526
Roy's test	1.0000	0.0478	0.0512	0.0514	0.7074

Table 5.119. *Error typeIII For m=7, p=7, n=70*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0190	0.0182	0.0148	0.0002
Pillai's test	1.0000	0.0090	0.0110	0.0076	0.0000
Hotelling -Lawley test	1.0000	0.0462	0.0450	0.0416	0.0114
Roy's test	1.0000	0.0548	0.0516	0.0474	0.8728

Table 5.120. *Error typeIII For m=7, p=7, n=120*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0326	0.0384	0.0386	0.0096
Pillai's test	1.0000	0.0264	0.0312	0.0312	0.0022
Hotelling -Lawley test	1.0000	0.0540	0.0562	0.0592	0.0406
Roy's test	1.0000	0.0452	0.0490	0.0510	0.8526

Table 5.121. *Error typeIII For m=7, p=7, n=170*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0402	0.0420	0.0390	0.0230
Pillai's test	1.0000	0.0354	0.0368	0.0344	0.0118
Hotelling -Lawley test	1.0000	0.0570	0.0606	0.0510	0.0514
Roy's test	1.0000	0.0490	0.0510	0.0454	0.8598

Table 5.122. *Error typeIII For m=7, p=7, n=220*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0444	0.0436	0.0430	0.0286
Pillai's test	1.0000	0.0408	0.0396	0.0390	0.0170
Hotelling -Lawley test	1.0000	0.0582	0.0558	0.0544	0.0524
Roy's test	1.0000	0.0504	0.0496	0.0476	0.8506

Table 5.123. *Error typeIII For  $m=7, p=7, n=270$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0462	0.0448	0.0458	0.0362
Pillai's test	1.0000	0.0448	0.0406	0.0418	0.0242
Hotelling -Lawley test	1.0000	0.0560	0.0542	0.0562	0.0542
Roy's test	1.0000	0.0514	0.0488	0.0516	0.8516

Table 5.124. *Error typeIII For  $m=7, p=7, n=320$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0416	0.0440	0.0432	0.0362
Pillai's test	1.0000	0.0388	0.0406	0.0410	0.0264
Hotelling -Lawley test	1.0000	0.0482	0.0522	0.0512	0.0516
Roy's test	1.0000	0.0438	0.0482	0.0472	0.8436

Table 5.125. *Error typeIII For  $m=7, p=7, n=370$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0458	0.0476	0.0462	0.0394
Pillai's test	1.0000	0.0442	0.0460	0.0043	0.0282
Hotelling -Lawley test	1.0000	0.0524	0.0526	0.0534	0.0534
Roy's test	1.0000	0.0486	0.0502	0.0496	0.8536



Table 5.126. *Error typeIII For  $m=7, p=7, n=420$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0434	0.0408	0.0458	0.0352
Pillai's test	1.0000	0.0422	0.0396	0.0434	0.0276
Hotelling -Lawley test	1.0000	0.0502	0.0474	0.0538	0.0468
Roy's test	1.0000	0.0480	0.0448	0.0498	0.8484

Table 5.127. *Error typeIII For  $m=7, p=7, n=470$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0492	0.0418	0.0460	0.0412
Pillai's test	1.0000	0.0474	0.0406	0.0448	0.0320
Hotelling -Lawley test	1.0000	0.0558	0.0450	0.0506	0.0514
Roy's test	1.0000	0.0518	0.0438	0.0482	0.8446

Table 5.128. *Error typeIII For  $m=7, p=7, n=520$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0510	0.0440	0.0472	0.0396
Pillai's test	1.0000	0.0484	0.0422	0.0464	0.0328
Hotelling -Lawley test	1.0000	0.0570	0.0486	0.0512	0.0498
Roy's test	1.0000	0.0542	0.0458	0.0494	0.8470

Table 5.129. *Error typeIII For m=7, p=7, n=570*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0454	0.0476	0.0430	0.0436
Pillai's test	1.0000	0.0440	0.0452	0.0420	0.0376
Hotelling -Lawley test	1.0000	0.0506	0.0524	0.0476	0.0514
Roy's test	1.0000	0.0472	0.0490	0.0452	0.8506

Table 5.130. *Error typeIII For m=7, p=7, n=620*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0478	0.0456	0.0460	0.0384
Pillai's test	1.0000	0.0464	0.0444	0.0448	0.0324
Hotelling -Lawley test	1.0000	0.0544	0.0498	0.0512	0.0472
Roy's test	1.0000	0.0502	0.0474	0.0484	0.8408

Table 5.131. *Error typeIII For m=7, p=7, n=670*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0472	0.0518	0.0484	0.0450
Pillai's test	1.0000	0.0460	0.0502	0.0468	0.0382
Hotelling -Lawley test	1.0000	0.0522	0.0544	0.0528	0.0522
Roy's test	1.0000	0.0494	0.0524	0.0500	0.8390

Table 5.132. *Error typeIII For m=7, p=7, n=720*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0482	0.0482	0.0480	0.0392
Pillai's test	1.0000	0.0468	0.0474	0.0476	0.0348
Hotelling -Lawley test	1.0000	0.0526	0.0516	0.0510	0.0466
Roy's test	1.0000	0.0498	0.0492	0.0492	0.8506

Table 5.133. *Error typeIII For m=7, p=7, n=770*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0518	0.0468	0.0508	0.0432
Pillai's test	1.0000	0.0504	0.0454	0.0496	0.0382
Hotelling -Lawley test	1.0000	0.0560	0.0488	0.0548	0.0494
Roy's test	1.0000	0.0538	0.0478	0.0522	0.8466

Table 5.134. *Error typeIII For  $m=7, p=7, n=820$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0470	0.0518	0.0494	0.0478
Pillai's test	1.0000	0.0466	0.0510	0.0490	0.0428
Hotelling -Lawley test	1.0000	0.0490	0.0544	0.0520	0.0552
Roy's test	1.0000	0.0478	0.0530	0.0506	0.8502

Table 5.135. *Error typeIII For  $m=7, p=7, n=870$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0494	0.0478	0.0526	0.0404
Pillai's test	1.0000	0.0482	0.0462	0.0514	0.0356
Hotelling -Lawley test	1.0000	0.0524	0.0498	0.0552	0.0472
Roy's test	1.0000	0.0512	0.0490	0.0534	0.8528

Table 5.136. *Error typeIII For  $m=7, p=7, n=920$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0476	0.0502	0.0422	0.0458
Pillai's test	1.0000	0.0464	0.0490	0.0412	0.0418
Hotelling -Lawley test	1.0000	0.0502	0.0544	0.0448	0.0526
Roy's test	1.0000	0.0490	0.0520	0.0432	0.8450

