RESISTANT DIMENSION REDUCTION

by

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DISSERTATION APPROVAL

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Regression is the study of the conditional distribution $y|\boldsymbol{x}$ of the response y given a vector $\boldsymbol{x}$ of the predictors, and dimension reduction (DR) methods attempt to find linear combinations $\boldsymbol{\beta}_1'\boldsymbol{x}$, ..., $\boldsymbol{\beta}_d'\boldsymbol{x}$ such that the response y is independent of $\boldsymbol{x}$ given the d linear combinations and $d \geq 1$ is as small as possible. A 1D regression model has $d = 1$, and the conditional distribution $y|\boldsymbol{\beta}'\boldsymbol{x}$ is of primary interest. Many of the most used statistical procedures, including multiple linear regression and generalized linear models, are special cases of 1D regression.

Existing DR methods such as ordinary least squares and sliced inverse regression often perform poorly in the presence of outliers. Also the DR theory usually assumes that the predictors satisfy the condition of linearly related predictors: e.g., for 1D regression $E[\boldsymbol{x}|\boldsymbol{\beta}'\boldsymbol{x}]$ must be a linear function of $\boldsymbol{\beta}'\boldsymbol{x}$. This dissertation develops outlier resistant DR methods that can give useful results when the assumption of linearly related predictors is violated.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION AND LITERATURE REVIEW

1.1 INTRODUCTION

Regression is the study of the conditional distribution $y|\boldsymbol{x}$ of the response variable y given the $(p-1)\times 1$ predictor vector $\boldsymbol{x} = (x_1, x_2, \cdots, x_{p-1})'$. Dimension reduction (DR) searches for a lower dimensional $d \times 1$ vector of predictors that carries all the information relevant to the regression.

Resistant dimension reduction is closely related to the structural dimension of the regression. We are trying to find a lower dimensional predictor $\boldsymbol{w}$, without any loss of information on the conditional distribution $y|\boldsymbol{x}$. The purpose of dimension reduction is to make $y \perp\!\!\!\perp \boldsymbol{x}|(\beta_1^T\boldsymbol{x}, \beta_2^T\boldsymbol{x}, \cdots, \beta_d^T\boldsymbol{x})$ with d be as small as possible, and d is called the structural dimension. The problem is trivial if $d = p-1$. However if $d < p-1$, we made an improvement. As a result, the response y depends on the $p-1$ dimensional predictor $\boldsymbol{x}$ only through the lower dimensional $d \times 1$ vector $\boldsymbol{w} = (\beta_1^T\boldsymbol{x}, \beta_2^T\boldsymbol{x}, \cdots, \beta_d^T\boldsymbol{x})'$. The future study can be done on the simplified data.

Two important DR methods are ordinary least squares (OLS) and sliced inverse regression (SIR), which was proposed in Li (1991) [16]. One remarkable difference between SIR and OLS is that SIR reverses the role of the dependent variable y and the predictor $\boldsymbol{x}$. In this method we regress $\boldsymbol{x}$ versus y to study $y|\boldsymbol{x}$. This is called inverse regression. SIR could be implemented one-dimensionally if we regress each entry x_i of $\boldsymbol{x}$ versus y separately. This is a benefit of using SIR. Several other DR methods have been suggested, including principal Hessian directions (PHD), which was introduced in Li (1992) [17]; and sliced average variance estimation (SAVE), which was introduced in Cook and Weisberg (1991) [10]. See Cook and Li for additional DR methods [8] [11]. We will discuss these methods later.

1.2 LITERATURE REVIEW

DR methods often work well if the predictors follow an elliptically contoured distribution. The definition of this distribution will be introduced in Section 2.2. If $\boldsymbol{x}$ does not satisfy this specific distribution, applying DR methods to a subset $(y_M, \boldsymbol{x}_M)$ of the data with the $\boldsymbol{x}_M$'s distribution closer to being elliptically contoured can be an effective method for making DR methods such as OLS and SIR resistant to the presence of strong nonlinearities. See Li and Duan (1989, p. 1011) [15], Brillinger (1991) [3], Cook and Nachtsheim (1994) [9], Cook (1998, p. 152) [6] and Li, Cook and Nachtsheim (2004) [18]. Outlier resistance is also studied by Gather, Hilker and Becker (2001, 2002) [12] [13], Heng-Hui (2001) [14], and Olive (2002) [21].

1.3 DISSERTATION OVERVIEW

The dissertation is organized as follows.

In Chapter 1, we introduce dimension reduction and give a review of the literature.

In Chapter 2, some important notation and definitions are introduced. A major part in this chapter is the theory about the *dimension reduction* (DR) methods ordinary least squares (OLS) and sliced inverse regression (SIR). For each method, we study two problems in this chapter. First, we want to know the asymptotic covariance matrix of the estimated coefficients $\hat{\boldsymbol{\beta}}$ from both methods. Second, what are the test statistics for the hypothesis testing problem $H_0 : \boldsymbol{A}\boldsymbol{\beta} = \boldsymbol{0}$ versus $H_1 : \boldsymbol{A}\boldsymbol{\beta} \neq \boldsymbol{0}$ for some full rank matrix $\boldsymbol{A}$? For the OLS estimator, there are two test statistics. We present a theorem to show that they are asymptotically equivalent.

DR methods often work well if the predictors follow an elliptically contoured distribution, otherwise we can apply the DR method to a subset $(y_M, \boldsymbol{x}_M)$ of the original data. This is the idea of the resistant dimension reduction method. In Chapter 3, the related theory about resistant dimension reduction is introduced.

We first introduce some topics which are related to obtaining the appropriate subset $(y_M, \boldsymbol{x}_M)$ of the data. These topics include ellipsoidal trimming, the median ball algorithm, the DD plot, and the EY plot. Similar to Chapter 2, we are also interested in two resistant DR methods, OLS and SIR. For the resistant OLS estimator $\hat{\boldsymbol{\beta}}_M$, we introduce the theory about its asymptotic covariance matrix; for the resistant SIR estimator $\hat{\boldsymbol{\beta}}_{Mi}$, we introduce the asymptotic covariance matrix of $\boldsymbol{A}\hat{\boldsymbol{\beta}}_{Mi}$ for some full rank matrix $\boldsymbol{A}$. We also introduce the corresponding test statistics for testing $H_0 : \boldsymbol{A\beta} = \mathbf{0}$ versus $H_1 : \boldsymbol{A\beta} \neq \mathbf{0}$ for both methods.

We present all the simulations in Chapter 4. There are three different types of simulations in this chapter. For the first type of simulation, we want to study the behavior of the estimated resistant coefficients $\hat{\boldsymbol{\beta}}_M$ for both OLS and SIR. For the OLS estimator, we also want to know their standard errors. We run our simulations on the different $1D$ models ($d = 1$) and predictors. We run the second type of simulation on a $2D$ model ($d = 2$). For this model, we also compare our results from resistant DR methods using different predictor distributions. The third type of simulation is related to the hypothesis testing problem. We test three different matrices $\boldsymbol{A}$. For each $\boldsymbol{A}$, we run our simulations on the different $1D$ models and predictor distributions.

For $1D$ models, the results suggest that OLS outperforms SIR, and that SIR works best when the predictors $\boldsymbol{x}$ follow a multivariate normal distribution. The OLS test can be implemented using standard output originally meant for multiple linear regression.

CHAPTER 2

DIMENSION REDUCTION THEORY

In this chapter, we introduce some theoretical results relevant to *dimension reduction* (DR) methods. In Section 2.1, kD regression and DR methods are introduced; in Section 2.2, some basic definitions and theory are stated; in Section 2.3, some theoretical results about the ordinary least squares (OLS) estimator are presented; and in Section 2.4, some theory about the sliced inverse regression (SIR) estimator is presented.

2.1 INTRODUCTION

Let $\boldsymbol{B} = (\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \cdots, \boldsymbol{\beta}_k)$ be a $(p-1) \times k$ matrix. An important regression model states that y is independent of $\boldsymbol{x}$ given $\boldsymbol{B}'\boldsymbol{x}$, denoted by

$$y \perp\!\!\!\perp \boldsymbol{x} | \boldsymbol{B}'\boldsymbol{x}. \tag{2.1}$$

Equivalently, $y \perp\!\!\!\perp \boldsymbol{x} | \boldsymbol{\beta}_1'\boldsymbol{x}, \boldsymbol{\beta}_2'\boldsymbol{x}, \cdots, \boldsymbol{\beta}_k'\boldsymbol{x}$. We can always find such a matrix $\boldsymbol{B}$ by letting $\boldsymbol{B}$ be a $(p-1) \times (p-1)$ identity matrix.

The structural dimension d of the regression is the smallest number of the linear combinations needed to make model (2.1) hold. Here d is an integer between 0 and $p-1$. The regression is also said to have dD structure or to be a dD regression model. Most regression problems have $0D, 1D$, or $2D$ structures. See Cook and Weisberg (1999) [11].

If $d = 0$, then y is independent of $\boldsymbol{x}$, written $y \perp\!\!\!\perp \boldsymbol{x}$.

If $d = 1$, then $y|\boldsymbol{x}$ depends on a single linear combination $\boldsymbol{\beta}'\boldsymbol{x}$, written

$$y \perp\!\!\!\perp \boldsymbol{x} | \boldsymbol{\beta}'\boldsymbol{x}. \tag{2.2}$$

A very important $1D$ regression model, introduced by Li and Duan (1989)

[15], is

$$y = g(\alpha + \boldsymbol{\beta}'\boldsymbol{x}, e), \tag{2.3}$$

where g is a bivariate function and the error e has zero mean and finite variance σ^2 and is independent of $\boldsymbol{x}$. There are many important models with $1D$ structure. For example, an additive error single index model is

$$y = m(\alpha + \boldsymbol{\beta}'\boldsymbol{x}) + e, \tag{2.4}$$

where the error e has zero mean and finite variance σ^2. The multiple linear regression (MLR) model

$$y = \alpha + \boldsymbol{\beta}'\boldsymbol{x} + e, \tag{2.5}$$

is a special case of the single index model where m is the identity function. Generalized linear models (GLMs) are also $1D$ models.

2.2 DEFINITIONS AND NOTATION

Definition. [23] Let $\boldsymbol{x}$ be a $(p-1) \times 1$ random vector with the density function

$$f(\boldsymbol{z}) = k_{p-1}|\boldsymbol{V}|^{-1/2} g[(\boldsymbol{z} - \boldsymbol{\mu})'\boldsymbol{V}^{-1}(\boldsymbol{z} - \boldsymbol{\mu})], \tag{2.6}$$

where k_{p-1} is a constant, then we say that $\boldsymbol{x}$ has an *elliptically contoured distribution* or *elliptically symmetric distribution* and denote it as $\boldsymbol{x} \sim \text{EC}_{p-1}(\boldsymbol{\mu}, \boldsymbol{V}, g)$. Also, the characteristic function of $\boldsymbol{x}$ is

$$\phi_{\boldsymbol{x}}(\boldsymbol{t}) = exp(i\boldsymbol{t}'\boldsymbol{\mu})\Psi(\boldsymbol{t}'\boldsymbol{V}\,\boldsymbol{t}) \tag{2.7}$$

for some function Ψ.

Let $c_{\boldsymbol{x}} = -2\Psi'(0)$. If $\boldsymbol{x} \sim \text{EC}(\boldsymbol{\mu}, \boldsymbol{V}, g)$ and the second moments of $\boldsymbol{x}$ exist, then we have $E(\boldsymbol{x}) = \boldsymbol{\mu}$ and $\text{Cov}(\boldsymbol{x}) = c_{\boldsymbol{x}}\boldsymbol{V}$ [23].

Definition. Linearity Condition (LC):

We say a $(p-1)\times 1$ random vector $\boldsymbol{x}$ satisfies the linearity condition if for any $(p-1)\times 1$ vector $\boldsymbol{b}$, there exists some constants $(c_0, c_1, \cdots, c_k)$ such that $E(\boldsymbol{b}'\boldsymbol{x}|\boldsymbol{B}'\boldsymbol{x}) = c_0 + c_1\boldsymbol{\beta}_1'\boldsymbol{x} + \cdots + c_k\boldsymbol{\beta}_k'\boldsymbol{x}$, where $\boldsymbol{B} = (\boldsymbol{\beta}_1, \cdots, \boldsymbol{\beta}_k)$. In other words, $\boldsymbol{x}$ satisfies the linearity condition if $E(\boldsymbol{b}'\boldsymbol{x}|\boldsymbol{B}'\boldsymbol{x} = \boldsymbol{M})$ is a linear function of $\boldsymbol{M}$ for any $\boldsymbol{b} \in R^{p-1}$.

If $\boldsymbol{x} \sim \mathrm{EC}(\boldsymbol{\mu}, \boldsymbol{V}, g)$ with second moments where $\boldsymbol{V}$ is nondegenerate, then the LC holds [15].

Many methods for regression estimate the unknown coefficients α and $\boldsymbol{\beta}$ by minimizing some criterion function $Q(a + \boldsymbol{b}'\boldsymbol{x}, y)$ with respect to $(a, \boldsymbol{b})$ over some domain Ω. If the expectation of $Q(a + \boldsymbol{b}'\boldsymbol{x}, y)$ is well-defined, then we could define the following minimization problem for the large sample:

$$\text{minimize} \quad L(a, \boldsymbol{b}) = EQ(a + \boldsymbol{b}'\boldsymbol{x}, y). \tag{2.8}$$

Let $(\alpha^*, \boldsymbol{\beta}^*)$ be the population solution for the problem (2.8) and let $(\hat{\alpha}, \hat{\boldsymbol{\beta}})$ be the solution of the sampled minimization problem of (2.8). Under some extra regularity conditions, we have [15]

$$(\hat{\alpha}, \hat{\boldsymbol{\beta}}) \overset{a.s.}{\to} (\alpha^*, \boldsymbol{\beta}^*). \tag{2.9}$$

Consider the model (2.3). If the true function g is unknown, the following theorem says that the regression coefficient $\boldsymbol{\beta}^*$ is proportional to the true coefficient $\boldsymbol{\beta}$ in the model:

Theorem 2.2.1. *[15] Let the domain $\Omega = \{(a, \boldsymbol{b})|L(a + \boldsymbol{b}'\boldsymbol{x}) < \infty\}$ and assume that Ω is a nonempty convex set. Let $\boldsymbol{x}$ satisfy LC. Also assume that*

Condition 1: $L(a + \boldsymbol{b}'\boldsymbol{x})$ is convex in $(a, \boldsymbol{b})$ almost surely.

Condition 2: The minimization problem (2.8) has a unique solution $(\alpha^*, \boldsymbol{\beta}^*)$.

Then

$$\boldsymbol{\beta}^* = c\boldsymbol{\beta}, \tag{2.10}$$

where c is some scalar.

Assume the criterion function $L(\cdot)$ is smooth enough to have derivatives. Let $L_1(\cdot)$ denote the first partial derivative of $L(\cdot)$ with respect to $(a, \boldsymbol{b})$ and $L_{11}(\cdot)$ denote the second partial derivative of $L(\cdot)$ with respect to $(a, \boldsymbol{b})$. Suppose both L_1 and L_{11} exist and are continuous. Li and Duan (1989) also gave a general form of the asymptotic covariance matrix for the estimator $\hat{\boldsymbol{\beta}}$ for the 1D model (2.3).

Theorem 2.2.2. *[15] (p. 1031) Assume $\boldsymbol{x} \sim EC(\boldsymbol{\mu}, \boldsymbol{V})$, where the covariance matrix $\boldsymbol{\Sigma_x} \equiv \mathrm{Cov}(\boldsymbol{x}) = c_{\boldsymbol{x}}\boldsymbol{V}$ is nonsingular, then under regularity conditions, we have*

$$\sqrt{n}(\hat{\boldsymbol{\beta}} - c\boldsymbol{\beta}) \xrightarrow{L} N_{p-1}(\mathbf{0}, \boldsymbol{C}), \tag{2.11}$$

where

$$\boldsymbol{C} = \phi\, \eta \boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1} + k\,(c\boldsymbol{\beta})'(c\boldsymbol{\beta}), \tag{2.12}$$

$$\phi = \frac{E[L_1(\alpha^* + c\boldsymbol{\beta}'\boldsymbol{x})^2\, \Gamma(\boldsymbol{x})]}{E[L_{11}(\alpha^* + c\boldsymbol{\beta}'\boldsymbol{x})\, \Gamma(\boldsymbol{x})]}, \tag{2.13}$$

$$\eta = \frac{p-2}{E[L_{11}(\alpha^* + c\boldsymbol{\beta}'\boldsymbol{x})\, \Gamma(\boldsymbol{x})]}, \tag{2.14}$$

and

$$\Gamma = (\boldsymbol{x} - \boldsymbol{\mu})'\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}(\boldsymbol{x} - \boldsymbol{\mu}) - \frac{(c\boldsymbol{\beta}'(\boldsymbol{x} - \boldsymbol{\mu}))^2}{(c\boldsymbol{\beta})'\boldsymbol{\Sigma_x}(c\boldsymbol{\beta})}, \tag{2.15}$$

and c, k are some constants.

In addition, if we have the condition $\boldsymbol{A\beta} = \mathbf{0}$, where $\boldsymbol{A}$ is a $q \times (p-1)$ matrix with rank $q \le p-1$, Li and Duan (1989, p. 1032) have the following theorem for the asymptotic covariance matrix of $\boldsymbol{A}\hat{\boldsymbol{\beta}}$.

Theorem 2.2.3. *Consider the model (2.3). Given $\boldsymbol{A\beta} = \mathbf{0}$ for some $\boldsymbol{A}$ and under the same conditions of Theorem 2.2.2, then*

$$\sqrt{n}(\boldsymbol{A}\hat{\boldsymbol{\beta}} - c\boldsymbol{A\beta}) \xrightarrow{L} N_q(\mathbf{0}, \boldsymbol{AC}\,\boldsymbol{A}') \tag{2.16}$$

if the inverse link function g is known, while

$$\sqrt{n}(\boldsymbol{A}\hat{\boldsymbol{\beta}} - c\boldsymbol{A}\boldsymbol{\beta}) \xrightarrow{L} N_q(\boldsymbol{0}, \phi\, \boldsymbol{A}\boldsymbol{C}\, \boldsymbol{A}') \tag{2.17}$$

under inverse link violation, where c is some constant. Here $\boldsymbol{C}$ is defined in (2.12) and ϕ is defined in (2.13).

2.3 THEORETICAL RESULTS FOR OLS

2.3.1 Theoretical Results For The OLS Estimator

In this section we will introduce theoretical results for the OLS estimator. For OLS the minimization problem is a special case of (2.8),

$$\text{Minimize} \quad Q_{OLS}(a + \boldsymbol{b}'\boldsymbol{x}) = \|r(a, \boldsymbol{b})\|^2, \tag{2.18}$$

where the "residual" $r(a, \boldsymbol{b}) = y - a - \boldsymbol{b}'\boldsymbol{x}$. Suppose $\text{Cov}(\boldsymbol{x})$ and $\text{Cov}(\boldsymbol{x}, y)$ exist and let $\boldsymbol{\Sigma}_{\boldsymbol{x}} = \text{Cov}(\boldsymbol{x})$ and $\boldsymbol{\Sigma}_{\boldsymbol{x}y} = \text{Cov}(\boldsymbol{x}, y)$. Let $(\alpha_{ols}, \boldsymbol{\beta}_{ols})$ be the population OLS coefficients, then

$$\alpha_{ols} = Ey - \boldsymbol{\beta}_{ols}' E\boldsymbol{x}, \tag{2.19}$$

and

$$\boldsymbol{\beta}_{ols} = \boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1} \boldsymbol{\Sigma}_{\boldsymbol{x}y}. \tag{2.20}$$

Then we will define the OLS population "residual" $r(\alpha_{ols}, \boldsymbol{\beta}_{ols})$ as

$$r(\alpha_{ols}, \boldsymbol{\beta}_{ols}) = y - \alpha_{ols} - \boldsymbol{\beta}_{ols}'\boldsymbol{x}. \tag{2.21}$$

Using the definitions (2.19) and (2.20), (2.21) is equivalent to

$$r(\alpha_{ols}, \boldsymbol{\beta}_{ols}) = y - Ey - \boldsymbol{\beta}_{ols}'(\boldsymbol{x} - E\boldsymbol{x}). \tag{2.22}$$

Let the data be $(y_i, \boldsymbol{x}_i)$ for $i = 1, \cdots, n$ and let $(\hat{\alpha}_{ols}, \hat{\boldsymbol{\beta}}_{ols})$ denote the estimator of $(\alpha_{ols}, \boldsymbol{\beta}_{ols})$. The sample minimization problem is to minimize $\sum_{i=1}^n r_i^2$, where $r_i \equiv r_i(\hat{\alpha}_{ols}, \hat{\boldsymbol{\beta}}_{ols}) = y_i - \hat{\alpha}_{ols} - \hat{\boldsymbol{\beta}}_{ols}'\boldsymbol{x}_i$.

Consider the multiple linear regression model

$$\boldsymbol{Y} = \boldsymbol{X}\boldsymbol{\eta} + \boldsymbol{e}, \tag{2.23}$$

where $\boldsymbol{Y} = (y_1, \cdots, y_n)', \boldsymbol{\eta} = (\alpha, \boldsymbol{\beta}')'$, and $\boldsymbol{X}$ is the $n \times p$ design matrix with the ith row $(1, \boldsymbol{x}_i')$. Suppose $\boldsymbol{X}'\boldsymbol{X}$ is positive-definite, then the OLS estimator $\hat{\boldsymbol{\eta}}_{ols} = (\hat{\alpha}_{ols}, \hat{\boldsymbol{\beta}}_{ols}')' = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{Y}$. Define $\hat{\boldsymbol{\Sigma}}_{\boldsymbol{x}} = \frac{1}{n-1}\sum_{i=1}^{n}(\boldsymbol{x}_i - \bar{\boldsymbol{x}})(\boldsymbol{x}_i - \bar{\boldsymbol{x}})'$ and $\hat{\boldsymbol{\Sigma}}_{\boldsymbol{x}y} = \frac{1}{n}\sum_{i=1}^{n}(\boldsymbol{x}_i - \bar{\boldsymbol{x}})(y_i - \bar{y})$. Following Seber and Lee (2003, p. 99 - 106) [26], we have

$$(\boldsymbol{X}'\boldsymbol{X})^{-1} = \begin{pmatrix} \frac{1}{n} + \bar{\boldsymbol{x}}'\boldsymbol{D}^{-1}\bar{\boldsymbol{x}} & -\bar{\boldsymbol{x}}'\boldsymbol{D}^{-1} \\ -\boldsymbol{D}^{-1}\bar{\boldsymbol{x}} & \boldsymbol{D}^{-1} \end{pmatrix}, \tag{2.24}$$

where $\bar{\boldsymbol{x}} = \frac{1}{n}\sum_{i=1}^{n}\boldsymbol{x}_i$ and

$$\begin{aligned} \boldsymbol{D} &= (n-1)\hat{\boldsymbol{\Sigma}}_{\boldsymbol{x}}, \\ \boldsymbol{D}^{-1} &= \frac{\hat{\boldsymbol{\Sigma}}_{\boldsymbol{x}}^{-1}}{n-1}. \end{aligned} \tag{2.25}$$

Then

$$\begin{aligned} (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{Y} &= \begin{pmatrix} \frac{1}{n} + \bar{\boldsymbol{x}}'\boldsymbol{D}^{-1}\bar{\boldsymbol{x}} & -\bar{\boldsymbol{x}}'\boldsymbol{D}^{-1} \\ -\boldsymbol{D}^{-1}\bar{\boldsymbol{x}} & \boldsymbol{D}^{-1} \end{pmatrix} \begin{pmatrix} 1 & \cdots & 1 \\ \boldsymbol{x}_1 & \cdots & \boldsymbol{x}_n \end{pmatrix} \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{n} + \bar{\boldsymbol{x}}'\boldsymbol{D}^{-1}\bar{\boldsymbol{x}} & -\bar{\boldsymbol{x}}'\boldsymbol{D}^{-1} \\ -\boldsymbol{D}^{-1}\bar{\boldsymbol{x}} & \boldsymbol{D}^{-1} \end{pmatrix} \begin{pmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} \boldsymbol{x}_i y_i \end{pmatrix} \\ &= \begin{pmatrix} \bar{y} - \bar{\boldsymbol{x}}'\boldsymbol{D}^{-1}[\sum_{i=1}^{n}\boldsymbol{x}_i y_i - \bar{\boldsymbol{x}}\sum_{i=1}^{n} y_i] \\ \boldsymbol{D}^{-1}[\sum_{i=1}^{n}\boldsymbol{x}_i y_i - \bar{\boldsymbol{x}}\sum_{i=1}^{n} y_i] \end{pmatrix}. \end{aligned} \tag{2.26}$$

According to (2.25),

$$\begin{aligned}
\boldsymbol{D}^{-1}[\sum_{i=1}^{n}\boldsymbol{x}_i y_i - \bar{\boldsymbol{x}}\sum_{i=1}^{n} y_i] &= \frac{\hat{\boldsymbol{\Sigma}}_{\boldsymbol{x}}^{-1}}{n-1}[\sum_{i=1}^{n}\boldsymbol{x}_i y_i - \bar{\boldsymbol{x}}\sum_{i=1}^{n} y_i] \\
&= \frac{n}{n-1}\hat{\boldsymbol{\Sigma}}_{\boldsymbol{x}}^{-1}(\frac{1}{n}\sum \boldsymbol{x}_i y_i - \bar{\boldsymbol{x}}\frac{1}{n}\sum_{i=1}^{n} y_i) \\
&= \frac{n}{n-1}\hat{\boldsymbol{\Sigma}}_{\boldsymbol{x}}^{-1}(\frac{1}{n}\sum \boldsymbol{x}_i y_i - \bar{\boldsymbol{x}}\bar{y}) \\
&= \frac{n}{n-1}\hat{\boldsymbol{\Sigma}}_{\boldsymbol{x}}^{-1}\hat{\boldsymbol{\Sigma}}_{\boldsymbol{x}y}.
\end{aligned} \tag{2.27}$$

Therefore, the OLS estimators $(\hat{\alpha}_{ols}, \hat{\boldsymbol{\beta}}_{ols})$ are

$$\begin{pmatrix} \hat{\alpha}_{ols} \\ \\ \hat{\boldsymbol{\beta}}_{ols} \end{pmatrix} = \begin{pmatrix} \bar{y} - \hat{\boldsymbol{\beta}}'_{ols}\bar{\boldsymbol{x}} \\ \\ \frac{n}{n-1}\hat{\boldsymbol{\Sigma}}_{\boldsymbol{x}}^{-1}\hat{\boldsymbol{\Sigma}}_{\boldsymbol{x}y} \end{pmatrix}. \tag{2.28}$$

Recall (2.19) and (2.20), we have

$$\begin{aligned}
\hat{\alpha}_{ols} &\xrightarrow{L} \alpha_{ols}, \\
\hat{\boldsymbol{\beta}}_{ols} &\xrightarrow{L} \boldsymbol{\beta}_{ols},
\end{aligned} \tag{2.29}$$

when $n \to \infty$ if the vectors $(y_i, \boldsymbol{x}_i')'$ are iid such that $\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}$ and $\boldsymbol{\Sigma}_{\boldsymbol{x}y}$ exist.

Consider the 1D model (2.3). Chen and Li (1998) gave a special case of Theorem 2.2.2 for $\hat{\boldsymbol{\beta}}_{ols}$.

Theorem 2.3.1. *[5] Under regularity conditions,*

$$\sqrt{n}(\hat{\boldsymbol{\beta}}_{ols} - c\boldsymbol{\beta}) \xrightarrow{L} N_{p-1}(\boldsymbol{0}, \boldsymbol{C}_{ols}), \tag{2.30}$$

where c is some constant, and

$$\boldsymbol{C}_{ols} = \boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1} E[(y - \alpha_{ols} - \boldsymbol{\beta}'_{ols}\boldsymbol{x})^2(\boldsymbol{x} - E\boldsymbol{x})(\boldsymbol{x} - E\boldsymbol{x})'] \,\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}. \tag{2.31}$$

In addition, if $y - \alpha_{ols} - \boldsymbol{\beta}'_{ols}\boldsymbol{x} \perp\!\!\!\perp \boldsymbol{x}$, then

$$\boldsymbol{C}_{ols} = \tau^2\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}, \tag{2.32}$$

where

$$\tau^2 = E[(y - \alpha_{ols} - \boldsymbol{\beta}'_{ols}\boldsymbol{x})^2]. \tag{2.33}$$

Chen and Li (1998) also gave results about the asymptotic covariance matrix of $\boldsymbol{A}\hat{\boldsymbol{\beta}}_{ols}$ for some matrix $\boldsymbol{A}$:

Theorem 2.3.2. *Under the same regularity conditions as Theorem 2.3.1, for some* $q \times (p-1)$ *matrix* $\boldsymbol{A}$, *where the rank of* $\boldsymbol{A}$ *is* $q \leq (p-1)$, *we have*

$$\sqrt{n}(\boldsymbol{A}\hat{\boldsymbol{\beta}}_{ols} - c\boldsymbol{A}\boldsymbol{\beta}) \xrightarrow{L} N_q(\boldsymbol{0}, \boldsymbol{A}\boldsymbol{C}_{ols}\boldsymbol{A}'), \tag{2.34}$$

where c *is some constant and* $\boldsymbol{C}_{ols}$ *is defined in (2.31).*

Let $r = r(\alpha_{ols}, \boldsymbol{\beta}_{ols})$ *and let* $\boldsymbol{a}'_i$ *be the* i*th row of* $\boldsymbol{A}$ *where* $i = 1, \cdots, q$. *In addition, if* $\boldsymbol{A}\boldsymbol{\beta} = \boldsymbol{0}$ *and if* $\mathrm{Cov}(r^2, (\boldsymbol{a}'_i\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}(\boldsymbol{x} - E\boldsymbol{x}))^2) = 0$, *then (2.34) could be simplified as the following*

$$\sqrt{n}(\boldsymbol{A}\hat{\boldsymbol{\beta}}_{ols} - c\boldsymbol{A}\boldsymbol{\beta}) \xrightarrow{L} N_q(\boldsymbol{0}, \tau^2\boldsymbol{A}\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}\boldsymbol{A}'), \tag{2.35}$$

where τ^2 *is defined in (2.33).*

Proof. By (2.31), the asymptotic covariance

$$\begin{aligned}
\mathrm{Cov}(\boldsymbol{a}'_i\hat{\boldsymbol{\beta}}_{ols}) &= n^{-1}\boldsymbol{a}'_i\,\mathrm{Cov}(\hat{\boldsymbol{\beta}}_{ols})\,\boldsymbol{a}_i \\
&= n^{-1}\boldsymbol{a}'_i\,\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}E[r^2(\boldsymbol{x} - E\boldsymbol{x})(\boldsymbol{x} - E\boldsymbol{x})']\,\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}\,\boldsymbol{a}_i \\
&= n^{-1}E[r^2\boldsymbol{a}'_i\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}(\boldsymbol{x} - E\boldsymbol{x})(\boldsymbol{x} - E\boldsymbol{x})'\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}\boldsymbol{a}_i] \\
&= n^{-1}E[r^2(\boldsymbol{a}'_i\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}(\boldsymbol{x} - E\boldsymbol{x}))^2].
\end{aligned}$$

Since $\boldsymbol{a}'_i\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}(\boldsymbol{x} - E\boldsymbol{x})$ is a scalar, we have $(\boldsymbol{a}'_i\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}(\boldsymbol{x} - E\boldsymbol{x}))' = \boldsymbol{a}'_i\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}(\boldsymbol{x} - E\boldsymbol{x})$.

Since $Ewz = EwEz + \mathrm{Cov}(w, z)$,

$$\mathrm{Cov}(\boldsymbol{a}'_i\hat{\boldsymbol{\beta}}_{ols}) = n^{-1}Er^2E[(\boldsymbol{a}'_i\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}(\boldsymbol{x} - E\boldsymbol{x}))^2] + n^{-1}\mathrm{Cov}(r^2, (\boldsymbol{a}'_i\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}(\boldsymbol{x} - E\boldsymbol{x}))^2).$$

Since $\mathrm{Cov}(r^2, (\boldsymbol{a}_i'\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}(\boldsymbol{x} - E\boldsymbol{x}))^2) = 0$,

$$\begin{aligned}\mathrm{Cov}(\boldsymbol{a}_i'\hat{\boldsymbol{\beta}}_{ols}) &= n^{-1}\tau^2\boldsymbol{a}_i'\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}E[(\boldsymbol{x} - E\boldsymbol{x})(\boldsymbol{x} - E\boldsymbol{x})']\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}\boldsymbol{a}_i \\ &= n^{-1}\tau^2\boldsymbol{a}_i'\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}\boldsymbol{\Sigma}_{\boldsymbol{x}}\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}\boldsymbol{a}_i \\ &= n^{-1}\tau^2\boldsymbol{a}_i'\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}\boldsymbol{a}_i.\end{aligned}$$

Hence $\mathrm{Cov}(\boldsymbol{A}\hat{\boldsymbol{\beta}}_{ols}) = n^{-1}\tau^2\boldsymbol{A}\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}\boldsymbol{A}'$. □

In addition, if the predictor $\boldsymbol{x}$ has the multivariate normal distribution with mean $\boldsymbol{\mu}$ and the nonsingular covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{x}}$, then the asymptotic covariance matrix of the regression coefficient $\hat{\boldsymbol{\beta}}_{ols}$ takes the following form. See [1] and [2].

Theorem 2.3.3. *Consider the model (2.4). If* $E(\boldsymbol{x}'\boldsymbol{x}\,|m(\alpha + \boldsymbol{\beta}'\boldsymbol{x})|^2\,)$ *is finite, then under regularity conditions*

$$\sqrt{n}(\hat{\boldsymbol{\beta}}_{ols} - c\boldsymbol{\beta}) \xrightarrow{L} N_{p-1}(\boldsymbol{0}, \boldsymbol{C}_{bols}), \tag{2.36}$$

where

$$c = \frac{1}{Var(\alpha + \boldsymbol{\beta}'\boldsymbol{x})}\mathrm{Cov}(m(\alpha + \boldsymbol{\beta}'\boldsymbol{x}), \alpha + \boldsymbol{\beta}'\boldsymbol{x}), \tag{2.37}$$

the covariance matrix

$$\boldsymbol{C}_{bols} = \sigma^2\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1} + \boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}E(h(\boldsymbol{x})^2(\boldsymbol{x} - \boldsymbol{\mu})(\boldsymbol{x} - \boldsymbol{\mu})')\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}, \tag{2.38}$$

where

$$h(\boldsymbol{x}) = m(\alpha + \boldsymbol{\beta}'\boldsymbol{x}) - \alpha^* - \boldsymbol{\beta}^{*'}\boldsymbol{x}, \tag{2.39}$$

and

$$\alpha^* = Ey - \boldsymbol{\beta}^*E\boldsymbol{x} \quad \textit{and} \quad \boldsymbol{\beta}^* = c\,\boldsymbol{\beta}. \tag{2.40}$$

Notice that Theorem 2.3.3 and Theorem 2.3.1 are both results about the asymptotic covariance matrix of $\hat{\boldsymbol{\beta}}_{ols}$. A natural question we may ask is what is the relationship between them? We give the answer as the following theorem.

Theorem 2.3.4. *Suppose the regularity conditions are satisfied. If the single index model (2.4) holds and* $\boldsymbol{x} \sim N_{p-1}(\boldsymbol{\mu}, \boldsymbol{\Sigma}_{\boldsymbol{x}})$*, then (2.31) and (2.38) are equivalent.*

Proof. Recall that $\sigma^2 = Ee^2 = \text{Var}(e)$. According to (2.31), (2.39), and (2.22), the asymptotic covariance matrix

$$\begin{aligned}
\text{Cov}(\hat{\boldsymbol{\beta}}_{ols}) &= n^{-1}\boldsymbol{C}_{ols} = n^{-1}\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1} E[r^2(\boldsymbol{x} - E\boldsymbol{x})(\boldsymbol{x} - E\boldsymbol{x})']\,\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1} \\
&= n^{-1}\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1} E[(y - \alpha_{ols} - \boldsymbol{\beta}'_{ols}\boldsymbol{x})^2(\boldsymbol{x} - E\boldsymbol{x})(\boldsymbol{x} - E\boldsymbol{x})']\,\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1} \\
&= n^{-1}\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1} E[(m(\alpha + \boldsymbol{\beta}'\boldsymbol{x}) + e - Ey - \boldsymbol{\beta}'_{ols}(\boldsymbol{x} - E\boldsymbol{x}))^2(\boldsymbol{x} - E\boldsymbol{x})(\boldsymbol{x} - E\boldsymbol{x})']\,\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1} \\
&= n^{-1}\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1} E[(m(\alpha + \boldsymbol{\beta}'\boldsymbol{x}) - Ey - \boldsymbol{\beta}'_{ols}(\boldsymbol{x} - E\boldsymbol{x}))^2(\boldsymbol{x} - E\boldsymbol{x})(\boldsymbol{x} - E\boldsymbol{x})']\,\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1} \\
&\quad + n^{-1}\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1} E[e^2(\boldsymbol{x} - E\boldsymbol{x})(\boldsymbol{x} - E\boldsymbol{x})']\,\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1} \\
&\quad + n^{-1}\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1} E[2e(m(\alpha + \boldsymbol{\beta}^T\boldsymbol{x}) - Ey - \boldsymbol{\beta}'_{ols}(\boldsymbol{x} - E\boldsymbol{x}))(\boldsymbol{x} - E\boldsymbol{x})(\boldsymbol{x} - E\boldsymbol{x}')]\,\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}
\end{aligned}$$

$$\begin{aligned}
&= n^{-1}\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1} E[(m(\alpha + \boldsymbol{\beta}'\boldsymbol{x}) - Ey - \boldsymbol{\beta}'_{ols}(\boldsymbol{x} - E\boldsymbol{x}))^2(\boldsymbol{x} - E\boldsymbol{x})(\boldsymbol{x} - E\boldsymbol{x})']\,\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1} \\
&\quad + n^{-1}\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}\sigma^2\boldsymbol{\Sigma}_{\boldsymbol{x}}\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1} + 0 \qquad (\text{since } Ee = 0 \text{ and } e \perp\!\!\!\perp \boldsymbol{x}) \\
&= n^{-1}\sigma^2\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1} + n^{-1}\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1} E[(m(\alpha + \boldsymbol{\beta}'\boldsymbol{x}) - \alpha_{ols} - \boldsymbol{\beta}'_{ols}\boldsymbol{x})^2(\boldsymbol{x} - E\boldsymbol{x})(\boldsymbol{x} - E\boldsymbol{x})']\,\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1} \\
&= n^{-1}\sigma^2\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1} + n^{-1}\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1} E(h(\boldsymbol{x})^2(\boldsymbol{x} - \boldsymbol{\mu})(\boldsymbol{x} - \boldsymbol{\mu})')\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}.
\end{aligned}$$

□

If we have $\boldsymbol{x} \sim N_{p-1}(\boldsymbol{\mu}, \boldsymbol{\Sigma}_{\boldsymbol{x}})$ and the MLR model (2.5) holds, then the multiplicative constant c in (2.37) and the function $h(\boldsymbol{x})$ in (2.39) could be simplified as

$$\begin{aligned}
c &= \frac{1}{\text{Var}(\alpha + \boldsymbol{\beta}'\boldsymbol{x})}\,\text{Cov}(\alpha + \boldsymbol{\beta}'\boldsymbol{x}, \alpha + \boldsymbol{\beta}'\boldsymbol{x}) = 1, \\
h(\boldsymbol{x}) &= \alpha + \boldsymbol{\beta}'\boldsymbol{x} - \alpha - \boldsymbol{\beta}'\boldsymbol{x} = 0.
\end{aligned} \tag{2.41}$$

Hence we can get the familiar least squares theory

$$\sqrt{n}(\hat{\boldsymbol{\beta}}_{ols} - c\boldsymbol{\beta}) \xrightarrow{L} N_{p-1}(\boldsymbol{0}, \boldsymbol{C}_{ols}), \tag{2.42}$$

where $c = 1$ and

$$\boldsymbol{C}_{ols} = \sigma^2 \boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}. \tag{2.43}$$

Recall the τ^2 in (2.33). If the MLR model holds, then $\tau^2 = \sigma^2$ and formula (2.32) is equivalent to (2.43). Therefore, we have

$$\text{Cov}(\hat{\boldsymbol{\beta}}_{ols}) = n^{-1}\,\tau^2\,\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1} = n^{-1}\,\sigma^2\,\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}. \tag{2.44}$$

When $m(\cdot)$ is unknown, estimating τ^2 and $y - \alpha_{ols} - \boldsymbol{\beta}'_{ols}\boldsymbol{x}$ is easier than estimating σ^2.