Table 5.137. *Error typeIII For  $m=8, p=8, n=80$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0080	0.0080	0.0074	0.0000
Pillai's test	1.0000	0.0032	0.0036	0.0028	0.0000
Hotelling -Lawley test	1.0000	0.0300	0.0320	0.0246	0.0006
Roy's test	1.0000	0.0526	0.0510	0.0464	0.9574

Table 5.138. *Error typeIII For  $m=8, p=8, n=130$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0312	0.0338	0.0384	0.0072
Pillai's test	1.0000	0.0246	0.0248	0.0310	0.0012
Hotelling -Lawley test	1.0000	0.0538	0.0562	0.0582	0.0312
Roy's test	1.0000	0.0476	0.0492	0.0524	0.9496

Table 5.139. *Error typeIII For  $m=8, p=8, n=180$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0362	0.0442	0.0366	0.0186
Pillai's test	1.0000	0.0324	0.0384	0.0298	0.0060
Hotelling -Lawley test	1.0000	0.0532	0.0610	0.0552	0.0412
Roy's test	1.0000	0.0462	0.0542	0.0452	0.9386

Table 5.140. *Error typeIII For m=8, p=8, n=230*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0440	0.0424	0.0420	0.0232
Pillai's test	1.0000	0.0404	0.0378	0.0376	0.0110
Hotelling -Lawley test	1.0000	0.0594	0.0554	0.0566	0.0468
Roy's test	1.0000	0.0518	0.0496	0.0502	0.9348

Table 5.141. *Error typeIII For m=8, p=8, n=280*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0420	0.0472	0.0446	0.0236
Pillai's test	1.0000	0.0390	0.0426	0.0406	0.0148
Hotelling -Lawley test	1.0000	0.0536	0.0626	0.0570	0.0450
Roy's test	1.0000	0.0470	0.0554	0.0502	0.9378

Table 5.142. *Error typeIII For m=8, p=8, n=330*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0430	0.0424	0.0500	0.0292
Pillai's test	1.0000	0.0394	0.0406	0.0464	0.0174
Hotelling -Lawley test	1.0000	0.0528	0.0518	0.0592	0.0418
Roy's test	1.0000	0.0466	0.0468	0.0536	0.9308

Table 5.143. *Error typeIII For m=8, p=8, n=380*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0446	0.0432	0.0472	0.0298
Pillai's test	1.0000	0.0416	0.0412	0.0452	0.0204
Hotelling -Lawley test	1.0000	0.0534	0.0520	0.0560	0.0446
Roy's test	1.0000	0.0484	0.0466	0.0514	0.9406

Table 5.144. *Error typeIII For m=8, p=8, n=430*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0508	0.0446	0.0476	0.0390
Pillai's test	1.0000	0.0478	0.0432	0.0462	0.0286
Hotelling -Lawley test	1.0000	0.0552	0.0510	0.0550	0.0536
Roy's test	1.0000	0.0526	0.0478	0.0504	0.9318

Table 5.145. *Error typeIII For m=8, p=8, n=480*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0470	0.0464	0.0486	0.0384
Pillai's test	1.0000	0.0446	0.0450	0.0462	0.0296
Hotelling -Lawley test	1.0000	0.0528	0.0506	0.0574	0.0526
Roy's test	1.0000	0.0492	0.0480	0.0540	0.9376

Table 5.146. *Error typeIII For  $m=8, p=8, n=530$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0426	0.0406	0.0456	0.0346
Pillai's test	1.0000	0.0406	0.0394	0.0434	0.0272
Hotelling -Lawley test	1.0000	0.0492	0.0472	0.0544	0.0464
Roy's test	1.0000	0.0442	0.0442	0.0500	0.9342

Table 5.147. *Error typeIII For  $m=8, p=8, n=580$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0486	0.0516	0.0414	0.0386
Pillai's test	1.0000	0.0474	0.0500	0.0390	0.0310
Hotelling -Lawley test	1.0000	0.0538	0.0560	0.0446	0.0492
Roy's test	1.0000	0.0504	0.0536	0.0428	0.9296



Table 5.148. *Error typeIII For m=8, p=8, n=630*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0470	0.0468	0.0466	0.0392
Pillai's test	1.0000	0.0456	0.0458	0.0456	0.0312
Hotelling -Lawley test	1.0000	0.0516	0.0510	0.0518	0.0496
Roy's test	1.0000	0.0484	0.0488	0.0486	0.9296

Table 5.149. *Error typeIII For m=8, p=8, n=680*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0476	0.0382	0.0498	0.0394
Pillai's test	1.0000	0.0454	0.0366	0.0490	0.0328
Hotelling -Lawley test	1.0000	0.0508	0.0440	0.0544	0.0474
Roy's test	1.0000	0.0494	0.0412	0.0520	0.9414

Table 5.150. *Error typeIII For m=8, p=8, n=730*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0436	0.0494	0.0484	0.0396
Pillai's test	1.0000	0.0420	0.0472	0.0474	0.0346
Hotelling -Lawley test	1.0000	0.0490	0.0540	0.0520	0.0498
Roy's test	1.0000	0.0466	0.0520	0.0498	0.9348

Table 5.151. *Error typeIII For m=8, p=8, n=780*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0536	0.0426	0.0478	0.0390
Pillai's test	1.0000	0.0516	0.0416	0.0470	0.0316
Hotelling -Lawley test	1.0000	0.0580	0.0452	0.0516	0.0462
Roy's test	1.0000	0.0554	0.0448	0.0490	0.9298

Table 5.152. *Error typeIII For m=8, p=8, n=830*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0502	0.0514	0.0448	0.0404
Pillai's test	1.0000	0.0486	0.0502	0.0440	0.0364
Hotelling -Lawley test	1.0000	0.0544	0.0542	0.0492	0.0472
Roy's test	1.0000	0.0520	0.0526	0.0468	0.9292

Table 5.153. *Error typeIII For m=8, p=8, n=880*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0464	0.0504	0.0486	0.0478
Pillai's test	1.0000	0.0454	0.0496	0.0478	0.0406
Hotelling -Lawley test	1.0000	0.0512	0.0528	0.0528	0.0532
Roy's test	1.0000	0.0486	0.0514	0.0512	0.9292

Table 5.154. *Error typeIII For m=8, p=8, n=930*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0456	0.0470	0.0494	0.0428
Pillai's test	1.0000	0.0450	0.0460	0.0476	0.0374
Hotelling -Lawley test	1.0000	0.0492	0.0512	0.0534	0.0502
Roy's test	1.0000	0.0480	0.0484	0.0510	0.9296

Table 5.155. *Error typeIII For m=8, p=8, n=980*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0502	0.0462	0.0432	0.0450
Pillai's test	1.0000	0.0486	0.0452	0.0426	0.0390
Hotelling -Lawley test	1.0000	0.0530	0.0502	0.0470	0.0516
Roy's test	1.0000	0.0520	0.0482	0.0450	0.9394

Table 5.156. *Error typeIII For m=8, p=8, n=1030*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0526	0.0452	0.0512	0.0412
Pillai's test	1.0000	0.0514	0.0434	0.0502	0.0374
Hotelling -Lawley test	1.0000	0.0570	0.0484	0.0548	0.0480
Roy's test	1.0000	0.0538	0.0468	0.0528	0.9348

Table 5.157. *Error typeIII For m=8, p=8, n=1080*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0478	0.0502	0.0506	0.0462
Pillai's test	1.0000	0.0474	0.0500	0.0496	0.0410
Hotelling -Lawley test	1.0000	0.0498	0.0536	0.0530	0.0522
Roy's test	1.0000	0.0488	0.0516	0.0520	0.9340

Table 5.158. *Error typeIII For m=8, p=8, n=1130*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0508	0.0464	0.0488	0.0462
Pillai's test	1.0000	0.0500	0.0454	0.0482	0.0410
Hotelling -Lawley test	1.0000	0.0526	0.0496	0.0520	0.0522
Roy's test	1.0000	0.0518	0.0474	0.0500	0.9340

Table 5.159. *Error typeIII For m=8, p=8, n=1180*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0496	0.0430	0.0416	0.0388
Pillai's test	1.0000	0.0490	0.0424	0.0410	0.0364
Hotelling -Lawley test	1.0000	0.0520	0.0450	0.0442	0.0440
Roy's test	1.0000	0.0504	0.0442	0.0428	0.9328

Table 5.160. *Error typeIII For m=8, p=8, n=1230*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0514	0.0578	0.0488	0.0496
Pillai's test	1.0000	0.0500	0.0572	0.0480	0.0442
Hotelling -Lawley test	1.0000	0.0538	0.0602	0.0512	0.0538
Roy's test	1.0000	0.0528	0.0588	0.0496	0.9320

Table 5.161. *Error typeIII For m=8, p=8, n=1280*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0442	0.0480	0.0514	0.0512
Pillai's test	1.0000	0.0430	0.0476	0.0512	0.0466
Hotelling -Lawley test	1.0000	0.0466	0.0520	0.0536	0.0564
Roy's test	1.0000	0.0448	0.0494	0.0524	0.9354

Table 5.162. *Error typeIII For m=9, p=9, n=100*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0108	0.0086	0.0072	0.0000
Pillai's test	1.0000	0.0052	0.0040	0.0042	0.0000
Hotelling -Lawley test	1.0000	0.0320	0.0282	0.0262	0.0002
Roy's test	1.0000	0.0510	0.0474	0.0440	0.9846

Table 5.163. *Error typeIII For m=9, p=9, n=150*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0346	0.0342	0.0340	0.0048
Pillai's test	1.0000	0.0290	0.0266	0.0264	0.0006
Hotelling -Lawley test	1.0000	0.0588	0.0568	0.0582	0.0288
Roy's test	1.0000	0.0528	0.0506	0.0506	0.9750

Table 5.164. *Error typeIII For m=9, p=9, n=200*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0358	0.0396	0.0436	0.0110
Pillai's test	1.0000	0.0318	0.0336	0.0366	0.0036
Hotelling -Lawley test	1.0000	0.0530	0.0568	0.0604	0.0340
Roy's test	1.0000	0.0458	0.0510	0.0538	0.9790

Table 5.165. *Error typeIII For m=9, p=9, n=250*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0408	0.0472	0.0448	0.0194
Pillai's test	1.0000	0.0360	0.0424	0.0400	0.0076
Hotelling -Lawley test	1.0000	0.0540	0.0598	0.0592	0.0428
Roy's test	1.0000	0.0464	0.0536	0.0516	0.9774

Table 5.166. *Error typeIII For m=9, p=9, n=300*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0428	0.0446	0.0498	0.0262
Pillai's test	1.0000	0.0400	0.0400	0.0460	0.0148
Hotelling -Lawley test	1.0000	0.0532	0.0570	0.0616	0.0438
Roy's test	1.0000	0.0474	0.0504	0.0554	0.9768

Table 5.167. *Error typeIII For m=9, p=9, n=350*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0422	0.0498	0.0442	0.0314
Pillai's test	1.0000	0.0400	0.0472	0.0404	0.0198
Hotelling -Lawley test	1.0000	0.0532	0.0598	0.0560	0.0500
Roy's test	1.0000	0.0468	0.0542	0.0488	0.9768

Table 5.168. *Error typeIII For m=9, p=9, n=400*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0452	0.0492	0.0460	0.0294
Pillai's test	1.0000	0.0428	0.0458	0.0426	0.0184
Hotelling -Lawley test	1.0000	0.0540	0.0560	0.0550	0.0508
Roy's test	1.0000	0.0500	0.0520	0.0512	0.9782

Table 5.169. *Error typeIII For m=9, p=9, n=450*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0454	0.0496	0.0464	0.0290
Pillai's test	1.0000	0.0418	0.0474	0.0440	0.0218
Hotelling -Lawley test	1.0000	0.0534	0.0572	0.0538	0.0462
Roy's test	1.0000	0.0488	0.0536	0.0488	0.9760

Table 5.170. *Error typeIII For m=9, p=9, n=500*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0414	0.0418	0.0466	0.0332
Pillai's test	1.0000	0.0386	0.0404	0.0436	0.0244
Hotelling -Lawley test	1.0000	0.0484	0.0478	0.0530	0.0472
Roy's test	1.0000	0.0438	0.0444	0.0488	0.9730



Table 5.171. *Error typeIII For m=9, p=9, n=550*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0510	0.0470	0.0520	0.0352
Pillai's test	1.0000	0.0490	0.0440	0.0504	0.0280
Hotelling -Lawley test	1.0000	0.0570	0.0530	0.0574	0.0486
Roy's test	1.0000	0.0534	0.0500	0.0538	0.9732

Table 5.172. *Error typeIII For m=9, p=9, n=600*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0452	0.0504	0.0474	0.0426
Pillai's test	1.0000	0.0432	0.0486	0.0456	0.0330
Hotelling -Lawley test	1.0000	0.0492	0.0574	0.0534	0.0560
Roy's test	1.0000	0.0466	0.0520	0.0498	0.9748

Table 5.173. *Error typeIII For m=9, p=9, n=650*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0466	0.0450	0.0480	0.0356
Pillai's test	1.0000	0.0452	0.0432	0.0468	0.0260
Hotelling -Lawley test	1.0000	0.0510	0.0522	0.0538	0.0482
Roy's test	1.0000	0.0480	0.0482	0.0510	0.9754