2.3.2 Theoretical Results For Testing Problem

In this section we will consider the hypotheses testing problem

$$H_0 : \boldsymbol{A\beta} = \boldsymbol{0} \;\text{ versus }\; H_1 : \boldsymbol{A\beta} \neq \boldsymbol{0}, \tag{2.45}$$

where $\boldsymbol{A}$ is a full rank $q \times (p-1)$ matrix with $q \leq (p-1)$.

Later we will introduce two test statistics based on the OLS estimator $\hat{\boldsymbol{\beta}}_{ols}$ and discuss their relationship. Then the main new theoretical results will be presented.

We will first examine the χ^2 test statistic introduced by Li and Duan (1989). Let the data be $(y_i, \boldsymbol{x}_i)$ for $i = 1, \cdots, n$. A natural way to test (2.45) is using $\boldsymbol{A}\hat{\boldsymbol{\beta}}_{ols}$ as the test statistic, and H_0 will be rejected if $\boldsymbol{A}\hat{\boldsymbol{\beta}}_{ols}$ is sufficiently different from $\boldsymbol{0}$. Because we do not treat every element in $\boldsymbol{A\beta}$ equally, the quadratic $(\boldsymbol{A}\hat{\boldsymbol{\beta}}_{ols})(\text{Cov}(\boldsymbol{A}\hat{\boldsymbol{\beta}}_{ols}))^{-1}(\boldsymbol{A}\hat{\boldsymbol{\beta}}_{ols})'$ which considers the precision of each entry $\hat{\boldsymbol{\beta}}_{ols,i}$ of $\hat{\boldsymbol{\beta}}_{ols}$ should be used. To create the test statistic, we will define

$$\overline{\boldsymbol{x}} = \frac{1}{n}\sum_{i=1}^{n} \boldsymbol{x}_i, \tag{2.46}$$

$$\hat{\boldsymbol{\Sigma}}_{\boldsymbol{x}} = \frac{1}{n-1}\sum_{i=1}^{n}(\boldsymbol{x}_i - \overline{\boldsymbol{x}})(\boldsymbol{x}_i - \overline{\boldsymbol{x}})', \tag{2.47}$$

and

$$\hat{\tau}^2 = \text{MSE} = \frac{1}{n-p}\sum_{i=1}^{n} r_i^2, \tag{2.48}$$

where r_i is defined in Section 2.3. Recall (2.35), the asymptotic covariance matrix of $\boldsymbol{A}\hat{\boldsymbol{\beta}}_{ols}$ is $n^{-1}\tau^2\boldsymbol{A}\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}\boldsymbol{A}'$ when H_0 is true. This result suggests that the following test statistic for the testing problem (2.45) is

$$W_0 = \frac{n(\boldsymbol{A}\hat{\boldsymbol{\beta}}_{ols})'(\boldsymbol{A}\hat{\boldsymbol{\Sigma}}_{\boldsymbol{x}}^{-1}\boldsymbol{A}')^{-1}(\boldsymbol{A}\hat{\boldsymbol{\beta}}_{ols})}{\hat{\tau}^2}. \tag{2.49}$$

If $H_0 : \boldsymbol{A}\boldsymbol{\beta} = \boldsymbol{0}$ is true, then

$$W_0 \xrightarrow{L} \chi^2_q \quad \text{when} \quad n \to \infty. \tag{2.50}$$

Therefore we will reject H_0 if $W_0 \geq \chi^2_q(1-\alpha)$, where α denotes the type I error.

Next we will introduce the F test statistic for (2.45). Let $\tilde{\boldsymbol{A}} = (\boldsymbol{0}_q, \boldsymbol{A})$. Using the notation in Section 2.3.1, the testing problem (2.45) is equivalent to testing

$$H_0 : \tilde{\boldsymbol{A}}\boldsymbol{\eta} = \boldsymbol{0} \;\text{ versus }\; H_1 : \tilde{\boldsymbol{A}}\boldsymbol{\eta} \neq \boldsymbol{0}. \tag{2.51}$$

Seber and Lee (2003, p. 99 - 106)[26] derive the usual MLR F-test for a more generalized testing problem

$$H_0 : \tilde{\boldsymbol{A}}\boldsymbol{\eta} = \boldsymbol{c} \;\text{ versus }\; H_1 : \tilde{\boldsymbol{A}}\boldsymbol{\eta} \neq \boldsymbol{c}, \tag{2.52}$$

as

$$F_0 = \frac{(SSE_R - SSE)/q}{SSE/(n-p)} = \frac{(\tilde{\boldsymbol{A}}\hat{\boldsymbol{\eta}} - \boldsymbol{c})'[\tilde{\boldsymbol{A}}(\boldsymbol{X}'\boldsymbol{X})^{-1}\tilde{\boldsymbol{A}}']^{-1}(\tilde{\boldsymbol{A}}\hat{\boldsymbol{\eta}} - \boldsymbol{c})}{q\,MSE}. \tag{2.53}$$

Here $\boldsymbol{c}$ is some vector. Obviously, problem (2.51) is a special case of (2.52) with $\boldsymbol{c} = \boldsymbol{0}$.

Next we will present our main new theorem to show that either (2.49) or (2.53) can be used for testing (2.45).

Theorem 2.3.5. *Assume that the 1D model (2.2) holds and that under $H_0 : \boldsymbol{A}\boldsymbol{\beta} = \boldsymbol{0}$ (2.50) holds. Then the test statistic F_0 which is defined by (2.53) satisfies*

$$F_0 = \frac{(n-1)W_0}{nq} \xrightarrow{L} \frac{\chi^2_q}{q}. \tag{2.54}$$

as $n \to \infty$.

Proof. According to (2.24) and (2.25),

$$[\tilde{\boldsymbol{A}}(\boldsymbol{X}'\boldsymbol{X})^{-1}\tilde{\boldsymbol{A}}']^{-1} = (\boldsymbol{A}\boldsymbol{D}^{-1}\boldsymbol{A}')^{-1} = (n-1)(\boldsymbol{A}\hat{\boldsymbol{\Sigma}}_x^{-1}\boldsymbol{A}')^{-1}. \tag{2.55}$$

With the condition $\boldsymbol{c} = \boldsymbol{0}$ and the previous work, the test statistic for (2.53) satisfies

$$F_0 = \frac{(n-1)(\boldsymbol{A}\hat{\boldsymbol{\beta}}_{ols})'(\boldsymbol{A}\hat{\boldsymbol{\Sigma}}_x^{-1}\boldsymbol{A}')^{-1}(\boldsymbol{A}\hat{\boldsymbol{\beta}}_{ols})}{q\, MSE} = \frac{(n-1)W_0}{nq}. \tag{2.56}$$

Hence the result follows by (2.50). □

Thus Theorem 2.3.5 shows that hypotheses testing can be done using OLS software originally meant for MLR. If H_0 is true, the MLR model holds, and the errors e_i are iid $N(0, \sigma^2)$, then $F_0 \sim F_{q,n-p}$. Recall that if some statistic $T_n \sim F_{q,n-p}$, then $T_n \xrightarrow{L} \chi^2_q/q$ as $n \to \infty$. The OLS software is easier to use than the chi–square test (2.49). Tests developed for parametric models such as the deviance tests for GLMs will often have more power than the model free OLS tests. Simonoff and Tsai (2002) [27] suggest tests for single index models while Cook (2004) [7] develops model free tests for model (2.1).

To use the OLS output, the assumption that OLS is a useful estimator for the 1D model needs to be checked. Methods for checking OLS are suggested by Olive and Hawkins (2005) [24] who showed that variable selection methods, originally meant for MLR and based on OLS and the Mallow's C_p criterion, can also be used for 1D models. Since the C_p statistic is a one to one function of the F statistic for testing the submodel, Theorem 2.3.5 provides additional support for using OLS for variable selection for 1D models. Li, Cook and Nachtsheim (2005) [19] suggest model free methods of variable selection for model (2.1).

2.4 THEORETICAL RESULTS FOR SIR

2.4.1 Theoretical Results For The SIR Estimator

In this section we will introduce another DR method SIR and the related results about SIR. In SIR the following eigenvalue decomposition is conducted to find the SIR directions $\boldsymbol{\beta}_i$ [5].

$$\boldsymbol{\Sigma}_{E(\boldsymbol{x}|y)}\,\boldsymbol{\beta}_i = \lambda_i \boldsymbol{\Sigma}_{\boldsymbol{x}}\,\boldsymbol{\beta}_i, \tag{2.57}$$

where $\boldsymbol{\Sigma}_{E(\boldsymbol{x}|y)} = \text{Cov}(E(\boldsymbol{x}|y))$.

There are at most $p-1$ nonzero eigenvalues λ_i and $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{p-1}$. The ith SIR direction $\boldsymbol{\beta}_i$ corresponds to the ith largest eigenvalue λ_i. For a dD model, use $\boldsymbol{B} = [\boldsymbol{\beta}_1, \cdots, \boldsymbol{\beta}_d]$.

For SIR, our interest is to find the asymptotic covariance matrix $\boldsymbol{C}_{iSIR}$ of the estimated SIR directions. In general, $\boldsymbol{C}_{iSIR}$ does not have a simple formula and the asymptotics for SIR are complicated and hard to explain. The following result, given in Chen and Li (1998), is useful.

Theorem 2.4.1. *For model (2.1), let $\boldsymbol{A}$ be a full rank $q \times (p-1)$ matrix such that $\boldsymbol{A}\boldsymbol{\beta} = \boldsymbol{0}$, then under regularity conditions*

$$\sqrt{n}(\hat{\boldsymbol{\beta}}_{iSIR} - c_i\boldsymbol{\beta}) \xrightarrow{L} N_{p-1}(\boldsymbol{0}, \boldsymbol{C}_{iSIR}), \tag{2.58}$$

and

$$\sqrt{n}(\boldsymbol{A}\hat{\boldsymbol{\beta}}_{iSIR} - c_i\boldsymbol{A}\boldsymbol{\beta}) \xrightarrow{L} N_q(\boldsymbol{0}, \boldsymbol{A}\boldsymbol{C}_{iSIR}\boldsymbol{A}'), \tag{2.59}$$

where $\boldsymbol{A}\hat{\boldsymbol{C}}_{iSIR}\boldsymbol{A}' = \frac{1-\hat{\lambda}_i}{\hat{\lambda}_i}\,\boldsymbol{A}\hat{\boldsymbol{\Sigma}}_{\boldsymbol{x}}^{-1}\boldsymbol{A}'$.

For the $1D$ regression (2.3), let $\hat{\boldsymbol{\beta}}_{SIR} = \hat{\boldsymbol{\beta}}_{1SIR}$.

Comparing definitions (2.58) and (2.43), we can see the estimated asymptotic covariance matrices $\boldsymbol{A}\hat{\boldsymbol{\beta}}_{OLS}$ and $\boldsymbol{A}\hat{\boldsymbol{\beta}}_{iSIR}$ for the SIR directions are proportional to each other.

2.4.2 Theoretical Results For Testing Problem

In this section we will discuss the test statistics for the testing problem (2.45) based on the SIR estimator $\hat{\boldsymbol{\beta}}_{iSIR}$. According to (2.59), the estimated asymptotic covariance matrix of $\boldsymbol{A}\hat{\boldsymbol{\beta}}_{iSIR}$ is $\dfrac{1-\hat{\lambda}_i}{n\hat{\lambda}_i}\boldsymbol{A}\hat{\boldsymbol{\Sigma}}_{\boldsymbol{x}}^{-1}\boldsymbol{A}'$ when H_0 is true. Therefore the test statistic for (2.45) could be defined similarly to (2.49) as

$$W_{SIR} = n\,(\boldsymbol{A}\hat{\boldsymbol{\beta}}_{iSIR})'\,[\boldsymbol{A}\hat{\boldsymbol{\Sigma}}_{\boldsymbol{x}}^{-1}\boldsymbol{A}']^{-1}(\boldsymbol{A}\hat{\boldsymbol{\beta}}_{iSIR})/(\frac{1-\hat{\lambda}_i}{\hat{\lambda}_i}). \tag{2.60}$$

In addition if $H_0 : \boldsymbol{A\beta} = \boldsymbol{0}$ is true, we have

$$W_{SIR} \xrightarrow{L} \chi_q^2 \quad \text{when} \quad n \to \infty. \tag{2.61}$$

This means W_{SIR} has an approximate χ_q^2 distribution when n is large and if H_0 is true. Hence we will reject H_0 if $W_{SIR} > \chi_q^2(1-\alpha)$, where α denotes the type I error.

CHAPTER 3

RESISTANT DIMENSION REDUCTION

3.1 INTRODUCTION

Existing DR methods such as OLS and SIR often perform poorly in the presence of outliers. Also the DR theory usually assumes that the predictors satisfy LC. In this section we will introduce outlier resistant DR methods that can give useful results when the assumption of linearly related predictors is violated. In Section 3.2, the ellipsoidal trimming method and the median ball algorithm are introduced; in Section 3.3, two examples are given to explain the application of the DD plots; in Section 3.4, the trimmed EY plots of several DR methods are compared; in Section 3.5, the related theorems about resistant DR methods are introduced; and in Section 3.6, we will discuss the theory for the testing problem (2.45) based on the trimmed data set.

3.2 ELLIPSOIDAL TRIMMING

3.2.1 Introduction

As stated in the last chapter, one of the most important assumptions in the DR literature is that the predictor $\boldsymbol{x}$ should satisfy LC. In addition, if $\boldsymbol{x} \sim EC(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ then the related theorems could be simplified. If the original predictor $\boldsymbol{x}$ does not have an EC distribution, ellipsoidal trimming can remove a portion of relatively remote data from the predictor and make the trimmed predictor's distribution closer to being EC. In this section we will introduce this method and the median ball algorithm which performs the trimming.

To perform ellipsoidal trimming, we first need to compute an estimator $(T, \boldsymbol{C})$ from the original predictors. Let T be a $(p-1) \times 1$ location estimator and let $\boldsymbol{C}$ be a $(p-1)\times(p-1)$ symmetric positive definite dispersion estimator. Then use $(T, \boldsymbol{C})$ to create the Mahalanobis distance D_i for each $\boldsymbol{x}_i$ based on the following formula

$$D_i^2 \equiv D_i^2(T, \boldsymbol{C}) = (\boldsymbol{x}_i - T)'\boldsymbol{C}^{-1}(\boldsymbol{x}_i - T), \tag{3.1}$$

where $i = 1, \cdots, n$.

Let $D_{(1)} \leq D_{(2)} \leq \cdots \leq D_{(n)}$. Consider the hyper ellipsoid $\{\boldsymbol{x} : (\boldsymbol{x} - T)'\boldsymbol{C}^{-1}(\boldsymbol{x} - T) \leq D_{(k)}^2\}$. The ith observed case $(y_i, \boldsymbol{x}_i)$ will be trimmed if $D_i > D_{(k)}$. Then a resistant DR estimator $\hat{\boldsymbol{\beta}}$ of $\boldsymbol{\beta}$ is computed by applying the DR method, eg SIR or OLS, to the remaining cases.

3.2.2 Median Ball Algorithm

The median ball algorithm (MBA) (Olive (2004) [22]) is used to compute the estimator $(T, \boldsymbol{C})$. We need to introduce a definition before we give a detailed explanation about this algorithm.

Definition. The pair $(T_{0i}, \boldsymbol{C}_{0i})$ is called a start if it is an initial trial fit and the pair $(T_{ki}, \boldsymbol{C}_{ki})$ is called an attractor if it is a final fit generated by some algorithm from the start.

The following statements are the steps of the median ball algorithm:

1. Let the classical sample mean $\overline{\boldsymbol{x}}_{0,1} = \dfrac{1}{n}\sum_{i=1}^{n}\boldsymbol{x}_i$ and covariance $\boldsymbol{S}_{0,1} = \dfrac{1}{n-1}\sum_{i=1}^{n}(\boldsymbol{x}_i - \overline{\boldsymbol{x}}_{0,1})(\boldsymbol{x}_i - \overline{\boldsymbol{x}}_{0,1})'$, and let $(T_{0,1}, \boldsymbol{C}_{0,1}) = (\overline{\boldsymbol{x}}_{0,1}, \boldsymbol{S}_{0,1})$ be the first start. Then compute the squared Mahalanobis distances $D_i^2(T_{0,1}, \boldsymbol{C}_{0,1})$ for all $\boldsymbol{x}_i$ and let $md1 = \text{MED}(D_i^2(T_{0,1}, \boldsymbol{C}_{0,1}))$ where $i = 1, \cdots, n$ and $\text{MED}(W)$ means the median of W.

2. Let $\overline{\boldsymbol{x}}_{1,1}$ and $\boldsymbol{S}_{1,1}$ be the classical sample mean and covariance of the set of those $\boldsymbol{x}_i$ whose squared distances $D_i^2(T_{0,1}, \boldsymbol{C}_{0,1})$ are less than or equal to $md1$. Now we have the second start $(T_{1,1}, \boldsymbol{C}_{1,1}) = (\overline{\boldsymbol{x}}_{1,1}, \boldsymbol{S}_{1,1})$. Then recalculate $md1 = \text{MED}(D_i^2(T_{1,1}, \boldsymbol{C}_{1,1}))$.

3. Repeat step 2 for k times to get the sequence $(T_{1,1}, \boldsymbol{C}_{1,1}), \cdots, (T_{k,1}, \boldsymbol{C}_{k,1})$ where $(T_{k,1}, \boldsymbol{C}_{k,1})$ is the first attractor.

4. Let the ith row of $\boldsymbol{W}$ be $\boldsymbol{x}_i'$, the coordinatewise median $\boldsymbol{\nu} = \text{MED}(\boldsymbol{W})$, and $\boldsymbol{I}$ be a $(p-1) \times (p-1)$ identity matrix. Then compute the squared Mahalanobis distances $D_i^2(\boldsymbol{\nu}, \boldsymbol{I})$ for all $\boldsymbol{x}_i$ where $i = 1, \cdots, n$.

5. Let $md = \text{MED}(D_i^2(\boldsymbol{\nu}, \boldsymbol{I})), i = 1, \cdots, n$. Let $T_{0,2}$ and $\boldsymbol{C}_{0,2}$ be the mean and covariance of the set of those $\boldsymbol{x}_i$ whose squared distances $D_i^2(\boldsymbol{\nu}, \boldsymbol{I})$ are less than or equal to md. Here $(T_{0,2}, \boldsymbol{C}_{0,2})$ is the second start. Then compute the squared Mahalanobis distances $D_i^2(T_{0,2}, \boldsymbol{C}_{0,2})$ for all $\boldsymbol{x}_i$ and let $md2 = \text{MED}(D_i^2(T_{0,2}, \boldsymbol{C}_{0,2}))$ where $i = 1, \cdots, n$.

6. Let $\overline{\boldsymbol{x}}_{1,2}$ and $\boldsymbol{S}_{1,2}$ be the classical sample mean and covariance of the set of those $\boldsymbol{x}_i$ whose squared distances $D_i^2(T_{0,2}, \boldsymbol{C}_{0,2})$ are less than or equal to $md2$. Now we have $(T_{1,2}, \boldsymbol{C}_{1,2}) = (\overline{\boldsymbol{x}}_{1,2}, \boldsymbol{S}_{1,2})$. Then recalculate $md2 = \text{MED}(D_i^2(T_{1,2}, \boldsymbol{C}_{1,2}))$.

7. Repeat step 6 for k times to get the corresponding sequence $(T_{1,2}, \boldsymbol{C}_{1,2}), \cdots, (T_{k,2}, \boldsymbol{C}_{k,2})$ where $(T_{k,2}, \boldsymbol{C}_{k,2})$ is the second attractor.

8. Let $(T_a, \boldsymbol{C}_a)$ be $(T_{k,1}, \boldsymbol{C}_{k,1})$ if $|\boldsymbol{C}_{k,1}| \leq |\boldsymbol{C}_{k,2}|$ or $(T_{k,2}, \boldsymbol{C}_{k,2})$ otherwise, where $|\cdot|$ denotes the determinant of a matrix. Let

$$
\begin{aligned}
T_{mba} &= T_a, \\
\boldsymbol{C}_{mba} &= \frac{\boldsymbol{C}_a}{\chi^2_{p-1,0.5}} \text{MED}(D_i^2(T_a, \boldsymbol{C}_a)),
\end{aligned}
\tag{3.2}
$$

where $\chi^2_{p-1,0.5}$ is the 50th percentile of a chi-square distribution with $p-1$ degrees of freedom.

Then $(T_{mba}, \boldsymbol{C}_{mba})$ is the MBA estimator. We will use $(T_{mba}, \boldsymbol{C}_{mba})$ to perform ellipsoidal trimming. MBA works well if there are outliers in the data and it is one of the fastest robust estimators.

3.3 DD PLOT

In this section we will introduce the DD plot. Prior to that, we need to introduce some definitions.

Definition. [23] Let the data be $\boldsymbol{x}_1, \cdots, \boldsymbol{x}_n$, and let the $(p-1) \times 1$ vector

$$T_M = \overline{\boldsymbol{x}} = \frac{1}{n}\sum_{i=1}^{n} \boldsymbol{x}_i,$$

and the $(p-1) \times (p-1)$ matrix

$$\boldsymbol{C}_M = \boldsymbol{S} = \frac{1}{n-1}\sum_{i=1}^{n}(\boldsymbol{x}_i - T_M)(\boldsymbol{x}_i - T_M)'.$$

The *classical Mahalanobis distance* MD_i for each $\boldsymbol{x}_i$ is defined as

$$MD_i = MD_i(T_M, \boldsymbol{C}_M) = \sqrt{(\boldsymbol{x}_i - T_M)'\boldsymbol{C}_M^{-1}(\boldsymbol{x}_i - T_M)}. \tag{3.3}$$

Definition. [23] Suppose $(T_A, \boldsymbol{C}_A)$ is a consistent estimator for $(\boldsymbol{\mu}, a_A\boldsymbol{\Sigma})$, where a_A is some positive constant. Let $(T_R, \boldsymbol{C}_R) = (T_A, \boldsymbol{C}_A/\gamma^2)$ be the scaled algorithm estimator for some to be determined constant $\gamma > 0$. Then the *robust Mahalanobis distance* RD_i for each $\boldsymbol{x}_i$ is defined as

$$RD_i = RD_i(T_R, \boldsymbol{C}_R) = \sqrt{(\boldsymbol{x}_i - T_R)'\boldsymbol{C}_R^{-1}(\boldsymbol{x}_i - T_R)} = \gamma D_i(T_A, \boldsymbol{C}_A) \tag{3.4}$$

Definition. [23] The *DD plot* is a plot of the classical Mahalanobis distance MD_i versus the robust Mahalanobis distance RD_i.

In our study, $(T_R, \boldsymbol{C}_R)$ is obtained using the median ball algorithm. That means the robust Mahalanobis distances RD_i used in the DD plot were based on the MBA estimator of the location and dispersion.

The DD plot has several important applications such as detecting multivariate outliers. Olive (2002) [21] shows that the plotted points in the DD plot will follow the identity line with zero intercept and unit slope if the data distribution is multivariate normal (MVN), and will follow a line with zero intercept but non–unit slope if the

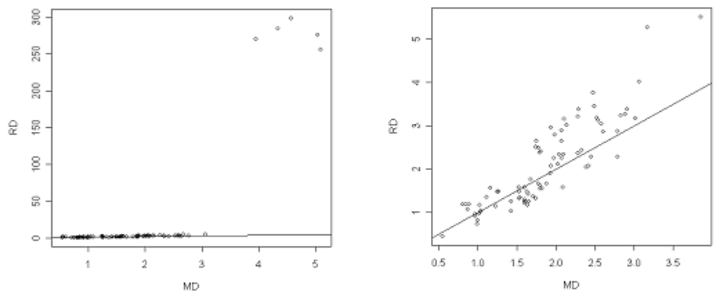
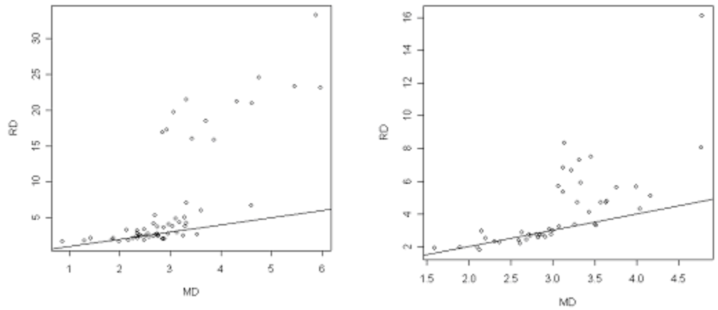


Figure 3.1. DD Plots of Buxton Data

data distribution is elliptically contoured but not MVN. Delete $M\%$ of the cases with the largest MBA distances so that the remaining cases follow the identity line (or some line through the origin) closely. Then apply the DR method on those remaining cases. We will illustrate this application of the DD plot using the following two examples.

Example 1: The data set we used in this example was given by Buxton in 1920 [4]. This data set is a set of 5 measurements of the head length, nasal height, bigonal breadth, cephalic index, and height of the 88 men in Cyprus. The first four variables were used as predictors to predict the dependent variable height and we will use the first four predictors to make the DD plot. There are five outliers, numbered from 61 to 65, which had head lengths over five feet but were only about 0.75 inches tall.

The result is shown in Figure 3.1. The left plot is the DD plot of the original data set. We can see that the majority of the data is clustering about the identity line except the five outlying points which are far away from the other points. The five outliers are the cases with huge head lengths. After removing these cases, we made another DD plot which is shown as the right plot. Then most of the cases are clustering about the identity line. This suggests that the trimmed data set



DDplot With Outliers **DDplot With Outliers Removed**

Figure 3.2. DD Plots of Schaaffhausen Data

distribution is approximately multivariate normal.

Example 2: The data set we used in this example was given by Schaaffhausen in 1878 [25]. This data set has 10 measurements on 47 humans (cases 1 to 47) and 13 apes (cases 48 to 60). The 9 predictors are head length, head breadth, head height, lower jaw length, face length, upper jaw length, height of lower jaw, eye width, and traverse diagonal length and they will be used to make the DD plot. The dependent variable is the cranial capacity. The 13 apes are the outliers in this dataset and we want to detect them.

The result is shown in Figure 3.2. Again, we put the DD plot of the original data set on the left. There are 13 points far away from the majority of the data in this plot. We found that they are the measurements of the 13 apes by examining their case numbers. After dropping these 13 outliers we made another DD plot and put it on the right side. Many of the points are above the identity line, suggesting that more trimming is needed before the predictor distribution is approximately multivariate normal.

3.4 TRIMMED VIEWS

In this section we will compare the EY plots (also called trimmed views), defined below, obtained from several DR methods.

Definition. [23] Suppose the 1D model (2.2) holds, then $y \perp\!\!\!\perp \boldsymbol{x}|\boldsymbol{\beta}'\boldsymbol{x}$. Then $y \perp\!\!\!\perp \boldsymbol{x}|a + c\boldsymbol{\beta}'\boldsymbol{x}$ for any constants a and $c \neq 0$. The term $a + c\boldsymbol{\beta}'\boldsymbol{x}$ is called the *sufficient predictor* (SP). The term $\tilde{\alpha} + \tilde{\boldsymbol{\beta}}'\boldsymbol{x}$ is called the *estimated sufficient predictor* (ESP) where $\tilde{\boldsymbol{\beta}}$ is some estimator of $c\boldsymbol{\beta}$ for some constant c.

Definition. [23] An *EY plot* is a plot of any ESP versus y.

We will give an example using the four DR methods OLS, SIR, PHD, and SAVE. The MBA algorithm will be used to trim $M\%$ of the cases where $M = 0\%, 10\%, \cdots, 90\%$ before we perform the DR method. Then the EY plot will be made for each M. For the methods PHD, SIR, and SAVE, we let the number of slices $h = 4$. By comparing all the EY plots, we will keep a record of the best M and the corresponding estimated regression coefficient $\hat{\boldsymbol{\beta}}$ for each method. The best plot had the smoothest mean function, visually. The results are shown in Table 3.1.

Let $\boldsymbol{\beta} = (\beta_1, \beta_2, \beta_3)' = (1, 2, 3)'$ and $\boldsymbol{x}_i = (x_{i1}, x_{i2}, x_{i3})'$. The true model we used in this example is

$$y_i = \frac{\sin(\boldsymbol{\beta}'\boldsymbol{x}_i)}{\boldsymbol{\beta}'\boldsymbol{x}_i}, \tag{3.5}$$

where $i = 1, \cdots, 250$.

Here the predictors $\boldsymbol{x}$ are coming from the data file "lsinc.lsp" [23]. They are not from the EC distribution. Let $\hat{\boldsymbol{\beta}} = (\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3)$ be the estimator of $\boldsymbol{\beta}$ and let $|\text{corr}(SP, ESP)| = |\text{corr}(\boldsymbol{x}'\boldsymbol{\beta}, \boldsymbol{x}'\hat{\boldsymbol{\beta}})|$.

Table 3.1 gives the 0% trimming and the best trimming regression coefficients $\hat{\boldsymbol{\beta}}$ obtained from all the DR methods. Obviously trimming greatly improved the OLS, SAVE, and PHD estimators. The 0% trimming ESP is not highly correlated

DR Method	M	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\lvert\text{corr}(SP, ESP)\rvert$
OLS	0%	0.00447	-0.000865	0.00232	0.5015
OLS	90%	0.0421	0.0810	0.1910	0.978
SIR	0%	-0.1926	-0.5305	-0.8255	0.996
SIR	20%	-0.3000	-0.5632	-0.7700	0.998
SAVE	0%	0.8180	-0.4443	0.3654	0.292
SAVE	50%	-0.2549	-0.5175	-0.8168	0.9996
PHD	0%	0.9934	0.0378	-0.1085	0.345
PHD	60%	-0.2777	-0.5236	-0.8054	0.9998

Table 3.1. Comparison Of Eight ESPs Using Dataset "lsinc.lsp"

with SP except for the SIR estimator, while all the correlations between the best trimmed ESP and SP are above 0.95.

We also give the EY plots of each method in Figure 3.3. As shown in this figure, trimming was effective, especially for the methods SIR, SAVE, and PHD. They have the best EY plots. We can clearly recognize the true model through these three methods. Comparing all the nonzero trimmed EY plots, SIR used the smallest amount of trimming while OLS used 90% trimming. For the 0% trimmed EY plots, SIR works best but SAVE and OLS completely failed.

Now, we repeat our example on another dataset "sinc.lsp" [23]. Similar to the previous dataset "lsinc.lsp," the predictors $\boldsymbol{x}$ are not from the EC distribution. The results are shown in Table 3.2.

As we can see, trimming greatly improved the OLS and SAVE estimators for this dataset. The best trimmed ESPs are highly correlated with SP for the SIR, SAVE, and PHD estimators.

The EY plots are given in Figure 3.4. The results are similar to Figure 3.3. All the best trimmed EY plots are better than the 0% trimmed views, except for the OLS

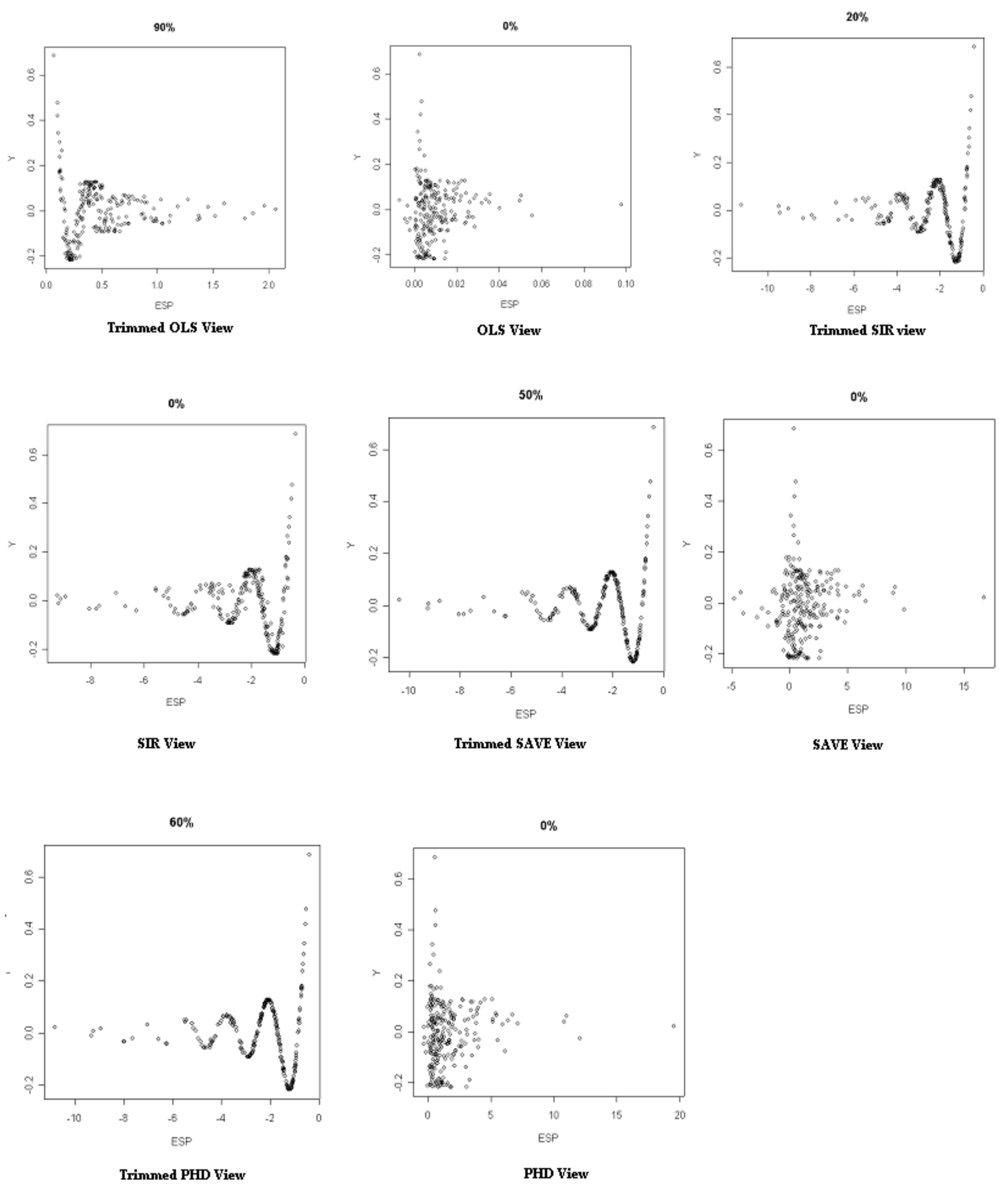


Figure 3.3. EY Plots of Four DR Methods Using Dataset “lsinc.lsp”

DR Method	M	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\lvert\text{corr}(SP, ESP)\rvert$
OLS	0%	0.0032	0.0011	0.0047	0.4148
OLS	90%	0.0321	0.0366	0.2329	0.8757
SIR	0%	-0.4066	-0.3916	-0.8254	0.8504
SIR	10%	0.3032	0.5003	0.8110	0.9918
SAVE	0%	0.0845	-0.7280	0.6804	0.4214
SAVE	60%	-0.2116	-0.5657	-0.7970	0.9927
PHD	0%	0.9995	0.0097	-0.0316	0.8832
PHD	60%	-0.2928	-0.6154	-0.7318	0.9651

Table 3.2. Comparison Of Eight ESPs Using Dataset "sinc.lsp"

estimator. The EY plots of the SIR, SAVE, and PHD estimators clearly displayed the true model and the SIR estimator used the smallest amount of trimming. For the 0% trimmed views, the SAVE and OLS estimators completely failed.

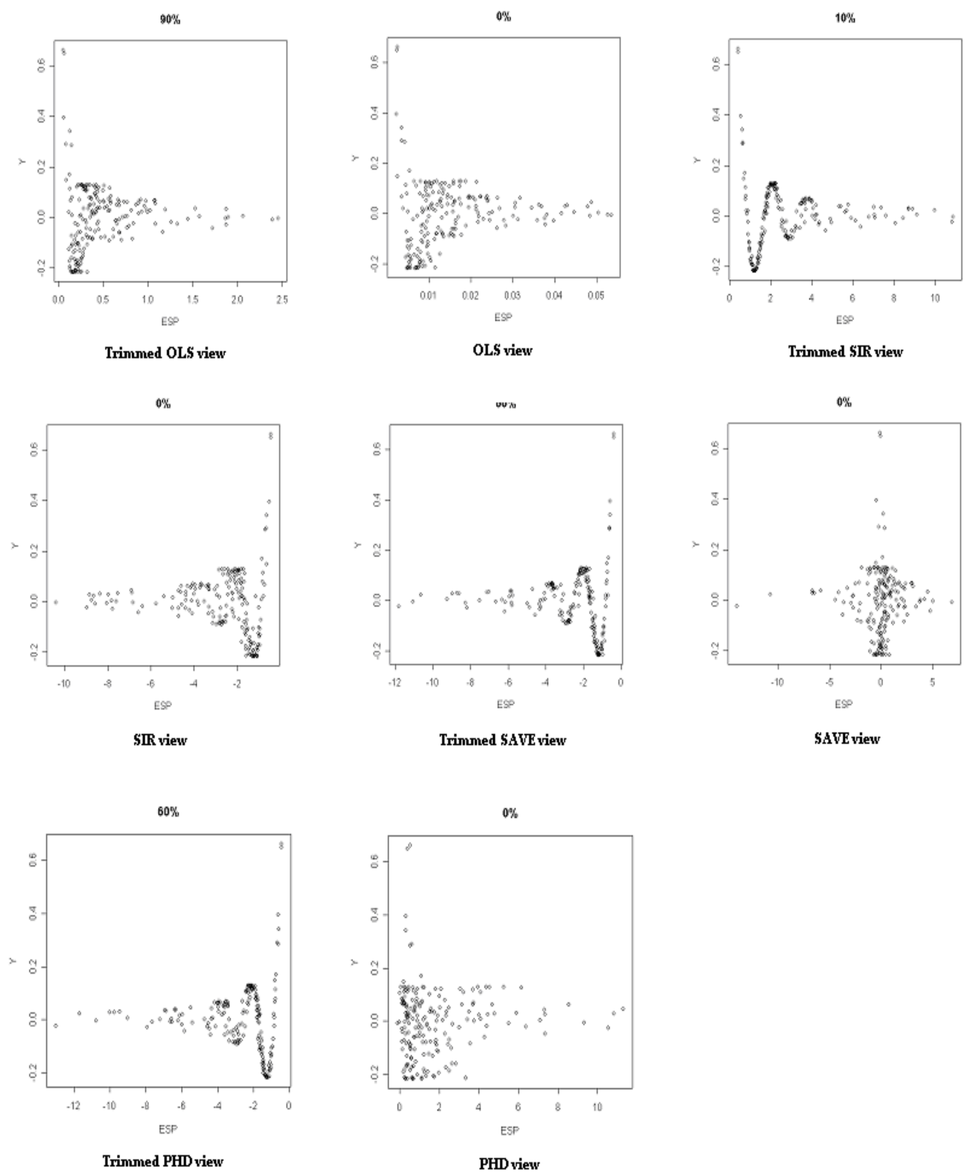


Figure 3.4. EY Plots of Four DR Methods Using Dataset “sinc.lsp”

3.5 RELATED THEOREMS

In this section, we will introduce some theorems about resistant DR methods.

Consider the 1D model (2.3). Let $(\boldsymbol{x}_{Mi}, y_{Mi}), i = 1, \cdots, n_M$ be the cases that remain after trimming $M\%$ of the data. The criterion function is $Q(a + \boldsymbol{b}'\boldsymbol{x}_{Mi}, y_{Mi})$ where $i = 1, \cdots, n_M$. Then the minimization problem is

$$\text{Minimize} \quad L(a + \boldsymbol{b}'\boldsymbol{x}_{Mi}, y_{Mi}) = EQ(a + \boldsymbol{b}'\boldsymbol{x}_{Mi}, y_{Mi}). \tag{3.6}$$

Suppose the above problem (3.6) has a proper solution and let $(\alpha_M, \boldsymbol{\beta}_M)$ be the population solution. We are interested in the relationship between $(\alpha_M, \boldsymbol{\beta}_M)$ and the true coefficients $(\alpha, \boldsymbol{\beta})$. Recall Theorem 2.2.1. If the trimmed data set satisfies LC, we have a good reason to expect that $(\alpha_M, \boldsymbol{\beta}_M)$ has the same property as $(\alpha^*, \boldsymbol{\beta}^*)$ which is the proper solution of the minimization problem (2.8).