Table 5.174. *Error typeIII For m=9, p=9, n=700*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0514	0.0502	0.0476	0.0406
Pillai's test	1.0000	0.0500	0.0486	0.0466	0.0328
Hotelling -Lawley test	1.0000	0.0574	0.0552	0.0514	0.0498
Roy's test	1.0000	0.0536	0.0534	0.0494	0.9772

Table 5.175. *Error typeIII For m=9, p=9, n=750*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0468	0.0532	0.0488	0.0390
Pillai's test	1.0000	0.0456	0.0516	0.0476	0.0316
Hotelling -Lawley test	1.0000	0.0534	0.0600	0.0532	0.0492
Roy's test	1.0000	0.0496	0.0562	0.0506	0.9780

Table 5.176. *Error typeIII For m=9, p=9, n=800*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0464	0.0480	0.0518	0.0382
Pillai's test	1.0000	0.0450	0.0476	0.0508	0.0322
Hotelling -Lawley test	1.0000	0.0500	0.0538	0.0560	0.0482
Roy's test	1.0000	0.0484	0.0508	0.0540	0.9786

Table 5.177. *Error typeIII For m=9, p=9, n=850*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0510	0.0440	0.0470	0.0378
Pillai's test	1.0000	0.0494	0.0428	0.0458	0.0312
Hotelling -Lawley test	1.0000	0.0562	0.0496	0.0510	0.0468
Roy's test	1.0000	0.0528	0.0460	0.0492	0.9752

Table 5.178. *Error typeIII For m=9, p=9, n=900*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0452	0.0458	0.0536	0.0460
Pillai's test	1.0000	0.0434	0.0446	0.0514	0.0378
Hotelling -Lawley test	1.0000	0.0502	0.0490	0.0588	0.0566
Roy's test	1.0000	0.0468	0.0470	0.0548	0.9752

Table 5.179. *Error typeIII For m=9, p=9, n=950*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0412	0.0486	0.0452	0.0394
Pillai's test	1.0000	0.0398	0.0472	0.0436	0.0338
Hotelling -Lawley test	1.0000	0.0452	0.0520	0.0496	0.0482
Roy's test	1.0000	0.0430	0.0508	0.0468	0.9776

Table 5.180. *Error typeIII For m=9, p=9, n=1000*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0504	0.0490	0.0488	0.0382
Pillai's test	1.0000	0.0496	0.0482	0.0474	0.0314
Hotelling -Lawley test	1.0000	0.0534	0.0522	0.0512	0.0468
Roy's test	1.0000	0.0516	0.0506	0.0502	0.9748

Table 5.181. *Error typeIII For m=9, p=9, n=1050*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0538	0.0488	0.0470	0.0452
Pillai's test	1.0000	0.0530	0.0472	0.0466	0.0398
Hotelling -Lawley test	1.0000	0.0562	0.0514	0.0514	0.0518
Roy's test	1.0000	0.0550	0.0496	0.0500	0.9754

Table 5.182. *Error typeIII For m=9, p=9, n=1100*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0514	0.0484	0.0502	0.0482
Pillai's test	1.0000	0.0508	0.0476	0.0488	0.0408
Hotelling -Lawley test	1.0000	0.0532	0.0522	0.0538	0.0550
Roy's test	1.0000	0.0528	0.0506	0.0510	0.9794

Table 5.183. *Error typeIII For m=9, p=9, n=1150*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0480	0.0486	0.0520	0.0410
Pillai's test	1.0000	0.0468	0.0482	0.0514	0.0356
Hotelling -Lawley test	1.0000	0.0506	0.0510	0.0556	0.0482
Roy's test	1.0000	0.0494	0.0496	0.0534	0.9780

Table 5.184. *Error typeIII For m=9, p=9, n=1200*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0494	0.0496	0.0526	0.0460
Pillai's test	1.0000	0.0484	0.0488	0.0516	0.0412
Hotelling -Lawley test	1.0000	0.0518	0.0516	0.0578	0.0524
Roy's test	1.0000	0.0500	0.0504	0.0548	0.9714

Table 5.185. *Error typeIII For m=9, p=9, n=1250*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0544	0.0490	0.0534	0.0404
Pillai's test	1.0000	0.0538	0.0480	0.0528	0.0358
Hotelling -Lawley test	1.0000	0.0576	0.0508	0.0558	0.0484
Roy's test	1.0000	0.0558	0.0494	0.0544	0.9756

Table 5.186. *Error typeIII For m=9, p=9, n=1300*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0496	0.0538	0.0548	0.0452
Pillai's test	1.0000	0.0494	0.0530	0.0536	0.0404
Hotelling -Lawley test	1.0000	0.0522	0.0564	0.0568	0.0516
Roy's test	1.0000	0.0510	0.0544	0.0558	0.9742

Table 5.187. *Error typeIII For m=10, p=10, n=110*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0002	0.0010	0.0012	0.0000
Pillai's test	1.0000	0.0000	0.0002	0.0002	0.0000
Hotelling -Lawley test	1.0000	0.0042	0.0068	0.0050	0.0000
Roy's test	1.0000	0.0464	0.0504	0.0460	0.9958

Table 5.188. *Error typeIII For  $m=10, p=10, n=160$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0296	0.0284	0.0320	0.0020
Pillai's test	1.0000	0.0226	0.0210	0.0228	0.0004
Hotelling -Lawley test	1.0000	0.0536	0.0530	0.0522	0.0124
Roy's test	1.0000	0.0504	0.0496	0.0492	0.9962

Table 5.189. *Error typeIII For  $m=10, p=10, n=210$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0414	0.0420	0.0368	0.0084
Pillai's test	1.0000	0.0362	0.0358	0.0332	0.0018
Hotelling -Lawley test	1.0000	0.0614	0.0604	0.0536	0.0302
Roy's test	1.0000	0.0528	0.0544	0.0468	0.9928

Table 5.190. *Error typeIII For  $m=10, p=10, n=260$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0406	0.0430	0.0400	0.0152
Pillai's test	1.0000	0.0360	0.0388	0.0344	0.0048
Hotelling -Lawley test	1.0000	0.0564	0.0594	0.0558	0.0378
Roy's test	1.0000	0.0478	0.0524	0.0498	0.9938

Table 5.191. *Error typeIII For  $m=10, p=10, n=310$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0390	0.0366	0.0508	0.0228
Pillai's test	1.0000	0.0366	0.0328	0.0450	0.0090
Hotelling -Lawley test	1.0000	0.0534	0.0526	0.0658	0.0448
Roy's test	1.0000	0.0456	0.0438	0.0576	0.9930

Table 5.192. *Error typeIII For  $m=10, p=10, n=360$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0432	0.0422	0.0464	0.0198
Pillai's test	1.0000	0.0384	0.0400	0.0436	0.0116
Hotelling -Lawley test	1.0000	0.0544	0.0520	0.0542	0.0446
Roy's test	1.0000	0.0492	0.0468	0.0506	0.9942

Table 5.193. *Error typeIII For  $m=10, p=10, n=410$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0460	0.0438	0.0460	0.0264
Pillai's test	1.0000	0.0438	0.0402	0.0422	0.0138
Hotelling -Lawley test	1.0000	0.0552	0.0526	0.0552	0.0452
Roy's test	1.0000	0.0490	0.0480	0.0498	0.9948



Table 5.194. *Error typeIII For m=10, p=10, n=460*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0484	0.0428	0.0490	0.0290
Pillai's test	1.0000	0.0464	0.0398	0.0460	0.0188
Hotelling -Lawley test	1.0000	0.0576	0.0522	0.0592	0.0478
Roy's test	1.0000	0.0524	0.0470	0.0540	0.9936

Table 5.195. *Error typeIII For m=10, p=10, n=510*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0488	0.0476	0.0450	0.0342
Pillai's test	1.0000	0.0462	0.0448	0.0430	0.0228
Hotelling -Lawley test	1.0000	0.0560	0.0556	0.0534	0.0500
Roy's test	1.0000	0.0528	0.0518	0.0470	0.9934

Table 5.196. *Error typeIII For m=10, p=10, n=560*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0478	0.0560	0.0434	0.0322
Pillai's test	1.0000	0.0448	0.0550	0.0416	0.0214
Hotelling -Lawley test	1.0000	0.0562	0.0646	0.0504	0.0506
Roy's test	1.0000	0.0516	0.0592	0.0470	0.9940

Table 5.197. *Error typeIII For m=10, p=10, n=610*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0466	0.0480	0.0474	0.0356
Pillai's test	1.0000	0.0440	0.0458	0.0456	0.0262
Hotelling -Lawley test	1.0000	0.0558	0.0552	0.0556	0.0500
Roy's test	1.0000	0.0530	0.0508	0.0502	0.9912

Table 5.198. *Error typeIII For m=10, p=10, n=660*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0502	0.0484	0.0468	0.0332
Pillai's test	1.0000	0.0488	0.0458	0.0458	0.0242
Hotelling -Lawley test	1.0000	0.0582	0.0546	0.0540	0.0454
Roy's test	1.0000	0.0534	0.0510	0.0494	0.9916

Table 5.199. *Error typeIII For m=10, p=10, n=710*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0478	0.0500	0.0500	0.0376
Pillai's test	1.0000	0.0462	0.0484	0.0492	0.0290
Hotelling -Lawley test	1.0000	0.0530	0.0552	0.0556	0.0520
Roy's test	1.0000	0.0506	0.0522	0.0528	0.9930

Table 5.200. *Error typeIII For m=10, p=10, n=760*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0428	0.0462	0.0450	0.0368
Pillai's test	1.0000	0.0412	0.0452	0.0436	0.0274
Hotelling -Lawley test	1.0000	0.0474	0.0516	0.0518	0.0480
Roy's test	1.0000	0.0444	0.0478	0.0470	0.9910

Table 5.201. *Error typeIII For m=10, p=10, n=810*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0488	0.0446	0.0486	0.0420
Pillai's test	1.0000	0.0476	0.0440	0.0466	0.0332
Hotelling -Lawley test	1.0000	0.0542	0.0518	0.0532	0.0554
Roy's test	1.0000	0.0514	0.0486	0.0510	0.9910

Table 5.202. *Error typeIII For  $m=10, p=10, n=860$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0516	0.0422	0.0540	0.0390
Pillai's test	1.0000	0.0506	0.0414	0.0518	0.0306
Hotelling -Lawley test	1.0000	0.0566	0.0476	0.0588	0.0506
Roy's test	1.0000	0.0538	0.0446	0.0552	0.9950

Table 5.203. *Error typeIII For  $m=10, p=10, n=910$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0454	0.0484	0.0494	0.0382
Pillai's test	1.0000	0.0442	0.0474	0.0478	0.0304
Hotelling -Lawley test	1.0000	0.0514	0.0526	0.0524	0.0504
Roy's test	1.0000	0.0468	0.0494	0.0504	0.9946

Table 5.204. *Error typeIII For  $m=10, p=10, n=960$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0486	0.0524	0.0468	0.0392
Pillai's test	1.0000	0.0474	0.0508	0.0456	0.0314
Hotelling -Lawley test	1.0000	0.0520	0.0548	0.0516	0.0478
Roy's test	1.0000	0.0498	0.0530	0.0488	0.9924

Table 5.205. *Error typeIII For  $m=10, p=10, n=1010$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0532	0.0522	0.0498	0.0448
Pillai's test	1.0000	0.0524	0.0512	0.0494	0.0368
Hotelling -Lawley test	1.0000	0.0584	0.0560	0.0540	0.0532
Roy's test	1.0000	0.0554	0.0542	0.0510	0.9918

Table 5.206. *Error typeIII For  $m=10, p=10, n=1060$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0422	0.0508	0.0454	0.0378
Pillai's test	1.0000	0.0414	0.0498	0.0442	0.0316
Hotelling -Lawley test	1.0000	0.0454	0.0560	0.0494	0.0458
Roy's test	1.0000	0.0440	0.0530	0.0472	0.9920

Table 5.207. *Error typeIII For  $m=10, p=10, n=1110$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0510	0.0486	0.0400	0.0436
Pillai's test	1.0000	0.0504	0.0482	0.0392	0.0372
Hotelling -Lawley test	1.0000	0.0548	0.0530	0.0434	0.0528
Roy's test	1.0000	0.0520	0.0502	0.0412	0.9906

Table 5.208. *Error typeIII For m=10, p=10, n=1160*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0542	0.0528	0.0524	0.0436
Pillai's test	1.0000	0.0514	0.0526	0.0508	0.0382
Hotelling -Lawley test	1.0000	0.0582	0.0560	0.0550	0.0536
Roy's test	1.0000	0.0558	0.0534	0.0532	0.9940

Table 5.209. *Error typeIII For m=10, p=10, n=1210*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0522	0.0526	0.0492	0.0428
Pillai's test	1.0000	0.0514	0.0518	0.0482	0.0376
Hotelling -Lawley test	1.0000	0.0568	0.0550	0.0532	0.0500
Roy's test	1.0000	0.0544	0.0538	0.0510	0.9908

Table 5.210. *Error typeIII For m=10, p=10, n=1260*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0490	0.0468	0.0518	0.0402
Pillai's test	1.0000	0.0488	0.0464	0.0510	0.0352
Hotelling -Lawley test	1.0000	0.0512	0.0502	0.0550	0.0464
Roy's test	1.0000	0.0504	0.0476	0.0530	0.9904