Theorem 3.5.1. *Under regularity conditions, if $\boldsymbol{x}_M$ satisfies LC, then*

$$\boldsymbol{\beta}_M = k_M \boldsymbol{\beta}, \tag{3.7}$$

where k_M is some constant.

Let the OLS estimator of $(\alpha_M, \boldsymbol{\beta}_M)$ be $(\hat{\alpha}_M, \hat{\boldsymbol{\beta}}_M)$ and the SIR estimator of $\boldsymbol{\beta}_{Mi} = k_{Mi}\boldsymbol{\beta}_i$ be $\hat{\boldsymbol{\beta}}_{Mi}$. The strong consistency of these estimators can be attained under regularity conditions in a manner similar to the estimators with 0% trimming.

Now we will discuss the asymptotic covariance matrices of these estimators.

Theorem 3.5.2. *Under regularity conditions, we have*

$$\sqrt{n_M}(\hat{\boldsymbol{\beta}}_M - k_M \boldsymbol{\beta}) \xrightarrow{L} N_{p-1}(\mathbf{0}, \boldsymbol{C}_M), \tag{3.8}$$

where

$$\boldsymbol{C}_M = \boldsymbol{\Sigma}_M^{-1} E[r_M^2 (\boldsymbol{x}_M - E\boldsymbol{x}_M)(\boldsymbol{x}_M - E\boldsymbol{x}_M)'] \, \boldsymbol{\Sigma}_M^{-1}, \tag{3.9}$$

$\boldsymbol{\Sigma}_M = \text{Cov}(\boldsymbol{x}_M)$*, and the population OLS "residual" r_M is defined as*

$$r_M = y_M - \alpha_M - \boldsymbol{\beta}_M' \boldsymbol{x}_M. \tag{3.10}$$

Under the same regularity conditions, for some $q \times (p-1)$ matrix $\boldsymbol{A}$, where the rank of $\boldsymbol{A}$ is $q \leq (p-1)$, we have

$$\sqrt{n_M}(\boldsymbol{A}\hat{\boldsymbol{\beta}}_M - k_M \boldsymbol{A}\boldsymbol{\beta}) \xrightarrow{L} N_q(\boldsymbol{0}, \boldsymbol{A}\boldsymbol{C}_M\boldsymbol{A}'). \tag{3.11}$$

Now we can express the previous two equations in terms of n :

$$\sqrt{n}(\hat{\boldsymbol{\beta}}_M - k_M\boldsymbol{\beta}) \xrightarrow{L} N_{p-1}(\boldsymbol{0}, \frac{n}{n_M}\boldsymbol{C}_M), \tag{3.12}$$

$$\sqrt{n}(\boldsymbol{A}\hat{\boldsymbol{\beta}}_M - k_M \boldsymbol{A}\boldsymbol{\beta}) \xrightarrow{L} N_q(\boldsymbol{0}, \frac{n}{n_M}\boldsymbol{A}\boldsymbol{C}_M\boldsymbol{A}'). \tag{3.13}$$

In addition, if the MLR model (2.5) holds and $y - \alpha_M - \boldsymbol{\beta}'_M\boldsymbol{x}_M \perp\!\!\!\perp \boldsymbol{x}_M$, we have

$$\text{Cov}(\hat{\boldsymbol{\beta}}_M) = n_M^{-1}\,\tau_M^2\,\boldsymbol{\Sigma}_M^{-1} = n_M^{-1}\,\sigma^2\,\boldsymbol{\Sigma}_M^{-1}, \tag{3.14}$$

where $\tau_M^2 = E(y_M - \alpha_M - \boldsymbol{\beta}'_M\boldsymbol{x}_M)^2$.

By Chen and Li, if we also have $\boldsymbol{A}\boldsymbol{\beta} = \boldsymbol{0}$ for some $q \times (p-1)$ matrix $\boldsymbol{A}$, then

$$\sqrt{n_M}(\boldsymbol{A}\hat{\boldsymbol{\beta}}_M - k_M \boldsymbol{A}\boldsymbol{\beta}) \xrightarrow{L} N_q(\boldsymbol{0}, \tau_M^2\boldsymbol{A}\boldsymbol{\Sigma}_M^{-1}\boldsymbol{A}'). \tag{3.15}$$

Now we will introduce a theorem about the SIR estimators $\hat{\boldsymbol{\beta}}_{Mi}$.

Theorem 3.5.3. *Under regularity conditions, if we have $\boldsymbol{A}\boldsymbol{\beta}_i = \boldsymbol{0}$ for some $q \times (p-1)$ matrix $\boldsymbol{A}$, then*

$$\sqrt{n_M}(\hat{\boldsymbol{\beta}}_{Mi} - k_{Mi}\boldsymbol{\beta}_i) \xrightarrow{L} N_{p-1}(\boldsymbol{0}, \boldsymbol{C}_{Mi}), \tag{3.16}$$

and

$$\sqrt{n_M}(\boldsymbol{A}\hat{\boldsymbol{\beta}}_{Mi} - k_{Mi}\boldsymbol{A}\boldsymbol{\beta}_i) \xrightarrow{L} N_q(\boldsymbol{0}, \boldsymbol{A}\boldsymbol{C}_{Mi}\boldsymbol{A}'), \tag{3.17}$$

where

$$\boldsymbol{A}\hat{\boldsymbol{C}}_{Mi}\boldsymbol{A}' = \frac{1 - \hat{\lambda}_{Mi}}{\hat{\lambda}_{Mi}}\boldsymbol{A}\hat{\boldsymbol{\Sigma}}_M^{-1}\boldsymbol{A}'. \tag{3.18}$$

Similarly, we have the following expression in terms of n :

$$\sqrt{n}(\hat{\boldsymbol{\beta}}_{Mi} - k_{Mi}\boldsymbol{\beta}_i) \xrightarrow{L} N_{p-1}(\boldsymbol{0}, \frac{n}{n_M}\boldsymbol{C}_{Mi}). \tag{3.19}$$

3.6 THEORETICAL RESULTS FOR TESTING PROBLEM

In this section we will consider the hypothesis testing problem (2.45) based on the trimmed data set.

1. The OLS estimator based on the data remaining after trimming.

Suppose we perform ellipsoidal trimming on the original data and let $(\boldsymbol{x}_{Mi}, y_{Mi})$ for $i = 1, \cdots, n_M$ be the data that is not trimmed, and let $\hat{\boldsymbol{\beta}}_M$ be the corresponding OLS regression coefficient. Then according to (2.56) and (3.15), we can define the two test statistics as the follows:

$$F_M = \frac{(n_M - 1)(\boldsymbol{A}\hat{\boldsymbol{\beta}}_M)'(\boldsymbol{A}\hat{\boldsymbol{\Sigma}}_M^{-1}\boldsymbol{A}')^{-1}(\boldsymbol{A}\hat{\boldsymbol{\beta}}_M)}{q\,\hat{\tau}_M^2} \tag{3.20}$$

and

$$W_M = \frac{n_M(\boldsymbol{A}\hat{\boldsymbol{\beta}}_M)'(\boldsymbol{A}\hat{\boldsymbol{\Sigma}}_M^{-1}\boldsymbol{A}')^{-1}(\boldsymbol{A}\hat{\boldsymbol{\beta}}_M)}{\hat{\tau}_M^2}, \tag{3.21}$$

where $\hat{\tau}_M^2 = \dfrac{\|\boldsymbol{y}_M - \hat{\boldsymbol{y}}_M\|^2}{n_M - p}$.

Under H_0, we expect $F_M \approx F_{q,n_M-p}$ and $W_M \approx \chi_q^2$. Hence H_0 will be rejected for large values of F_M and W_M. We can also express one of the test statistics as a function of the other as

$$F_M = \frac{(n_M - 1)W_M}{q\,n_M}. \tag{3.22}$$

2. The SIR estimator based on the data remaining after trimming.

We also do the χ^2 SIR test based on the trimmed data. Let $\hat{\boldsymbol{\beta}}_{Mi}$ be the SIR coefficients for the trimmed data. Then the corresponding χ^2 test statistic is

$$W_{SM} = n_M\,(\boldsymbol{A}\hat{\boldsymbol{\beta}}_{Mi})'\,[\,\boldsymbol{A}\hat{\boldsymbol{\Sigma}}_M^{-1}\boldsymbol{A}']^{-1}(\boldsymbol{A}\hat{\boldsymbol{\beta}}_{Mi})/(\frac{1-\hat{\lambda}_{Mi}}{\hat{\lambda}_{Mi}}), \tag{3.23}$$

where $\hat{\lambda}_{Mi}$ and $\hat{\boldsymbol{\beta}}_{Mi}$ are calculated from

$$\hat{\boldsymbol{\Sigma}}_{E(\boldsymbol{x}_M|y_M)}\,\hat{\boldsymbol{\beta}}_{Mi} = \hat{\lambda}_{Mi}\hat{\boldsymbol{\Sigma}}_M\,\hat{\boldsymbol{\beta}}_{Mi}\,, \tag{3.24}$$

where $\hat{\boldsymbol{\Sigma}}_{E(\boldsymbol{x}_M|y_M)} = \hat{\mathrm{Cov}}(E(\boldsymbol{x}_M|y_M))$.

If H_0 holds, then $W_{SM} \approx \chi_q^2$ and H_0 will be rejected if $W_{SM} > \chi_q^2(1-\alpha)$, where α denotes the type I error.

CHAPTER 4

SIMULATIONS

4.1 INTRODUCTION

We will give all the simulation results in this chapter. In Section 4.2, we will introduce the regression models and the predictor distributions used in the simulation; in Section 4.3, we will use two resistant DR methods to estimate the coefficients and their covariances; and in Section 4.4, we will discuss the testing problem (2.45) for two resistant DR estimators. For all simulations, the number of runs $nruns = 1000$. Simulations were done in R.

4.2 REGRESSION MODELS AND PREDICTOR DISTRIBUTIONS

In this section, we will discuss the models and predictor distributions which we will use in the simulation.

1. Regression Models.

The MLR model is the most important $1D$ model, and we also pick 6 nonlinear single index models for the comparison. Let the sufficient predictor

$$SP = \alpha + \boldsymbol{x}'\boldsymbol{\beta}. \tag{4.1}$$

Then the 7 single index models used in the simulation are

1. $y = SP + e$ (MLR),
2. $y = (SP)^2 + e$,
3. $y = exp(SP) + e$,
4. $y = (SP)^3 + e$,
5. $y = \dfrac{\sin(SP)}{SP} + 0.01e$,

6. $y = SP + \sin(SP) + 0.1e$,

7. $y = \sqrt{|SP|} + 0.1e$,

where the error $e \sim N(0,1)$ is independent of $\boldsymbol{x}$.

2. Distributions of $\boldsymbol{x}$.

We choose 8 different distributions for $\boldsymbol{x}_i = (x_{i1}, \cdots, x_{i(p-1)})'$, where the $(p-1) \times 1$ vectors $\boldsymbol{x}_i$ are i.i.d. for $i = 1, \cdots, n$.

1. $\boldsymbol{x}_i \sim N_{p-1}(\mathbf{0}, \boldsymbol{I})$.

2. $\boldsymbol{x}_i \sim 0.6N_{p-1}(\mathbf{0}, \boldsymbol{I}) + 0.4N_{p-1}(\mathbf{0}, 25\boldsymbol{I})$.

3. $\boldsymbol{x}_i \sim 0.4N_{p-1}(\mathbf{0}, \boldsymbol{I}) + 0.6N_{p-1}(\mathbf{0}, 25\boldsymbol{I})$.

4. $\boldsymbol{x}_i \sim 0.9N_{p-1}(\mathbf{0}, \boldsymbol{I}) + 0.1N_{p-1}(\mathbf{0}, 25\boldsymbol{I})$.

5. $\boldsymbol{x}_i \sim LN(\mathbf{0}, \boldsymbol{I})$.

   Here LN stands for the lognormal distribution. We construct the predictor by letting $\boldsymbol{x}_i = exp(\boldsymbol{z}_i)$, where $\boldsymbol{z}_i \sim N_{p-1}(\mathbf{0}, \boldsymbol{I})$.

6. $\boldsymbol{x}_i \sim MVT_3$.

   Here MVT_3 stands for the multivariate t distribution with 3 degrees of freedom. The predictor $\boldsymbol{x}_i$ is constructed by letting $\boldsymbol{x}_i = \dfrac{\boldsymbol{z}_i}{\sqrt{w_i/3}}$, where $\boldsymbol{z}_i \sim N_{p-1}(\mathbf{0}, \boldsymbol{I})$ and the scalar random variable $w_i \sim \chi^2_3$ is independent of $\boldsymbol{x}_i$. The $\boldsymbol{x}_i$ has first moments but not second moments [20].

7. $\boldsymbol{x}_i \sim MVT_5$.

   This type of $\boldsymbol{x}_i$ has a multivariate t distribution with 5 degrees of freedom. We construct it by letting $\boldsymbol{x}_i = \dfrac{\boldsymbol{z}_i}{\sqrt{w_i/5}}$, where $\boldsymbol{z}_i \sim N_{p-1}(\mathbf{0}, \boldsymbol{I})$ and the scalar random variable $w_i \sim \chi^2_5$ is independent of $\boldsymbol{x}_i$. Now $\boldsymbol{x}_i$ has both first and second moments [20].

8. $\boldsymbol{x}_i \sim MVT_{19}$.

Here $\boldsymbol{x}_i = \dfrac{\boldsymbol{z}_i}{\sqrt{w_i/19}}$, where $\boldsymbol{z}_i \sim N_{p-1}(\boldsymbol{0}, \boldsymbol{I})$ and the scalar random variable $w_i \sim \chi^2_{19}$ is independent of $\boldsymbol{x}_i$. It has the multivariate t distribution with 19 degrees of freedom and has both first and second moments. As the degrees of the freedom of the random vector which has the multivariate t distribution go to ∞, the joint distribution of these random vectors tends to a multivariate normal distribution [20]. Therefore, $\boldsymbol{x}_i$ behaves more like the standard multivariate normal distribution.

All these 8 distributions except the lognormal distribution are elliptically contoured distributions.

4.3 COEFFICIENT ESTIMATION USING RESISTANT DR METHODS

4.3.1 Introduction

In this section, we will use two DR methods, OLS and SIR, to find the estimator $\hat{\boldsymbol{\beta}}$ of the coefficients $c\boldsymbol{\beta}$ for the 7 single index models introduced in Section 4.2. Let the entire data be $(\boldsymbol{x}_i, y_i)$ for $i = 1, \cdots, n$, the dimension of the coefficients $p - 1 = 4$, and the true coefficient $\boldsymbol{\beta} = (1, 1, 1, 1)'$. Also we let the predictor $\boldsymbol{x}_i \sim N_{p-1}(\boldsymbol{0}, \boldsymbol{I})$ for $i = 1, \cdots, n$. In the simulation we will use the MBA algorithm to trim $M = 0\%$ to 90% of the data and let $(\boldsymbol{x}_{Mi}, y_{Mi})$ for $i = 1, \cdots, n_M$ be the data remaining after trimming.

Let $\hat{\boldsymbol{\beta}}_M = (\hat{\beta}_{M1}, \cdots, \hat{\beta}_{M(p-1)})'$ be the OLS estimator and $\hat{\boldsymbol{\beta}}_{M,SIR} = (\hat{\beta}_{M1,SIR}, \cdots, \hat{\beta}_{M(p-1),SIR})'$ be the SIR estimator. For $i = 1, \cdots, p-1$, let $SD = SD(\hat{\boldsymbol{\beta}}_{Mi})$ be the sample standard deviation of the OLS estimators $\hat{\boldsymbol{\beta}}_{Mi,1}, \cdots, \hat{\boldsymbol{\beta}}_{Mi,1000}$ from the simulation, let the standard error of $\hat{\boldsymbol{\beta}}_{Mi}$ be

$$SE(\hat{\boldsymbol{\beta}}_{Mi}) = \sqrt{MSE_M}\sqrt{(\boldsymbol{X}'_M\boldsymbol{X}_M)^{-1}_{i+1,i+1}} \approx \frac{1}{\sqrt{n_M}}\sqrt{\hat{\tau}^2_M(\hat{\boldsymbol{\Sigma}}_M)^{-1}_{ii}}, \qquad (4.2)$$

where $MSE_M = \hat{\tau}_M^2 = \dfrac{\|\boldsymbol{y}_M - \hat{\boldsymbol{y}}_M\|^2}{n_M - p}$ and $\boldsymbol{X}_M$ is the analog of $\boldsymbol{X}$ which is defined in Section 2.3.1, and let the standard error of $\hat{\boldsymbol{\beta}}_{Mi}$ based on the theorem introduced by Chen and Li (1998) be

$$SE_{cl}(\hat{\boldsymbol{\beta}}_{Mi}) = \sqrt{n_M^{-1}(\hat{\boldsymbol{C}}_M)_{ii}}, \tag{4.3}$$

where $\boldsymbol{C}_M$ is defined in (3.9).

For the OLS estimator, we keep a record of $\hat{\beta}_{Mi}, SE(\hat{\beta}_{Mi})$, and $SE_{cl}(\hat{\beta}_{Mi})$ for each run; while for the SIR estimator, we keep a record of $\hat{\beta}_{Mi,SIR}$ for each run. Then the sample means of all the previous values coming from 1000 runs will be calculated. We use $\overline{\boldsymbol{\beta}}_M, \overline{SE}, \overline{SE}_{cl}$, and $\overline{\boldsymbol{\beta}}_{M,SIR}$ to denote the corresponding sample means. In this section, we are interested in the following questions.

- How large should the sample size n be for $\overline{\boldsymbol{\beta}}_M$ or $\overline{\boldsymbol{\beta}}_{M,SIR}$ to be approximately equal to $c\boldsymbol{\beta}$ for some constant c, where $\boldsymbol{\beta} = (1,1,1,1)'$? For the OLS MLR model, c should be 1.

- For the OLS estimator, what is the relationship between $\overline{SE}, \overline{SE}_{cl}$ and SD for large n?

4.3.2 The OLS Estimator

In this section, we will estimate the OLS coefficients $\hat{\boldsymbol{\beta}}_M$ and its standard errors for the 7 single index models given in Section 4.2.

Normal Distributed Predictor $\boldsymbol{x}$

In this section, the predictor $\boldsymbol{x} \sim N_4(\boldsymbol{0}, \boldsymbol{I})$ and we will show the results in two tables.

1. MLR Model.

The model we used here is $y = SP + e$, which is the MLR model introduced in Section 4.2. We already defined the covariance matrix of $\hat{\boldsymbol{\beta}}_M$ as (2.44) in Section

2.3.1 for the entire data and as (3.14) in Section 3.5 for the data after trimming. In the previous two equations, we have $\mathrm{Cov}(\hat{\boldsymbol{\beta}}_M) = n_M^{-1}\sigma^2\boldsymbol{\Sigma}_M^{-1}, M = 0\%, 10\%, \cdots, 90\%$. Here $M = 0\%$ means 0% trimming.

In the notation of Section 2.3.1, the covariance matrix of $\hat{\boldsymbol{\beta}}_M$ of the MLR model could also take the following form

$$\mathrm{Cov}(\hat{\boldsymbol{\eta}}_{ols}) = \sigma^2(\boldsymbol{X}'\boldsymbol{X})^{-1}, \tag{4.4}$$

for 0% trimming where $\boldsymbol{\eta}_{ols} = (\alpha_{ols}, \boldsymbol{\beta}'_{ols})'$.

Now let us compare (4.2) with (4.4). We know MSE is an unbiased estimator for σ^2. Hence (4.2) should be close to the standard error of $\hat{\beta}_{Mi}$ when n is large. That means for large n, $SE_{cl}(\hat{\boldsymbol{\beta}}_M)$ and $SE(\hat{\boldsymbol{\beta}}_M)$ should be close to each other.

For nonzero percent trimming, we will also compare $SE_{cl}(\hat{\boldsymbol{\beta}}_M)$ and $SE(\hat{\boldsymbol{\beta}}_M)$ and expect they are getting closer for large n.

All the results for the MLR model are shown in Table 4.1. Column "M" denotes the trimming percentage, column"n" means the size of entire data, the last three columns are the multiple of $\sqrt{n}$ with the standard errors and the standard deviation of $\hat{\boldsymbol{\beta}}_{ols}$. By multiplying $\sqrt{n}$ to each value, we can make the standard errors and the standard deviation based on different n comparable. Recall (3.12) and (3.14), we have

$$\sqrt{n}\,\overline{SE}_{cl} \approx \sqrt{\frac{n}{n_M}\,\mathrm{diag}(\boldsymbol{C}_M)} = \sqrt{\frac{1}{1-M}\,\mathrm{diag}(\boldsymbol{C}_M)}, \tag{4.5}$$

and

$$\sqrt{n}\,\overline{SE} \approx \sqrt{\frac{n}{n_M}\tau_M^2\,\mathrm{diag}(\boldsymbol{\Sigma}_M^{-1})} = \sqrt{\frac{1}{1-M}\,\tau_M^2\,\mathrm{diag}(\boldsymbol{\Sigma}_M^{-1})}, \tag{4.6}$$

where $\boldsymbol{C}_M$ is defined in (3.9) and "diag(W)" means the main diagonal elements of the matrix W. Therefore, $\sqrt{n}\,\overline{SE}_{cl}$ calculated on different n should be close as long as the trimming percentage M is the same. Similar remarks apply to $\sqrt{n}\,\overline{SE}$. Furthermore, we hope that $\sqrt{n}\,\overline{SE}_{cl}$ and $\sqrt{n}\,\overline{SE}$ will be close for each M when n is getting large.

M	n	$\overline{\boldsymbol{\beta}}_M$	$\sqrt{n}\,\overline{SE}_{cl}$	$\sqrt{n}\,\overline{SE}$	$\sqrt{n}\,SD$
0	60	1,.99,1,1	.96,.96,.96,.96	1.04,1.05,1.05,1.04	1.02,1.06,1.04,1.04
	500	1,1,1,1	.99,.99,.99,1	1,1,1,1	1.04,1.01,1.03,1.01
	1000	1,1,1,1	1,1,1,1	1,1,1,1	1,1,.98,1.03
10	60	1,.99,1,1	1.11,1.12,1.1,1.11	1.21,1.22,1.21,1.21	1.21,1.23,1.21,1.22
	500	1,1,1,1	1.15,1.15,1.15,1.15	1.16,1.16,1.16,1.16	1.23,1.18,1.18,1.2
	1000	1,1,1,1	1.15,1.16,1.16,1.15	1.16,1.16,1.16,1.16	1.16,1.18,1.13,1.17
20	60	1,1,1,1	1.27,1.29,1.27,1.28	1.41,1.42,1.41,1.4	1.41,1.47,1.41,1.41
	500	1,1,1,1	1.31,1.31,1.31,1.31	1.33,1.33,1.33,1.33	1.39,1.35,1.35,1.36
	1000	1,1,1,1	1.31,1.32,1.32,1.31	1.32,1.32,1.32,1.32	1.31,1.32,1.28,1.31
30	60	.99,.99,1,1	1.49,1.51,1.49,1.5	1.68,1.69,1.67,1.67	1.66,1.69,1.71,1.67
	500	1,1,1,1	1.51,1.5,1.5,1.5	1.53,1.52,1.52,1.52	1.58,1.55,1.56,1.57
	1000	1,1,1,1	1.5,1.51,1.51,1.5	1.51,1.52,1.52,1.51	1.5,1.54,1.46,1.51
40	60	.99,.99,1,1	1.8,1.81,1.79,1.79	2.05,2.06,2.03,2.04	1.98,2.05,2.09,2.02
	500	1,1,1,1	1.75,1.75,1.74,1.74	1.78,1.77,1.77,1.77	1.8,1.88,1.82,1.79
	1000	1,1,1,1	1.74,1.75,1.75,1.74	1.76,1.76,1.76,1.76	1.78,1.78,1.71,1.75
50	60	.99,1,1,.99	2.13,2.13,2.1,2.12	2.47,2.47,2.44,2.46	2.5,2.49,2.56,2.45
	500	1,1,1,1	2.08,2.07,2.06,2.07	2.11,2.11,2.1,2.1	2.13,2.19,2.2,2.1
	1000	1,1,1,1	2.06,2.06,2.07,2.06	2.08,2.08,2.08,2.08	2.09,2.09,2.06,2.06
60	60	.98,.99,.98,.99	2.65,2.63,2.6,2.62	3.22,3.19,3.16,3.18	3.44,3.22,3.31,3.33
	500	1,1,1,1	2.51,2.51,2.5,2.5	2.57,2.56,2.56,2.56	2.66,2.58,2.66,2.52
	1000	1,1,1,1	2.5,2.5,2.5,2.49	2.53,2.53,2.53,2.52	2.52,2.51,2.54,2.54
70	60	.99,.99,1,1	3.27,3.27,3.21,3.24	4.27,4.26,4.19,4.23	4.57,4.41,4.4,4.58
	500	1,1.01,1,.99	3.2,3.2,3.18,3.19	3.29,3.28,3.28,3.28	3.37,3.3,3.27,3.31
	1000	1,1,1,1	3.19,3.18,3.19,3.18	3.23,3.23,3.23,3.22	3.17,3.16,3.26,3.25
80	60	.99,.98,1.02,1.04	4.03,4.12,4.03,4.06	6.37,6.43,6.3,6.37	7.02,7.16,7.11,7.32
	500	1,1,1,1	4.41,4.41,4.41,4.43	4.62,4.59,4.6,4.62	4.56,4.55,4.45,4.64
	1000	1,.99,1,1	4.42,4.43,4.42,4.41	4.51,4.51,4.52,4.51	4.52,4.452,4.54,4.59
90	60	1,.75,.97,1.08	5.1,5.51,5.37,5.39	19.08,20.74,19.92,19.78	29.3,47.45,44.49,36.94
	500	1.01,.99,.99,.99	7.49,7.47,7.44,7.46	8.16,8.12,8.11,8.16	8.06,8.27,8.54,8.08
	1000	1,1,1.01,.99	7.56,7.61,7.6,7.54	7.9,7.9,7.92,7.89	7.81,8.02,7.76,7.59

Table 4.1. Results Of OLS Estimators For The MLR Model Based On Type 1 $\boldsymbol{x}$

The results in Table 4.1 are as expected. First of all, all $\hat{\boldsymbol{\beta}}_M$ except $M = 90\%$ estimate the true coefficients $(1,1,1,1)'$ even for small n. Second of all, $SE(\hat{\boldsymbol{\beta}}_M)$ and $SE_{cl}(\hat{\boldsymbol{\beta}}_M)$ are approximately equal to each other for each M when n is large. They both estimate $(1,1,1,1)'$ for the 0% trimming. Also $SE(\hat{\boldsymbol{\beta}}_M)$ increases with M and results for $n = 500$ and $n = 1000$ are close. Last but not least, $SD(\hat{\boldsymbol{\beta}}_M)$ is close to the two standard errors for large n.

2. Nonlinear Models.

In this section we will run our simulations on the 6 nonlinear models which were introduced in Section 4.2. We want to know whether the trimming improves the result if the result of the 0% trimming is not good. Therefore for each model, we keep a record of the results based on $0\%, 10\%$, and the trimming corresponding to the best result. We consider the result as the best if all the entries of both $\hat{\boldsymbol{\beta}}_M$ and $SE_{cl}(\hat{\boldsymbol{\beta}}_M)$ are approximately equal. If there are several similar results which were based on the different trimming percentages M, we will pick the result which corresponded to the smallest M as the best one. In column "M" of Table 4.2, we denote them by $0, 10$, and B correspondingly. The subscript of those values denotes the type of the model we used. The number before B denotes the trimming percentage corresponding to the best result. For example, $1B_4$ means the best result for the type 4 model is obtained by trimming 10% of the original data.

M	n	$\overline{\boldsymbol{\beta}}_M$	$\sqrt{n}\,\overline{SE}_{cl}$	$\sqrt{n}\,\overline{SE}$	$\sqrt{n}\,SD$
0_2	60	1.96,1.97,1.94,2	6.66,6.71,6.63,6.71	5.76,5.77,5.78,5.76	8.1, 7.98,8.02,8.09
	500	2,2.01,2,2	7.81,7.79,7.76,7.8	5.71,5.72,5.72,5.72	7.87,8,8.02,7.88
	1000	1.99,1.99,1.99,1.99	7.9,7.9,7.93,7.87	5.73,5.73,5.73,5.73	8.33,8.29,8.65,8.09
10_2	60	1.97,2,1.96,2	6.07,6.12,6.06,6.12	5.57,5.61,5.56,5.56	7.69,7.58,7.32,7.37
	500	1.99,2,2,1.99	6.19,6.18,6.22,6.21	5.14,5.15,5.14,5.14	6.94,6.88,6.94,6.93
	1000	1.99,1.99,1.99,1.99	6.19,6.2,6.2,6.19	5.13,5.13,5.13,5.13	6.66,6.78,6.69,6.41
$0B_2$	60	1.96,1.97,1.94,2	6.66,6.71,6.63,6.71	5.76,5.77,5.78,5.76	8.1, 7.98,8.02,8.09
$0B_2$	500	2,2.01,2,2	7.81,7.79,7.76,7.8	5.71,5.72,5.72,5.72	7.87,8,8.02,7.88
$0B_2$	1000	1.99,1.99,1.99,1.99	7.9,7.9,7.93,7.87	5.73,5.73,5.73,5.73	8.33,8.29,8.65,8.09
0_3	60	20.75,21.05,20.92,21.3	84.5,85.16,85.46,86.68	67.27,67.8,67.93,67.37	494.8,459.2,472,422.8
	500	20.83,20.49,21.3,21.07	167,160.3,177.3,172.4	96.8,96.7,96.81,96.9	759,641.3,1207,1046
	1000	19.86,19.74,19.81,19.54	169.4,166,166.4,160.5	95.17,95.36,95.2,95.17	314.2,330.4,262.9,294.6
10_3	60	12.72,12.63,12.55,13.05	42.84,42.39,42.18,44.63	38.3,38.29,38.01,37.93	90.51,75.83,86.95,91.48
	500	10.85,10.85,10.93,10.86	41.65,41.6,42.31,41.75	31.82,31.85,31.85,31.84	54.11,53.4,54.92,52.69
	1000	10.7,10.73,10.71,10.69	41.32,41.7,41.46,41.15	31.17,31.2,31.21,31.16	49.63,50.89,47.86,48.57
$2B_3$	60	10.05,10.39,10.16,10.18	31.59,32.34,31.68,31.62	30.16,30.05,29.76,29.85	61.06,60.26,63.75,57.62
$1B_3$	500	10.85,10.85,10.93,10.86	41.65,41.6,42.31,41.75	31.82,31.85,31.85,31.84	54.11,53.4,54.92,52.69
$2B_3$	1000	8.6,8.6,8.6,8.6	28.6,28.8,28.6,28.7	22.9,22.9,23,22.9	39.3,38.9,38.2,38.8
0_4	60	14.64,14.8,14.53,14.87	27.76,28.01,27.59,28.39	24.23,24.32,24.32,24.25	38.92,38.5,38.54,39.18
	500	14.93,14.93,14.9,14.9	36.37,36,36.07,36.25	25.41,25.46,25.45,25.49	38.84,38.19,37.51,38.75
	1000	14.92,14.92,14.95,14.88	37.28,37.02,37.39,36.79	25.53,25.55,25.53,25.55	39.27,39.99,39.38,39.48
10_4	60	12.48,12.61,12.44,12.66	23.63,23.93,23.41,24.25	21.95,22.03,21.85,21.86	33.3,30.43,31.11,32.35
	500	11.9,11.91,11.96,11.91	24.31,24.25,24.54,24.35	19.92,19.94,19.93,19.93	29.54,29.12,29.9,29.15

Table 4.2. Results Of OLS Estimators For The Nonlinear Models Based On Type 1 $\boldsymbol{x}$

M	n	$\overline{\boldsymbol{\beta}}_M$	$\sqrt{n}\,\overline{SE}_{cl}$	$\sqrt{n}\,\overline{SE}$	$\sqrt{n}\,SD$
	1000	11.85,11.88,11.87,11.86	24.27,24.34,24.28,24.23	19.77,19.78,19.79,19.76	28.57,29.17,27.39,27.91
$0B_4$	60	14.64,14.8,14.53,14.87	27.76,28.01,27.59,28.39	24.23,24.32,24.32,24.25	38.92,38.5,38.54,39.18
$0B_4$	500	14.93,14.93,14.9,14.9	36.37,36,36.07,36.25	25.41,25.46,25.45,25.49	38.84,38.19,37.51,38.75
$0B_4$	1000	14.92,14.92,14.95,14.88	37.28,37.02,37.39,36.79	25.53,25.55,25.53,25.55	39.27,39.99,39.38,39.48
0_5	60	-.11,-.11,-.11,-.11	.37,.37,.37,.37	.35,.35,.35,.35	.43,.42,.43,.43
	500	-.11,-.11,-.11,-.11	.4,.4,.4,.4	.34,.34,.34,.34	.4,.41,.41,.4
	1000	-.11,-.11,-.11,-.11	.4,.4,.4,.4	.34,.34,.34,.34	.41,.4,.42,.4
10_5	60	-.13,-.13,-.12,-.13	.39,.39,.39,.39	.38,.38,.38,.38	.46,.47,.47,.46
	500	-.13,-.13,-.13,-.13	.41,.41,.41,.41	.36,.36,.36,.36	.45,.45,.47,.46
	1000	-.13,-.13,-.13,-.13	.41,.41,.41,.41	.36,.36,.36,.36	.44,.44,.46,.44
$0B_5$	60	-.13,-.13,-.12,-.13	.39,.39,.39,.39	.38,.38,.38,.38	.46,.47,.47,.46
$0B_5$	500	-.13,-.13,-.13,-.13	.41,.41,.41,.41	.36,.36,.36,.36	.45,.45,.47,.46
$0B_5$	1000	-.13,-.13,-.13,-.13	.41,.41,.41,.41	.36,.36,.36,.36	.44,.44,.46,.44
0_6	60	1.08,1.07,1.08,1.07	.7,.7,.7,.7	.72,.72,.72,.72	.77,.76,.77,.78
	500	1.07,1.07,1.07,1.07	.73,.73,.73,.73	.69,.69,.69,.69	.73,.74,.75,.74
	1000	1.07,1.07,1.07,1.07	.73,.73,.73,.73	.69,.69,.69,.69	.75,.75,.71,.73
10_6	60	1.11,1.1,1.11,1.1	.8,.81,.8,.8	.82,.82,.81,.81	.89,.88,.9,.89
	500	1.11,1.11,1.11,1.11	.83,.83,.83,.83	.78,.79,.78,.78	.88,.87,.92,.91
	1000	1.11,1.1,1.11,1.11	.83,.83,.83,.83	.78,.78,.78,.78	.89,.91,.88,.87
$0B_6$	60	1.08,1.07,1.08,1.07	.7,.7,.7,.7	.72,.72,.72,.72	.77,.76,.77,.78
$0B_6$	500	1.07,1.07,1.07,1.07	.73,.73,.73,.73	.69,.69,.69,.69	.73,.74,.75,.74
$0B_6$	1000	1.07,1.07,1.07,1.07	.73,.73,.73,.73	.69,.69,.69,.69	.75,.75,.71,.73
0_7	60	.14,.14,.14,.14	.48,.48,.48,.48	.47,.47,.47,.47	.55,.54,.55,.55
	500	.14,.14,.14,.14	.52,.52,.52,.52	.45,.45,.45,.46	.51,.53,.53,.53
	1000	.14,.14,.14,.14	.52,.52,.52,.52	.45,.45,.45,.45	.53,.53,.56,.52
10_7	60	.15,.15,.15,.15	.51,.51,.51,.51	.51,.52,.51,.51	.6,.61,.6,.59
	500	.16,.16,.16,.16	.53,.53,.53,.53	.49,.49,.49,.49	.58,.58,.59,.59
	1000	.16,.16,.16,.16	.53,.53,.53,.53	.49,.49,.49,.49	.55,.57,.58,.55
$0B_7$	60	.14,.14,.14,.14	.48,.48,.48,.48	.47,.47,.47,.47	.55,.54,.55,.55
$0B_7$	500	.14,.14,.14,.14	.52,.52,.52,.52	.45,.45,.45,.46	.51,.53,.53,.53
$0B_7$	1000	.14,.14,.14,.14	.52,.52,.52,.52	.45,.45,.45,.45	.53,.53,.56,.52

Table 4.2. (Continued)

Similar to Table 4.1, the predictor in Table 4.2 is $\boldsymbol{x}_i \sim N_4(\boldsymbol{0}, \boldsymbol{I})$, while the models we used here are all nonlinear. Comparing these two tables, there are several things to notice.

1) In Table 4.2, usually $\hat{\boldsymbol{\beta}}_M \approx c\boldsymbol{\beta}$ where c is some constant which depends on n, M, and the model, whereas $c = 1$ in Table 4.1.

2) The best results for nonlinear models are mostly obtained at 0% trimming. This is the same as the results for the MLR model. That means the 0% trimming OLS regression works well no matter which $1D$ model we used if $\boldsymbol{x}_i \sim N_4(\boldsymbol{0}, \boldsymbol{I})$.

3) In Table 4.2 both $SE(\hat{\boldsymbol{\beta}}_M)$ and $SE_{cl}(\hat{\boldsymbol{\beta}}_M)$ estimate $a(1,1,1,1)'$ where a is some constant which depends on n, M, the model, and the formula. But unlike the results for the MLR model, $SE(\hat{\boldsymbol{\beta}}_M) \not\approx SE_{cl}(\hat{\boldsymbol{\beta}}_M)$ even for large n. This difference is reasonable, because (4.2) can not be used to estimate the standard error of $\hat{\boldsymbol{\beta}}_{ols}$ for the nonlinear model.

4) Similar to Table 4.1, most of the sample standard deviations $SD(\hat{\boldsymbol{\beta}}_M)$ are getting close to $SE_{cl}(\hat{\boldsymbol{\beta}}_M)$ for large n, although the two values are not as close as in Table 4.1.