Table 5.211. *Error typeIII For  $m=10, p=10, n=1310$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0564	0.0510	0.0494	0.0430
Pillai's test	1.0000	0.0560	0.0502	0.0484	0.0394
Hotelling -Lawley test	1.0000	0.0596	0.0550	0.0524	0.0496
Roy's test	1.0000	0.0574	0.0528	0.0508	0.9924

Table 5.212. *Error typeIII For  $m=10, p=10, n=1360$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0492	0.0432	0.0452	0.0416
Pillai's test	1.0000	0.0488	0.0418	0.0436	0.0362
Hotelling -Lawley test	1.0000	0.0530	0.0462	0.0486	0.0462
Roy's test	1.0000	0.0498	0.0444	0.0464	0.9928

Table 5.213. *Error typeIII For  $m=10, p=10, n=1410$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0552	0.0474	0.0536	0.0396
Pillai's test	1.0000	0.0536	0.0464	0.0524	0.0350
Hotelling -Lawley test	1.0000	0.0578	0.0504	0.0560	0.0468
Roy's test	1.0000	0.0558	0.0484	0.0546	0.9922

Table 5.214. *Error typeIII For  $m=10, p=10, n=1460$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0478	0.0486	0.0472	0.0472
Pillai's test	1.0000	0.0466	0.0480	0.0460	0.0432
Hotelling -Lawley test	1.0000	0.0500	0.0498	0.0496	0.0536
Roy's test	1.0000	0.0480	0.0488	0.0490	0.9934

Table 5.215. *Error typeIII For  $m=10, p=10, n=1510$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0504	0.0508	0.0492	0.0446
Pillai's test	1.0000	0.0486	0.0500	0.0482	0.0396
Hotelling -Lawley test	1.0000	0.0530	0.0552	0.0512	0.0532
Roy's test	1.0000	0.0514	0.0526	0.0502	0.9940



Table 5.216. *Error typeIII For m=10, p=10, n=1560*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0500	0.0544	0.0422	0.0406
Pillai's test	1.0000	0.0488	0.0536	0.0416	0.0372
Hotelling -Lawley test	1.0000	0.0526	0.0566	0.0442	0.0488
Roy's test	1.0000	0.0514	0.0552	0.0432	0.9944

Table 5.217. *Error typeIII For m=10, p=10, n=1610*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0496	0.0522	0.0502	0.0488
Pillai's test	1.0000	0.0476	0.0514	0.0500	0.0438
Hotelling -Lawley test	1.0000	0.0524	0.0550	0.0536	0.0540
Roy's test	1.0000	0.0510	0.0534	0.0516	0.9922

## 5.4 ERROR TYPE IV

Table 5.218. *Error typeIV For  $m=2, p=2, n=20$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	0.5492	0.0266			0.0266
Pillai's test	0.3426	0.0136			0.0136
Hotelling -Lawley test	0.7590	0.0522			0.0522
Roy's test	0.7266	0.0468			0.0468

Table 5.219. *Error typeIV For  $m=2, p=2, n=70$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0446			0.0446
Pillai's test	1.0000	0.0396			0.0396
Hotelling -Lawley test	1.0000	0.0528			0.0528
Roy's test	1.0000	0.0512			0.0512

Table 5.220. *Error typeIV For  $m=2, p=2, n=120$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0430			0.0430
Pillai's test	1.0000	0.0410			0.0410
Hotelling -Lawley test	1.0000	0.0476			0.0476
Roy's test	1.0000	0.0456			0.0456

Table 5.221. *Error typeIV For  $m=2, p=2, n=170$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0484			0.0484
Pillai's test	1.0000	0.0464			0.0464
Hotelling -Lawley test	1.0000	0.0544			0.0544
Roy's test	1.0000	0.0528			0.0528

Table 5.222. *Error typeIV For  $m=2, p=2, n=220$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0446			0.0446
Pillai's test	1.0000	0.0438			0.0438
Hotelling -Lawley test	1.0000	0.0480			0.0480
Roy's test	1.0000	0.0472			0.0472

Table 5.223. *Error typeIV For  $m=2, p=2, n=270$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0476			0.0476
Pillai's test	1.0000	0.0464			0.0464
Hotelling -Lawley test	1.0000	0.0504			0.0504
Roy's test	1.0000	0.0504			0.0504

Table 5.224. *Error typeIV For  $m=3, p=3, n=30$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	0.8446	0.0250		0.0258	0.0202
Pillai's test	0.7280	0.0126		0.0156	0.0056
Hotelling -Lawley test	0.9432	0.0594		0.0592	0.0560
Roy's test	0.9290	0.0518		0.0502	0.1670

Table 5.225. *Error typeIV For  $m=3, p=3, n=80$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0390		0.0434	0.0366
Pillai's test	1.0000	0.0344		0.0358	0.0276
Hotelling -Lawley test	1.0000	0.0496		0.0568	0.0532
Roy's test	1.0000	0.0468		0.0534	0.1576

Table 5.226. *Error typeIV For  $m=3, p=3, n=130$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0470		0.0432	0.0450
Pillai's test	1.0000	0.0432		0.0416	0.0380
Hotelling -Lawley test	1.0000	0.0550		0.0500	0.0550
Roy's test	1.0000	0.0520		0.0474	0.1636

Table 5.227. *Error typeIV For  $m=3, p=3, n=180$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0390		0.0444	0.0458
Pillai's test	1.0000	0.0374		0.0432	0.0398
Hotelling -Lawley test	1.0000	0.0448		0.0508	0.0520
Roy's test	1.0000	0.0424		0.0488	0.1488

Table 5.228. *Error typeIV For  $m=3, p=3, n=230$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0524		0.0514	0.0500
Pillai's test	1.0000	0.0498		0.0498	0.0464
Hotelling -Lawley test	1.0000	0.0558		0.0560	0.0560
Roy's test	1.0000	0.0552		0.0542	0.1702

Table 5.229. *Error typeIV For  $m=3, p=3, n=280$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0510		0.0462	0.0478
Pillai's test	1.0000	0.0492		0.0448	0.0452
Hotelling -Lawley test	1.0000	0.0536		0.0494	0.0524
Roy's test	1.0000	0.0526		0.0484	0.1648

Table 5.230. *Error typeIV For  $m=3, p=3, n=330$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0506		0.0446	0.0476
Pillai's test	1.0000	0.0490		0.0434	0.0454
Hotelling -Lawley test	1.0000	0.0540		0.0482	0.0514
Roy's test	1.0000	0.0526		0.0470	0.1584

Table 5.231. *Error typeIV For  $m=4, p=4, n=40$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	0.9628	0.0228	0.0252	0.0252	0.0090
Pillai's test	0.9196	0.0106	0.0118	0.0146	0.0016
Hotelling -Lawley test	0.9892	0.0536	0.0614	0.0600	0.0486
Roy's test	0.9864	0.0446	0.0526	0.0518	0.3524