Nonnormal Distributed Predictor $\boldsymbol{x}$

Now we will run the simulation for the 7 nonnormal predictor distributions which were introduced in Section 4.2. The results which are based on the different type of predictors are displayed in the different tables. For each type of predictor, we run the simulation for all the 7 models which are introduced in Section 4.2. Similar to Table 4.2, for each model, we keep a record of the results based on $0\%, 10\%$, and the trimming corresponding to the best result. Here we use the same method to choose the "best" result as used for Table 4.2. Similarly, we use column "M" to denote the trimming percentage and the type of model. For example, 10_1 means the results in this row are obtained by using the type 1 model and trimming 10% of the original dataset, $4B_2$ means the results in this row are the best for the type 2 model and the specific n and they are obtained by trimming 40% of the original data. Since the predictors do not have the normal distribution, $SE(\hat{\boldsymbol{\beta}}_M)$ is not appropriate to estimate the standard error of $\hat{\boldsymbol{\beta}}_M$ and we will only compare $SE_{cl}(\hat{\boldsymbol{\beta}}_M)$ with $SD(\hat{\boldsymbol{\beta}}_M)$. Similarly, we multiply the two values by $\sqrt{n}$ and denote them as $\sqrt{n}\,\overline{SE}_{cl_i}$ and $\sqrt{n}\,SD_i$ in all tables, where the subscript i means the type of predictor we used.

n	M	$\overline{\boldsymbol{\beta}}_{M_2}$	$\sqrt{n}\,\overline{SE}_{cl_2}$	$\sqrt{n}\,SD_2$
60	0_1	1,1,1,1	.29,.29,.29,.30	.35,.34,.35,.36
500		1,1,1,1	.3,.3,.3,.3	.31,.31,.3,.3
1000		1,1,1,1	.31,.3,.31,.31	.31,.31,.3,.31
60	10_1	1,1,1,1	.4,.4,.4,.4	.5,.48,.49,.5
500		1,1,1,1	.42,.42,.42,.42	.42,.42,.42,.43
1000		1,1,1,1	.42,.42,.42,.42	.43,.42,.43,.42
60	$0B_1$	1,1,1,1	.29,.29,.29,.30	.35,.34,.35,.36
500	$0B_1$	1,1,1,1	.3,.3,.3,.3	.31,.31,.3,.3
1000	$0B_1$	1,1,1,1	.31,.3,.31,.31	.31,.31,.3,.31
60	0_2	1.91,1.75,1.39,2.05	43.69,43.85,43.74,43.61	62.49,62.49,62.44,63.26
500		2.07,1.92,2.04,2.05	61.76,62.31,62.41,61.6	68.15,65.41,66.82,65.64
1000		1.99,2.02,1.93,1.98	64.16,64.11,64.26,64.3	65.2,63.56,63.56,63.1
60	10_2	1.77,2,2.01,1.92	35.76,35.98,35.51,35.86	50.81,53.22,50.72,48.41
500		1.99,2,1.92,2.01	44.09,44.22,43.86,44.03	47.01,45.74,46.3,45.85
1000		2.01,2,1.99,2.01	44.39,44.31,44.51,44.57	44.72,44.14,45.95,44.84
60	$4B_2$	2.08,2.06,2.07,2.1	9.67,9.64,9.52,9.65	16.21,17.76,16.18,16.57
500	$3B_2$	1.95,1.97,1.97,1.97	27.44,27.44,27.73,27.94	30.15,31.12,30.91,31.1
1000	$1B_2$	2.01,2,1.99,2.01	44.39,44.31,44.51,44.57	44.72,44.14,45.95,44.84
60	0_3	2.7e12,1.5e12,1.8e12,1.3e12	1.5e13,9.2e12,1.1e13,1.1e13	3.6e14,1.7e14,2.6e14,2.2e14
500		2.4e13,2e13,3.1e13,1.7e13	4.9e14,4.1e14,6.2e14,3.7e14	9.4e15,6.4e15,1.2e16,5.3e15
1000		2.7e14,4.7e14,3.5e14,2.5e14	8.3e15,1.4e16,1.1e16,7.5e15	2.3e17,4.3e17,3.1e17,1.9e17
60	10_3	7.6e8,1.7e9,8.4e8,9e8	7.9e9,1e10,8.3e9,6e9	1.3e11,3.5e11,1.6e11,1.9e11
500		1e7,1e7,1.1e7,1.1e7	1.9e8,1.8e8,2.1e8,1.9e8	8.4e8,7.8e8,1.1e9, 8.7e8
1000		6.5e6,6.4e6,6.7e6,6.5e6	1.5e8,1.5e8,1.6e8,1.5e8	3.8e8,4.2e8,4.1e8,3.7e8
60	$6B_3$	8.03,8.05,8.45,8.31	26.19,26.02,26.83,25.86	54.46,58.05,77.37,59.29
500	$5B_3$	9.06,9.05,9.04,9.08	37.56,37,37.31,37.41	55.35,54.02,53.35,55.03
1000	$5B_3$	9.08,9.03,9.03,9.07	38.24,37.96,38.01,38.36	54.79,54.85,54.61,55.11
60	0_4	262.3,261.2,257.4,259.9	592.8,594.5,587.3,598.4	1056,1036,1004,1047
500		283.4,284.7,285.5,283.6	1001,1007,1024,992	1107,1131,1142,1141
1000		285.7,285.7,285.5,286.8	1091,1090,1089,1094	1199,1198,1169,1225
60	10_4	160.1,161.7,158.3,161.5	371.8,373.5,363.9,369.1	639.9,683.6,629.8,640.8
500		157.4,157.5,156.4,157.1	454.6,455.7,454.2,453.8	570.2,567.2,555.3,573.9
1000		156.3,156.3,156.6,156.7	456,455.6,460.2,461.1	566.2,555,573.8,561.8
60	$5B_4$	11.39,11.32,11.53,11.5	24.85,24.94,25,25.09	42.68,46.66,46.57,42.4
500	$4B_4$	15.05,15,15,14.95	45.5,45.26,45.73,44.82	82.52,79.14,82.71,79.11
1000	$4B_4$	14.54,14.57,14.54,14.54	43,43.37,43.14,43.17	66.92,67.82,66.25,65.51
60	0_5	-.01,-.01,-.01,-.01	.13,.13,.13,.13	.15,.16,.15,.15
500		-.01,-.01,-.01,-.01	.13,.13,.13,.13	.14,.13,.13,.13
1000		-.01,-.01,-.01,-.01	.13,.13,.13,.13	.13,.12,.12,.12
60	10_5	-.01,-.02,-.02,-.02	.18,.18,.19,.18	.22,.23,.22,.21
500		-.01,-.01,-.01,-.01	.18,.18,.18,.19	.19,.19,.2,.19
1000		-.01,-.01,-.01,-.01	.19,.18,.19,.19	.18,.18,.18,.19

Table 4.3. Results Of OLS Estimators Based On Type 2 $\boldsymbol{x}$

n	M	$\overline{\boldsymbol{\beta}}_{M_2}$	$\sqrt{n}\,\overline{SE}_{cl_2}$	$\sqrt{n}\,SD_2$
60	$0B_5$	-.01,-.01,-.01,-.01	.13,.13,.13,.13	.15,.16,.15,.15
500	$0B_5$	-.01,-.01,-.01,-.01	.13,.13,.13,.13	.14,.13,.13,.13
1000	$0B_5$	-.01,-.01,-.01,-.01	.13,.13,.13,.13	.13,.12,.12,.12
60	0_6	1,1.01,1.01,1	.21,.21,.21,.21	.24,.25,.25,.25
500		1,1,1,1	.22,.22,.22,.22	.22,.22,.23,.22
1000		1,1,1,1	.22,.22,.22,.22	.22,.22,.22,.23
60	10_6	1.01,1.01,1.01,1.01	.29,.29,.29,.29	.34,.36,.35,.34
500		1.01,1.01,1.01,1.01	.3,.3,.3,.3	.3,.31,.31,.29
1000		1.01,1.01,1.01,1.01	.3,.3,.3,.3	.3,.3,.3,.31
60	$0B_6$	1,1.01,1.01,1	.21,.21,.21,.21	.24,.25,.25,.25
500	$0B_6$	1,1,1,1	.22,.22,.22,.22	.22,.22,.23,.22
1000	$0B_6$	1,1,1,1	.22,.22,.22,.22	.22,.22,.22,.23
60	0_7	.02,.02,.02,.02	.42,.42,.43,.43	.53,.55,.53,.54
500		.02,.02,.02,.02	.49,.49,.49,.49	.54,.52,.51,.5
1000		.02,.02,.02,.02	.49,.49,.49,.49	.5,.49,.49,.48
60	10_7	.03,.03,.03,.03	.49,.49,.49,.49	.62,.65,.63,.61
500		.03,.03,.03,.03	.56,.56,.55,.56	.59,.58,.59,.58
1000		.03,.03,.03,.03	.56,.56,.56,.56	.56,.56,.58,.57
60	$0B_7$	.02,.02,.02,.02	.42,.42,.43,.43	.53,.55,.53,.54
500	$0B_7$	.02,.02,.02,.02	.49,.49,.49,.49	.54,.52,.51,.5
1000	$0B_7$	.02,.02,.02,.02	.49,.49,.49,.49	.5,.49,.49,.48

Table 4.3. (Continued)

In Table 4.3, the predictor $\boldsymbol{x}_i \sim 0.6N_4(\mathbf{0}, \boldsymbol{I}) + 0.4N_4(\mathbf{0}, 25\boldsymbol{I})$.

1) For the MLR model, $\hat{\boldsymbol{\beta}}_M \approx (1,1,1,1)'$. Also $SE_{cl}(\hat{\boldsymbol{\beta}}_M)$ and $SD(\hat{\boldsymbol{\beta}}_M)$ are close to each other, both of them are estimating $a(1,1,1,1)'$ for some constant a. The best results are obtained at 0% trimming.

2) The results for type 5, 6, and 7 models are similar to the MLR model except that $\hat{\boldsymbol{\beta}}_M \approx c(1,1,1,1)'$ for some constant c.

3) For the type 2, 3, and 4 models, a large amount of trimming often greatly improved the results, such as 40% or 50% trimming. But even for the best results, $SE_{cl}(\hat{\boldsymbol{\beta}}_M)$ and $SD(\hat{\boldsymbol{\beta}}_M)$ are not close when the model is type 3 or 4.

n	M	$\overline{\boldsymbol{\beta}}_{M_3}$	$\sqrt{n}\,\overline{SE}_{cl_3}$	$\sqrt{n}\,SD_3$
60	0_1	1,1,1,1	.24,.24,.24,.24	.29,.27,.27,.29
500		1,1,1,1	.25,.25,.25,.25	.26,.26,.25,.26
1000		1,1,1,1	.25,.25,.25,.25	.26,.26,.25,.26
60	10_1	1,1,1,1	.3,.3,.3,.3	.36,.35,.35,.36
500		1,1,1,1	.32,.32,.32,.32	.32,.32,.31,.32

Table 4.4. Results Of OLS Estimators Based On Type 3 $\boldsymbol{x}$

n	M	$\overline{\boldsymbol{\beta}}_{M_3}$	$\sqrt{n}\,\overline{SE}_{cl_3}$	$\sqrt{n}\,SD_3$
1000		1,1,1,1	.32,.32,.32,.32	.32,.32,.33,.31
60	$0B_1$	1,1,1,1	.24,.24,.24,.24	.29,.27,.27,.29
500	$0B_1$	1,1,1,1	.25,.25,.25,.25	.26,.26,.25,.26
1000	$0B_1$	1,1,1,1	.25,.25,.25,.25	.26,.26,.25,.26
60	0_2	1.91,1.74,1.8,2.15	39.6,39.95,39.69,39.7	53.95,50.94,51.69,53.06
500		2.01,1.93,2.06,2.06	50.73,51.19,51.25,50.58	54.47,53.83,54.54,52.9
1000		1.99,2.05,1.98,2	52.39,52.18,52.2,52.52	53.6,52.3,53.92,53.76
60	10_2	2.06,1.96,1.86,2.03	34.67,34.71,34.56,35.11	45.66,44.22,44.79,45.61
500		1.97,1.98,2,2.03	38.69,38.82,38.53,38.79	40.68,39.76,39.91,39.8
1000		2.03,2.01,1.99,1.99	38.69,38.44,38.6,38.83	39.83,40.1,39.98,39.58
60	$7B_2$	2.07,1.97,2.05,2.09	7.68,7.74,7.57,7.7	13.61,13.76,13.64,13.59
500	$3B_2$	1.93,1.96,1.95,2.01	34.23,34.34,34.49,34.58	36.5,36.8,37.08,37.01
1000	$2B_2$	2.01,2.02,2.02,2.01	35.89,35.68,35.7,35.94	36.94,36.76,37.52,36.63
60	0_3	3.1e12,2.1e12,1.8e12,1.9e12	1.7e13,1.3e13,1.1e13,1.2e13	3.3e14,2.2e14,1.9e14,2e14
500		2.2e13,1.9e13,2.6e13,1.8e13	4.6e14,4e14,5.5e14,3.8e14	6.7e15,5e15,8.1e15,4.3e15
1000		5.2e14,5.6e14,4.7e14,3.3e14	1.6e16,1.7e16,1.4e16,1e16	3.4e17,3.7e17,2.8e17,1.7e17
60	10_3	8.8e10,8.7e10,3.9e10,6.8e10	4.9e11,5.3e11,2.9e11,4.3e11	1.7e13,1.8e13,7.7e12,1.2e13
500		1.7e8,1.9e8,2.4e8,2.2e8	3.4e9,3.7e9,4.9e9,4.4e9	4.2e10,5.1e10,8.7e10,6.6e10
1000		8.4e7,8.6e7,8.2e7,9.3e7	2.1e9,2.2e9,2e9,2.4e9	1.4e10,1.5e10,1.1e10,1.8e10
60	$8B_3$	7.17,6.92,7.14,7.57	22.72,22.31,22.64,22.53	77.29,70.31,79.06,97.52
500	$7B_3$	7.98,8,8.02,8.06	37.73,37.84,38.19,38.65	56.52,57.82,60.13,61.25
1000	$7B_3$	8.01,7.95,7.98,8	38.86,38.68,38.69,39.02	56.38,54.48,55.6,55.35
60	0_4	282.5,278.6,277,282.3	587.4,587.4,580.2,590.8	921.1,931.3,856.6,963.4
500		291.9,294,293.7,292.1	859.7,866.6,877.9,850.8	933,957.9,964.9,957.2
1000		294.8,294.2,293.9,296.1	932.9,924.9,924.5,935.7	1015,1024, 970.6,1032
60	10_4	203.4,201.1,199.1,205	411.1,413.8,406,422.9	681.3,669.7,657.6,691.9
500		193.7,193.8,192.5,193.7	453.4,455.7,453.7,457.3	584.2,587.6,586,588.9
1000		192.3,191.6,191.7, 192.8	452.2,448.6,451.2,454.1	603.3,602.2,605.2,619.7
60	$7B_4$	11.45,11.22,11.33,11.75	28.73,29.12,29.01,29.41	68.43,78.94,63.33,69.86
500	$6B_4$	14.58,14.6, 14.48,14.54	49.47,49.61,49.14,49.45	87.86,84.25,81.49,86.33
1000	$6B_4$	14.19,14.22,14.21,14.19	48.55,48.47,47.93,48.26	75,76.87,76.58,71.92
60	0_5	-.004,-.004,-.005,-.004	.09,.09,.09,.09	.1,.11,.1,.1
500		-.004,-.004,-.004,-.004	.09,.09,.09,.09	.09,.09,.1,.09
1000		-.004,-.004,-.004,-.004	.09,.09,.09,.09	.09,.09,.09,.09
60	10_5	-.007,-.006,-.007,-.007	.12,.12,.12,.12	.14,.15,.14,.14
500		-.006,-.006,-.006,-.006	.12,.12,.12,.12	.12,.12,.13,.13
1000		-.006,-.006,-.006,-.006	.12,.12,.12,.12	.12,.12,.12,.12
60	$0B_5$	-.004,-.004,-.005,-.004	.09,.09,.09,.09	.1,.11,.1,.1
500	$0B_5$	-.004,-.004,-.004,-.004	.09,.09,.09,.09	.09,.09,.1,.09
1000	$0B_5$	-.004,-.004,-.004,-.004	.09,.09,.09,.09	.09,.09,.09,.09
60	0_6	1,1,1,1	.17,.18,.18,.18	.19,.2,.2,.2
500		1,1,1,1	.18,.18,.18,.18	.18,.18,.19,.18
1000		1,1,1,1	.18,.18,.18,.18	.19,.18,.18,.19
60	10_6	1,1,1,1	.22,.22,.22,.22	.24,.25,.26,.25
500		1,1,1,1	.23,.23,.23,.23	.23,.23,.23,.23
1000		1,1,1,1	.23,.23,.23,.23	.24,.22,.23,.23

Table 4.4. (Continued)

n	M	$\overline{\boldsymbol{\beta}}_{M_3}$	$\sqrt{n}\,\overline{SE}_{cl_3}$	$\sqrt{n}\,SD_3$
60	$0B_6$	1,1,1,1	.17,.18,.18,.18	.19,.2,.2,.2
500	$0B_6$	1,1,1,1	.18,.18,.18,.18	.18,.18,.19,.18
1000	$0B_6$	1,1,1,1	.18,.18,.18,.18	.19,.18,.18,.19
60	0_7	.02,.01,.02,.02	.33,.34,.34,.34	.41,.4,.39,.4
500		.02,.02,.02,.02	.37,.37,.37,.37	.4,.39,.39,.38
1000		.02,.02,.02,.02	.37,.37,.37,.37	.38,.36,.38,.37
60	10_7	.02,.02,.02,.02	.38,.38,.38,.38	.46,.47,.45,.45
500		.02,.02,.02,.02	.41,.41,.41,.41	.42,.41,.42,.42
1000		.02,.02,.02,.02	.41,.4,.41,.41	.41,.41,.41,.42
60	$1B_7$	.02,.02,.02,.02	.38,.38,.38,.38	.46,.47,.45,.45
500	$0B_7$	.02,.02,.02,.02	.37,.37,.37,.37	.4,.39,.39,.38
1000	$0B_7$	.02,.02,.02,.02	.37,.37,.37,.37	.38,.36,.38,.37

Table 4.4. (Continued)

In Table 4.4, the predictor $\boldsymbol{x}_i \sim 0.4N_4(\mathbf{0}, \boldsymbol{I}) + 0.6N_4(\mathbf{0}, 25\boldsymbol{I})$.

1) The results for the MLR model are almost the same as those in Table 4.3.

2) The results for the type 5, 6, and 7 models are almost the same as those in Table 4.3 except that the best results for the type 7 model are obtained at 10% trimming when n is small.

3) For the type 2, 3, and 4 models, the best results are obtained by using larger amounts of trimming compared to the results in Table 4.3, for example 60% or 70%. Also $SE_{cl}(\hat{\boldsymbol{\beta}}_M)$ and $SD(\hat{\boldsymbol{\beta}}_M)$ are not close for type 3 and 4 models even for the best results.

n	M	$\overline{\boldsymbol{\beta}}_{M_4}$	$\sqrt{n}\,\overline{SE}_{cl_4}$	$\sqrt{n}\,SD_4$
60	0_1	1,1,1,1	.57,.56,.57,.57	.72,.68,.69,.72
500		1,1,1,1	.53,.53,.54,.53	.56,.56,.56,.56
1000		1,1,1,1	.54,.54,.54,.54	.54,.57,.54,.55
60	10_1	1,1,1,1	.98,.99,.98,.98	1.12,1.07,1.07,1.11
500		1,1,1,1	1.04,1.04,1.04,1.04	1.08,1.04,1.05,1.04
1000		1,1,1,1	1.05,1.05,1.05,1.05	1.04,1.06,1.02,1.08
60	$0B_1$	1,1,1,1	.57,.56,.57,.57	.72,.68,.69,.72
500	$0B_1$	1,1,1,1	.53,.53,.54,.53	.56,.56,.56,.56
1000	$0B_1$	1,1,1,1	.54,.54,.54,.54	.54,.57,.54,.55
60	0_2	2,2.17,1.4,1.51	32.38,32.61,32.46,33.15	71.56,75.4,72.07,74.14
500		2.06,2.05,2,2.01	89.11,89.57,89.07,88.78	102.4,105.1,103.7,102
1000		2.1,2.05,1.96,2.13	98.44,97.37,98.69,99.52	107.4,107.7,107.1,110.7
60	10_2	2.07,1.96,2,2.08	9.24,9.25,9.43,9.42	18.03,19.27,21.04,19.66
500		1.99,1.98,2.01,2	9.02,9.05,9.03,9.02	9.55,10.01,9.77,9.75

Table 4.5. Results Of OLS Estimators Based On Type 4 $\boldsymbol{x}$

n	M	$\overline{\boldsymbol{\beta}}_{M_4}$	$\sqrt{n}\,\overline{SE}_{cl_4}$	$\sqrt{n}\,SD_4$
1000		2,2.01,2,2	8.65,8.63,8.63,8.68	9.64,9.06,9.54,9.3
60	$2B_2$	2.02,2,2.05,2.06	6.39,6.37,6.3,6.34	7.95,7.66,7.86,7.81
500	$1B_2$	1.99,1.98,2.01,2	9.02,9.05,9.03,9.02	9.55,10.01,9.77,9.75
1000	$1B_2$	2,2.01,2,2	8.65,8.63,8.63,8.68	9.64,9.06,9.54,9.3
60	0_3	9.8e10,3.8e10,1.1e11,1.3e11	4.4e11,3.2e11,4.1e11,4.6e11	1.3e13,7.5e12,1.6e13,2e13
500		4.2e12,7e12,4.9e12,5.7e12	7.9e13,1.2e14,8.7e13,1e14	2.1e15,3.7e15,2e15,2.6e15
1000		5.8e13,4.9e13,5.4e13,8.2e13	1.7e15,1.5e15,1.5e15,2.3e15	3.7e16,3.5e16,3.3e16,5e16
60	10_3	3.7e4,4.7e4,3.3e4,4.6e4	1.7e5,2.1e5,1.7e5,2e5	4e6,5.5e6,3.9e6,5.3e6
500		51.23,54.06,51.73,51.68	770.2,819.9,763.3,768.3	5993,5961,5723,6113
1000		32.4,32.81,33.76,34.77	476.2,497.8,515,552.9	4659,4198,4623,4927
60	$5B_3$	6.34,6.39,6.46,6.34	16.67,17.08,17.16,16.71	31.77,35.06,36.22,31.51
500	$2B_3$	10.66,10.63,10.67,10.67	42.74,42.53,42.87,42.65	55.05,54.27,54.57,55.88
1000	$2B_3$	10.42,10.43,10.46,10.44	40.9,41.07,41.59,41.1	50.3,50.95,51.38,51.16
60	0_4	148.8,153.9,146.8,150.2	414,417.7,409.8,413.2	1202,1198,1187,1201
500		214.5,216.5,216.2,214	1347,1350,1362,1335	1741,1815,1853,1766
1000		221.8,220.3,221.9,224.8	1592,1558,1584,1632	1973,1978,1933,2123
60	10_4	22.91,23.45,24.4,24.68	60.66,61.19,62.14,62.4	226.7,253,295.1,267.6
500		15.65,15.68,15.7,15.66	46.08,46.4,46.64,46.32	79.7,83.79,79.35,82.5
1000		15.21,15.21,15.2,15.22	42.94,42.9,43.07,43.47	69.19,70.58,69.82,68.93
60	$2B_4$	12.59,12.43,12.62,12.68	24.94,24.68,24.98,25.19	35.12,35.11,37.07,37.14
500	$1B_4$	15.65,15.68,15.7,15.66	46.08,46.4,46.64,46.32	79.7,83.79,79.35,82.5
1000	$1B_4$	15.21,15.21,15.2,15.22	42.94,42.9,43.07,43.47	69.19,70.58,69.82,68.93
60	0_5	-.05,-.05,-.05,-.05	.27,.27,.27,.27	.39,.39,.38,.39
500		-.03,-.03,-.03,-.03	.3,.3,.3,.3	.31,.32,.33,.31
1000		-.03,-.03,-.03,-.03	.3,.3,.3,.3	.31,.33,.3,.31
60	10_5	-.1,-.1,-.1,-.1	.38,.38,.38,.38	.46,.46,.46,.45
500		-.11,-.11,-.11,-.11	.42,.42,.42,.42	.44,.49,.46,.44
1000		-.11,-.11,-.11,-.11	.43,.42,.42,.42	.46,.45,.45,.45
60	$0B_5$	-.05,-.05,-.05,-.05	.27,.27,.27,.27	.39,.39,.38,.39
500	$0B_5$	-.03,-.03,-.03,-.03	.3,.3,.3,.3	.31,.32,.33,.31
1000	$0B_5$	-.03,-.03,-.03,-.03	.3,.3,.3,.3	.31,.33,.3,.31
60	0_6	1.03,1.03,1.03,1.03	.42,.42,.42,.42	.53,.54,.52,.53
500		1.02,1.02,1.02,1.02	.4,.4,.4,.4	.42,.41,.41,.43
1000		1.02,1.02,1.02,1.02	.41,.4,.4,.41	.43,.42,.41,.39
60	10_6	1.07,1.08,1.07,1.07	.71,.71,.72,.71	.8,.81,.8,.8
500		1.07,1.07,1.07,1.07	.76,.76,.77,.76	.8,.81,.78,.79
1000		1.07,1.07,1.07,1.07	.77,.77,.77,.77	.76,.77,.78,.76
60	$0B_6$	1.03,1.03,1.03,1.03	.42,.42,.42,.42	.53,.54,.52,.53
500	$0B_6$	1.02,1.02,1.02,1.02	.4,.4,.4,.4	.42,.41,.41,.43
1000	$0B_6$	1.02,1.02,1.02,1.02	.41,.4,.4,.41	.43,.42,.41,.39
60	0_7	.07,.07,.06,.06	.51,.51,.52,.52	.84,.87,.85,.87
500		.05,.05,.05,.05	.87,.87,.86,.87	.92,.95,.95,.93
1000		.05,.05,.05,.05	.9,.9,.91,.9	.93,.97,.94,.94
60	10_7	.13,.13,.13,.13	.51,.51,.52,.51	.63,.65,.64,.62
500		.14,.13,.14,.14	.56,.56,.55,.56	.57,.62,.59,.57
1000		.14,.14,.14,.14	.55,.55,.55,.55	.6,.57,.59,.58

Table 4.5. (Continued)

n	M	$\overline{\boldsymbol{\beta}}_{M_4}$	$\sqrt{n}\,\overline{SE}_{cl_4}$	$\sqrt{n}\,SD_4$
60	$1B_7$	.13,.13,.13,.13	.51,.51,.52,.51	.63,.65,.64,.62
500	$0B_7$	.05,.05,.05,.05	.87,.87,.86,.87	.92,.95,.95,.93
1000	$0B_7$	.05,.05,.05,.05	.9,.9,.91,.9	.93,.97,.94,.94

Table 4.5. (Continued)

In Table 4.5, the predictor $\boldsymbol{x}_i \sim 0.9N_4(\boldsymbol{0}, \boldsymbol{I}) + 0.1N_4(\boldsymbol{0}, 25\boldsymbol{I})$.

1) The results for the MLR model are similar to the results in Table 4.3.

2) The results for the type 5, 6, and 7 models are similar to the results in Table 4.3 except the best results for the type 7 model are obtained at 10% trimming when n is small.

3) For the type 2, 3, and 4 models, the best results are obtained by using smaller amount of trimming compared to the results in Table 4.3 and Table 4.4, for example 10% or 20%. $SE_{cl}(\hat{\boldsymbol{\beta}}_M)$ and $SD(\hat{\boldsymbol{\beta}}_M)$ are not close for all three models even for the best results.

n	M	$\overline{\boldsymbol{\beta}}_{M_5}$	$\sqrt{n}\,\overline{SE}_{cl_5}$	$\sqrt{n}\,SD_5$
60	0_1	1,1,1,1	.48,.48,.48,.49	.59,.61,.62,.62
500		1,1,1,1	.46,.46,.46,.46	.51,.5,.49,.5
1000		1,1,1,1	.46,.46,.46,.46	.49,.48,.48,.48
60	10_1	1,.99,1,1	.76,.77,.76,.78	.91,.92,.9,.93
500		1,1,1,1	.81,.81,.81,.81	.86,.81,.83,.84
1000		1,1,1,1	.81,.82,.82,.82	.81,.83,.81,.81
60	$0B_1$	1,1,1,1	.48,.48,.48,.49	.59,.61,.62,.62
500	$0B_1$	1,1,1,1	.46,.46,.46,.46	.51,.5,.49,.5
1000	$0B_1$	1,1,1,1	.46,.46,.46,.46	.49,.48,.48,.48
60	0_2	20.7,21.1,20.9,21	16.7,17.2,17.1,17	52.1,57.27,52.87,51.13
500		25.5,24.9,24.8,25	68.6,63.9,63.7,65.2	166.8,147.8,155.2,185.2
1000		26.2,25.9,26.1,25.9	101.2,95.6,96.2,94.8	230,245.1,237.1,255.1
60	10_2	15,15.1,15,15	8.3,8.7,8.4,8.5	14.32,15.25,14.95,14.86
500		14.9,14.9,14.9,14.9	9.4,9.3,9.4,9.4	13.36,13.34,14.31,13.96
1000		14.8,14.8,14.8,14.8	9.3,9.3,9.3,9.3	14.5,13.58,13.5,13.71
60	$1B_2$	15,15.1,15,15	8.3,8.7,8.4,8.5	14.32,15.25,14.95,14.86
500	$1B_2$	14.9,14.9,14.9,14.9	9.4,9.3,9.4,9.4	13.36,13.34,14.31,13.96
1000	$1B_2$	14.8,14.8,14.8,14.8	9.3,9.3,9.3,9.3	14.5,13.58,13.5,13.71
60	0_3	3e35,6e34,1.5e34,-7.7e33	7.7e34,1.7e35,1.5e35,2.5e35	7.4e37,1.5e37,3.6e36,1.9e36
500		1e48,2.8e49,-4.8e49,4.8e50	3.7e50,3.8e50,7.1e50,1.9e51	7e50,2e52,3.4e52,3.4e53
1000		5e78,4e78,5.2e78,2.3e80	1.4e80,1.1e80,1.4e80,11e81	5e81,4e81,5.2e81,2.3e83
60	10_3	2.3e6,2.2e6,1.6e6,4.9e6	1.1e7,9.7e6,1.2e7,1.3e7	5.3e8,2.6e8,2e8,9.9e8

Table 4.6. Results Of OLS Estimators Based On Type 5 $\boldsymbol{x}$

n	M	$\overline{\boldsymbol{\beta}}_{M_5}$	$\sqrt{n}\,\overline{SE}_{cl_5}$	$\sqrt{n}\,SD_5$
500		4.8e4,4.7e4,4.8e4,4.7e4	5.5e5,5.3e5,5.4e5,5.3e5	2.1e6,1.8e6,1.8e6,2e6
1000		3.7e4,3.7e4,3.6e4,3.6e4	4.1e6,4.1e6,4.1e6,4e6	7.9e5,7.9e5,7.6e5,7.8e5
60	$8B_3$	139.7,130.2,141,133.9	291.1,283,295.8,298.9	1063,856,1440, 951.9
500	$8B_3$	121,121.4,120.7,120.8	327.4,327.7,329,326.9	498,509.3,496.5,494.8
1000	$8B_3$	119.1,119,118.9,119.2	334.4,333.9,334.2,335.1	463.4,468.6,467.2,464.8
60	0_4	417.9,438.5,427.8,425.2	696.8,738.5,724.6,714.2	3570,3705,3315,2946
500		689.9,642.4,643.3,676.8	4058,3679,3692,3796	14439,11525,13077,19442
1000		753.8,741.7,743.5,741.7	6664,6299,6345,6168	22698,27299,28397,38975
60	10_4	187.8,192.1,190.6,190.2	212.2,223.1,215.7,217	397.1,432.9,425,428.7
500		183.1,183,182.7,182.7	241,238.6,239.9,239.4	351.4,351,377.4,368
1000		181.7,181.8,181.6,181	237.3,237.6,237.8,235.9	379,354.7,350.2,356.2
60	$4B_4$	95.2,95.4,95.5,95.6	90.4,90.4,90.6,90.7	153.9,157.1,158,158.4
500	$3B_4$	109.4,109.5,109.6,109.5	106.3,105.8,106.1,106.2	155.8,154.5,158.8,155.1
1000	$2B_4$	134.6,134.6,134.5,134.4	140.9,141.7,140.9,140.7	222.1,217.4,208.6,212.8
60	0_5	.01,.01,.01,.01	.05,.05,.05,.05	.06,.06,.07,.07
500		.01,.01,.01,.01	.05,.05,.05,.05	.06,.06,.06,.06
1000		.01,.01,.01,.01	.05,.05,.05,.05	.06,.06,.06,.06
60	10_5	.02,.02,.02,.02	.09,.09,.09,.09	.1,.1,.11,.1
500		.02,.02,.02,.02	.09,.09,.09,.09	.1,.1,.11,.1
1000		.02,.02,.02,.02	.09,.09,.09,.09	.1,.1,.1,.1
60	$0B_5$	.01,.01,.01,.01	.05,.05,.05,.05	.06,.06,.07,.07
500	$0B_5$	.01,.01,.01,.01	.05,.05,.05,.05	.06,.06,.06,.06
1000	$0B_5$	.01,.01,.01,.01	.05,.05,.05,.05	.06,.06,.06,.06
60	0_6	1.05,1.05,1.05,1.05	.36,.36,.36,.36	.46,.47,.48,.47
500		1.04,1.04,1.04,1.04	.36,.36,.37,.37	.41,.42,.42,.4
1000		1.04,1.04,1.04,1.04	.37,.38,.37,.37	.41,.41,.41,.42
60	10_6	1.1,1.09,1.1,1.09	.56,.56,.55,.56	.68,.69,.71,.69
500		1.08,1.09,1.09,1.09	.61,.61,.61,.61	.64,.66,.7,.67
1000		1.08,1.08,1.08,1.08	.61,.61,.61,.61	.69,.67,.65,.67
60	$0B_6$	1.05,1.05,1.05,1.05	.36,.36,.36,.36	.46,.47,.48,.47
500	$0B_6$	1.04,1.04,1.04,1.04	.36,.36,.37,.37	.41,.42,.42,.4
1000	$0B_6$	1.04,1.04,1.04,1.04	.37,.38,.37,.37	.41,.41,.41,.42
60	0_7	.17,.17,.17,.17	.08,.08,.08,.08	.15,.15,.15,.15
500		.16,.16,.16,.16	.17,.16,.16,.17	.28,.27,.27,.28
1000		.16,.16,.16,.16	.22,.21,.21,.21	.33,.34,.34,.33
60	10_7	.19,.19,.19,.19	.09,.09,.09,.09	.12,.12,.12,.12
500		.19,.19,.19,.19	.1,.1,.1,.1	.11,.11,.12,.12
1000		.19,.19,.19,.19	.1,.1,.1,.1	.12,.11,.11,.11
60	$0B_7$	.17,.17,.17,.17	.08,.08,.08,.08	.15,.15,.15,.15
500	$0B_7$	.16,.16,.16,.16	.17,.16,.16,.17	.28,.27,.27,.28
1000	$0B_7$	.16,.16,.16,.16	.22,.21,.21,.21	.33,.34,.34,.33

Table 4.6. (Continued)

In Table 4.6, the predictor $\boldsymbol{x}_i \sim LN(\mathbf{0}, \boldsymbol{I})$.

1) The results for the MLR model are similar to the results in Table 4.3.

2) For the type 5, 6, and 7 models, $\hat{\boldsymbol{\beta}}_M \approx c(1,1,1,1)'$, $SE_{cl}(\hat{\boldsymbol{\beta}}_M) \approx a(1,1,1,1)'$, and $SD(\hat{\boldsymbol{\beta}}_M) \approx g(1,1,1,1)'$ for some constants c,a, and g. When the model is type 6 or 7, the two values $SE_{cl}(\hat{\boldsymbol{\beta}}_M)$ and $SD(\hat{\boldsymbol{\beta}}_M)$ are close for the 10% trimming but not for the "best" results. All "best" results are obtained at 0% trimming.

3) For the type 3 and 4 models, nonzero percentage trimming greatly improved the results. For the type 3 model, 80% trimming gave the best results. Compared to the other models, $\hat{\boldsymbol{\beta}}_M$, $SE_{cl}(\hat{\boldsymbol{\beta}}_M)$, and $SD(\hat{\boldsymbol{\beta}}_M)$ are very large and $SE_{cl}(\hat{\boldsymbol{\beta}}_M)$ and $SD(\hat{\boldsymbol{\beta}}_M)$ are much further apart for the type 3 and 4 models. For the type 2 model, the best results are obtained by trimming 10% data and $SE_{cl}(\hat{\boldsymbol{\beta}}_M)$ and $SD(\hat{\boldsymbol{\beta}}_M)$ are also not close.

n	M	$\overline{\boldsymbol{\beta}}_{M_6}$	$\sqrt{n}\,\overline{SE}_{cl_6}$	$\sqrt{n}\,SD_6$
60	0_1	1,1,1,1	.62,.61,.61,.61	.72,.72,.72,.72
500		1,1,1,1	.6,.6,.6,.59	.61,.63,.63,.6
1000		1,1,1,1	.59,.59,.59,.59	.58,.62,.59,.6
60	10_1	1,1,.99,1	.87,.86,.86,.85	.98,.98,.99,.97
500		1,1,1,1	.91,.92,.91,.92	.92,.96,.92,.95
1000		1,1,1,1	.92,.91,.91,.91	.86,.91,.92,.93
60	$0B_1$	1,1,1,1	.62,.61,.61,.61	.72,.72,.72,.72
500	$0B_1$	1,1,1,1	.6,.6,.6,.59	.61,.63,.63,.6
1000	$0B_1$	1,1,1,1	.59,.59,.59,.59	.58,.62,.59,.6
60	0_2	1.54,1.85,1.8,1.55	22.1,21.9,22.4,22.2	68.09,64.09,67.43,63.63
500		2.19,1.99,2.19,1.5	90.8,92.8,91.1,93.7	297.1,278.3,259.9,258
1000		1.93,2.57,2.52,2.18	136.2,137,136.7,134.5	377.7,433.1,402.5,390.9
60	10_2	1.9,2.01,1.94,1.94	10.44,10.5,10.58,10.49	14.5,14.78,14.31,15.42
500		1.98,1.98,1.98,1.98	11.88,11.79,11.83,11.82	12,12.22,12.53,12.36
1000		1.98,2.01,2,2.01	11.97,11.97,12.01,12	12.03,12.08,12,12.12
60	$2B_2$	1.96,2.05,1.98,2.03	8.22,8.21,8.26,8.16	10.6,11.31,11.08,10.87
500	$2B_2$	1.98,1.98,1.98,1.98	8.74,8.75,8.76,8.76	9.47,8.82,9.6,9.34
1000	$2B_2$	2,2,2.01,2.01	8.79,8.82,8.79,8.8	9.47,9.28,9.44,9.55
60	0_3	7.1e35,7.5e35,1.6e35,3.9e35	9e35,1.2e36,1.7e36,1.3e36	1.8e38,1.8e38,4e37,9.6e37
500		4.6e53,2.5e53,5.7e53,8.5e52	2e54,1.5e54,2.4e54,2e54	3.2e56,1.8e56,4e56,6e55
1000		1.1e66,7.5e66,4e66,6.1e65	1.4e67,3.8e67,2.3e67,1.5e67	1.1e69,7.5e69,4e69,6.1e68
60	10_3	48.5,308,223.3,531.7	1282,1522,1227,2249	24407,33572,18358,88332
500		62.5,60.8,61.5,61.1	643.2,621.8,633.4,624.6	1230,1099,1115,1153
1000		58.4,58.9,58.2,58.9	626,633.7,617.4,632.3	916.3,929.9,832.2,962.8
60	$4B_3$	9.37,9.49,9.34,9.62	31.98,31.9,31.39,31.5	80.46,76.81,74.15,84.36
500	$3B_3$	11.12,11.13,11.14,11.11	50.51,50.64,50.64,50.71	67.38,66.71,64.51,65
1000	$3B_3$	11.08,11.09,11.1,11.09	51.64,51.43,51.32,51.28	65.12,67.41,66.48,64.27
60	0_4	113.8,115.1,117,116.9	325.1,320.1,340.2,330.6	3648,3553,2788,2809

Table 4.7. Results Of OLS Estimators Based On Type 6 $\boldsymbol{x}$

n	M	$\overline{\boldsymbol{\beta}}_{M_6}$	$\sqrt{n}\,\overline{SE}_{cl_6}$	$\sqrt{n}\,SD_6$
500		335,346.9,311,324.6	2984,3080,2870,3140	37110,24811,27794,20996
1000		398.3,451.9,383,393.5	5630,5922,5812,5533	38263,50943,45302,52062
60	10_4	23.6,24.3,24.1,24.3	54,55.4,54.5,55.3	101.3,113.1,103.4,132.2
500		22,21.9,21.9,21.9	62.6,61.9,62.5,62.2	78.39,74.22,73.66,72.46
1000		22,22.1,22.1,22.1	63.3,63.5,63.3,63.5	73.84,75.83,72.76,76.91
60	$2B_4$	16.4,16.7,16.7,16.7	35.5,36.4,36.1,35.7	54.75,58.64,58.18,56.2
500	$2B_4$	15.2,15.2,15.3,15.3	38,37.8,38,38.1	46.31,43.76,44.9,45.75
1000	$1B_4$	22,22.1,22.1,22.1	63.3,63.5,63.3,63.5	73.84,75.83,72.76,76.91
60	0_5	-.04,-.04,-.04,-.04	.29,.29,.29,.29	.35,.37,.36,.37
500		-.03,-.03,-.03,-.03	.31,.31,.31,.31	.35,.36,.35,.36
1000		-.03,-.03,-.03,-.03	.32,.32,.32,.32	.37,.36,.37,.35
60	10_5	-.07,-.08,-.07,-.08	.38,.38,.38,.38	.45,.45,.46,.45
500		-.07,-.07,-.07,-.07	.41,.41,.42,.41	.44,.41,.44,.44
1000		-.07,-.07,-.07,-.07	.42,.41,.42,.42	.43,.42,.44,.42
60	$0B_5$	-.04,-.04,-.04,-.04	.29,.29,.29,.29	.35,.37,.36,.37
500	$0B_5$	-.03,-.03,-.03,-.03	.31,.31,.31,.31	.35,.36,.35,.36
1000	$0B_5$	-.03,-.03,-.03,-.03	.32,.32,.32,.32	.37,.36,.37,.35
60	0_6	1.03,1.02,1.03,1.02	.46,.45,.45,.46	.53,.55,.52,.51
500		1.02,1.02,1.02,1.02	.44,.44,.44,.44	.48,.46,.47,.46
1000		1.02,1.02,1.02,1.02	.44,.44,.44,.44	.45,.45,.46,.49
60	10_6	1.05,1.05,1.04,1.04	.65,.64,.64,.65	.72,.75,.72,.71
500		1.04,1.04,1.04,1.04	.67,.67,.67,.67	.68,.68,.7,.68
1000		1.04,1.04,1.04,1.04	.67,.67,.67,.67	.67,.67,.69,.7
60	$0B_6$	1.05,1.05,1.04,1.04	.65,.64,.64,.65	.53,.55,.52,.51
500	$0B_6$	1.04,1.04,1.04,1.04	.67,.67,.67,.67	.48,.46,.47,.46
1000	$0B_6$	1.04,1.04,1.04,1.04	.67,.67,.67,.67	.45,.45,.46,.49
60	0_7	.07,.07,.07,.07	.50,.49,.50,.49	.7,.71,.69,.7
500		.06,.06,.06,.06	.74,.74,.75,.75	.91,.95,.91,.94
1000		.06,.06,.06,.06	.83,.82,.83,.82	1.05,1,1.03,.97
60	10_7	.11,.11,.11,.11	.53,.52,.53,.52	.63,.63,.66,.64
500		.11,.11,.11,.11	.58,.57,.57,.57	.6,.58,.62,.61
1000		.11,.11,.11,.11	.58,.58,.58,.58	.59,.58,.6,.59
60	$1B_7$	.11,.11,.11,.11	.53,.52,.53,.52	.63,.63,.66,.64
500	$1B_7$	.11,.11,.11,.11	.58,.57,.57,.57	.6,.58,.62,.61
1000	$1B_7$	.11,.11,.11,.11	.58,.58,.58,.58	.59,.58,.6,.59

Table 4.7. (Continued)

In Table 4.7, the predictor $\boldsymbol{x}_i \sim MVT_3$.

1) The results for the MLR model are similar to the results in Table 4.3.

2) For the type 5, 6, and 7 models, $\hat{\boldsymbol{\beta}}_M \approx c(1,1,1,1)'$, $SE_{cl}(\hat{\boldsymbol{\beta}}_M) \approx a(1,1,1,1)'$, and $SD(\hat{\boldsymbol{\beta}}_M) \approx g(1,1,1,1)'$ for some constants c,a, and g. When the model is type 5 or 6, the two values $SE_{cl}(\hat{\boldsymbol{\beta}}_M)$ and $SD(\hat{\boldsymbol{\beta}}_M)$ are not close even for the best results. The best results are obtained at 0% trimming for the type 5 and 6 models, and at

10% trimming for the type 7 model.

3) For the type 2, 3, and 4 models, the best results are obtained at nonzero percentage trimming. When the model is type 2 or 4, we need to trim about 20% data; when the model is type 3, we need to trim about 40% data. All $SE_{cl}(\hat{\boldsymbol{\beta}}_M)$ and $SD(\hat{\boldsymbol{\beta}}_M)$ are not close to each other.

n	M	$\overline{\boldsymbol{\beta}}_{M_7}$	$\sqrt{n}\,\overline{SE}_{cl_7}$	$\sqrt{n}\,SD_7$
60	0_1	1,1,1,1	.76,.76,.76,.75	.86,.85,.92,.9
500		1,1,1,1	.77,.77,.77,.77	.78,.77,.77,.79
1000		1,1,1,1	.77,.77,.77,.77	.74,.74,.76,.8
60	10_1	1,1,1,1	.97,.95,.98,.96	1.1,1.06,1.16,1.14
500		.97,.95,.98,.96	1.01,1.01,1.01,1.01	1.03,1.02,1.03,1.05
1000		1,1,1,1	1.01,1.01,1.01,1.01	.99,.99,1,1.04
60	$0B_1$	1,1,1,1	.76,.76,.76,.75	.86,.85,.92,.9
500	$0B_1$	1,1,1,1	.77,.77,.77,.77	.78,.77,.77,.79
1000	$0B_1$	1,1,1,1	.77,.77,.77,.77	.74,.74,.76,.8
60	0_2	1.84,1.9,1.88,1.79	13.5,13.51,13.58,13.56	27.19,26.45,26.65,25.78
500		1.99,1.89,1.95,1.99	29.72,29.67,28.57,28.34	47.58,44.54,44.31,41.12
1000		2.02,1.95,2.01,2.03	35.83,35.31,35.31,34.86	52,58.13,51.28,51.91
60	10_2	2,2,2.02,2.03	8.32,8.11,8.27,8.11	10.86,10.67,10.96,10.45
500		1.99,2,1.98,1.99	8.96,8.93,8.98,8.97	9.55,9.04,9.23,9.27
1000		2.01,2.01,2.02,2.01	9,8.96,8.95,8.96	9.66,9.22,9.43,9.25
60	$1B_2$	2,2,2.02,2.03	8.32,8.11,8.27,8.11	10.86,10.67,10.96,10.45
500	$1B_2$	1.99,2,1.98,1.99	8.96,8.93,8.98,8.97	9.55,9.04,9.23,9.27
1000	$1B_2$	2.01,2.01,2.02,2.01	9,8.96,8.95,8.96	9.66,9.22,9.43,9.25
60	0_3	2.1e6,2.6e6,2e6,1.6e5	7.6e6,7.8e6,6.2e6,5.9e6	3.8e8,3.7e8,2.e8,1.3e8
500		1.3e12,1.4e12,8.1e11,1.9e12	1.9e13,2.1e13,1.4e13,2.8e13	7.4e14,8.8e14,4.2e14,1.2e15
1000		3.3e14,9.7e14,3.8e14,5.3e14	7.9e15,2.3e16,9e15,1.2e16	3.3e17,9.7e17,3.8e17,5.3e17
60	10_3	31.34,40.54,39.76,44.18	193.3,192.4,188.4,208	1181,1451,1057,1855
500		23.86,23.68,23.76,23.79	161.8,159.4,162.7,161	237.2,236.2,235.6,246.3
1000		23.55,23.39,23.53,23.45	166.2,165,167.1,165.8	225.8,219.4,222.6,219
60	$4B_3$	8.25,8.28,8.32,8.4	24.86,25.46,25.77,25.44	48.09,48.25,49.54,55.79
500	$3B_3$	9.19,9.16,9.14,9.15	35.81,35.41,35.47,35.46	47.82,46.76,48.74,46.54
1000	$2B_3$	12.61,12.61,12.63,12.56	60.32,60.23,60.41,60.15	78.95,78.63,79.16,80.19
60	0_4	38.36,38.83,39.05,37.53	102.2,104.9,104.5,102.8	455.1,413.5,461.1,378
500		49.04,48.72,47.91,46.44	382.5,372.1,355.3,346.3	1187,1027,1152, 934.9
1000		50.95,51.22,51,50.1	531.8,535.4,521.9,516	1186,1635,1320,1370
60	10_4	17.71,17.59,17.84,17.54	37.32,36.65,37.86,36.71	58.65,58.8,59.5,60.66
500		16.63,16.63,16.6,16.62	41.08,40.76,41.11,40.94	50.67,49.17,49.29,49.81
1000		16.62,16.56,16.6,16.56	41.64,41.49,41.58,41.55	49.91,49.48,49.78,48.89
60	$2B_4$	13.98,13.77,13.89,13.93	28.82,28.42,28.77,28.58	44.99,43.88,44.64,43.04
500	$1B_4$	16.63,16.63,16.6,16.62	41.08,40.76,41.11,40.94	50.67,49.17,49.29,49.81
1000	$1B_4$	16.62,16.56,16.6,16.56	41.64,41.49,41.58,41.55	49.91,49.48,49.78,48.89
60	0_5	-.07,-.07,-.07,-.07	.35,.34,.34,.34	.4,.4,.42,.41

Table 4.8. Results Of OLS Estimators Based On Type 7 $\boldsymbol{x}$

n	M	$\overline{\boldsymbol{\beta}}_{M_7}$	$\sqrt{n}\,\overline{SE}_{cl_7}$	$\sqrt{n}\,SD_7$
500		-.06,-.06,-.06,-.06	.38,.38,.38,.38	.4,.39,.4,.4
1000		-.06,-.06,-.06,-.06	.39,.39,.38,.38	.39,.4,.39,.39
60	10_5	-.1,-.1,-.1,-.1	.4,.39,.39,.39	.46,.45,.48,.47
500		-.09,-.1,-.09,-.1	.43,.43,.43,.43	.46,.43,.46,.46
1000		-.1,-.1,-.1,-.1	.43,.43,.43,.43	.45,.44,.44,.44
60	$2B_5$	-.12,-.12,-.12,-.12	.42,.42,.42,.42	.52,.51,.5,.5
500	$0B_5$	-.06,-.06,-.06,-.06	.38,.38,.38,.38	.4,.39,.4,.4
1000	$1B_5$	-.1,-.1,-.1,-.1	.43,.43,.43,.43	.45,.44,.44,.44
60	0_6	1.04,1.04,1.04,1.04	.56,.55,.56,.56	.63,.61,.66,.64
500		1.04,1.03,1.04,1.03	.57,.57,.57,.57	.62,.58,.6,.59
1000		1.03,1.04,1.03,1.04	.57,.57,.57,.57	.57,.57,.57,.59
60	10_6	1.06,1.06,1.06,1.06	.72,.71,.72,.72	.8,.77,.81,.79
500		1.06,1.06,1.06,1.06	.74,.75,.74,.75	.78,.75,.78,.75
1000		1.06,1.06,1.06,1.06	.75,.75,.75,.75	.74,.74,.76,.78
60	$0B_6$	1.04,1.04,1.04,1.04	.56,.55,.56,.56	.63,.61,.66,.64
500	$1B_6$	1.06,1.06,1.06,1.06	.74,.75,.74,.75	.78,.75,.78,.75
1000	$1B_6$	1.06,1.06,1.06,1.06	.75,.75,.75,.75	.74,.74,.76,.78
60	0_7	.1,.1,.1,.1	.51,.5,.51,.51	.64,.63,.67,.64
500		.09,.09,.09,.09	.65,.65,.64,.64	.71,.7,.7,.7
1000		.09,.09,.09,.09	.68,.68,.68,.67	.71,.74,.7,.7
60	10_7	.13,.13,.13,.13	.53,.52,.52,.52	.63,.61,.63,.6
500		.13,.13,.13,.13	.56,.56,.56,.56	.59,.57,.58,.59
1000		.13,.13,.13,.13	.56,.56,.56,.56	.59,.57,.58,.57
60	$0B_7$	.1,.1,.1,.1	.51,.5,.51,.51	.64,.63,.67,.64
500	$1B_7$	.13,.13,.13,.13	.56,.56,.56,.56	.59,.57,.58,.59
1000	$1B_7$	.13,.13,.13,.13	.56,.56,.56,.56	.59,.57,.58,.57

Table 4.8. (Continued)

In Table 4.8, the predictor $\boldsymbol{x}_i \sim MVT_5$.

1) The results for the MLR model are similar to the results in Table 4.3.

2) For the type 5, 6, and 7 models, $SE_{cl}(\hat{\boldsymbol{\beta}}_M)$ and $SD(\hat{\boldsymbol{\beta}}_M)$ are close for the best results when n is large. The best results are often obtained at 10% trimming.

3) For the type 2, 3, and 4 models, the best results are obtained at nonzero percentage trimming. When the model is type 3, we need to trim a larger percentage of data to get the best results compared with the other two models. $SE_{cl}(\hat{\boldsymbol{\beta}}_M)$ and $SD(\hat{\boldsymbol{\beta}}_M)$ are not close especially for models 3 and 4.

n	M	$\overline{\boldsymbol{\beta}}_{M_8}$	$\sqrt{n}\,\overline{SE}_{cl_8}$	$\sqrt{n}\,SD_8$
60	0_1	1,1,1.01,1	.91,.91,.91,.9	.99,1.04,.98,.99
500		1,1,1,1	.95,.94,.94,.94	.94,.97,.95,.97

Table 4.9. Results Of OLS Estimators Based On Type 8 $\boldsymbol{x}$

n	M	$\overline{\boldsymbol{\beta}}_{M_8}$	$\sqrt{n}\,\overline{SE}_{cl_8}$	$\sqrt{n}\,SD_8$
1000		1,1,1,1	.94,.94,.94,.95	.91,.94,.95,.97
60	10_1	1,1,1.01,.99	1.08,1.07,1.07,1.08	1.21,1.2,1.17,1.21
500		1,1,1,1	1.12,1.12,1.11,1.12	1.15,1.14,1.13,1.15
1000		1,1,1,1	1.12,1.12,1.12,1.12	1.09,1.12,1.15,1.15
60	$0B_1$	1,1,1.01,1	.91,.91,.91,.9	.99,1.04,.98,.99
500	$0B_1$	1,1,1,1	.95,.94,.94,.94	.94,.97,.95,.97
1000	$0B_1$	1,1,1,1	.94,.94,.94,.95	.91,.94,.95,.97
60	0_2	1.96,1.98,2,1.97	7.75,7.66,7.73,7.84	10.05,9.85,9.79,10.45
500		1.99,2,2.01,1.99	9.87,10.03,9.95,10.01	10.42,10.45,10.34,10.47
1000		2,1.99,1.99,1.99	10.29,10.4,10.4,10.38	10.22,10.63,10.81,10.67
60	10_2	1.98,1.97,2,2	6.58,6.47,6.59,6.57	8.29,8.12,7.81,7.9
500		1.99,2.01,2.01,2	6.81,6.8,6.8,6.81	7.24,7.33,7.3,7.22
1000		2.01,2,2,2	6.75,6.75,6.74,6.76	7.05,7.35,7.28,7.31
60	$1B_2$	1.98,1.97,2,2	6.58,6.47,6.59,6.57	8.29,8.12,7.81,7.9
500	$1B_2$	1.99,2.01,2.01,2	6.81,6.8,6.8,6.81	7.24,7.33,7.3,7.22
1000	$1B_2$	2.01,2,2,2	6.75,6.75,6.74,6.76	7.05,7.35,7.28,7.31
60	0_3	49.63,67.36,46.86,48.89	249.3,332.2,229.7,247.1	2788,7318,2299,2793
500		58.44,60.53,54.99,55.97	838,880.4,768.1,787	6378,5931,5564,4263
1000		50.12,48.92,53.3,55.43	854.7,824.7,947.7,1011	2745,3205,4457,6535
60	10_3	16.04,15.86,16.16,15.99	60.18,59.19,61.51,60.62	162.1,200,179.9,157.5
500		12.82,12.89,12.93,12.87	57.83,58.09,58.51,57.82	74.61,74.01,72.92,73.77
1000		12.56,12.52,12.54,12.52	55.54,55.58,55.49,55.32	67.27,68.9,69.47,69.93
60	$3B_3$	9.17,9.36,9.21,9.33	29.31,29.47,29.19,28.94	61.92,66.45,59.87,66.19
500	$2B_3$	9.55,9.54,9.56,9.51	34.58,34.45,34.81,34.49	47.27,48.54,47.18,46.78
1000	$1B_3$	12.56,12.52,12.54,12.52	55.54,55.58,55.49,55.32	67.27,68.9,69.47,69.93
60	0_4	17.26,17.11,17.27,17.4	35.77,35.82,36,36.85	58.92,61.3,56.05,57.37
500		17.83,17.97,17.94,17.91	53.66,54.9,54.05,54.13	61.96,65.62,62.47,63.79
1000		18.07,18.07,18.1,18.07	57.8,57.76,58.49,58.35	64.24,65.03,66.78,69.46
60	10_4	13.55,13.45,13.6,13.6	26.55,26.05,26.96,26.55	38.59,37.59,36.49,37.1
500		12.91,12.96,12.97,12.95	27.78,27.88,28.03,27.81	31.53,31.88,32.51,32.58
1000		12.87,12.83,12.84,12.84	27.59,27.55,27.53,27.5	31.04,32.11,32.06,32.27
60	$1B_4$	13.55,13.45,13.6,13.6	26.55,26.05,26.96,26.55	38.59,37.59,36.49,37.1
500	$1B_4$	12.91,12.96,12.97,12.95	27.78,27.88,28.03,27.81	31.53,31.88,32.51,32.58
1000	$1B_4$	12.87,12.83,12.84,12.84	27.59,27.55,27.53,27.5	31.04,32.11,32.06,32.27
60	0_5	-.09,-.1,-.1,-.1	.37,.37,.37,.37	.42,.42,.42,.43
500		-.09,-.09,-.09,-.09	.4,.41,.4,.41	.42,.41,.41,.4
1000		-.09,-.09,-.09,-.09	.41,.41,.41,.41	.41,.42,.42,.42
60	10_5	-.12,-.12,-.12,-.12	.4,.39,.39,.4	.48,.47,.46,.45
500		-.12,-.12,-.12,-.12	.42,.42,.41,.42	.45,.46,.45,.42
1000		-.12,-.12,-.12,-.12	.42,.42,.42,.42	.44,.45,.45,.46
60	$0B_5$	-.09,-.1,-.1,-.1	.37,.37,.37,.37	.42,.42,.42,.43
500	$0B_5$	-.09,-.09,-.09,-.09	.4,.41,.4,.41	.42,.41,.41,.4
1000	$0B_5$	-.09,-.09,-.09,-.09	.41,.41,.41,.41	.41,.42,.42,.42
60	0_6	1.07,1.07,1.07,1.07	.67,.67,.67,.67	.75,.76,.73,.74
500		1.06,1.06,1.06,1.06	.69,.69,.69,.69	.68,.69,.73,.7
1000		1.06,1.06,1.06,1.06	.69,.69,.69,.69	.67,.69,.7,.69

Table 4.9. (Continued)

n	M	$\overline{\boldsymbol{\beta}}_{M_8}$	$\sqrt{n}\,\overline{SE}_{cl_8}$	$\sqrt{n}\,SD_8$
60	10_6	1.1,1.1,1.1,1.09	.78,.78,.79,.78	.88,.89,.86,.9
500		1.09,1.09,1.09,1.09	.81,.81,.81,.81	.81,.83,.87,.83
1000		1.09,1.09,1.09,1.09	.81,.81,.81,.81	.8,.84,.82,.83
60	$0B_6$	1.07,1.07,1.07,1.07	.67,.67,.67,.67	.75,.76,.73,.74
500	$0B_6$	1.06,1.06,1.06,1.06	.69,.69,.69,.69	.68,.69,.73,.7
1000	$0B_6$	1.06,1.06,1.06,1.06	.69,.69,.69,.69	.67,.69,.7,.69
60	0_7	.13,.13,.13,.13	.49,.48,.48,.49	.57,.56,.56,.58
500		.12,.13,.13,.12	.54,.54,.54,.54	.56,.56,.55,.55
1000		.13,.12,.12,.12	.54,.55,.55,.55	.56,.56,.56,.56
60	10_7	.15,.15,.15,.15	.52,.51,.51,.52	.62,.6,.59,.59
500		.15,.15,.15,.15	.54,.54,.54,.54	.58,.59,.58,.55
1000		.15,.15,.15,.15	.54,.54,.54,.54	.56,.57,.57,.58
60	$0B_7$	.13,.13,.13,.13	.49,.48,.48,.49	.57,.56,.56,.58
500	$1B_7$	.15,.15,.15,.15	.54,.54,.54,.54	.58,.59,.58,.55
1000	$1B_7$	.15,.15,.15,.15	.54,.54,.54,.54	.56,.57,.57,.58

Table 4.9. (Continued)

In Table 4.9, the predictor $\boldsymbol{x}_i \sim MVT_{19}$.

1) The results for the MLR model are similar to the results in Table 4.3.

2) For the type 5, 6, and 7 models, $SE_{cl}(\hat{\boldsymbol{\beta}}_M)$ and $SD(\hat{\boldsymbol{\beta}}_M)$ are close for the best results when n is large. The best results are often obtained at 0% trimming.

3) For the type 2 and 4 model, the best results are obtained at 10% trimming. When the model is type 3, we need to trim a larger percentage of data to get the best results compared with the other two models. $SE_{cl}(\hat{\boldsymbol{\beta}}_M)$ and $SD(\hat{\boldsymbol{\beta}}_M)$ are not close especially for models 3 and 4.

4.3.3 The SIR Estimator

The resistant DR method we will use in this section is SIR. There are two SIR algorithms that we will use. One is denoted as SSIR, another is denoted as WSIR. The SSIR algorithm was obtained from STATLIB and written by Thomas Koetter, while the WSIR algorithm was due to Weisberg (2002) [28]. We need compare $\hat{\boldsymbol{\beta}}_{SSIR}$ and $\hat{\boldsymbol{\beta}}_{WSIR}$ for different methods. For both algorithms, we let the number of slices $h = 4$.

The MLR Model And The Normal Distributed Predictor $\boldsymbol{x}$

The results in this section are based on the MLR model and $\boldsymbol{x}_i \sim N_4(\mathbf{0}, \boldsymbol{I})$. We show them in Table 4.10. Column "$\overline{\boldsymbol{\beta}}_{SSIRi}$" represents the sample mean of $\hat{\boldsymbol{\beta}}_{SSIR}$ with type i predictors, column "$\overline{\boldsymbol{\beta}}_{WSIRi}$" represents the sample mean of $\hat{\boldsymbol{\beta}}_{WSIR}$ with type i predictors, column "M" denotes the trimming percentage, and column"n" means the size of the entire data.

M	n	$\overline{\boldsymbol{\beta}}_{SSIR1}$	$\overline{\boldsymbol{\beta}}_{WSIR1}$
0	60	-.24,-.25,-.25,-.25	.02,.02,.02,.02
	500	-.22,-.22,-.22,-.22	.02,.02,.02,.02
	1000	-.24,-.24,-.24,-.24	-.01,-.01,-.01,-.01
	2000	-.24,-.24,-.24,-.24	.01,.01,.01,.01
10	60	-.19,-.21,-.21,-.22	.03,.03,.03,.03
	500	-.22,-.22,-.22,-.22	-.003,-.003,-.002,-.003
	1000	-.24,-.24,-.24,-.24	.003,.004,.002,.003
	2000	-.24,-.24,-.24,-.24	.04,.04,.04,.04
20	60	-.16,-.19,-.19,-.19	.01,.01,.01,.01
	500	-.25,-.25,-.25,-.25	.02,.02,.02,.02
	1000	-.24,-.24,-.25,-.24	.003,.004,.004,.002
	2000	-.25,-.25,-.25,-.25	.01,.01,.01,.01
30	60	-.13,-.17,-.16,-0.16	.03,.02,.02,.03
	500	-.26,-.26,-.26,-.26	.03,.03,.03,.03
	1000	-.24,-.24,-.24,-.24	-.01,-.01,-.02,-.02
	2000	-.23,-.23,-.23,-.23	.01,.005,.005,.004
40	60	-.13,-.16,-.16,-.16	.01,.005,.002,.002
	500	-.25,-.26,-.26,-.26	.005,.007,.006,.006
	1000	-.25,-.25,-.25,-.25	.01,.01,.01,.01
	2000	-.29,-.29,-.29,-.29	-.002,-.004,-.001,-.001
50	60	-.09,-.16,-.17,-.15	.02,.01,.02,.01
	500	-.24,-.25,-.24,-.24	-.01,-.01,-.02,-.02
	1000	-.25,-.25,-.25,-.25	.006,.005,.005,.004
	2000	-.26,-.26,-.26,-.26	-.002,-.002,-.002,-.003
60	60	-.002,-.1,-.09,-.08	.02,-.002,.03,-.004
	500	-.25,-.26,-.26,-.26	-.004,-.003,-.005,-.007
	1000	-.22,-.22,-.22,-.22	-.007,-.005,-.009,-.011
	2000	-.28,-.28,-.28,-.28	-.008,-.007,-.007,-.006
70	60	.04,-.1,-.09,-.07	.04,.01,.01,.02
	500	-.21,-.23,-.23,-.22	.03,.03,.02,.03
	1000	-.25,-.26,-.26,-.26	-.03,-.04,-.03,-.03
	2000	-.22,-.22,-.22,-.22	-.008,-.008,-.004,-.008
80	60	.1,-.08,-.05,-.04	.007,-.0003,.017,.007
	500	-.17,-.21,-.21,-.21	.03,.02,.02,.03
	1000	-.21,-.22,-.23,-.23	-.02,-.02,-.02,-.02
	2000	-.2,-.21,-.21,-.21	-.03,-.03,-.03,-.03

Table 4.10. Results Of SIR Estimators For The MLR Model

M	n	$\overline{\boldsymbol{\beta}}_{SSIR1}$	$\overline{\boldsymbol{\beta}}_{WSIR1}$
90	60	.12,.001,.009,.14	.004,-.002,-.02,-.005
	500	-.07,-.17,-.16,-.16	-.005,-.0002,.004,-.012
	1000	-.16,-.2,-.2,-.2	-.005,-.01,-.001,-.005
	2000	-.22,-.24,-.24,-.24	.01,.01,.01,.01

Table 4.10. (Continued)

We can see that both $\hat{\boldsymbol{\beta}}_{SSIR}$ and $\hat{\boldsymbol{\beta}}_{WSIR}$ estimate $c\boldsymbol{\beta}$ when n is large, where c is some constant which depends on n, M, and the algorithm. But for $\hat{\boldsymbol{\beta}}_{WSIR}$, the constant c is very close to 0.

Nonnormal Distributed Predictor $\boldsymbol{x}$

The results in this section are based on the nonnormal predictor distributions and are shown in Table 4.11 to Table 4.14. For each type of distribution, we ran the simulations on 7 different models which were introduced in Section 4.2. Similar to Table 4.2, we keep a record of the results based on $0\%, 10\%$, and the trimming corresponding to the best result. In column "M", we denote them by $0, 10$, and B correspondingly. Similarly, the subscript denotes the type of model we used, and the number before B denotes the trimming percentage corresponding to the best result. Column "$\overline{\boldsymbol{\beta}}_{SSIRi}$" represents the sample mean of $\hat{\boldsymbol{\beta}}_{SSIR}$ with type i predictors, and column "$\overline{\boldsymbol{\beta}}_{WSIRi}$" represents the sample mean of $\hat{\boldsymbol{\beta}}_{WSIR}$ with type i predictors.

n	M	$\overline{\boldsymbol{\beta}}_{SSIR1}$	M	$\overline{\boldsymbol{\beta}}_{SSIR2}$	M	$\overline{\boldsymbol{\beta}}_{WSIR1}$	M	$\overline{\boldsymbol{\beta}}_{WSIR2}$
60	0_1	-.24,-.25,-.25,-.25	0_1	.07,.01,.02,.01	0_1	.02,.02,.02,.02	0_1	.004,.01,.01,.01
500		-.22,-.22,-.22,-.22		.1,.09,.09,.09		.02,.02,.02,.02		.01,.01,.01,.01
1000		-.24,-.24,-.24,-.24		.08,.08,.08,.08		-.01,-.01,-.01,-.01		-.02,-.02,-.02,-.02
2000		-.24,-.24,-.24,-.24		.08,.07,.07,.07		.01,.01,.01,.01		-.01,-.01,-.01,-.01
60	10_1	-.19,-.21,-.21,-.22	10_1	.08,.01,.03,.02	10_1	.03,.03,.03,.03	10_1	-.01,-.004,-.01,-.01
500		-.22,-.22,-.22,-.22		.12,.11,.11,.12		-.003,-.003,-.002,-.003		-.01,-.02,-.02,-.02
1000		-.24,-.24,-.24,-.24		.12,.12,.12,.12		.003,.004,.002,.003		.02,.02,.02,.02
2000		-.24,-.24,-.24,-.24		.09,.09,.09,.09		.04,.04,.04,.04		.005,.003,.003,.005
60	$0B_1$	-.24,-.25,-.25,-.25	$2B_1$	.08,.02,.03,.03	$0B_1$	.02,.02,.02,.02	$0B_1$	.004,.01,.01,.01
500	$0B_1$	-.22,-.22,-.22,-.22	$0B_1$	.1,.09,.09,.09	$0B_1$	.02,.02,.02,.02	$0B_1$	.01,.01,.01,.01
1000	$0B_1$	-.24,-.24,-.24,-.24	$0B_1$	.08,.08,.08,.08	$0B_1$	-.01,-.01,-.01,-.01	$0B_1$	-.02,-.02,-.02,-.02

Table 4.11. Results Of SIR Estimators Based On Type 1&2 $\boldsymbol{x}$

n	M	$\overline{\boldsymbol{\beta}}_{SSIR1}$	M	$\overline{\boldsymbol{\beta}}_{SSIR2}$	M	$\overline{\boldsymbol{\beta}}_{WSIR1}$	M	$\overline{\boldsymbol{\beta}}_{WSIR2}$
2000	$0B_1$	-.24,-.24,-.24,-.24	$1B_1$	.09,.09,.09,.09	$0B_1$	.01,.01,.01,.01	$0B_1$	-.01,-.01,-.01,-.01
60	0_2	.15,-.06,-.02,-.004	0_2	.21,-.15,-.01,-.03	0_2	.03,.04,.03,.03	0_2	.02,.01,-.01,-.02
500		.11,.06,.06,.06		.19,-.17,-.04,-.02		-.004,-.01,-.004,-.01		.03,.01,.02,.03
1000		.08,.04,.04,.05		.19,-.13,-.001,-.03		.01,.01,.01,.01		-.01,.01,.004,.004
2000		.1,.08,.08,.08		.2,-.07,.004,-.01		.01,.01,.01,.01		.02,.02,.001,.02
60	10_2	.14,-.04,-.005,-.008	10_2	.21,-.17,-.02,-.02	10_2	.02,.02,.02,.02	10_2	.001,.005,.01,.02
500		.07,.05,.05,.05		.2,-.11,.02,-.001		-.01,-.01,.0002,-.002		.005,-.006,-.01,.0002
1000		.09,.07,.07,.07		.15,-.09,-.03,-.03		.02,.02,.02,.01		-.01,.003,.01,.001
2000		.07,.06,.06,.06		.14,-.01,.04,.01		-.01,-.01,-.01,-.01		.02,.01,-.003,.01
60		N/A		N/A	$1B_2$	.02,.02,.02,.02	$2B_2$	.01,-.001,.01,.01
500	$4B_2$	.05,.03,.03,.03	$6B_2$	.05,.02,.02,.02	$0B_2$	-.004,-.01,-.004,-.01	$2B_2$	-.001,-.004,.002,.01
1000	$4B_2$	.04,.03,.03,.03	$5B_2$	.09,.08,.08,.08	$0B_2$	.01,.01,.01,.01	$2B_2$	-.01,-.01,.01,-.01
2000	$1B_2$	.07,.06,.06,.06	$6B_2$	.08,.07,.07,.07	$0B_2$	.01,.01,.01,.01	$2B_2$	.01,.02,.01,.01
60	0_3	.12,-.06,-.06,-.04	0_3	.2,-.09,-.04,-.03	0_3	.01,.01,.01,.01	0_3	.004,.01,.001,.0004
500		.11,-.02,-.004,-.02		.14,-.05,-.04,-.03		.02,.01,.01,.02		.01,.01,.01,.01
1000		.12,.002,.01,.001		.13,-.06,-.04,-.06		-.01,-.01,-.01,-.01		.003,.0003,.002,.002
2000		.1,.01,-.002,-.003		.13,-.04,-.04,-.03		.02,.02,.02,.02		.01,.01,.01,.01
60	10_3	.12,-.06,-.04,-.03	10_3	.18,-.11,-.02,-.03	10_3	.01,.02,.02,.02	10_3	.01,.01,.01,.01
500		.04,-.02,-.02,-.02		.08,-.08,-.07,-.06		.02,.01,.01,.02		-.01,-.01,-.01,-.01
1000		.05,.02,.02,.02		.08,-.04,-.04,-.03		.01,.01,.01,.01		-.02,-.02,-.02,-.02
2000		.09,.06,.06,.07		.09,.01,.02,.01		.01,.01,.01,.01		.004,.004,.003,.005
60		N/A		N/A	$0B_3$	.01,.01,.01,.01	$1B_3$	.01,.01,.01,.01
500	$4B_3$	.08,.05,.04,.04	$6B_3$	.08,.03,.01,.02	$0B_3$	.02,.01,.01,.02	$0B_3$	.01,.01,.01,.01
1000	$1B_3$	.05,.02,.02,.02	$6B_3$	.07,.03,.03,.03	$0B_3$	-.01,-.01,-.01,-.01	$1B_3$	-.02,-.02,-.02,-.02
2000	$2B_3$	.09,.07,.07,.07	$5B_3$	.1,.07,.07,.08	$0B_3$	.02,.02,.02,.02	$0B_3$	.01,.01,.01,.01
60	0_4	.09,-.01,-.02,-.01	0_4	.13,-.03,-.02,-.01	0_4	.04,.04,.04,.04	0_4	-.002,.001,-.004,-.002
500		.07,.02,.02,.03		.1,.04,.04,.04		.03,.03,.03,.03		-.02,-.02,-.02,-.02
1000		.07,.05,.05,.05		.1,.06,.07,.06		-.03,-.03,-.03,-.03		.02,.02,.01,.01
2000		.09,.06,.06,.07		.1,.07,.07,.07		.02,.02,.02,.02		.01,.01,.01,.01
60	10_4	.1,-.01,-.01,-.003	10_4	.13,-.02,-.01,-.003	10_4	.02,.01,.02,.02	10_4	3.7e-5,3.7e-3, 7.2e-3,5.7e-3
500		.07,.04,.04,.03		.06,.03,.03,.03		.02,.02,.02,.02		.01,.01,.01,.01
1000		.09,.08,.08,.07		.09,.06,.07,.06		.01,.01,.01,.01		-.01,-.01,-.01,-.01
2000		.06,.06,.06,.06		.08,.06,.07,.07		.01,.01,.01,.01		.02,.01,.02,.02
60		N/A		N/A	$0B_4$	.04,.04,.04,.04	$2B_4$	.02,.01,.02,.02
500	$2B_4$	.08,.05,.05,.05	$1B_4$	.06,.03,.03,.03	$0B_4$	.03,.03,.03,.03	$0B_4$	-.02,-.02,-.02,-.02
1000	$1B_4$	.1,.08,.08,.08	$2B_4$	.07,.05,.05,.05	$0B_4$	-.03,-.03,-.03,-.03	$1B_4$	-.01,-.01,-.01,-.01
2000	$1B_4$	.06,.06,.06,.06	$1B_4$	.08,.06,.07,.07	$0B_4$	.02,.02,.02,.02	$0B_4$	.01,.01,.01,.01
60	0_5	.09,-.08,-.05,-.05	0_5	.23,-.24,-.03,-.05	0_5	.04,.04,.04,.03	0_5	.03,.02,-.01,-.01
500		.11,.08,.08,.08		.18,-.14,-.04,-.04		.02,.01,.01,.02		.01,-.02,-.01,.003

Table 4.11. (Continued)

n	M	$\overline{\boldsymbol{\beta}}_{SSIR1}$	M	$\overline{\boldsymbol{\beta}}_{SSIR2}$	M	$\overline{\boldsymbol{\beta}}_{WSIR1}$	M	$\overline{\boldsymbol{\beta}}_{WSIR2}$
1000		.14,.13,.13,.13		.13,-.06,-.04,-.03		.01,.01,.005,.01		-.02,.01,-.0005,-.01
2000		.09,.09,.09,.09		.09,-.02,-.02,-.02		.02,.02,.02,.02		-.001,.01,-.005,-.01
60	10_5	.13,-.02,.01,.004	10_5	.25,-.2,-.02,-.01	10_5	.02,.004,.01,.02	10_5	.003,.02,-.01,.01
500		.06,.04,.05,.05		.16,-.09,-.04,-.05		.02,.02,.02,.01		.01,-.002,.01,.02
1000		.06,.05,.05,.05		.1,-.04,-.03,-.03		.01,.01,.01,.01		-.01,.01,-.01,-.0002
2000		.07,.07,.07,.06		.05,-.02,-.03,-.01		.02,.02,.02,.02		.01,.01,-.001,-.002
60		N/A		N/A	$0B_5$	.04,.04,.04,.03	$4B_5$	.02,.03,.03,.03
500	$1B_5$	.06,.04,.05,.05	$6B_5$	.06,.03,.05,.04	$0B_5$	.02,.01,.01,.02	$4B_5$	.01,.01,.01,.01
1000	$0B_5$	.14,.13,.13,.13	$5B_5$	.08,.07,.07,.07	$1B_5$	.01,.01,.01,.01	$4B_5$	-.03,-.03,-.03,-.03
2000	$0B_5$	.09,.09,.09,.09	$4B_5$	.11,.1,.1,.1	$0B_5$	.02,.02,.02,.02	$4B_5$	-.01,-.01,-.01,-.01
60	0_6	.02,-.01,-.02,-.02	0_6	.1,.05,.06,.05	0_6	.02,.03,.02,.03	0_6	-.004,-.002,-.01,-.002
500		.04,.03,.03,.03		.1,.08,.08,.08		.01,.01,.01,.01		-.003,-.005,-.002,-.002
1000		.02,.02,.02,.02		.1,.1,.1,.1		.04,.04,.04,.04		.01,.01,.01,.01
2000		.05,.05,.05,.05		.1,.1,.1,.1		.01,.01,.01,.01		-.02,-.02,-.02,-.01
60	10_6	.05,.02,.02,.02	10_6	.07,.01,.03,.02	10_6	.03,.03,.03,.04	10_6	.02,.02,.02,.02
500		.06,.06,.06,.06		.1,.1,.1,.09		.004,.0001,.004,.0005		-.004,-.007,-.005,-.004
1000		.04,.04,.04,.04		.1,.09,.09,.1		.01,.01,.01,.01		.0005,-.001,-.001,-.003
2000		.04,.04,.04,.04		.08,.08,.08,.08		.02,.02,.01,.01		-.01,-.01,-.01,-.01
60	$1B_6$	.05,.02,.02,.02	$0B_6$	.1,.05,.06,.05	$1B_6$	.03,.03,.03,.04	$1B_6$	.02,.02,.02,.02
500	$1B_6$	.06,.06,.06,.06	$1B_6$	.1,.1,.1,.09	$0B_6$	.01,.01,.01,.01	$0B_6$	-.003,-.005,-.002,-.002
1000	$0B_6$	.02,.02,.02,.02	$0B_6$	.1,.1,.1,.1	$0B_6$	.04,.04,.04,.04	$0B_6$	.01,.01,.01,.01
2000	$0B_6$	.05,.05,.05,.05	$0B_6$	.1,.1,.1,.1	$0B_6$	.01,.01,.01,.01	$1B_6$	-.01,-.01,-.01,-.01
60	0_7	.1,-.09,-.06,-.05	0_7	.24,-.24,-.02,-.03	0_7	.02,.04,.02,.03	0_7	.003,.01,-.01,-.02
500		.05,.01,.01,.01		.25,-.19,.003,-.0005		.01,.01,.02,.02		.02,-.001,.004,.02
1000		.08,.06,.06,.06		.21,-.17,-.05,-.03		.006,.003,-.003,.001		-.01, .01,.001,.01
2000		.07,.06,.06,.06		.19,-.13,-.02,-.04		.04,.03,.04,.04		-.0002,-.005,-.01,-.002
60	10_7	.13,-.07,-.01,-.02	10_7	.25,-.23,-.02,-.005	10_7	.02,.01,.02,.03	10_7	.004,.001,.002,.02
500		.1,.07,.07,.07		.23,-.18,-.001,.004		-.001,-.003,.001,.001		.02,.02,.02,.01
1000		.09,.08,.08,.08		.2,-.15,-.04,-.04		.02,.02,.01,.01		-.01,.01,.002,.004
2000		.09,.08,.08,.08		.15,-.11,-.05,-.06		.02,.02,.02,.02		-.001,-.01,-.01,-.01
60		N/A		N/A	$1B_7$	.02,.01,.02,.03	$4B_7$	.02,.01,.02,.01
500	$2B_7$	.08,.06,.06,.06	$5B_7$	.08,.04,.03,.03	$1B_7$	-.001,-.003,.001,.001	$6B_7$	.046,.045,.044,.045
1000	$1B_7$	.09,.08,.08,.08	$5B_7$	.08,.06,.06,.06	$1B_7$	.02,.02,.01,.01	$5B_7$	-.01,-.02,-.01,-.01
2000	$1B_7$	.09,.08,.08,.08	$6B_7$	.07,.06,.06,.06	$1B_7$	.02,.02,.02,.02	$4B_7$	-.01,-.01,-.01,-.01

Table 4.11. (Continued)

The results of Table 4.11 are based on the type 1 predictors $\boldsymbol{x}_i \sim N_4(\mathbf{0}, \boldsymbol{I})$ and the type 2 predictors $\boldsymbol{x}_i \sim 0.6N_4(\mathbf{0}, \boldsymbol{I}) + 0.4N_4(\mathbf{0}, 25\boldsymbol{I})$. We will discuss the results separately.