Table 5.232. *Error typeIV For  $m=4, p=4, n=90$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0434	0.0398	0.0420	0.0316
Pillai's test	1.0000	0.0362	0.0338	0.0364	0.0188
Hotelling -Lawley test	1.0000	0.0600	0.0564	0.0546	0.0552
Roy's test	1.0000	0.0534	0.0502	0.0492	0.3342

Table 5.233. *Error typeIV For  $m=4, p=4, n=140$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0428	0.0398	0.0466	0.0388
Pillai's test	1.0000	0.0382	0.0356	0.0424	0.0298
Hotelling -Lawley test	1.0000	0.0522	0.0496	0.0544	0.0560
Roy's test	1.0000	0.0500	0.0452	0.0516	0.3248

Table 5.234. *Error typeIV For  $m=4, p=4, n=190$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0420	0.0488	0.0452	0.0396
Pillai's test	1.0000	0.0398	0.0454	0.0422	0.0334
Hotelling -Lawley test	1.0000	0.0472	0.0566	0.0530	0.0498
Roy's test	1.0000	0.0442	0.0534	0.0504	0.3242

Table 5.235. *Error typeIV For  $m=4, p=4, n=240$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0470	0.0520	0.0440	0.0448
Pillai's test	1.0000	0.0460	0.0490	0.0418	0.0366
Hotelling -Lawley test	1.0000	0.0518	0.0604	0.0482	0.0542
Roy's test	1.0000	0.0494	0.0564	0.0458	0.3274



Table 5.236. *Error typeIV For  $m=4, p=4, n=290$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0484	0.0512	0.0482	0.0454
Pillai's test	1.0000	0.0458	0.0494	0.0476	0.0412
Hotelling -Lawley test	1.0000	0.0528	0.0558	0.0534	0.0528
Roy's test	1.0000	0.0504	0.0544	0.0514	0.3320

Table 5.237. *Error typeIV For  $m=4, p=4, n=340$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0480	0.0396	0.0442	0.0414
Pillai's test	1.0000	0.0462	0.0390	0.0422	0.0376
Hotelling -Lawley test	1.0000	0.0516	0.0426	0.0478	0.0490
Roy's test	1.0000	0.0504	0.0408	0.0466	0.3180

Table 5.238. *Error typeIV For  $m=4, p=4, n=390$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0450	0.0490	0.0452	0.043
Pillai's test	1.0000	0.0434	0.0478	0.0440	0.0404
Hotelling -Lawley test	1.0000	0.0476	0.0524	0.0476	0.0490
Roy's test	1.0000	0.0468	0.0512	0.0458	0.3238

Table 5.239. *Error typeIV For  $m=4, p=4, n=440$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0470	0.0472	0.0508	0.0488
Pillai's test	1.0000	0.0456	0.0462	0.0490	0.0464
Hotelling -Lawley test	1.0000	0.0496	0.0504	0.0546	0.0554
Roy's test	1.0000	0.0482	0.0490	0.0532	0.3168

Table 5.240. *Error typeIV For  $m=5, p=5, n=50$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	0.9906	0.0222	0.0256	0.0214	0.0050
Pillai's test	0.9750	0.0116	0.0150	0.0110	0.0000
Hotelling -Lawley test	0.9984	0.0568	0.0560	0.0520	0.0344
Roy's test	0.9976	0.0518	0.0514	0.0468	0.5542

Table 5.241. *Error typeIV For  $m=5, p=5, n=100$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0354	0.0434	0.0416	0.0244
Pillai's test	1.0000	0.0286	0.0358	0.0354	0.0122
Hotelling -Lawley test	1.0000	0.0546	0.0608	0.0624	0.0508
Roy's test	1.0000	0.0452	0.0538	0.0528	0.5384

Table 5.242. *Error typeIV For m=5, p=5, n=150*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0448	0.0378	0.0456	0.0318
Pillai's test	1.0000	0.0416	0.0338	0.0410	0.0208
Hotelling -Lawley test	1.0000	0.0548	0.0470	0.0576	0.0506
Roy's test	1.0000	0.0500	0.0432	0.0524	0.5302

Table 5.243. *Error typeIV For m=5, p=5, n=200*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0448	0.0494	0.0454	0.0398
Pillai's test	1.0000	0.0408	0.0454	0.0426	0.0284
Hotelling -Lawley test	1.0000	0.0546	0.0596	0.0520	0.0530
Roy's test	1.0000	0.0512	0.0542	0.0490	0.5274

Table 5.244. *Error typeIV For m=5, p=5, n=250*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0506	0.0472	0.0482	0.0382
Pillai's test	1.0000	0.0484	0.0454	0.0448	0.0328
Hotelling -Lawley test	1.0000	0.0562	0.0556	0.0536	0.0526
Roy's test	1.0000	0.0536	0.0516	0.0500	0.5238

Table 5.245. *Error typeIV For  $m=5, p=5, n=300$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0434	0.0468	0.0470	0.0408
Pillai's test	1.0000	0.0416	0.0452	0.0444	0.0336
Hotelling -Lawley test	1.0000	0.0480	0.0518	0.0524	0.0490
Roy's test	1.0000	0.0462	0.0498	0.0506	0.5240

Table 5.246. *Error typeIV For  $m=5, p=5, n=350$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0432	0.0506	0.0392	0.0386
Pillai's test	1.0000	0.0406	0.0484	0.0370	0.0322
Hotelling -Lawley test	1.0000	0.0492	0.0560	0.0432	0.0472
Roy's test	1.0000	0.0464	0.0528	0.0414	0.5158

Table 5.247. *Error typeIV For  $m=5, p=5, n=400$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0480	0.0504	0.0486	0.0398
Pillai's test	1.0000	0.0446	0.0488	0.0462	0.0358
Hotelling -Lawley test	1.0000	0.0528	0.0540	0.0516	0.0466
Roy's test	1.0000	0.0504	0.0528	0.0496	0.5304

Table 5.248. *Error typeIV For m=5, p=5, n=450*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0504	0.0464	0.0486	0.0484
Pillai's test	1.0000	0.0490	0.0446	0.0484	0.0424
Hotelling -Lawley test	1.0000	0.0540	0.0492	0.0516	0.0556
Roy's test	1.0000	0.0522	0.0480	0.0498	0.5276

Table 5.249. *Error typeIV For m=5, p=5, n=500*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0460	0.0488	0.0476	0.0436
Pillai's test	1.0000	0.0444	0.0478	0.0466	0.0384
Hotelling -Lawley test	1.0000	0.0504	0.0506	0.0520	0.0474
Roy's test	1.0000	0.0476	0.0498	0.0504	0.5188