For the type 1 $\boldsymbol{x}$:

1) If the model is MLR, all the $\hat{\boldsymbol{\beta}}_{SSIR}$ and the $\hat{\boldsymbol{\beta}}_{WSIR}$ are estimating some constant multiple of the true coefficient $\boldsymbol{\beta}$. The best results are obtained at 0% trimming.

2) If the method is SSIR, for models such as 2, 3, 4, 5, and 7, the best results are often obtained at nonzero percentage trimming. When n is small, these best results are not close to $c\,(1,1,1,1)'$ for some constant c. If $n = 60$, we can not find a good result for any of the ten values of M. For this case, we denote it by "N/A". Whereas for the method WSIR 0% trimming works well for most of the models.

For the type 2 $\boldsymbol{x}$:

1) If the model is MLR or type 6, the $\hat{\boldsymbol{\beta}}_{SSIR}$ and the $\hat{\boldsymbol{\beta}}_{WSIR}$ are estimating the true coefficient $\boldsymbol{\beta}$ multiplied by some constant when n is large. Most of the best results for both methods are obtained at 0% trimming.

2) For models 2, 3, 5, and 7, the results for the method SSIR are never good for 0% and 10% trimming. Actually even the best results we could get are usually not close to $c\,(1,1,1,1)'$ for some constant c. For $n > 60$, the best results are obtained at high percentage trimming such as 70% or 80%. When $n = 60$, none of the results can be called the best no matter what M was used. For this case, we denote it by "N/A". The method WSIR has much better results. In general it needs a smaller amount of trimming to obtain the best results, and most of the best results are estimating $c\,(1,1,1,1)'$ for some constant c.

3) For the type 4 model, we only need to trim 10% or 20% to get the best results for the method SSIR, while 0% to 20% for the method WSIR. Similar to other models, the method WSIR has better results than the SSIR.

Overall, the type 1 $\boldsymbol{x}$ has the better results than the type 2 $\boldsymbol{x}$ and the method WSIR has the better results than SSIR. In addition the $\hat{\boldsymbol{\beta}}_{WSIR}$ are smaller than the $\hat{\boldsymbol{\beta}}_{SSIR}$.

n	M	$\overline{\boldsymbol{\beta}}_{SSIR3}$	M	$\overline{\boldsymbol{\beta}}_{SSIR4}$	M	$\overline{\boldsymbol{\beta}}_{WSIR3}$	M	$\overline{\boldsymbol{\beta}}_{WSIR4}$
60	0_1	.06,.01,.01,.02	0_1	.09,-.02,.001,.005	0_1	.002,.01,.002,.003	0_1	-.01, 0.01,.002,.01
500		.09,.08,.08,.09		.05,.02,.02,.03		.005,.001,.005,.005		-.01,-.01,-.01,-.01
1000		.12,.12,.12,.12		.05,.04,.04,.04		-.01,-.01,-.01,-.01		-.01,-.01,-.01,-.01
2000		.12,.12,.12,.12		.05,.05,.04,.05		-.02,-.02,-.02,-.02		.01,.01,.01,.01
60	10_1	.09,.04,.05,.05	10_1	.08,.02,.01,.01	10_1	-.02,-.02,-.02,-.02	10_1	.01,.01,.001,.01
500		.1,.09,.1,.09		.07,.06,.06,.06		-.003,-.01,-.01,-.005		.01,.01,.01,.01
1000		.1,.1,.1,.1		.09,.08,.08,.08		-.0004,-.002,-.002,.0001		.004,.003,.004,.005
2000		.09,.09,.09,.09		.06,.06,.06,.06		-.001,.001,-.0002,.0004		.02,.02,.02,.02
60	$0B_1$	.06,.01,.01,.02	$1B_1$	.08,.02,.01,.01	$1B_1$	-.02,-.02,-.02,-.02	$2B_1$	-.02,-.01,-.02,-.01
500	$2B_1$	.09,.09,.09,.09	$1B_1$	.07,.06,.06,.06	$2B_1$	.02,.02,.02,.02	$0B_1$	-.01,-.01,-.01,-.01
1000	$0B_1$	.12,.12,.12,.12	$3B_1$	.07,.07,.07,.07	$0B_1$	-.01,-.01,-.01,-.01	$0B_1$	-.01,-.01,-.01,-.01
2000	$0B_1$	.12,.12,.12,.12	$1B_1$	.06,.06,.06,.06	$0B_1$	-.02,-.02,-.02,-.02	$0B_1$	.01,.01,.01,.01
60	0_2	.22,-.14,-.04,-.03	0_2	.23,-.13,.01,.03	0_2	-.01,-.005,-.02,-.02	0_2	.005,.01,-.02,.005
500		.21,-.13,-.02,-.02		.18,-.19,-.02,-.02		.001,-.007,-.01,-.006		.02,.006,.004,.009
1000		.17,-.12,-.03,-.04		.19,-.15,-.03,-.04		-.008,.02,-.02,.01		.005,.02,.02,.02
2000		.15,-.09,-.03,-.05		.19,-.16,-.02,-.03		-.006,.01,.005,.005		.02,.02,.02,.02
60	10_2	.18,-.17,-.03,-.03	10_2	.18,-.06,.01,.01	10_2	.01,-.02,-.01,-.01	10_2	.02,.01,.01,.01
500		.17,-.11,.01,-.01		.09,-.02,.01,.02		.006,.01,.01,.003		.02,.02,.02,.02
1000		.15,-.07,-.003,.01		.08,.02,.03,.02		-.01,.01,.01,.01		-.001,-.006,-.005,.0004
2000		.1,-.04,-.003,-.01		.1,.07,.07,.07		.02,.02,.01,.01		-.01,-.01,-.01,-.01
60		N/A		N/A	$4B_2$	.03,.02,.002,.01	$1B_2$	.02,.01,.01,.01
500	$8B_2$	.08,.04,.04,.04	$2B_2$	.06,.04,.03,.04	$5B_2$	.03,.03,.02,.02	$1B_2$	.02,.02,.02,.02
1000	$7B_2$	.04,.01,.01,.02	$2B_2$	.11,.1,.09,.1	$5B_2$	-.01,-.02,-.02,-.01	$2B_2$	-.01,-.01,-.015,-.01
2000	$7B_2$	.08,.07,.06,.07	$2B_2$	.1,.09,.09,.09	$5B_2$	.007,.007,.007,.008	$0B_2$	.02,.02,.02,.02
60	0_3	.16,-.12,-.05,-.05	0_3	.21,-.16,-.03,-.04	0_3	.007,.01,.005,.003	0_3	.005,.02,.007,.01
500		.12,-.07,-.06,-.06		.17,-.11,-.06,-.04		-.02,-.02,-.02,-.02		-.005,-.008,-.01,-.005
1000		.12,-.07,-.06,-.06		.16,-.08,-.05,-.05		.004,.002,.004,.003		-.02,-.03,-.02,-.03
2000		.1,-.06,-.05,-.06		.16,-.06,-.02,-.004		.02,.02,.02,.02		.007,.007,.008,.006
60	10_3	.17,-.08,-.04,-.03	10_3	.16,-.05,-.04,-.02	10_3	.01,.008,.002,.004	10_3	.002,-.002,-.008,-.003
500		.1,-.03,-.02,-.01		.12,-.02,-.02,-.01		-.02,-.02,-.02,-.02		.007,.007,.009,.008
1000		.1,-.004,-.01,-.02		.09,-.02,-.02,-.02		.002,.0002,.0002,.001		.005,.003,.002,.002
2000		.08,-.01,-.01,-.01		.07,-.01,-.01,-.01		.01,.01,.01,.01		-.004,-.005,-.004,-.003
60		N/A		N/A	$0B_3$	.007,.01,.005,.003	$3B_3$	-.002,-.004,-.005,-.006
500		N/A	$7B_3$	.07,.03,.02,.03	$0B_3$	-.02,-.02,-.02,-.02	$3B_3$	-.015,-.017,-.014,-.018
1000	$8B_3$	.1,.05,.05,.05	$6B_3$	.07,.05,.05,.05	$0B_3$	.004,.002,.004,.003	$2B_3$	-.03,-.03,-.03,-.03
2000	$7B_3$	.09,.06,.05,.06	$4B_3$	.09,.07,.07,.07	$0B_3$	.02,.02,.02,.02	$2B_3$	-.01,-.01,-.01,-.01
60	0_4	.1,-.03,-.02,-.02	0_4	.2,-.05,.01,.01	0_4	.007,.008,.004,.002	0_4	.002,.02,.006,.003
500		.09,.05,.06,.06		.09,-.04,-.03,-.03		.007,.005,.005,.003		1.9e-3,-5.5e-5, -2.5e-3,1.3e-3
1000		.07,.04,.04,.04		.09,-.004,.01,-.003		.01,.01,.01,.01		.004,.001,.002,.005
2000		.1,.07,.08,.07		.08,.03,.03,.03		-.01,-.01,-.01,-.01		.02,.02,.02,.02
60	10_4	.09,-.03,-.01,-.02	10_4	.12,-.03,-.01,-.01	10_4	-.001,-.004,-.008,-.008	10_4	.02,.008,.008,.008
500		.06,.05,.04,.04		.07,.02,.02,.02		-.01,-.01,-.02,-.02		-.002,-.004,-.003,-.003
1000		.1,.09,.09,.09		.09,.05,.05,.05		.01,.01,.01,.01		.001,-.00004,-.0004,-.0008
2000		.08,.07,.07,.07		.08,.06,.06,.06		-.01,-.02,-.02,-.01		.003,.002,.003,.003
60		N/A		N/A	$0B_4$	.007,.008,.004,.002	$3B_4$	.01,.003,.005,.006
500	$1B_4$	.06,.05,.04,.04	$3B_4$	.07,.04,.04,.04	$0B_4$	.007,.005,.005,.003	$1B_4$	-.002,-.004,-.003,-.003
1000	$1B_4$	.1,.09,.09,.09	$2B_4$	.08,.06,.06,.06	$0B_4$	.01,.01,.01,.01	$2B_4$	-.02,-.02,-.02,-.02
2000	$1B_4$	.08,.07,.07,.07	$2B_4$	.08,.08,.08,.08	$0B_4$	-.01,-.01,-.01,-.01	$0B_4$	.02,.02,.02,.02
60	0_5	.25,-.27,-.01,-.03	0_5	.17,-.17,-.02,-.02	0_5	-.01,.009,.009,.006	0_5	.01,.02,-.005,.01
500		.21,-.15,-.02,-.03		.12,-.01,.003,-.004		.01,-.01,.02,-.004		-.006,-.01,-.01,-.01
1000		.16,-.1,-.06,-.05		.08,.01,.01,.01		-.004,.03,-.0002,.03		-.007,-.002,-.002,-.004
2000		.11,-.07,-.05,-.05		.08,.03,.03,.03		-.02,-.02,-.02,-.02		-.006,-.003,-.01,-.007
60	10_5	.25,-.25,.01,-.02	10_5	.13,-.06,-.01,-.02	10_5	-.006,-.03,-.04,-.01	10_5	.009,.005,.002,.02
500		.18,-.12,-.05,-.05		.1,.07,.07,.07		.006,.02,.008,.003		.01,.02,.02,.01
1000		.15,-.06,-.03,-.03		.08,.06,.06,.06		-.01,-.009,-.02,-.009		.003,-.004,.0006,.0003
2000		.09,-.04,-.04,-.03		.05,.04,.04,.04		-.0001,-.008,-.006,-.0004		.02,.02,.02,.02
60		N/A		N/A	$0B_5$	-.01,.009,.009,.006	$2B_5$	.009,.002,.002,.006

Table 4.12. Results Of SIR Estimators Based On Type 3&4 $\boldsymbol{x}$

n	M	$\overline{\boldsymbol{\beta}}_{SSIR3}$	M	$\overline{\boldsymbol{\beta}}_{SSIR4}$	M	$\overline{\boldsymbol{\beta}}_{WSIR3}$	M	$\overline{\boldsymbol{\beta}}_{WSIR4}$
500	$7B_5$	.06,.02,.02,.02	$2B_5$	.07,.05,.05,.05	$1B_5$	.006,.02,.008,.003	$2B_5$	.002,.001,.007,.002
1000	$7B_5$	.06,.04,.05,.04	$2B_5$	.08,.07,.07,.07	$1B_5$	-.01,-.009,-.02,-.009	$2B_5$	-.02,-.02,-.02,-.02
2000	$7B_5$	.09,.08,.08,.08	$2B_5$	.03,.03,.03,.03	$0B_5$	-.02,-.02,-.02,-.02	$1B_5$	.02,.02,.02,.02
60	0_6	.08,.03,.03,.03	0_6	.09,-.02,-.004,.001	0_6	.02,.02,.02,.02	0_6	-.008,.002,-.0004,-.002
500		.12,.11,.11,.11		.06,.04,.04,.04		.009,.008,.009,.007		-.0009,-.002,-.005,-.003
1000		.11,.1,.1,.1		.05,.04,.04,.04		.01,.01,.01,.01		-.02,-.02,-.02,-.02
2000		.07,.07,.07,.07		.03,.03,.03,.03		-.007,-.008,-.007,-.007		.01,.01,.01,.01
60	10_6	.06,.01,.01,.01	10_6	.04,-.01,-.004,-.01	10_6	.003,.003,-.005,-.003	10_6	.007,.0002,-.005,-.004
500		.05,.04,.05,.05		.05,.05,.05,.05		.002,-.0006,.0005,.002		-.01,-.01,-.01,-.01
1000		.12,.12,.12,.12		.05,.04,.04,.04		-.003,-.003,-.004,-.005		-.004,-.004,-.004,-.004
2000		.08,.08,.08,.08		.05,.05,.05,.05		-.01,-.01,-.01,-.009		.02,.02,.02,.02
60	$1B_6$	.06,.01,.01,.01	$2B_6$	.03,-.01,-.01,-.02	$0B_6$	.02,.02,.02,.02	$4B_6$	-.01,-.02,-.01,-.01
500	$1B_6$	.05,.04,.05,.05	$1B_6$	.05,.05,.05,.05	$0B_6$	.009,.008,.009,.007	$1B_6$	-.01,-.01,-.01,-.01
1000	$1B_6$	.12,.12,.12,.12	$2B_6$	.06,.06,.06,.06	$0B_6$	.01,.01,.01,.01	$0B_6$	-.02,-.02,-.02,-.02
2000	$0B_6$	.07,.07,.07,.07	$0B_6$	.03,.03,.03,.03	$0B_6$	-.007,-.008,-.007,-.007	$0B_6$	.01,.01,.01,.01
60	0_7	.23,-.24,-.001,.002	0_7	.22,-.17,-.003,-.02	0_7	-.004,-.004,-.03,-.02	0_7	-.01,.01,-.004,.004
500		.24,-.2,-.02,-.02		.25,-.21,-.03,-.02		.005,.008,.007,.01		.01,.001,.006,.01
1000		.2,-.12,-.05,-.04		.22,-.2,.02,-.02		-.02,.003,-.006,.007		.01,.006,.02,.01
2000		.16,-.1,-.04,-.03		.26,-.2,-.01,.01		.001,.02,.007,.01		-.006,.003,-.007,-.006
60	10_7	.24,-.25,-.01,-.02	10_7	.12,-.09,-.02,-.03	10_7	.008,-.02,-.002,-.02	10_7	.008,.004,.01,.02
500		.23,-.17,-.01,.004		.07,.04,.04,.04		-.0002,.007,.004,.002		-.002,.002,-.0009,-.007
1000		.22,-.12,.005,-.01		.08,.07,.07,.06		-.004,.006,-.004,-.004		-.007,-.006,-.007,-.008
2000		.17,-.06,-.01,-.01		.07,.06,.06,.06		.02,.01,.007,.006		-.02,-.02,-.02,-.02
60		N/A		N/A	$6B_7$	.03,.03,.01,.01	$2B_7$	.01,.009,.005,.009
500	$7B_7$	.08,.01,.02,.02	$1B_7$	.07,.04,.04,.04	$6B_7$	.02,.03,.03,.02	$3B_7$	-.02,-.02,-.02,-.03
1000	$7B_7$	.06,.03,.03,.02	$1B_7$	.08,.07,.07,.06	$5B_7$	-.02,-.02,-.02,-.02	$2B_7$	-.02,-.02,-.02,-.02
2000	$6B_7$	.09,.07,.07,.08	$1B_7$	.07,.06,.06,.06	$6B_7$	-.02,-.02,-.02,-.02	$1B_7$	-.02,-.02,-.02,-.02

Table 4.12. (Continued)

The results of Table 4.12 are based on the type 3 predictors $\boldsymbol{x}_i \sim 0.4N_4(\mathbf{0}, \boldsymbol{I}) + 0.6N_4(\mathbf{0}, 25\boldsymbol{I})$ and the type 4 predictors $\boldsymbol{x}_i \sim 0.9N_4(\mathbf{0}, \boldsymbol{I}) + 0.1N_4(\mathbf{0}, 25\boldsymbol{I})$.

For the type 3 $\boldsymbol{x}$:

1) If the method is SSIR and the model is MLR or type 6, when n is large the best results are obtained at 0% trimming and the $\hat{\boldsymbol{\beta}}_{SSIR} \approx c(1, 1, 1, 1)'$ for some c. If the method is WSIR, the results are even better. For the type 6 model, the best results are obtained at 0% and all the $\hat{\boldsymbol{\beta}}_{WSIR} \approx c(1, 1, 1, 1)'$. For the MLR model, 10% or 20% trimming improved the results when n is small and all the $\hat{\boldsymbol{\beta}}_{WSIR} \approx c(1, 1, 1, 1)'$.

2) If the method is SSIR, for models such as 2, 3, 5, and 7, we need to trim 70% or 80% of the data to get the best results but even the best results are not close to $c\,(1, 1, 1, 1)'$ unless n is very large. The results for the method WSIR are much better for these 4 models. The trimming percentages needed are smaller. For the

type 3 model, even 0% works well. The $\hat{\boldsymbol{\beta}}_{WSIR}$ are estimating $c(1,1,1,1)'$ for some constant c when n is large.

3) If the model is the type 4, the best results are obtained at 10% for SSIR and 0% for WSIR. Both the $\hat{\boldsymbol{\beta}}_{SSIR}$ and the $\hat{\boldsymbol{\beta}}_{WSIR}$ are estimating $c(1,1,1,1)'$ when n is large.

For the type 4 $\boldsymbol{x}$:

1) For the model 1, 4, or 6, the results are similar to those for the type 3 $\boldsymbol{x}$ except that we need to trim a larger amount of the data to obtain the best results for most cases.

2) For the model 2, 3, 5, or 7 and the SSIR method, the results are similar to those for the type 3 $\boldsymbol{x}$ except that we only need to trim a smaller amount of data to get the best results. If the method is WSIR, the results depend on the model. Larger amounts of trimming are needed for the models 3 and 5 compared to the type 3 $\boldsymbol{x}$, whereas smaller amounts of trimming are needed for the models 2 and 7.

n	M	$\overline{\boldsymbol{\beta}}_{SSIR5}$	M	$\overline{\boldsymbol{\beta}}_{SSIR6}$	M	$\overline{\boldsymbol{\beta}}_{WSIR5}$	M	$\overline{\boldsymbol{\beta}}_{WSIR6}$
60	0_1	.09,-.07,-.02,-.04	0_1	.08,.01,.01,.02	0_1	.03,.01,.02,.02	0_1	.0003,-.002,-.005,.01
500		.12,-.02,.01,.001		.05,-.01,-.002,-.01		.02,.01,.01,.01		-.02,-.01,-.01,-.01
1000		.15,.01,.03,.05		.09,.03,.04,.04		.03,.02,.03,.02		-.003,.003,-.0004,.001
2000		.09,-.06,-.04,-.04		.07,.01,.02,.02		.05,.04,.04,.04		-.005,-.01,-.01,-.01
60	10_1	-.0004,-.06,-.06,-.06	10_1	.08,.03,.03,.03	10_1	.03,.02,.02,.03	10_1	-.005,-.01,-.003,-.01
500		-.01,-.02,-.02,-.02		.1,.1,.1,.1		-.02,-.03,-.03,-.03		.02,.02,.02,.02
1000		-.01,-.01,-.01,-.01		.09,.09,.09,.09		.004,.005,.003,.004		-.02,-.01,-.02,-.02
2000		-.002,-.01,-.01,-.01		.1,.1,.1,.1		-.03,-.03,-.03,-.03		.02,.02,.02,.02
60		N/A	$1B_1$	.08,.03,.03,.03	$1B_1$	.03,.02,.02,.03	$1B_1$	-.005,-.01,-.003,-.01
500	$1B_1$	-.01,-.02,-.02,-.02	$1B_1$	.1,.1,.1,.1	$2B_1$	-.02,-.02,-.02,-.02	$1B_1$	.02,.02,.02,.02
1000	$1B_1$	-.01,-.01,-.01,-.01	$1B_1$	.09,.09,.09,.09	$2B_1$	-.02,-.02,-.02,-.02	$2B_1$	-.01,-.01,-.01,-.01
2000	$2B_1$	-.04,-.04,-.04,-.04	$1B_1$	.1,.1,.1,.1	$1B_1$	-.03,-.03,-.03,-.03	$1B_1$	.02,.02,.02,.02
60	0_2	.16,-.14,-.03,-.02	0_2	.22,-.14,-.002,-.0006	0_2	.02,.02,.02,.01	0_2	-.002,.004,.003,-.001
500		.22,-.12,-.0008,.007		.27,-.14,.007,-.01		.05,.05,.04,.05		.01,.01,.02,.01
1000		.21,-.11,-.006,.004		.25,-.18,-.02,-.003		.05,.05,.05,.05		-.002,-.01,-.01,-.01
2000		.19,-.14,-.02,-.02		.25,-.15,-.01,-.002		.04,.04,.03,.04		.01,.01,.001,.01
60	10_2	.08,-.06,-.05,-.04	10_2	.15,-.14,-.01,-.04	10_2	.02,.01,.02,.02	10_2	-.02,-.01,-.01,-.02
500		.08,.03,.03,.03		.09,-.01,-.001,.005		-.004,-.006,-.008,-.007		.01,.02,.02,.02
1000		.02,-.005,-.007,-.006		.09,.03,.03,.03		.03,.03,.03,.03		-.03,-.03,-.03,-.04
2000		.05,.03,.03,.02		.04,.01,.01,.01		.01,.01,.01,.01		-.01,-.005,-.01,-.01
60		N/A		N/A	$3B_2$	.01,.01,.01,.01	$1B_2$	-.02,-.01,-.01,-.02
500	$3B_2$	.03,.01,.01,.01	$4B_2$	.08,.06,.05,.05	$3B_2$	.006,.004,.005,.004	$2B_2$	.02,.02,.02,.02
1000	$3B_2$	.04,.03,.03,.03	$4B_2$	.07,.05,.05,.05	$0B_2$	.05,.05,.05,.05	$1B_2$	-.03,-.03,-.03,-.04
2000	$2B_2$	.05,.04,.04,.04	$2B_2$	.09,.08,.08,.08	$1B_2$	.01,.01,.01,.01	$1B_2$	-.01,-.005,-.01,-.01

Table 4.13. Results Of SIR Estimators Based On Type 5&6 $\boldsymbol{x}$

n	M	$\overline{\boldsymbol{\beta}}_{SSIR5}$	M	$\overline{\boldsymbol{\beta}}_{SSIR6}$	M	$\overline{\boldsymbol{\beta}}_{WSIR5}$	M	$\overline{\boldsymbol{\beta}}_{WSIR6}$
60	0_3	.25,-.22,.03,.04	0_3	.19,-.12,-.03,-.03	0_3	.01,.004,.01,.004	0_3	.003,.01,.001,.001
500		.27,-.22,.07,.06		.23,-.1,-.01,-.04		.03,.03,.03,.03		.03,.03,.03,.03
1000		.28,-.22,.03,.02		.2,-.13,-.03,-.01		.07,.06,.06,.06		-.02,-.02,-.03,-.02
2000		.25,-.24,.08,.05		.21,-.13,-.01,-.03		.02,.01,.01,.02		.01,.01,.01,.01
60	10_3	.19,-.18,-.04,-.04	10_3	.15,-.09,-.04,-.04	10_3	.03,.02,.02,.03	10_3	.0002,-.001,-.001,.002
500		.13,-.08,-.06,-.07		.1,-.02,-.02,-.02		-.0004,-.0004,.002,-.002		.01,.01,.01,.01
1000		.12,-.08,-.05,-.05		.1,.006,-.005,.005		-.02,-.02,-.02,-.02		-.01,-.01,-.02,-.02
2000		.1,-.05,-.03,-.04		.07,-.01,-.006,.002		.0004,-.001,.001,-.0003		.01,.01,.01,.01
60		N/A		N/A	$0B_3$	.01,.004,.01,.004	$2B_3$	-.02,-.02,-.02,-.02
500		N/A	$6B_3$	.09,.04,.04,.03	$0B_3$	.03,.03,.03,.03	$0B_3$	.03,.03,.03,.03
1000		N/A	$6B_3$	.05,.02,.02,.02	$1B_3$	-.02,-.02,-.02,-.02	$0B_3$	-.02,-.02,-.03,-.02
2000	$4B_3$	.05,.008,.003,.008	$4B_3$	.08,.06,.05,.06	$2B_3$	.006,.006,.006,.006	$0B_3$	.01,.01,.01,.01
60	0_4	.18,-.17,-.02,.004	0_4	.17,-.07,-.02,-.01	0_4	.01,.004,.01,.001	0_4	-.02,-.01,-.02,-.02
500		.23,-.16,.02,.005		.2,-.07,.005,-.01		.01,.02,.006,.006		.005,-.001,.004,.01
1000		.25,-.16,.003,.008		.2,-.06,-.02,-.003		.07,.06,.06,.05		-.01,-.01,-.01,-.01
2000		.23,-.17,.01,.005		.18,-.06,-.03,-.03		.02,.01,.01,.02		3.3e-4,-2.1e-3, -9.6e-5,2.4e-3
60	10_4	.11,-.1,-.06,-.04	10_4	.11,-.04,-.02,-.03	10_4	.02,.01,.02,.02	10_4	-.01,-.01,-.004,-.005
500		.06,-.01,-.03,-.02		.1,.05,.05,.05		.001,.001,.003,-.0005		-.003,-.001,-.003,-.0005
1000		.07,.01,.01,.01		.04,.02,.02,.01		-.005,-.005,-.007,-.007		-.02,-.02,-.02,-.02
2000		.02,-.02,-.02,-.02		.08,.07,.07,.07		-.007,-.008,-.005,-.007		-.01,-.01,-.01,-.01
60		N/A		N/A	$1B_4$	.02,.01,.02,.02	$2B_4$	-.02,-.02,-.02,-.02
500	$6B_4$	.07,.04,.03,.02	$3B_4$	.06,.04,.04,.03	$2B_4$	.01,.01,.01,.01	$2B_4$	.01,.02,.01,.02
1000	$7B_4$	.04,.02,.02,.02	$3B_4$	.08,.07,.07,.07	$1B_4$	-.02,-.02,-.02,-.02	$1B_4$	-.02,-.02,-.02,-.02
2000	$6B_4$	.06,.05,.05,.05	$1B_4$	.08,.07,.07,.07	$2B_4$	.01,.01,.01,.01	$1B_4$	-.01,-.01,-.01,-.01
60	0_5	.18,-.14,-.01,-.02	0_5	.19,-.14,-.03,.001	0_5	.05,-.05,-.01,-.01	0_5	-.01,.01,-.002,.01
500		.19,-.15,-.01,-.002		.07,-.04,-.03,-.04		.03,.01,.01,.003		-.02,-.01,.003,-.02
1000		.13,-.08,-.05,-.02		.09,.03,.02,.02		.06,.05,.05,.05		-.004,-.01,-.002,-.01
2000		.13,.04,.04,.04		.1,.06,.06,.05		.03,.01,.01,.02		.02,.02,.02,.02
60	10_5	.16,-.14,-.02,-.02	10_5	.16,-.09,-.03,-.03	10_5	.06,-.003,-.002,.01	10_5	-.02,-.02,-.002,-.005
500		.14,-.13,-.05,-.05		.06,.02,.02,.01		.03,.005,-.0002,-.002		.02,.02,.02,.02
1000		.15,-.07,-.01,-.002		.08,.06,.06,.05		.01,.004,.01,.01		-.01,-.02,-.02,-.02
2000		.11,-.05,-.02,-.0003		.06,.04,.04,.04		.02,.01,.01,.01		-.003,.002,-.002,-.0004
60		N/A		N/A		N/A	$2B_5$	-.03,-.02,-.01,-.01
500		N/A	$6B_5$	.07,.05,.05,.06	$3B_5$	-.01,-.02,-.01,-.02	$1B_5$	.02,.02,.02,.02
1000		N/A	$6B_5$	.07,.07,.06,.07	$1B_5$	.01,.004,.01,.01	$1B_5$	-.01,-.02,-.02,-.02
2000	$6B_5$	.05,.03,.03,.03	$5B_5$	.1,.1,.1,.1	$3B_5$	-.01,-.01,-.01,-.01	$0B_5$	.02,.02,.02,.02
60	0_6	.09,-.09,-.02,-.03	0_6	.07,.004,.01,.02	0_6	.02,.02,.02,.01	0_6	-.01,-.02,-.02,-.02
500		.17,-.01,.04,.03		.09,.02,.04,.03		.03,.03,.02,.02		-.002,-.01,.001,-.003
1000		.13,-.03,-.01,.01		.11,.04,.06,.06		.05,.05,.04,.04		-.02,-.01,-.02,-.02
2000		.13,-.04,-.01,-.02		.11,.06,.07,.07		.04,.04,.03,.04		.01,.01,.01,.01
60	10_6	.02,-.04,-.04,-.04	10_6	.03,.01,.007,.01	10_6	.03,.01,.02,.02	10_6	-.01,-.01,-.02,-.01
500		.0004,-.01,-.01,-.01		.05,.04,.04,.04		-.0002,-.002,-.003,-.003		.01,.02,.01,.01
1000		.004,-.0005,-.001,-.002		.02,.01,.01,.01		.01,.01,.01,.01		-.03,-.04,-.04,-.04
2000		.02,.02,.02,.02		.03,.03,.03,.03		-.01,-.01,-.01,-.01		-.01,-.01,-.01,-.01
60		N/A	$1B_6$	.03,.01,.007,.01	$0B_6$	.02,.02,.02,.01	$0B_6$	-.01,-.02,-.02,-.02
500	$3B_6$	-.02,-.03,-.03,-.03	$3B_6$	.05,.05,.05,.05	$2B_6$	-.01,-.01,-.01,-.01	$2B_6$	.02,.02,.02,.02
1000	$2B_6$	-.01,-.01,-.01,-.01	$2B_6$	.05,.05,.05,.05	$1B_6$	.01,.01,.01,.01	$2B_6$	-.02,-.02,-.02,-.02
2000	$1B_6$	.02,.02,.02,.02	$1B_6$	.03,.03,.03,.03	$1B_6$	-.01,-.01,-.01,-.01	$0B_6$	.01,.01,.01,.01
60	0_7	.04,-.02,-.03,-.02	0_7	.19,-.16,-.02,-.05	0_7	.01,.01,.01,-.01	0_7	-1.3e-3,7.7e-3, 5.5e-5,-4.9e-3
500		.09,.06,.06,.05		.19,-.11,-.01,-.02		.04,.04,.04,.04		-.01,-.01,.005,-.01
1000		3.9e-2,-4.1e-3, 2.5e-5,5.6e-3		.2,-.12,-.04,.01		.05,.05,.04,.04		-.03,-.04,-.03,-.03
2000		.11,.08,.08,.08		.18,-.15,-.03,-.06		.03,.02,.02,.02		.02,.02,.02,.02
60	10_7	.004,-.02,-.02,-.02	10_7	.17,-.1,-.02,-.01	10_7	.03,.02,.02,.02	10_7	-.03,-.02,-.01,-.03
500		.007,.005,.005,.003		.09,.04,.03,.03		.003,.002,.002,.002		.02,.02,.02,.02
1000		.03,.03,.03,.03		.07,.04,.03,.04		.01,.01,.01,.01		-.03,-.03,-.03,-.04
2000		.03,.03,.03,.03		.06,.05,.05,.05		.04,.03,.03,.03		-.002,.0003,.0001,.002

Table 4.13. (Continued)

n	M	$\overline{\boldsymbol{\beta}}_{SSIR5}$	M	$\overline{\boldsymbol{\beta}}_{SSIR6}$	M	$\overline{\boldsymbol{\beta}}_{WSIR5}$	M	$\overline{\boldsymbol{\beta}}_{WSIR6}$
60	$2B_7$	-.001,-.03,-.03,-.04		N/A	$1B_7$	.03,.02,.02,.02	$1B_7$	-.03,-.02,-.01,-.03
500	$1B_7$	.007,.005,.005,.003	$3B_7$	.07,.04,.05,.04	$0B_7$	.04,.04,.04,.04	$1B_7$	.02,.02,.02,.02
1000	$1B_7$	.03,.03,.03,.03	$3B_7$	.08,.07,.07,.07	$1B_7$	.01,.01,.01,.01	$0B_7$	-.03,-.04,-.03,-.03
2000	$1B_7$	.03,.03,.03,.03	$1B_7$	.06,.05,.05,.05	$2B_7$	-.02,-.02,-.02,-.02	$0B_7$	.02,.02,.02,.02

Table 4.13. (Continued)

The results of Table 4.13 are based on the type 5 predictors $\boldsymbol{x}_i \sim LN(\boldsymbol{0}, \boldsymbol{I})$ and the type 6 predictors $\boldsymbol{x}_i \sim MVT_3$. Here we will only discuss the results for the type 5 $\boldsymbol{x}$. We will compare the results for the type 6 predictors with the type 7 and 8 predictors later.

The type 5 $\boldsymbol{x}$:

If the method is SSIR, in general the results for the models 1, 2, 6, and 7 are better than those for the other models. For these models, the $\hat{\boldsymbol{\beta}}_{SSIR} \approx c(1,1,1,1)'$ for some constant c when n is large. For the models 3 and 5, none of the results could be called the best unless n is very large.

For the method WSIR, the results are much better than those for the method SSIR especially for the models 3 and 5. In general, 10% to 20% trimming work well and $\hat{\boldsymbol{\beta}}_{WSIR}$ are estimating $c(1,1,1,1)'$ for some constant c when $n > 60$.

n	M	$\overline{\boldsymbol{\beta}}_{SSIR7}$	M	$\overline{\boldsymbol{\beta}}_{SSIR7}$	M	$\overline{\boldsymbol{\beta}}_{WSIR8}$	M	$\overline{\boldsymbol{\beta}}_{WSIR8}$
60	0_1	.11,.06,.06,.06	0_1	.07,.02,.03,.02	0_1	.02,.01,.01,.02	0_1	.02,.03,.01,.01
500		.11,.08,.09,.08		.04,.03,.03,.03		.04,.04,.04,.03		.001,.001,.001,.001
1000		.09,.07,.08,.07		.07,.07,.07,.07		.0005,.002,.001,.0003		-.04,-.04,-.04,-.04
2000		.08,.08,.08,.07		.09,.09,.08,.08		-.01,-.01,-.01,-.01		-.01,-.01,-.01,-.02
60	10_1	.07,.01,.02,.02	10_1	.1,.04,.04,.04	10_1	.02,.02,.01,.02	10_1	.005,.003,-.01,.001
500		.06,.05,.06,.05		.09,.07,.08,.08		.04,.04,.04,.04		.01,.01,.01,.01
1000		.1,.1,.1,.1		.09,.08,.08,.08		.01,.01,.01,.01		-.04,-.03,-.03,-.04
2000		.07,.06,.07,.06		.07,.06,.07,.06		-.02,-.02,-.02,-.02		-.002,-.002,-.001,-.002
60	$0B_1$	.11,.06,.06,.06	$0B_1$	.07,.02,.03,.02	$0B_1$	.02,.01,.01,.02	$0B_1$	.02,.03,.01,.01
500	$2B_1$	.1,.09,.09,.09	$0B_1$	.04,.03,.03,.03	$1B_1$	.04,.04,.04,.04	$0B_1$	.001,.001,.001,.001
1000	$1B_1$	.1,.1,.1,.1	$0B_1$	.07,.07,.07,.07	$1B_1$	.01,.01,.01,.01	$0B_1$	-.04,-.04,-.04,-.04
2000	$2B_1$	.11,.11,.11,.11	$0B_1$	.09,.09,.08,.08	$0B_1$	-.01,-.01,-.01,-.01	$0B_1$	-.01,-.01,-.01,-.02
60	0_2	.19,-.15,-.04,-.05	0_2	.15,-.1,-.02,-.02	0_2	.03,.02,.01,.02	0_2	-.004,.02,-.02,.005
500		.18,-.17,-.02,-.05		.09,-.01,-.01,-.01		.01,.02,.01,.02		.04,.04,.04,.04
1000		.21,-.14,-.02,-.03		.14,.05,.06,.06		.02,.02,.02,.02		-.02,-.02,-.02,-.02
2000		.22,-.14,-.01,-.04		.12,.05,.05,.05		-.01,-.02,-.02,-.02		-.01,-.01,-.01,-.01
60	10_2	.16,-.07,-.01,-.02	10_2	.12,-.06,-.01,-.02	10_2	.01,.01,.01,.02	10_2	.003,-.01,-.004,-.002
500		.1,.04,.05,.04		.09,.06,.06,.06		.01,.01,.02,.01		.03,.03,.03,.03
1000		.08,.05,.05,.05		.05,.04,.03,.04		.02,.03,.02,.02		-.01,-.02,-.02,-.02
2000		.08,.06,.06,.06		.04,.03,.04,.04		-.02,-.01,-.02,-.02		-.01,-.01,-.01,-.01

Table 4.14. Results Of SIR Estimators Based On Type 7&8 $\boldsymbol{x}$

n	M	$\overline{\boldsymbol{\beta}}_{SSIR7}$	M	$\overline{\boldsymbol{\beta}}_{SSIR7}$	M	$\overline{\boldsymbol{\beta}}_{WSIR8}$	M	$\overline{\boldsymbol{\beta}}_{WSIR8}$
60		N/A		N/A	$1B_2$	.01,.01,.01,.02	$2B_2$	.01,.01,.02,.01
500	$3B_2$	.08,.06,.05,.06	$2B_2$	.06,.04,.04,.04	$2B_2$	.02,.02,.02,.02	$0B_2$	.04,.04,.04,.04
1000	$3B_2$	.1,.09,.08,.08	$3B_2$	.06,.05,.05,.05	$0B_2$	.02,.02,.02,.02	$0B_2$	-.02,-.02,-.02,-.02
2000	$2B_2$	.09,.08,.08,.08	$1B_2$	.04,.03,.04,.04	$2B_2$	-.01,-.01,-.01,-.01	$0B_2$	-.01,-.01,-.01,-.01
60	0_3	.2,-.1,-.04,-.04	0_3	.15,-.06,-.05,-.04	0_3	.01,.002,-.0003,.01	0_3	.01,.02,.01,.02
500		.17,-.11,-.06,-.06		.11,-.04,-.05,-.03		.03,.03,.03,.03		.03,.03,.03,.02
1000		.18,-.11,-.02,-.03		.11,-.04,-.03,-.03		-.001,.003,.003,.002		-.04,-.04,-.04,-.04
2000		.15,-.11,-.03,-.06		.11,-.05,-.06,-.05		-.002,-.004,-.003,-.004		.01,.01,.01,.01
60	10_3	.14,-.06,-.05,-.04	10_3	.13,-.04,-.03,-.03	10_3	-.003,-.01,-.01,-.004	10_3	.001,.01,-.001,.01
500		.08,-.02,-.01,-.02		.06,-.02,-.01,-.02		.01,.01,.01,.01		.03,.03,.03,.03
1000		.09,.01,.003,.004		.09,.04,.04,.04		.02,.02,.02,.02		-.03,-.03,-.03,-.03
2000		.1,.05,.05,.04		.09,.05,.05,.04		-.01,-.01,-.01,-.01		-.02,-.02,-.02,-.02
60		N/A		N/A	$2B_3$	.01,.01,.01,.01	$0B_3$	.01,.02,.01,.02
500	$5B_3$	.07,.02,.03,.03	$4B_3$	.08,.03,.03,.03	$0B_3$	.03,.03,.03,.03	$1B_3$	.03,.03,.03,.03
1000	$4B_3$	.07,.03,.04,.03	$5B_3$	.08,.06,.06,.06	$1B_3$	.02,.02,.02,.02	$0B_3$	-.04,-.04,-.04,-.04
2000	$3B_3$	.09,.06,.07,.06	$4B_3$	.08,.06,.06,.06	$1B_3$	-.01,-.01,-.01,-.01	$0B_3$	.01,.01,.01,.01
60	0_4	.15,-.04,-.02,-.02	0_4	.1,-.04,-.02,-.02	0_4	.03,.03,.03,.03	0_4	.02,.03,.02,.02
500		.1,-.07,-.03,-.06		.07,.01,.01,.01		.02,.02,.02,.02		.01,.01,.01,.004
1000		.12,-.05,-.02,-.03		.09,.05,.06,.05		.003,.005,.004,.004		-.03,-.03,-.03,-.03
2000		.12,-.03,-.02,-.01		.07,.04,.03,.03		-.004,-.008,-.005,-.007		-.01,-.01,-.01,-.01
60	10_4	.09,-.02,-.02,-.02	10_4	.11,-.01,-.005,-.005	10_4	.01,.01,.01,.01	10_4	.004,.01,.003,.01
500		.07,.04,.03,.04		.07,.04,.04,.04		.02,.02,.02,.02		.04,.04,.04,.04
1000		.06,.04,.04,.03		.09,.08,.08,.07		.02,.02,.03,.03		-.03,-.03,-.03,-.03
2000		.09,.08,.08,.08		.05,.04,.04,.04		.001,.001,.001,.001		-.01,-.01,-.01,-.01
60		N/A	$5B_4$	.11,.02,.01,.03	$0B_4$	.03,.03,.03,.03	$3B_4$	.01,.01,.01,.01
500	$2B_4$	.1,.07,.07,.07	$3B_4$	.08,.06,.06,.06	$0B_4$	.02,.02,.02,.02	$1B_4$	.04,.04,.04,.04
1000	$2B_4$	.09,.07,.08,.07	$1B_4$	.09,.08,.08,.07	$0B_4$	.003,.005,.004,.004	$0B_4$	-.03,-.03,-.03,-.03
2000	$1B_4$	.09,.08,.08,.08	$1B_4$	.05,.04,.04,.04	$1B_4$	.001,.001,.001,.001	$0B_4$	-.01,-.01,-.01,-.01
60	0_5	.14,-.13,-.04,-.06	0_5	.1,-.09,-.07,-.06	0_5	.02,.02,.01,.01	0_5	-.01,.01,-.001,.02
500		.06,-.002,-.005,-.0001		.07,.03,.04,.04		.02,.02,.02,.02		.01,.005,.01,.004
1000		.1,.07,.06,.06		.06,.04,.04,.04		-.001,-.001,-.001,-.005		-.02,-.02,-.02,-.02
2000		.1,.08,.08,.07		.06,.05,.06,.05		-.02,-.01,-.01,-.02		-.01,-.01,-.005,-.01
60	10_5	.09,-.09,-.05,-.06	10_5	.11,-.07,-.03,-.04	10_5	-.004,.002,.0003,.02	10_5	9.6e-5,1.6e-2, 1.8e-2,8.5e-3
500		.09,.06,.06,.06		.09,.06,.06,.06		.03,.04,.04,.03		.03,.03,.03,.03
1000		.09,.08,.08,.08		.08,.06,.06,.06		.02,.02,.02,.02		-.03, -.03,-.03,-.03
2000		.08,.07,.07,.07		.06,.06,.06,.06		-.01,-.01,-.01,-.01		-.01,-.01,-.01,-.01
60		N/A		N/A	$0B_5$	.02,.02,.01,.01	$5B_5$	.01,.01,.01,.01
500	$2B_5$	.05,.03,.03,.03	$3B_5$	.11,.09,.09,.09	$0B_5$	.02,.02,.02,.02	$1B_5$	.03,.03,.03,.03
1000	$1B_5$	.09,.08,.08,.08	$2B_5$	.07,.06,.06,.06	$1B_5$	.02,.02,.02,.02	$0B_5$	-.02,-.02,-.02,-.02
2000	$2B_5$	.08,.08,.08,.08	$1B_5$	.06,.06,.06,.06	$1B_5$	-.01,-.01,-.01,-.01	$1B_5$	-.01,-.01,-.01,-.01
60	0_6	.07,.03,.02,.02	0_6	.05,.02,.03,.03	0_6	.02,.01,.01,.01	0_6	-.01,.002,-.01,-.002
500		.05,.04,.04,.04		.05,.05,.05,.05		.02,.02,.02,.01		.002,.001,.0003,.002
1000		.05,.03,.03,.03		.04,.04,.04,.04		.02,.02,.01,.02		-.03,-.03,-.03,-.03
2000		.06,.05,.06,.05		.03,.02,.02,.02		-.01,-.01,-.01,-.01		-.01,-.01,-.01,-.01
60	10_6	.05,.02,.03,.02	10_6	.05,.03,.02,.02	10_6	5.1e-3,-9.6e-5,9.7e-4,2e-3	10_6	-.01,.0002,-.002,-.01
500		.05,.05,.05,.05		.04,.03,.03,.03		.03,.03,.03,.03		.01,.01,.01,.01
1000		.04,.04,.04,.04		.004,.003,.002,.002		.01,.01,.01,.01		-.04,-.04,-.04,-.04
2000		.08,.08,.08,.08		.04,.04,.04,.04		-.001,.0004,.0002,.0004		.001,.002,.001,.001
60	$1B_6$	.05,.02,.03,.02	$0B_6$	.05,.02,.03,.03	$0B_6$	.02,.01,.01,.01	$5B_6$	.01,.02,.01,.02
500	$1B_6$	.05,.05,.05,.05	$0B_6$	.05,.05,.05,.05	$1B_6$	.03,.03,.03,.03	$1B_6$	.01,.01,.01,.01
1000	$1B_6$	.04,.04,.04,.04	$0B_6$	.04,.04,.04,.04	$1B_6$	.01,.01,.01,.01	$0B_6$	-.03,-.03,-.03,-.03
2000	$1B_6$	.08,.08,.08,.08	$1B_6$	.04,.04,.04,.04	$0B_6$	-.01,-.01,-.01,-.01	$0B_6$	-.01,-.01,-.01,-.01
60	0_7	.16,-.12,-.04,-.03	0_7	.16,-.06,-.02,-.02	0_7	.01,.001,.01,.003	0_7	-.0001,.02,-.01,.01
500		.09,-.002,.0008,-.003		.06,.03,.03,.03		.001,.01,-.002,.01		.01,.01,.01,.01
1000		.1,.02,.04,.04		.08,.07,.07,.07		.002,.01,.01,.01		-.01,-.01,-.01,-.01
2000		.08,.03,.04,.03		.1,.09,.09,.08		.001,.003,.005,.001		-.02,-.02,-.02,-.02
60	10_7	.12,-.08,-.04,-.04	10_7	.12,-.08,-.05,-.03	10_7	.004,.0002,.01,.03	10_7	-.02,-.02,-.01,-.003

Table 4.14. (Continued)

n	M	$\overline{\boldsymbol{\beta}}_{SSIR7}$	M	$\overline{\boldsymbol{\beta}}_{SSIR7}$	M	$\overline{\boldsymbol{\beta}}_{WSIR8}$	M	$\overline{\boldsymbol{\beta}}_{WSIR8}$
500		.06,.03,.03,.03		.09,.05,.05,.06		.01,.02,.02,.02		.03,.02,.02,.02
1000		.07,.06,.06,.06		.09,.07,.07,.07		.003,.01,.003,.003		-.01,-.02,-.02,-.02
2000		.07,.06,.06,.06		.06,.05,.05,.05		-.004,-.002,-.002,-.0003		.003,.01,.01,.01
60		N/A		N/A	$5B_7$	.01,.02,.01,.02	$4B_7$	.01,.02,.02,.02
500	$3B_7$	.04,.02,.02,.02	$2B_7$	.08,.05,.05,.05	$1B_7$	.01,.02,.02,.02	$0B_7$	.01,.01,.01,.01
1000	$1B_7$	.07,.06,.06,.06	$2B_7$	.09,.08,.08,.08	$2B_7$	.01,.01,.01,.01	$0B_7$	-.01,-.01,-.01,-.01
2000	$1B_7$	.07,.06,.06,.06	$1B_7$	.06,.05,.05,.05	$3B_7$	-.02,-.02,-.02,-.02	$0B_7$	-.02,-.02,-.02,-.02

Table 4.14. (Continued)

The results of Table 4.14 are based on the type 7 predictors $\boldsymbol{x}_i \sim MVT_5$ and the type 8 predictors $\boldsymbol{x}_i \sim MVT_{19}$. We will also discuss the results for the type 6 $\boldsymbol{x}$ here.

In general, the results for the type 8 $\boldsymbol{x}$ are better than those for the type 7 $\boldsymbol{x}$ and the results for the type 7 $\boldsymbol{x}$ are better than those for the type 6 $\boldsymbol{x}$ for most cases. We made this conclusion based on the trimming percentage for the best results. Similarly, the results for the method WSIR are better than those for the method SSIR.

If we compare the results in this section with the corresponding results in Section 4.3.2, we will find that the OLS method has better results than the SIR method in general.

4.3.4 $2D$ Simulation

In this section, we will run the simulation on a $2D$ model. As we said before, most regression problems have $0D, 1D$, or $2D$ structures. A general $2D$ model is

$$y|\boldsymbol{x} = g(\boldsymbol{x}) + \sigma(\boldsymbol{x})e, \tag{4.7}$$

where $y \perp\!\!\!\perp \boldsymbol{x} | \boldsymbol{\alpha}_1'\boldsymbol{x}, \boldsymbol{\alpha}_2'\boldsymbol{x}$.

Let the OLS population coefficient from regressing y on $\boldsymbol{x}$ be $\boldsymbol{\beta}_{ols} = \boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}\text{Cov}(\boldsymbol{x}, g(\boldsymbol{x}))$, then the previous $2D$ model (4.7) is equivalent to

$$y|\boldsymbol{x} = \beta_0 + \boldsymbol{\beta}_{ols}'\boldsymbol{x} + g_r(\boldsymbol{x}) + \sigma(\boldsymbol{x})e \tag{4.8}$$

where β_0 is the intercept and the residual regression function $g_r(\boldsymbol{x}) = g(\boldsymbol{x}) - \beta_0 - \boldsymbol{\beta}'_{ols}\boldsymbol{x}$ [6].

Let $r = y - \hat{\beta}_0 - \hat{\boldsymbol{\beta}}'_{ols}\boldsymbol{x}$ be the OLS residual, then a plot r versus $\boldsymbol{x}$ could help to explore the structure of $g_r(\boldsymbol{x})$ [6].

Let us consider a special case of model (4.7) as:

$$y|\boldsymbol{x} = \boldsymbol{\alpha}'_1\boldsymbol{x} + g_2(\boldsymbol{\alpha}'_2\boldsymbol{x}) + \sigma e, \tag{4.9}$$

where $\boldsymbol{\alpha}_1$ is independent of $\boldsymbol{\alpha}_2$ and $\boldsymbol{\alpha}'_2\boldsymbol{x}$ is a scalar.

Following Cook (1998, p. 56 - 57), the OLS population coefficient is

$$\boldsymbol{\beta}_{ols} = \boldsymbol{\alpha}_1 + \boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}\text{Cov}(\boldsymbol{x}, g_2(\boldsymbol{\alpha}'_2\boldsymbol{x})), \tag{4.10}$$

and the residual regression function is

$$g_r(\boldsymbol{x}) = g_2(\boldsymbol{\alpha}'_2\boldsymbol{x}) - \beta_0 - [\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}\text{Cov}(\boldsymbol{x}, g_2(\boldsymbol{\alpha}'_2\boldsymbol{x}))]'\boldsymbol{x}. \tag{4.11}$$

If $\boldsymbol{\Sigma}_{\boldsymbol{x}}^{-1}\text{Cov}(\boldsymbol{x}, g_2(\boldsymbol{\alpha}'_2\boldsymbol{x}))$ is a linear function of $\boldsymbol{\alpha}_2$, then the residual plot only has the $1D$ structure. Since $\text{Cov}(\boldsymbol{x}, g_2(\boldsymbol{\alpha}'_2\boldsymbol{x})) = E[g_2(\boldsymbol{\alpha}'_2\boldsymbol{x})E(\boldsymbol{x}|\boldsymbol{\alpha}'_2\boldsymbol{x})]$, that means the residual plot will have the $1D$ structure if $E(\boldsymbol{x}|\boldsymbol{\alpha}'_2\boldsymbol{x})$ is a linear combination of $\boldsymbol{\alpha}'_2\boldsymbol{x}$. As we stated in Section 2.2, if $\boldsymbol{x}$ has an elliptically contoured distribution then $E(\boldsymbol{x}|\boldsymbol{b}'\boldsymbol{x})$ is a linear function of $\boldsymbol{b}'\boldsymbol{x}$ for all $\boldsymbol{b}$. Therefore, if $\boldsymbol{x}$ has an elliptically contoured distribution, we can estimate the coefficient $\boldsymbol{\alpha}_2$ by regressing r on $\boldsymbol{x}$.

The model we used in this section is a type of (4.9). It is

$$y = \boldsymbol{\beta}'_1\boldsymbol{x} + (\boldsymbol{\beta}_2\boldsymbol{x}')^3 + e, \tag{4.12}$$

where $\boldsymbol{\beta}_1 = (1, 2, 3)'$, $\boldsymbol{\beta}_2 = (1, 1, 1)'$, and the error $e \sim N(0, \sigma^2)$ is independent of $\boldsymbol{x}_i$.

We will run two DR methods, OLS and SIR, on the previous model in the following pages.

The OLS Estimator

Next we will run two kinds of OLS regression on model (4.12). The first one is to regress y on $\boldsymbol{x}$, and the second one is to regress the residual of the first regression r on $\boldsymbol{x}$. Let $\hat{\boldsymbol{\beta}}_y$ be the coefficients from the regression of y on $\boldsymbol{x}$, and $\hat{\boldsymbol{\beta}}_r$ be the coefficients from the regression of r on $\boldsymbol{x}$. As we stated in Section 4.3.4, we hope that $\hat{\boldsymbol{\beta}}_r$ will estimate $c\boldsymbol{\beta}_2$ for some constant c. In this section, we will run the regression for 4 different types of predictors as introduced in Section 4.2 and 4 different sample sizes n. For each case, we let the number of runs $nruns = 1000$. For each run, we will use the MBA algorithm to trim $M = 0\%, \cdots, 90\%$ data. We will keep a record of $\hat{\boldsymbol{\beta}}_y$ based on the $0\%, 10\%$, and the adaptive trimming and $\hat{\boldsymbol{\beta}}_r$ based on the 10% and the adaptive trimming. The definition of the adaptive trimming is the same as the one defined in Section 4.4.2. If we trim 0% of the data, the obtained $\hat{\boldsymbol{\beta}}_r = \boldsymbol{0}$ theoretically. That is why we do not keep a record of the coefficients based on the 0% trimming. Then we will calculate the sample means of all the coefficients and show the results in Table 4.15. Column "Type" denotes the type of predictor distribution, column "n" means the sample size, and column "$\overline{\boldsymbol{\beta}}_{y_0}$" means the sample mean of $\hat{\boldsymbol{\beta}}_y$ based on the 0% trimming, etc.

Type	n	$\overline{\boldsymbol{\beta}}_{y_0}$	$\overline{\boldsymbol{\beta}}_{y_{10}}$	$\overline{\boldsymbol{\beta}}_{y_{adap}}$	$\overline{\boldsymbol{\beta}}_{r_{10}}$	$\overline{\boldsymbol{\beta}}_{r_{adap}}$
1	60	9.56,10.65,11.56	7.68,8.68,9.67	9.56,10.64,11.54	-1.89,-1.97,-1.9	-3.15,-3.26,-3.16
	500	9.92,10.92,11.98	7.29,8.27,9.29	9.92,10.92,11.98	-2.63,-2.65,-2.69	-2.54,-2.56,-2.6
	1000	9.95,10.94,11.97	7.22,8.21,9.23	9.95,10.94,11.97	-2.73,-2.73,-2.74	-2.62,-2.62,-2.64
	2000	10.01,11.02,11.99	7.2,8.2,9.19	10.01,11.02,11.99	-2.8,-2.82,-2.8	-2.67,-2.69,-2.67
2	60	2.44,2.7,2.95	2.39,2.65,2.97	2.55,2.79,3.05	-.04,-.05,0.02	3.54,3.87,4.21
	500	2.44,2.7,2.94	2.13,2.43,2.73	2.44,2.7,2.94	-.31,-.27,-.21	1.36,1.57,1.83
	1000	2.45,2.69,2.95	2.13,2.41,2.71	2.45,2.69,2.95	-.32,-.27,-.24	.79,.96,1.13
	2000	2.46,2.71,2.95	2.12,2.42,2.71	2.46,2.71,2.95	-.33,-.28,-.24	.39,.54,.68
5	60	4.42,4.91,5.33	5.41,6.19,6.72	4.92,5.41,5.88	.98,1.28,1.39	3.39,3.80,4.17
	500	4.05,4.42,4.8	5.35,6.02,6.72	4.14,4.52,4.91	1.3,1.6,1.92	1.93,2.27,2.64
	1000	3.97,4.36,4.68	5.37,6.03,6.7	4.01,4.42,4.73	1.4,1.67,2.02	1.68, 1.97,2.35
	2000	3.97,4.33,4.68	5.36,6.03,6.7	3.98,4.33,4.69	1.39,1.71,2.02	1.46,1.77,2.08

Table 4.15. Results Of 2D OLS Estimator

Type	n	$\overline{\boldsymbol{\beta}}_{y_0}$	$\overline{\boldsymbol{\beta}}_{y_{10}}$	$\overline{\boldsymbol{\beta}}_{y_{adap}}$	$\overline{\boldsymbol{\beta}}_{r_{10}}$	$\overline{\boldsymbol{\beta}}_{r_{adap}}$
8	60	9.13,10.21,11.07	7.49,8.61,9.53	9.1,10.2,11.04	-1.63,-1.59,-1.54	-3.2,-3.3,-3.19
	500	9.33,10.26,11.17	7.43,8.4,9.37	9.33,10.26,11.17	-1.89,-1.86,-1.8	-1.93,-1.89,-1.83
	1000	9.3,10.24,11.19	7.36,8.34,9.33	9.3,10.24,11.19	-1.93,-1.9,-1.86	-1.85,-1.82,-1.79
	2000	9.31,10.27,11.19	7.38,8.36,9.33	9.31,10.27,11.19	-1.93,-1.91,-1.86	-1.84,-1.82,-1.77

Table 4.15. (Continued)

The results of Table 4.15 are based on 4 different types of predictors.

1) For the type 1 predictor $\boldsymbol{x}_i \sim N_3(\boldsymbol{0}, \boldsymbol{I})$, both the 10% and the adaptive trimming coefficients $\hat{\boldsymbol{\beta}}_r$ from the regression of r on $\boldsymbol{x}$ are estimating $c(1,1,1)' = c\boldsymbol{\beta}_2$ for $n > 60$. The coefficients $\hat{\boldsymbol{\beta}}_y$ from the regression of y on $\boldsymbol{x}$ are not estimating $c\boldsymbol{\beta}_1$ or $c\boldsymbol{\beta}_2$ for any constant c for all trimmings. For all $n > 60$, the adaptive trimming's results are equal to those of the 0% trimming.

2) For the type 8 predictor $\boldsymbol{x}_i \sim MVT_{19}$, all $\hat{\boldsymbol{\beta}}_r$ are estimating $c\boldsymbol{\beta}_2$ for large n for some constant c, but all $\hat{\boldsymbol{\beta}}_y$ are not estimating either $c\boldsymbol{\beta}_1$ or $c\boldsymbol{\beta}_2$. However all $\hat{\boldsymbol{\beta}}_{y_{adap}}$ are equal to $\hat{\boldsymbol{\beta}}_{y_0}$ for large n.

3) The results based on the type 2 and 5 predictors are not good. The $\hat{\boldsymbol{\beta}}_r$ are not close to $c\boldsymbol{\beta}_2$.

The SIR Estimator

Now we will run two types of SIR, WSIR and SSIR, on the model (4.12). For both methods, we let the number of runs $nruns = 1000$ and the number of slices $h = 4$. Similarly, for each run we use the MBA algorithm to trim $M = 0\%, \cdots, 90\%$ data. For each case, we keep a record of the first two eigenvalues and coefficients based on the $0\%, 10\%$, and the adaptive trimming. Then we will calculate all the sample means and show all the results in Table 4.16 for the method WSIR and Table 4.17 for the method SSIR. Let $\hat{\lambda}$ denote the two eigenvalues. Column "Type" denotes the type of predictors, column "n" denotes the sample size, column "$\overline{\lambda}_0$" denotes the sample mean of the first two eigenvalues based on the 0% trimming, and column "$\overline{\boldsymbol{\beta}}_0$" denotes the sample mean of the coefficients $\hat{\boldsymbol{\beta}}$ corresponding to the first eigenvalue based on the 0% trimming, etc. As we noticed in the following tables, the second eigenvalue is much smaller than the first eigenvalue. This means both the SIR methods incorrectly suggest $1D$ structure. That is the reason why we only keep one direction of the coefficients.

Type	n	$\overline{\lambda}_0$	$\overline{\boldsymbol{\beta}}_0$	$\overline{\lambda}_{10}$	$\overline{\boldsymbol{\beta}}_{10}$	$\overline{\boldsymbol{\beta}}_{adap}$
1	60	.85,.08	.02,.02,.03	.85,.08	.01,.01,.01	.02,.02,.02
	500	.86,.03	-.01,-.02,-.02	.84,.03	-.01,-.01,-.01	-.01,-.02,-.02
	1000	.86,.03	-.004,-.006,-.007	.84,.03	.003,.004,.004	-.004,-.006,-.007
	2000	.86,.02	-.02,-.02,-.03	.84,.02	.005,.007,.009	-.02,-.02,-.03
2	60	.57,.08	-.03,-.03,-.04	.56,.07	.001,-.007,-.006	-.02,-.03,-.03
	500	.55,.02	.02,.03,.04	.54,.02	.02,.02,.03	.02,.03,.04
	1000	.55,.02	.0004,.0002,-.0004	.54,.02	-.004,-.004,-.006	.0004, .0002,-.0004
	2000	.55,.02	.004,.006,.007	.54,.02	-.01,-.01,-.01	.004,.006,.007
5	60	.71,.08	-.08,-.1,-.13	.61,.06	-.04,-.07,-.09	-.07,-.09,-.1
	500	.69,.02	-.08,-.09,-.12	.52,.01	-.08,-.11,-.14	-.08,-.1,-.12
	1000	.69,.02	-.05,-.07,-.08	.51,.01	-.07,-.1,-.13	-.05,-.07,-.08
	2000	.69,.02	-.05,-.08,-.09	.5,.01	-.06,-.08,-.11	-.05,-.07,-.09
8	60	.84,.08	-.001,.005,.003	.82,.08	.004,.007,.009	-.0001,.007,.006
	500	.84,.03	-.01,-.01,-.02	.82,.03	-.002,-.004,-.004	-.01,-.01,-.02
	1000	.84,.03	.0006,.001,.002	.82,.02	.03,.04,.04	.001,.001,.002
	2000	.84,.02	.01,.01,.01	.82,.02	.01,.02,.02	.01,.01,.01

Table 4.16. Results Of 2D WSIR Estimator

The results of Table 4.16 are based on the method WSIR.

1) For the type 1 $\boldsymbol{x}$, $\hat{\boldsymbol{\beta}}$ which are based on the 10% trimming are estimating $c\boldsymbol{\beta}_2$ for some constant c when n is less than 2000. However, the adaptive trimming's results are equal to those based on the 0% trimming for large n.

2) For the type 2 and 8 predictors, the adaptive trimming's results are also equal to those based on the 0% trimming when n is large. However, most of the $\hat{\boldsymbol{\beta}}$ are not estimating $c\boldsymbol{\beta}_2$ for some constant c except the following two cases. When $n = 2000$, $\hat{\boldsymbol{\beta}}_{10} = c(1,1,1)'$ for the type 2 $\boldsymbol{x}$, and $\hat{\boldsymbol{\beta}}_0 = c\boldsymbol{\beta}_2$ for the type 8 $\boldsymbol{x}$.

3) The results for the type 5 predictor are the worst compared to the other types of predictors. None of the $\hat{\boldsymbol{\beta}}$ is close to $c\boldsymbol{\beta}_2$ for some constant c.

Type	n	$\overline{\lambda}_0$	$\overline{\boldsymbol{\beta}}_0$	$\overline{\lambda}_{10}$	$\overline{\boldsymbol{\beta}}_{10}$	$\overline{\boldsymbol{\beta}}_{adap}$
1	60	.68,.06	-.11,-.17,-.19	.65,.06	-.12,-.18,-.2	-.11,-.17,-.18
	500	.59,.01	-.1,-.12,-.13	.44,.01	-.15,-.18,-.22	-.1,-.12,-.13
	1000	.6,.01	-.12,-.14,-.15	.39,.004	-.19,-.23,-.26	-.12,-.14,-.15
	2000	.61,.005	-.11,-.13,-.14	.35,.002	-.17,-.21,-.25	-.11,-.13,-.14
2	60	.38,.03	-.06,-.14,-.15	.47,.05	-.06,-.14,-.17	-.06,-.12,-.13
	500	.19,.002	-.09,-.12,-.13	.29,.01	-.09,-.12,-.14	-.09,-.11,-.12
	1000	.15,.001	-.08,-.1,-.11	.26,.004	-.12,-.14,-.16	-.08,-.1,-.11
	2000	.14,.0004	-.09,-.1,-.11	.22,.002	-.11,-.14,-.16	-.09,-.1,-.11
5	60	.52,.04	-.003,-.15,-.13	.58,.09	-.07,-.17,-.19	-.06,-.11,-.13
	500	.37,.003	-.05,-.14,-.14	.39,.02	-.14,-.17,-.2	-.08,-.13,-.14
	1000	.35,.001	-.04,-.1,-.12	.34,.02	-.15,-.18,-.21	-.07,-.1,-.12
	2000	.31,.001	-.07,-.11,-.13	.3,.01	-.17,-.2,-.23	-.08,-.11,-.12
8	60	.65,.06	-.12,-.18,-.19	.63,.06	-.14,-.2,-.23	-.11,-.16,-.18
	500	.48,.01	-.11,-.13,-.14	.42,.01	-.12,-.15,-.17	-.11,-.13,-.14
	1000	.45,.003	-.13,-.15,-.16	.37,.004	-.12,-.14,-.16	-.13,-.15,-.16
	2000	.42,.002	-.07,-.09,-.09	.33,.002	-.13,-.16,-.18	-.07,-.09,-.09

Table 4.17. Results Of 2D SSIR Estimator

The results in Table 4.17 are based on the method SSIR. In general, the results based on the method SSIR are worse than those based on the method WSIR. Most of the $\hat{\boldsymbol{\beta}}$ are not close to $c\boldsymbol{\beta}_2$ for some constant c.

4.4 TESTING PROBLEM

4.4.1 Three Testing Problems

In this section, we will discuss the three cases of the testing problems. For each case, we have a different F test statistic, but all the χ^2 test statistics take the same form. Therefore, for each case we will explain in detail the F test statistic.

1. Testing $H_0 : \boldsymbol{\beta} = \mathbf{0}$ versus $H_1 : \boldsymbol{\beta} \neq \mathbf{0}$

This testing problem is a special case of (2.45) with $\boldsymbol{A}$ being a $(p-1)\times(p-1)$ identity matrix. For the OLS estimator, if the regression was made based on the original dataset $(\boldsymbol{x}, y)$, then the F test statistic in (2.56) could be simplified as

$$F = \frac{MSR}{MSE} = \frac{\sum\limits_{i=1}^{n}(\hat{y}_i - \overline{y})^2/(p-1)}{\sum\limits_{i=1}^{n} r_i^2/(n-p)}; \tag{4.13}$$

where $r_i = y_i - \hat{y}_i$. If the regression was made based on the trimmed dataset $(\boldsymbol{x}_M, y_M)$, then the corresponding F test statistic is

$$F_M = \frac{\sum_{i=1}^{n_M} (\hat{y}_{M,i} - \overline{y}_M)^2/(p-1)}{\sum_{i=1}^{n_M} r_{M,i}^2/(n_M - p)}, \tag{4.14}$$

where $r_{M,i} = y_{M,i} - \hat{y}_{M,i}$.

If H_0 is true and the sample size large, then $F \approx F_{p-1,n-p}$, $F_M \approx F_{p-1,n_M-p}$, and $W \approx \chi^2_{p-1}$. As a result, H_0 will be rejected if $F > F_{p-1,n-p}(1-\alpha)$, $F_M > F_{p-1,n_M-p}(1-\alpha)$, or $W > \chi^2_{p-1}(1-\alpha)$, where α denotes the type I error.

2. Testing $H_0 : \beta_i = 0$ versus $H_1 : \beta_i \neq 0$

This testing problem is another special case of (2.45) with $\boldsymbol{A}$ being a $1 \times (p-1)$ vector with all entries zero except the ith one. For the OLS estimator, the Wald t test is equivalent to the F test since $t^2_{n-p} = F_{1,n-p}$. The t test statistic is

$$t = \frac{\hat{\beta}_{ols,i}}{se(\hat{\beta}_{ols,i})}. \tag{4.15}$$

The t_M test statistic for the trimmed dataset could be defined in the same way.

If H_0 holds, $t \approx t_{n-p}$ and $t_M \approx t_{n_m-p}$, therefore H_0 will be rejected if $|t| > t_{1-\alpha/2,n-p}$ or $|t_M| > t_{1-\alpha/2,n_M-p}$, where α denotes the type I error. When $n-p$ or $n_M - p$ is large, the Wald test converges to the Z test.

3. Testing $H_0 : \boldsymbol{\beta}_O = \boldsymbol{0}$ versus $H_1 : \boldsymbol{\beta}_O \neq \boldsymbol{0}$

Let $\boldsymbol{\beta} = (\boldsymbol{\beta}'_R, \boldsymbol{\beta}'_O)'$ and $\boldsymbol{x} = (\boldsymbol{x}'_R, \boldsymbol{x}'_O)'$, where $\boldsymbol{\beta}_O$ is a $j \times 1$ vector and $\boldsymbol{x}_O$ is a $n \times j$ matrix. Then let the full model be

$$SP = \alpha + \boldsymbol{x}'\boldsymbol{\beta} \tag{4.16}$$

and the reduced model be

$$SP = \alpha_R + \boldsymbol{x}'_R\boldsymbol{\beta}_R. \tag{4.17}$$

This testing problem is equivalent to testing the reduced model. It is also a special case of (2.45) with $\boldsymbol{A} = (\boldsymbol{0}\ \ \boldsymbol{I}_j)$ where $\boldsymbol{0}$ is a $j \times (p-1-j)$ matrix of zeroes.

For the OLS estimator, the F test statistic is

$$F = \frac{(SSE(R) - SSE(F))/(df_R - df_F)}{MSE(F)}, \tag{4.18}$$

where the subscript R denotes the reduced model and F denotes the full model, $df_R = n - j - p, df_F = n - p, SSE(R) = \sum_{i=1}^{n} r_{i,R}^2, r_{i,R} = y_i - \hat{y}_{R,i}, SSE(F) = \sum_{i=1}^{n} r_i^2$, and $MSE(F) = \frac{SSE(F)}{n-p}$. We can define the test statistic F_M for the trimmed dataset similarly. If H_0 holds, $F \approx F(j, n-p)$ and $F_M \approx F(j, n_M - p)$ and we will reject H_0 for large F values.

4.4.2 Testing Results

Prior to presenting the testing results, we will introduce the simulation process. First of all, for each run we generate n i.i.d. $(p-1)\times 1$ random vectors $\boldsymbol{x}_i$ using one of the distributions described in Section 4.2 and an $n \times 1$ vector of the random errors with the standard normal distribution. The dependent variable y was obtained according to one of the models described in Section 4.2. For the second and the third testing problem, we will use the MBA algorithm to drop $M\%$ of the cases, where $M = 0\%, 10\%, 20\%, \cdots, 90\%$.

Second of all, both OLS and SIR were performed on the data $(\boldsymbol{x}_M, y_M)$ remaining after trimming for each M. The corresponding test statistics which were introduced in Section 4.4.1 were also calculated for each M. At the same time we calculated the so called adaptive trimming percentage M_{adp} based on the following algorithm.

1. Let $\hat{\boldsymbol{\beta}}_M$ denote the regression coefficients based on the dataset $(\boldsymbol{x}_M, y_M)$, $\boldsymbol{\beta}$ denote the true coefficients of the model, and "Corr" represent the correlation. For each run, the expression $|\text{Corr}(\boldsymbol{x}_i'\boldsymbol{\beta}, \boldsymbol{x}_i'\hat{\boldsymbol{\beta}}_M)|$ where $M = 0\%, 10\%, 20\%, \cdots, 90\%$ was calculated.

2. Find the maximum value of $|\text{Corr}(\boldsymbol{x}_i'\boldsymbol{\beta}, \boldsymbol{x}_i'\hat{\boldsymbol{\beta}}_M)|$ and call the corresponding

trimming percentage M_m. Then calculate $|\text{Corr}(\boldsymbol{x}_i'\hat{\boldsymbol{\beta}}_{M_m}, \boldsymbol{x}_i'\hat{\boldsymbol{\beta}}_M)|$ for all M.

3. Find the smallest value of M which makes $|\text{Corr}(\boldsymbol{x}_i'\hat{\boldsymbol{\beta}}_{M_m}, \boldsymbol{x}_i'\hat{\boldsymbol{\beta}}_M)| > 0.95$, and call it M_{adp}. This is the adaptive trimming percentage.

4. Calculate all the corresponding test statistics on the adaptive trimming percentage M_{adp}.

Last, keep a record of all the rejection percentages of each test for $M = 0\%, \cdots, 90\%$ and for the adaptive trimming.

Now we will present the testing results for the three testing problems.

1. Test For $H_0 : \boldsymbol{\beta} = \mathbf{0}, H_1 : \boldsymbol{\beta} \neq \mathbf{0}$.

For this testing problem, the true coefficient is $\boldsymbol{\beta} = \mathbf{0}$. According to (4.1), we have $SP = \alpha$, which is a constant. Therefore, the dependent variable y for all the seven models introduced in Section 4.2 is equal to some constant plus the error. We can show that the rejection percentages for the different models are the same. Therefore, for this testing problem, the rejection percentages only depend on the predictor distribution $\boldsymbol{x}$.

The results are shown in Table (4.18). Column "$\boldsymbol{x}$" expresses the predictor distribution used in the simulation, column "F" expresses the rejection percentage made by the F test based on the OLS estimator, column "χ^2" expresses the rejection percentage made by the χ^2 test based on the OLS estimator, and columns "SSIR" and "WSIR" denote the rejection percentage made by the χ^2 test based on the two algorithms of the SIR estimator. For both algorithms, we let the number of slices $h = 2$. Larger values of h rejected too often.

$\boldsymbol{x}$	n	F	χ^2	SSIR	WSIR
Type 1	10	0.047	0.2	0.167	0.226
	50	0.053	0.076	0.071	0.08
	100	0.041	0.059	0.059	0.057
	500	0.05	0.051	0.043	0.048
Type 2	10	0.04	0.209	0.857	0.739

Table 4.18. Test For $H_0 : \boldsymbol{\beta} = \mathbf{0}, H_1 : \boldsymbol{\beta} \neq \mathbf{0}$

$\boldsymbol{x}$	n	F	χ^2	SSIR	WSIR
	50	0.055	0.08	0.966	0.898
	100	0.05	0.059	0.952	0.908
	500	0.045	0.046	0.953	0.93
Type 3	10	0.05	0.238	0.671	0.896
	50	0.053	0.071	0.891	0.947
	100	0.047	0.053	0.91	0.955
	500	0.05	0.052	0.93	0.93
Type 4	10	0.057	0.213	0.281	0.367
	50	0.052	0.067	0.446	0.444
	100	0.045	0.055	0.496	0.526
	500	0.048	0.049	0.598	0.599
Type 5	10	0.053	0.205	0.329	0.428
	50	0.055	0.074	0.582	0.606
	100	0.055	0.063	0.635	0.621
	500	0.061	0.061	0.71	0.709
Type 6	10	0.053	0.21	0.332	0.429
	50	0.049	0.076	0.399	0.379
	100	0.042	0.05	0.434	0.439
	500	0.036	0.038	0.477	0.472
Type 7	10	0.048	0.215	0.256	0.332
	50	0.05	0.065	0.2	0.224
	100	0.054	0.064	0.214	0.214
	500	0.047	0.049	0.202	0.197
Type 8	10	0.057	0.22	0.194	0.247
	50	0.056	0.087	0.107	0.121
	100	0.044	0.055	0.078	0.074
	500	0.06	0.06	0.074	0.077

Table 4.18. (Continued)

We noticed that there are several interesting results shown by Table 4.18.

1) The F test works very well for all the predictor distributions and dimension n's. Even for $n = 10$, the rejection percentage is close to 0.05, which is the type I error.

2) For large n, the χ^2 test is as effective as the F test no matter which distribution we used. If n is small, the χ^2 test rejected H_0 too often.

3) The quality of the χ^2 SIR test depends on the predictor distribution. It works well if $\boldsymbol{x}$ has normal distribution. For other distributions, the test rejected

H_0 too often.

2. Test For $H_0 : \beta_i = 0, H_1 : \boldsymbol{\beta}_i \neq 0$

Here the true coefficient $\boldsymbol{\beta} = [1, 0, 1, 1]'$, i.e. $\beta_2 = 0$ for the true model. The results of the different distributions were displayed in a different table for each distribution. In each table, column "Model" means the model used in the simulation, column "n" denotes the sample size, column "90%" to column "0%" gives the rejection percentage based on $M\%$ trimming, and column "ADAP" gives the rejection percentage based on the adaptive percentage trimming. There are four rows of results for each model and each n. Row "t" has the results of the Wald test, row "χ^2" has the results of the χ^2 test based on the OLS estimator, and rows "SSIR" and "WSIR" both give the results of the χ^2 test based on the SIR estimator. The results are shown in Table 4.19 to Table 4.26.