Table 5.250. *Error typeIV For  $m=5, p=5, n=550$*

	$b_1$	$b_2$	$b_{p-1}$	$b_p$	MANOVA F
Wilk's test	1.0000	0.0522	0.0530	0.0506	0.0488
Pillai's test	1.0000	0.0518	0.0510	0.0502	0.0432
Hotelling -Lawley test	1.0000	0.0552	0.0558	0.0554	0.0550
Roy's test	1.0000	0.0536	0.0546	0.0534	0.5242

**CHAPTER 6**  
**CONCLUSIONS**

Multivariate linear regression is a semiparametric method that is nearly as easy to use as multiple linear regression if  $m$  is small. The  $m$  response and residual plots should be made as well as the DD plot. Sample sizes are needed for determining when the tests and plots start to work well. Response and residual plots can look good for  $n \geq 10p$ , but for testing, may need  $n \geq k(m + p)^2$  where  $0.5 \leq k \leq 5$  even for well behave elliptically contoured error distributions. From the simulations needed  $n \geq a + b(m + p)^2$  or  $n \geq c(m + p)^2$  as shown below. These simulations used 5000 runs.

$$\begin{array}{l}
 \text{Hoteling Lawley's test-}H_0 \text{ true} \\
 \left| \begin{array}{l} \text{etype 1} \\ \text{etype 2} \\ \text{etype 3} \\ \text{etype 4} \end{array} \right| \left\| \begin{array}{l} 2.5 + 0.773(m + p)^2 \\ 13.7 + 0.773(m + p)^2 \\ 2.02 + .743(m + p)^2 \\ 6.07 + 0.752(m + p)^2 \end{array} \right\| \left| \begin{array}{l} 0.783(m + p)^2 \\ 0.825(m + p)^2 \\ 0.80(m + p)^2 \\ 0.775(m + p)^2 \end{array} \right|
 \end{array}$$

$$\begin{array}{l}
 \text{Wilk's test-}H_0 \text{ true} \\
 \left| \begin{array}{l} \text{etype 1} \\ \text{etype 2} \\ \text{etype 3} \\ \text{etype 4} \end{array} \right| \left\| \begin{array}{l} 2.5 + 2.66(m + p)^2 \\ -12 + 2.71(m + p)^2 \\ 27 + 2.70(m + p)^2 \\ 76.1 + 1.81(m + p)^2 \end{array} \right\| \left| \begin{array}{l} 2.67(m + p)^2 \\ 2.7(m + p)^2 \\ 2.80(m + p)^2 \\ 2.5(m + p)^2 \end{array} \right|
 \end{array}$$

<i>Pillai's test-Ho true</i>	etype 1	$104 + 3.17(m + p)^2$	$3.57(m + p)^2$
	etype 2	$95.4 + 3.58(m + p)^2$	$4.0(m + p)^2$
	etype 3	$73.5 + 3.78(m + p)^2$	$4.05(m + p)^2$
	etype 4	$80.4 + 3.87(m + p)^2$	$4.2(m + p)^2$

The *R* software was used to make plots and software. See R Development Core Team (2011). `mregsim` was used to simulate the tests of hypotheses, and `mregddsim` simulated the DD plots for various distributions. The function `mltreg` makes the response and residual plots and computes the  $F_j$ , MANOVA  $F$  and MANOVA partial  $F$  test pvalues while the function `ddplot4` makes the DD plots.



CHAPTER 7  
SUPPLEMENTARY MATERIAL

**Appendix, Technical Proofs:**

*Proof of Theorem 1.*

Using the Searle (1982, p. 333) identity  $tr(\mathbf{A}\mathbf{G}^T\mathbf{D}\mathbf{G}\mathbf{C}) = [\text{vec}(\mathbf{G})]^T[\mathbf{C}\mathbf{A} \otimes \mathbf{D}^T][\text{vec}(\mathbf{G})]$ , it follows that  $(n-p)U(\mathbf{L}) = tr[\hat{\Sigma}_\epsilon^{-1}\hat{\mathbf{B}}^T\mathbf{L}^T[\mathbf{L}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{L}^T]^{-1}\mathbf{L}\hat{\mathbf{B}}] = [\text{vec}(\mathbf{L}\hat{\mathbf{B}})]^T[\hat{\Sigma}_\epsilon^{-1} \otimes (\mathbf{L}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{L}^T)^{-1}][\text{vec}(\mathbf{L}\hat{\mathbf{B}})] = T$

where  $\mathbf{A} = \hat{\Sigma}_\epsilon^{-1}$ ,  $\mathbf{G} = \mathbf{L}\hat{\mathbf{B}}$ ,  $\mathbf{D} = [\mathbf{L}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{L}^T]^{-1}$ , and  $\mathbf{C} = \mathbf{I}$ . Hence (2.1) holds.

*Proof of Theorem 2.* By Su and Cook (2012),  $\sqrt{n} \text{vec}(\hat{\mathbf{B}} - \mathbf{B}) \xrightarrow{D} N_{pm}(\mathbf{0}, \Sigma_\epsilon \otimes \mathbf{W})$ .

Then under  $H_0$ ,

$$\sqrt{n} \text{vec}(\mathbf{L}\hat{\mathbf{B}}) = \sqrt{n} \begin{pmatrix} \mathbf{L}\hat{\beta}_1 \\ \mathbf{L}\hat{\beta}_2 \\ \vdots \\ \mathbf{L}\hat{\beta}_m \end{pmatrix} \xrightarrow{D} N_{rm}(\mathbf{0}, \Sigma_\epsilon \otimes \mathbf{L}\mathbf{W}\mathbf{L}^T),$$

and

$$n [\text{vec}(\mathbf{L}\hat{\mathbf{B}})]^T[\Sigma_\epsilon^{-1} \otimes (\mathbf{L}\mathbf{W}\mathbf{L}^T)^{-1}][\text{vec}(\mathbf{L}\hat{\mathbf{B}})] \xrightarrow{D} \chi_{rm}^2.$$

The above result also holds if  $\mathbf{W}$  and  $\Sigma_\epsilon$  are replaced by  $\hat{\mathbf{W}} = n(\mathbf{X}^T\mathbf{X})^{-1}$  and  $\hat{\Sigma}_\epsilon$ . Hence under  $H_0$  and using the proof of Theorem 1,

$$T = (n-p)U(\mathbf{L}) = [\text{vec}(\mathbf{L}\hat{\mathbf{B}})]^T[\hat{\Sigma}_\epsilon^{-1} \otimes (\mathbf{L}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{L}^T)^{-1}][\text{vec}(\mathbf{L}\hat{\mathbf{B}})] \xrightarrow{D} \chi_{rm}^2. \quad (7.1)$$

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