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
1. t	60	.053	.055	.049	.050	.048	.042	.043	.050	.048	.056	.055
χ^2		.320	.112	.092	.072	.066	.051	.056	.062	.061	.063	.062
SSIR		.000	.002	.007	.010	.012	.020	.030	.033	.046	.048	.027
WSIR		.180	.018	.012	.018	.023	.021	.028	.037	.043	.057	.046
t	150	.065	.073	.061	.056	.062	.051	.046	.05	.044	.043	.043
χ^2		.098	.081	.071	.068	.072	.052	.051	.054	.049	.046	.046
SSIR		.002	.001	.006	.012	.012	.011	.023	.026	.042	.057	.052
WSIR		.004	.001	.007	.013	.015	.020	.027	.032	.045	.056	.056
t	500	.05	.042	.047	.039	.055	.057	.056	.057	.049	.051	.051
χ^2		.062	.045	.051	.041	.058	.058	.056	.060	.053	.051	.051
SSIR		.000	.000	.002	.009	.011	.011	.016	.025	.029	.065	.065
WSIR		.000	.000	.002	.004	.008	.014	.013	.024	.035	.048	.048
2. t	60	.051	.051	.032	.026	.028	.027	.033	.038	.028	.051	.019
χ^2		.324	.100	.054	.046	.036	.032	.044	.042	.033	.056	.022
SSIR		.000	.007	.004	.004	.009	.011	.017	.016	.017	.066	.008
WSIR		.192	.022	.013	.008	.016	.020	.028	.036	.039	.050	.021
t	150	.044	.048	.034	.035	.026	.029	.038	.029	.030	.038	.033
χ^2		.078	.059	.043	.038	.033	.032	.041	.031	.032	.039	.034
SSIR		.000	.002	.002	.004	.007	.007	.007	.017	.027	.055	.009
WSIR		.005	.001	.004	.009	.010	.014	.020	.024	.046	.073	.025

Table 4.19. Test For $H_0 : \beta_i = 0, H_1 : \beta_i \neq 0$ With Type 1 $\boldsymbol{x}$

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
t	500	.039	.043	.03	.025	.029	.023	.014	.027	.025	.038	.038
χ^2		.046	.048	.03	.026	.029	.023	.014	.027	.025	.038	.038
SSIR		0	0	0	.004	.002	.001	.006	.014	.017	.051	.036
WSIR		.000	.001	.001	.005	.004	.009	.011	.017	.021	.054	.050
3. t	60	.050	.051	.023	.021	.018	.022	.023	.034	.022	.050	.019
χ^2		.326	.095	.042	.032	.026	.031	.027	.041	.027	.057	.024
SSIR		.000	.004	.005	.003	.006	.004	.009	.017	.021	.076	.004
WSIR		.220	.020	.012	.020	.018	.030	.043	.035	.040	.062	.058
t	150	.040	.034	.024	.027	.016	.020	.026	.018	.025	.044	.034
χ^2		.081	.047	.032	.029	.020	.022	.027	.022	.026	.048	.038
SSIR		.000	.000	.000	.001	.003	.003	.004	.009	.017	.049	.003
WSIR		.004	.003	.011	.009	.012	.019	.028	.026	.040	.056	.056
t	500	.029	.034	.022	.012	.010	.009	.009	.015	.010	.042	.038
χ^2		.036	.035	.024	.013	.010	.009	.009	.015	.010	.044	.039
SSIR		.000	.000	.000	.001	.000	.001	.001	.004	.015	.049	.006
WSIR		.000	.002	.006	.008	.007	.014	.010	.021	.034	.055	.055
4. t	60	.045	.049	.024	.021	.016	.023	.023	.034	.023	.052	.046
χ^2		.313	.083	.037	.039	.029	.029	.030	.042	.026	.060	.054
SSIR		.000	.002	.002	.003	.007	.009	.016	.016	.027	.069	.014
WSIR		.206	.021	.015	.016	.022	.030	.041	.036	.045	.053	.046
t	150	.041	.026	.020	.027	.023	.024	.029	.024	.029	.045	.045
χ^2		.074	.039	.024	.034	.027	.026	.032	.026	.036	.049	.049
SSIR		.000	.000	.001	.001	.004	.006	.010	.012	.020	.067	.021
WSIR		.006	.004	.006	.007	.009	.018	.027	.034	.041	.060	.060
t	500	.030	.028	.020	.018	.011	.014	.017	.018	.020	.047	.047
χ^2		.035	.031	.023	.019	.011	.014	.017	.018	.021	.049	.049
SSIR		.000	.000	.000	.001	.001	.001	.005	.007	.012	.057	.037
WSIR		.000	.001	.001	.007	.010	.013	.015	.019	.023	.055	.055
5. t	60	.054	.037	.019	.027	.022	.023	.030	.034	.030	.037	.019
χ^2		.318	.088	.041	.043	.027	.032	.036	.044	.035	.041	.021
SSIR		.000	.001	.006	.005	.009	.017	.021	.025	.026	.052	.016
WSIR		.223	.021	.017	.011	.010	.016	.019	.035	.043	.044	.014
t	150	.025	.017	.019	.023	.019	.019	.020	.022	.027	.037	.029
χ^2		.057	.025	.030	.026	.022	.024	.021	.023	.029	.044	.036
SSIR		.000	.002	.002	.006	.011	.016	.014	.014	.024	.044	.030
WSIR		.006	.001	.002	.003	.006	.005	.010	.014	.025	.055	.026
t	500	.024	.026	.020	.020	.018	.019	.012	.017	.027	.050	.050

Table 4.19. (Continued)

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
χ^2		.028	.031	.022	.021	.018	.019	.012	.018	.029	.050	.050
SSIR		.000	.000	.001	.005	.007	.009	.012	.018	.018	.044	.044
WSIR		.000	.002	.000	.002	.004	.008	.003	.014	.024	.055	.051
6. t	60	.057	.036	.028	.024	.023	.031	.037	.041	.032	.047	.047
χ^2		.314	.085	.047	.035	.040	.038	.048	.052	.037	.053	.053
SSIR		.000	.001	.011	.012	.016	.012	.027	.025	.032	.058	.054
WSIR		.220	.018	.021	.019	.030	.035	.029	.041	.050	.062	.058
t	150	.033	.029	.030	.030	.034	.038	.035	.032	.037	.040	.040
χ^2		.063	.034	.035	.035	.039	.040	.040	.034	.040	.040	.040
SSIR		.000	.003	.007	.010	.011	.009	.024	.025	.035	.058	.058
WSIR		.009	.005	.006	.005	.006	.019	.018	.028	.038	.054	.054
t	500	.029	.030	.023	.023	.023	.020	.027	.030	.030	.037	.037
χ^2		.043	.034	.024	.023	.024	.020	.027	.031	.030	.037	.037
SSIR		.000	.000	.001	.005	.008	.011	.011	.017	.028	.045	.045
WSIR		.001	.000	.002	.001	.008	.013	.014	.020	.024	.048	.048
7. t	60	.054	.050	.034	.033	.029	.030	.040	.045	.033	.034	.009
χ^2		.329	.093	.057	.051	.043	.041	.045	.053	.038	.039	.015
SSIR		.000	.003	.001	.007	.007	.017	.012	.021	.038	.061	.015
WSIR		.216	.018	.012	.011	.013	.016	.016	.030	.042	.056	.018
t	150	.051	.036	.030	.029	.031	.034	.033	.036	.038	.044	.036
χ^2		.080	.046	.035	.030	.033	.038	.034	.037	.040	.046	.038
SSIR		.001	.000	.000	.003	.011	.009	.010	.020	.037	.055	.018
WSIR		.008	.001	.001	.005	.007	.008	.011	.025	.040	.055	.015
t	500	.029	.034	.022	.017	.025	.029	.029	.030	.031	.046	.046
χ^2		.035	.036	.023	.018	.028	.029	.029	.033	.032	.047	.047
SSIR		.000	.000	.001	.002	.001	.004	.009	.013	.023	.057	.053
WSIR		.000	.000	.001	.001	.001	.004	.006	.017	.026	.061	.055

Table 4.19. (Continued)

Table 4.19 gives the results for $\boldsymbol{x}_i \sim N_4(\mathbf{0}, \boldsymbol{I})$. For all the models, the best results for all the tests are usually based on the 0% trimming, i.e. the original dataset, and many of the results based on the adaptive trimming are the same as those based on the 0% trimming. The t test and the χ^2 OLS test perform similarly. WSIR and SSIR are similar. Overall, all the tests work well for this predictor distribution.

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
1. t	60	.040	.047	.042	.058	.058	.047	.059	.042	.044	.049	.049
χ^2		.302	.097	.076	.078	.069	.068	.065	.052	.052	.060	.060
SSIR		.003	.022	.021	.028	.046	.080	.176	.331	.416	.540	.409
WSIR		.218	.036	.031	.046	.067	.092	.146	.255	.335	.462	.398
t	150	.058	.049	.046	.046	.052	.051	.054	.043	.035	.045	.045
χ^2		.091	.062	.061	.053	.062	.058	.055	.043	.045	.049	.049
SSIR		.003	.008	.014	.027	.030	.059	.148	.292	.433	.577	.564
WSIR		.017	.007	.013	.025	.029	.055	.119	.244	.338	.506	.505
t	500	.054	.044	.044	.046	.051	.041	.056	.052	.048	.049	.049
χ^2		.060	.051	.045	.049	.053	.043	.058	.052	.049	.051	.051
SSIR		.000	.006	.011	.018	.024	.047	.129	.230	.380	.549	.549
WSIR		.001	.003	.009	.013	.027	.046	.136	.220	.356	.468	.468
2. t	60	.058	.033	.032	.027	.043	.080	.180	.195	.177	.195	.049
χ^2		.310	.084	.054	.041	.058	.094	.209	.210	.192	.213	.057
SSIR		.001	.007	.004	.013	.038	.120	.369	.523	.603	.682	.166
WSIR		.228	.033	.033	.026	.047	.088	.171	.323	.446	.563	.056
t	150	.045	.033	.023	.024	.026	.070	.183	.182	.142	.166	.040
χ^2		.077	.045	.028	.030	.028	.076	.192	.191	.144	.172	.042
SSIR		.002	.004	.003	.014	.019	.077	.331	.475	.597	.685	.163
WSIR		.010	.012	.007	.010	.021	.067	.177	.328	.452	.576	.050
t	500	.032	.039	.031	.026	.021	.049	.179	.152	.141	.167	.051
χ^2		.043	.042	.033	.027	.021	.051	.179	.154	.142	.168	.051
SSIR		.001	.003	.002	.008	.013	.044	.347	.437	.574	.670	.227
WSIR		.001	.001	.005	.013	.027	.046	.163	.311	.417	.526	.137
3. t	60	.047	.028	.029	.032	.044	.070	.180	.228	.206	.216	.025
χ^2		.330	.088	.049	.042	.054	.089	.197	.248	.220	.220	.027
SSIR		.001	.007	.003	.014	.023	.126	.358	.549	.624	.675	.134
WSIR		.231	.040	.036	.057	.060	.108	.174	.264	.379	.485	.427
t	150	.046	.023	.018	.021	.016	.066	.180	.177	.167	.220	.024
χ^2		.076	.034	.025	.021	.020	.071	.189	.185	.174	.227	.027
SSIR		.000	.002	.000	.008	.011	.089	.363	.494	.571	.722	.130
WSIR		.008	.014	.014	.022	.040	.053	.133	.250	.357	.499	.497
t	500	.030	.029	.023	.014	.019	.037	.151	.096	.087	.177	.027
χ^2		.033	.029	.025	.014	.019	.038	.153	.096	.089	.177	.027
SSIR		.000	.002	.003	.000	.003	.043	.316	.392	.540	.687	.227
WSIR		.001	.005	.011	.020	.029	.059	.130	.229	.340	.476	.476
4. t	60	.045	.029	.032	.035	.047	.062	.138	.160	.159	.179	.123
χ^2		.325	.084	.055	.048	.054	.076	.153	.182	.167	.198	.138

Table 4.20. Test For $H_0 : \beta_i = 0, H_1 : \beta_i \neq 0$ With Type 2 $\boldsymbol{x}$

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
SSIR		.001	.005	.006	.018	.032	.123	.347	.531	.614	.705	.355
WSIR		.228	.039	.042	.051	.058	.102	.167	.271	.370	.466	.416
t	150	.044	.024	.023	.021	.025	.052	.131	.150	.138	.178	.173
χ^2		.074	.035	.027	.025	.027	.056	.138	.154	.142	.184	.179
SSIR		.001	.002	.002	.015	.016	.082	.314	.501	.592	.674	.475
WSIR		.009	.012	.011	.020	.033	.058	.134	.260	.367	.502	.501
t	500	.019	.035	.027	.017	.024	.034	.132	.107	.134	.206	.206
χ^2		.026	.037	.029	.018	.026	.035	.133	.107	.135	.207	.207
SSIR		.000	.000	.000	.002	.008	.046	.332	.490	.597	.687	.648
WSIR		.001	.000	.011	.015	.023	.055	.132	.212	.339	.461	.461
5. t	60	.050	.034	.027	.030	.045	.052	.098	.093	.082	.072	.028
χ^2		.322	.084	.054	.041	.063	.066	.107	.105	.094	.076	.036
SSIR		.002	.008	.016	.019	.034	.075	.205	.353	.456	.545	.043
WSIR		.240	.038	.034	.029	.039	.092	.179	.333	.446	.549	.046
t	150	.040	.025	.021	.020	.029	.061	.112	.111	.071	.064	.020
χ^2		.063	.039	.027	.024	.033	.068	.116	.116	.081	.068	.024
SSIR		.001	.009	.009	.014	.025	.075	.227	.371	.460	.558	.058
WSIR		.012	.006	.006	.011	.020	.068	.182	.329	.444	.572	.042
t	500	.026	.032	.030	.021	.031	.043	.094	.084	.064	.052	.030
χ^2		.033	.032	.033	.022	.032	.045	.095	.086	.066	.052	.031
SSIR		.000	.002	.008	.012	.015	.052	.202	.355	.441	.531	.157
WSIR		.002	.002	.004	.014	.029	.056	.170	.319	.431	.538	.150
6. t	60	.046	.034	.035	.037	.039	.045	.058	.066	.051	.072	.072
χ^2		.308	.084	.053	.050	.049	.054	.068	.074	.062	.079	.079
SSIR		.001	.013	.025	.032	.036	.086	.192	.343	.440	.569	.453
WSIR		.250	.044	.044	.056	.055	.107	.183	.280	.381	.460	.399
t	150	.043	.028	.023	.034	.032	.046	.054	.067	.049	.057	.057
χ^2		.072	.039	.030	.038	.040	.056	.064	.070	.050	.058	.058
SSIR		.002	.004	.011	.020	.032	.053	.161	.304	.404	.583	.575
WSIR		.007	.013	.018	.021	.033	.068	.130	.256	.362	.509	.508
t	500	.031	.045	.037	.030	.041	.037	.071	.060	.060	.053	.053
χ^2		.034	.048	.041	.031	.046	.038	.072	.061	.060	.054	.054
SSIR		.000	.002	.009	.013	.027	.042	.171	.238	.393	.530	.530
WSIR		.002	.004	.005	.013	.030	.053	.129	.218	.343	.473	.473
7. t	60	.050	.040	.038	.028	.041	.068	.102	.148	.132	.120	.035
χ^2		.294	.081	.062	.050	.057	.078	.122	.159	.137	.134	.043
SSIR		.000	.011	.013	.028	.041	.088	.218	.426	.558	.661	.072
WSIR		.243	.037	.031	.023	.041	.090	.185	.319	.446	.559	.049

Table 4.20. (Continued)

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
t	150	.036	.032	.027	.029	.026	.059	.141	.150	.119	.127	.027
χ^2		.067	.048	.036	.034	.030	.060	.147	.156	.125	.129	.030
SSIR		.002	.005	.006	.010	.024	.061	.257	.469	.588	.669	.060
WSIR		.007	.008	.010	.010	.028	.066	.191	.334	.441	.573	.051
t	500	.031	.033	.031	.028	.028	.048	.109	.115	.108	.116	.042
χ^2		.039	.038	.032	.029	.028	.050	.112	.117	.108	.117	.043
SSIR		.002	.001	.004	.011	.023	.043	.227	.461	.529	.693	.139
WSIR		.003	.001	.005	.006	.023	.047	.169	.318	.419	.533	.131

Table 4.20. (Continued)

The results in Table 4.20 are based on $\boldsymbol{x}_i \sim 0.6N_4(\mathbf{0}, \boldsymbol{I}) + 0.4N_4(\mathbf{0}, 25\boldsymbol{I})$. This distribution belongs to EC[23].

1) For the OLS estimator, both the t test and χ^2 test work well for the original dataset if the model is MLR. Also the results of the adaptive trimming and the 0% trimming are the same. If the model is nonlinear, such as model 2,3,4, or 7, the 0% trimming rejected too often. The results based on higher M are better, especially 40% and 50% trimming. The rejection rates based on these two trimming percentages are closer to 0.05, the type I error. In general, the adaptive trimming is better than the original dataset but OLS ADAP fails for model 4.

2) For the SIR estimator, the two methods WSIR and SSIR are similar. The χ^2 tests based on the original dataset are not applicable for all the models. Similar to the OLS estimator, most of the results based on the trimmed data are better, and the best results usually happened based on 40% and 50% trimming. The results of the adaptive trimming depend on the model. For some models, the adaptive trimming works well; but for other models, the rejection percentage of the adaptive trimming are as high as the one for 0% trimming.

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
1. t	60	.045	.053	.057	.049	.058	.055	.045	.050	.052	.049	.049
χ^2		.324	.103	.089	.074	.073	.063	.053	.061	.059	.054	.054
SSIR		.008	.028	.045	.106	.185	.299	.390	.466	.541	.601	.540
WSIR		.250	.062	.065	.108	.182	.281	.331	.436	.492	.564	.549
t	150	.043	.048	.038	.041	.044	.048	.051	.052	.040	.050	.050
χ^2		.082	.056	.048	.050	.050	.049	.052	.055	.045	.054	.054
SSIR		.007	.019	.034	.062	.153	.256	.348	.436	.521	.601	.600
WSIR		.017	.024	.035	.061	.152	.231	.317	.377	.471	.585	.584
t	500	.055	.054	.047	.046	.059	.055	.049	.050	.045	.045	.045
χ^2		.068	.059	.051	.047	.061	.056	.051	.051	.046	.046	.046
SSIR		.005	.010	.033	.046	.153	.233	.375	.446	.492	.576	.576
WSIR		.006	.015	.031	.053	.159	.212	.320	.419	.485	.536	.536
2. t	60	.049	.048	.047	.081	.104	.121	.131	.101	.079	.109	.039
χ^2		.295	.088	.086	.099	.134	.142	.147	.115	.088	.122	.053
SSIR		.007	.019	.054	.114	.269	.433	.528	.569	.636	.702	.246
WSIR		.260	.050	.052	.092	.185	.278	.392	.501	.541	.629	.119
t	150	.034	.037	.033	.056	.120	.110	.097	.076	.070	.108	.024
χ^2		.062	.054	.041	.067	.131	.116	.101	.077	.074	.108	.027
SSIR		.003	.013	.013	.098	.270	.407	.466	.555	.627	.712	.236
WSIR		.021	.019	.029	.068	.191	.290	.397	.498	.560	.638	.067
t	500	.033	.029	.034	.048	.129	.104	.076	.069	.070	.103	.022
χ^2		.041	.034	.035	.049	.133	.106	.077	.071	.070	.103	.022
SSIR		.002	.006	.016	.050	.279	.390	.469	.554	.601	.680	.337
WSIR		.002	.005	.026	.050	.191	.322	.410	.498	.545	.607	.123
3. t	60	.039	.043	.056	.082	.109	.165	.144	.115	.109	.116	.020
χ^2		.310	.089	.082	.101	.130	.185	.166	.134	.117	.125	.028
SSIR		.001	.024	.046	.118	.295	.466	.572	.607	.626	.687	.203
WSIR		.293	.075	.074	.123	.189	.280	.342	.462	.531	.588	.557
t	150	.040	.030	.029	.056	.108	.111	.086	.062	.075	.130	.013
χ^2		.067	.041	.032	.061	.120	.117	.094	.066	.079	.134	.017
SSIR		.006	.009	.012	.079	.285	.412	.496	.546	.610	.712	.191
WSIR		.022	.029	.031	.074	.159	.252	.327	.389	.463	.562	.561
t	500	.026	.013	.018	.039	.103	.043	.040	.034	.026	.123	.007
χ^2		.030	.014	.019	.042	.104	.044	.041	.034	.028	.123	.009
SSIR		.000	.001	.004	.027	.242	.310	.380	.432	.546	.698	.292
WSIR		.003	.014	.029	.054	.180	.250	.330	.439	.482	.541	.541

Table 4.21. Test For $H_0 : \beta_i = 0, H_1 : \beta_i \neq 0$ With Type 3 $\boldsymbol{x}$

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
4. t	60	.047	.046	.062	.083	.087	.113	.106	.091	.092	.112	.081
χ^2		.303	.089	.088	.099	.106	.134	.130	.101	.104	.118	.088
SSIR		.004	.023	.050	.126	.283	.434	.526	.589	.655	.692	.483
WSIR		.280	.074	.068	.112	.186	.275	.340	.438	.524	.588	.570
t	150	.044	.032	.027	.058	.096	.081	.071	.057	.062	.123	.120
χ^2		.073	.044	.033	.066	.104	.084	.075	.065	.066	.125	.122
SSIR		.007	.007	.018	.097	.294	.421	.518	.576	.642	.683	.590
WSIR		.024	.028	.028	.069	.152	.263	.337	.378	.465	.541	.539
t	500	.027	.020	.022	.048	.117	.069	.060	.079	.058	.104	.104
χ^2		.032	.020	.024	.049	.122	.071	.061	.081	.061	.105	.105
SSIR		.000	.005	.004	.055	.310	.407	.500	.569	.619	.675	.668
WSIR		.002	.007	.029	.058	.185	.249	.300	.408	.461	.537	.537
5. t	60	.047	.050	.057	.063	.096	.091	.069	.054	.036	.032	.027
χ^2		.321	.096	.091	.088	.111	.106	.082	.059	.046	.043	.043
SSIR		.002	.021	.047	.088	.198	.295	.402	.476	.522	.570	.068
WSIR		.272	.047	.058	.090	.196	.273	.390	.498	.568	.633	.100
t	150	.035	.037	.034	.048	.099	.079	.060	.061	.034	.032	.019
χ^2		.069	.056	.042	.052	.106	.083	.064	.063	.039	.036	.023
SSIR		.008	.019	.027	.085	.198	.321	.385	.482	.540	.618	.058
WSIR		.017	.013	.014	.066	.194	.295	.382	.505	.568	.631	.065
t	500	.026	.026	.033	.053	.104	.067	.059	.041	.033	.028	.025
χ^2		.031	.030	.036	.053	.106	.069	.059	.041	.033	.030	.025
SSIR		.001	.005	.012	.036	.211	.345	.424	.474	.529	.583	.139
WSIR		.001	.008	.022	.055	.182	.324	.414	.489	.547	.594	.145
6. t	60	.045	.043	.057	.058	.060	.048	.051	.059	.058	.066	.066
χ^2		.321	.093	.081	.086	.079	.058	.056	.065	.067	.070	.070
SSIR		.000	.027	.044	.093	.190	.320	.408	.490	.525	.605	.560
WSIR		.284	.079	.082	.133	.200	.284	.338	.449	.532	.586	.560
t	150	.035	.035	.037	.054	.054	.053	.065	.059	.057	.050	.050
χ^2		.074	.055	.042	.061	.067	.053	.068	.059	.061	.052	.052
SSIR		.006	.020	.023	.064	.169	.253	.358	.419	.520	.580	.576
WSIR		.021	.024	.031	.071	.171	.236	.344	.401	.473	.567	.567
t	500	.033	.027	.037	.047	.077	.066	.057	.069	.070	.056	.056
χ^2		.039	.030	.040	.051	.079	.066	.061	.071	.070	.056	.056
SSIR		.000	.010	.020	.058	.186	.253	.356	.425	.503	.574	.574
WSIR		.006	.010	.028	.050	.186	.251	.320	.420	.492	.537	.537
7. t	60	.043	.044	.042	.060	.071	.090	.093	.081	.060	.069	.022
χ^2		.292	.095	.074	.085	.094	.107	.107	.093	.070	.079	.043

Table 4.21. (Continued)

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
SSIR		.003	.021	.042	.088	.211	.350	.476	.547	.601	.656	.121
WSIR		.271	.047	.055	.102	.183	.276	.384	.493	.545	.625	.109
t	150	.027	.042	.042	.059	.089	.080	.076	.067	.059	.072	.016
χ^2		.062	.060	.051	.067	.098	.091	.082	.071	.063	.078	.018
SSIR		.009	.008	.020	.075	.219	.363	.466	.567	.596	.639	.089
WSIR		.017	.019	.023	.069	.186	.286	.400	.500	.568	.621	.070
t	500	.032	.040	.047	.048	.116	.096	.071	.053	.043	.071	.022
χ^2		.039	.042	.050	.048	.119	.099	.072	.054	.045	.072	.023
SSIR		.001	.008	.025	.043	.222	.403	.498	.531	.578	.645	.147
WSIR		.007	.006	.019	.053	.203	.321	.405	.475	.546	.596	.131

Table 4.21. (Continued)

The results in Table 4.21 are based on $\boldsymbol{x}_i \sim 0.4N_4(\mathbf{0}, \boldsymbol{I}) + 0.6N_4(\mathbf{0}, 25\boldsymbol{I})$. This distribution also belongs to EC[23]. Compared with those in Table 4.20, the results are very similar here except that the best results are usually obtained at 60% or 70% trimming.

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
1. t	60	.043	.048	.041	.050	.040	.049	.052	.051	.037	.056	.056
χ^2		.295	.108	.066	.074	.054	.064	.059	.063	.042	.063	.063
SSIR		.003	.011	.009	.016	.019	.036	.039	.044	.074	.413	.164
WSIR		.170	.023	.018	.014	.016	.032	.042	.043	.073	.265	.110
t	150	.049	.046	.041	.041	.041	.045	.054	.054	.050	.049	.049
χ^2		.086	.062	.053	.045	.047	.050	.056	.056	.053	.049	.049
SSIR		.002	.005	.006	.006	.013	.017	.032	.033	.072	.430	.282
WSIR		.001	.007	.006	.014	.014	.022	.032	.051	.059	.329	.271
t	500	.056	.056	.056	.054	.046	.044	.050	.059	.051	.048	.048
χ^2		.063	.062	.057	.058	.046	.045	.052	.060	.051	.050	.050
SSIR		.000	.001	.009	.008	.009	.015	.030	.030	.044	.412	.384
WSIR		.000	.001	.004	.004	.011	.016	.020	.033	.045	.314	.314
2. t	60	.053	.043	.036	.034	.024	.034	.029	.039	.112	.431	.056
χ^2		.325	.095	.058	.041	.031	.040	.041	.044	.114	.437	.059
SSIR		.002	.005	.005	.006	.008	.013	.023	.036	.177	.670	.072
WSIR		.193	.019	.014	.010	.022	.032	.030	.045	.073	.267	.030
t	150	.039	.044	.027	.023	.024	.024	.022	.034	.097	.456	.067

Table 4.22. Test For $H_0 : \beta_i = 0, H_1 : \beta_i \neq 0$ With Type 4 $\boldsymbol{x}$

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
χ^2		.074	.061	.037	.033	.027	.025	.025	.037	.101	.460	.070
SSIR		.002	.004	.004	.006	.005	.010	.013	.023	.126	.682	.082
WSIR		.005	.003	.006	.005	.007	.010	.016	.035	.077	.316	.035
t	500	.041	.032	.031	.024	.025	.023	.018	.024	.070	.452	.091
χ^2		.050	.037	.034	.027	.025	.025	.018	.025	.071	.454	.093
SSIR		.000	.000	.001	.006	.004	.002	.006	.020	.086	.681	.103
WSIR		.001	.000	.002	.004	.006	.011	.019	.019	.060	.351	.126
3. t	60	.047	.040	.033	.029	.021	.025	.029	.034	.107	.420	.020
χ^2		.325	.088	.058	.039	.028	.035	.038	.042	.114	.435	.025
SSIR		.000	.000	.001	.004	.002	.011	.018	.021	.133	.614	.039
WSIR		.202	.025	.024	.022	.036	.043	.047	.058	.093	.332	.140
t	150	.039	.040	.025	.021	.021	.017	.012	.025	.104	.463	.038
χ^2		.069	.055	.030	.025	.023	.019	.015	.029	.104	.466	.040
SSIR		.001	.000	.002	.002	.002	.003	.007	.011	.140	.693	.044
WSIR		.003	.009	.006	.012	.014	.016	.028	.032	.074	.380	.318
t	500	.036	.025	.017	.012	.011	.008	.013	.017	.085	.465	.059
χ^2		.044	.029	.018	.012	.012	.009	.013	.019	.087	.465	.060
SSIR		.000	.000	.000	.002	.000	.001	.002	.003	.078	.691	.067
WSIR		.000	.001	.004	.006	.011	.009	.016	.024	.054	.369	.369
4. t	60	.044	.040	.033	.026	.027	.031	.026	.030	.109	.390	.102
χ^2		.319	.083	.055	.039	.036	.033	.036	.044	.116	.407	.113
SSIR		.001	.003	.004	.003	.005	.014	.025	.035	.182	.667	.091
WSIR		.196	.022	.017	.018	.033	.032	.040	.053	.095	.333	.143
t	150	.032	.037	.027	.020	.022	.023	.017	.030	.123	.412	.259
χ^2		.058	.051	.035	.024	.029	.026	.017	.031	.128	.420	.267
SSIR		.000	.001	.001	.002	.006	.011	.009	.020	.132	.715	.156
WSIR		.004	.002	.008	.018	.014	.019	.030	.035	.063	.371	.311
t	500	.027	.023	.018	.016	.019	.010	.021	.028	.068	.399	.384
χ^2		.036	.025	.019	.017	.019	.010	.022	.028	.070	.402	.388
SSIR		.00	.001	.000	.001	.002	.001	.005	.013	.084	.649	.314
WSIR		.000	.000	.002	.003	.013	.011	.012	.031	.059	.371	.370
5. t	60	.056	.036	.036	.021	.028	.025	.027	.039	.066	.187	.023
χ^2		.362	.069	.056	.036	.035	.035	.036	.043	.069	.198	.025
SSIR		.000	.004	.008	.005	.008	.012	.023	.032	.079	.358	.029
WSIR		.211	.018	.009	.009	.017	.024	.028	.046	.080	.285	.025
t	150	.034	.026	.023	.016	.019	.025	.018	.034	.058	.196	.039
χ^2		.058	.039	.028	.021	.023	.027	.021	.036	.063	.198	.042
SSIR		.001	.000	.004	.010	.011	.015	.015	.029	.081	.410	.046

Table 4.22. (Continued)

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
WSIR		.009	.003	.005	.005	.012	.011	.017	.024	.071	.344	.021
t	500	.019	.021	.026	.026	.023	.023	.017	.022	.048	.170	.048
χ^2		.025	.024	.028	.026	.024	.023	.018	.022	.048	.171	.048
SSIR		.000	.001	.002	.007	.012	.011	.015	.023	.053	.424	.165
WSIR		.001	.002	.004	.004	.007	.008	.014	.019	.050	.352	.122
6. t	60	.063	.035	.037	.021	.029	.036	.035	.041	.048	.083	.083
χ^2		.324	.062	.055	.037	.039	.048	.046	.046	.054	.091	.091
SSIR		.000	.009	.014	.014	.019	.026	.031	.040	.093	.469	.164
WSIR		.227	.024	.012	.018	.036	.039	.039	.065	.094	.344	.155
t	150	.040	.023	.026	.024	.030	.032	.028	.044	.051	.088	.088
χ^2		.067	.040	.036	.027	.036	.036	.033	.048	.055	.091	.091
SSIR		.000	.003	.005	.005	.012	.014	.021	.031	.079	.473	.316
WSIR		.009	.006	.012	.009	.013	.016	.030	.040	.076	.386	.319
t	500	.028	.038	.034	.037	.024	.024	.036	.031	.041	.053	.053
χ^2		.034	.042	.036	.041	.025	.024	.037	.031	.041	.055	.055
SSIR		.000	.001	.003	.005	.009	.008	.012	.019	.058	.455	.416
WSIR		.001	.002	.001	.010	.013	.015	.023	.034	.045	.380	.380
7. t	60	.043	.042	.042	.036	.027	.036	.022	.034	.071	.260	.036
χ^2		.320	.097	.061	.052	.037	.047	.031	.042	.081	.267	.044
SSIR		.000	.003	.010	.010	.008	.018	.021	.046	.082	.500	.058
WSIR		.209	.019	.013	.007	.020	.022	.028	.047	.081	.272	.030
t	150	.036	.036	.030	.031	.023	.027	.028	.039	.056	.305	.053
χ^2		.071	.059	.040	.036	.024	.032	.029	.040	.056	.306	.053
SSIR		.002	.001	.005	.001	.006	.011	.014	.024	.075	.596	.048
WSIR		.005	.003	.006	.005	.009	.015	.015	.028	.075	.323	.032
t	500	.036	.031	.032	.036	.036	.026	.027	.029	.052	.331	.070
χ^2		.045	.037	.033	.037	.037	.026	.029	.030	.052	.335	.071
SSIR		.000	.001	.002	.003	.003	.009	.018	.019	.047	.658	.062
WSIR		.000	.001	.001	.005	.007	.011	.014	.015	.056	.362	.133

Table 4.22. (Continued)

The results in Table 4.22 are based on $\boldsymbol{x}_i \sim 0.9N_4(\mathbf{0}, \boldsymbol{I}) + 0.1N_4(\mathbf{0}, 25\boldsymbol{I})$. This distribution also belongs to EC[23]. Compared with Table 4.20 and Table 4.21, the results here are better in general.

1) OLS estimator: For all nonlinear models, both the t test and the χ^2 test based on 0% trimming do not work well. Ellipsoidal trimming again improves the results a lot. The best results are often obtained at 10% or 20% trimming. The results based on the adaptive trimming are better than 0% trimming. But compared

with the 10% or 20% trimming, the adaptive trimming rejection rate is often too high.

2) SIR estimator: Compared to the OLS estimator, the rejection percentages based on 0% trimming are too high although WSIR's results are smaller than SSIR. The other results are similar to the OLS estimator. The best results are based on 10% or 20% trimming, and the results of the adaptive trimming are better than 0% trimming.

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
1. t	60	.042	.044	.051	.060	.047	.049	.038	.040	.039	.038	.038
χ^2		.308	.105	.076	.074	.064	.060	.052	.050	.050	.041	.041
SSIR		.001	.002	.002	.004	.011	.021	.038	.096	.127	.355	.223
WSIR		.145	.004	.006	.013	.017	.024	.044	.083	.149	.289	.182
t	150	.058	.054	.057	.064	.051	.055	.043	.047	.053	.045	.045
χ^2		.095	.071	.068	.069	.053	.059	.044	.054	.055	.049	.049
SSIR		.000	.001	.000	.001	.002	.016	.029	.071	.131	.338	.242
WSIR		.004	.000	.001	.003	.009	.010	.031	.072	.127	.295	.238
t	500	.052	.045	.043	.044	.040	.046	.055	.049	.047	.054	.054
χ^2		.067	.050	.043	.044	.044	.047	.055	.050	.048	.054	.054
SSIR		.000	.000	.000	.000	.003	.008	.026	.059	.120	.344	.246
WSIR		.000	.000	.000	.001	.003	.008	.019	.054	.138	.318	.291
2. t	60	.048	.046	.043	.050	.039	.029	.032	.032	.027	.054	.055
χ^2		.315	.099	.074	.066	.052	.039	.038	.037	.033	.061	.062
SSIR		.000	.000	.002	.005	.013	.023	.033	.034	.155	.370	.131
WSIR		.190	.010	.007	.009	.014	.013	.034	.074	.162	.298	.171
t	150	.054	.044	.044	.034	.039	.034	.039	.027	.034	.046	.047
χ^2		.100	.056	.053	.041	.047	.040	.042	.028	.036	.049	.051
SSIR		.000	.001	.001	.001	.005	.009	.022	.050	.121	.330	.123
WSIR		.004	.004	.001	.001	.002	.008	.028	.080	.113	.320	.244
t	500	.053	.039	.036	.047	.059	.045	.043	.047	.035	.054	.052
χ^2		.061	.043	.038	.051	.060	.046	.044	.048	.038	.054	.052
SSIR		.000	.000	.000	.002	.000	.007	.011	.051	.091	.315	.125
WSIR		.001	.000	.000	.000	.002	.009	.026	.065	.143	.321	.296
3. t	60	.043	.042	.027	.028	.021	.019	.018	.030	.027	.066	.017
χ^2		.309	.090	.041	.039	.030	.028	.028	.033	.029	.071	.020
SSIR		.000	.002	.003	.000	.006	.003	.024	.055	.120	.335	.024

Table 4.23. Test For $H_0 : \beta_i = 0, H_1 : \beta_i \neq 0$ With Type 5 $\boldsymbol{x}$

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
WSIR		.191	.008	.005	.010	.017	.019	.033	.076	.156	.307	.181
t	150	.058	.032	.017	.020	.024	.023	.010	.017	.017	.042	.011
χ^2		.083	.042	.025	.027	.025	.026	.011	.019	.018	.043	.011
SSIR		.001	.000	.001	.001	.000	.000	.005	.016	.066	.291	.008
WSIR		.006	.001	.000	.002	.004	.011	.035	.064	.114	.316	.247
t	500	.046	.028	.018	.014	.023	.012	.010	.018	.006	.048	.013
χ^2		.052	.028	.020	.015	.024	.012	.011	.019	.007	.048	.013
SSIR		.000	.000	.000	.000	.001	.000	.000	.003	.018	.257	.004
WSIR		.000	.000	.000	.001	.002	.008	.028	.066	.139	.334	.310
4. t	60	.038	.035	.024	.028	.031	.024	.028	.029	.028	.059	.045
χ^2		.322	.089	.037	.046	.048	.034	.035	.035	.033	.067	.050
SSIR		.000	.002	.003	.005	.009	.014	.037	.062	.137	.357	.080
WSIR		.191	.008	.005	.010	.018	.019	.033	.076	.154	.308	.179
t	150	.054	.034	.026	.029	.030	.032	.022	.026	.025	.045	.034
χ^2		.084	.045	.033	.031	.034	.035	.028	.027	.026	.048	.034
SSIR		.000	.000	.000	.001	.003	.006	.020	.033	.081	.332	.056
WSIR		.006	.001	.000	.002	.004	.011	.035	.063	.116	.314	.250
t	500	.048	.033	.025	.031	.034	.039	.038	.048	.033	.053	.045
χ^2		.053	.035	.025	.037	.035	.040	.038	.050	.035	.053	.045
SSIR		.000	.000	.000	.000	.004	.000	.004	.032	.050	.285	.053
WSIR		.000	.000	.000	.001	.002	.007	.029	.064	.135	.337	.311
5. t	60	.035	.035	.027	.033	.039	.039	.036	.043	.042	.051	.012
χ^2		.315	.092	.043	.050	.053	.051	.046	.050	.047	.057	.022
SSIR		.000	.002	.002	.002	.006	.010	.035	.064	.123	.256	.020
WSIR		.187	.008	.006	.006	.006	.018	.028	.079	.143	.299	.019
t	150	.057	.037	.029	.033	.033	.041	.044	.038	.041	.043	.014
χ^2		.085	.046	.034	.039	.040	.045	.050	.039	.042	.044	.016
SSIR		.000	.000	.000	.002	.004	.007	.026	.057	.107	.270	.064
WSIR		.001	.001	.002	.002	.002	.007	.020	.060	.136	.342	.021
t	500	.051	.033	.031	.037	.051	.056	.071	.065	.048	.044	.047
χ^2		.054	.036	.032	.039	.052	.056	.072	.065	.048	.044	.048
SSIR		.000	.000	.000	.000	.003	.019	.033	.077	.127	.310	.234
WSIR		.000	.000	.000	.000	.003	.005	.037	.082	.141	.373	.094
6. t	60	.042	.054	.036	.041	.041	.029	.038	.045	.040	.040	.040
χ^2		.328	.101	.056	.059	.056	.042	.043	.049	.051	.046	.046
SSIR		.001	.001	.003	.001	.010	.017	.035	.081	.146	.375	.238
WSIR		.153	.010	.004	.014	.006	.014	.037	.075	.162	.293	.178
t	150	.049	.042	.034	.034	.036	.039	.043	.036	.040	.047	.047

Table 4.23. (Continued)

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
SSIR		.001	.001	.001	.001	.002	.012	.017	.057	.123	.344	.201
WSIR		.002	.000	.002	.002	.004	.012	.025	.074	.111	.322	.247
t	500	.055	.034	.040	.031	.036	.048	.055	.057	.041	.042	.042
χ^2		.064	.038	.042	.033	.038	.050	.055	.057	.042	.045	.045
SSIR		.000	.000	.000	.000	.002	.003	.018	.051	.122	.353	.247
WSIR		.000	.000	.000	.000	.002	.005	.030	.069	.141	.319	.290
7. t	60	.047	.049	.054	.063	.055	.044	.037	.041	.053	.051	.051
χ^2		.310	.103	.078	.079	.068	.057	.047	.046	.060	.061	.061
SSIR		.000	.003	.008	.008	.007	.014	.047	.095	.143	.338	.259
WSIR		.178	.011	.005	.011	.012	.026	.043	.071	.144	.300	.173
t	150	.056	.053	.058	.058	.053	.054	.046	.044	.051	.037	.037
χ^2		.096	.072	.070	.064	.057	.060	.049	.049	.057	.040	.040
SSIR		.000	.002	.000	.001	.007	.013	.026	.071	.120	.337	.314
WSIR		.003	.002	.001	.000	.005	.005	.034	.080	.118	.319	.250
t	500	.051	.046	.041	.052	.048	.048	.058	.047	.051	.061	.060
χ^2		.067	.054	.046	.054	.048	.049	.059	.047	.051	.061	.060
SSIR		.000	.000	.000	.000	.001	.007	.023	.070	.111	.327	.298
WSIR		.000	.000	.000	.000	.000	.009	.028	.071	.135	.348	.315

Table 4.23. (Continued)

The results in Table 4.23 are based on $\boldsymbol{x}_i \sim LN(\mathbf{0}, \boldsymbol{I})$. This distribution does not belong to EC[23]. To our surprise, the results in this table are even better than the previous two tables which are made based on EC data. For the OLS estimator, the best results are obtained from the original dataset. In general, the rejection percentages of the adaptive trimming are less or equal to the original dataset. For the SIR estimator, the original dataset still does not work well. 20% or 30% trimming had the best results. The two SIR methods gave similar results, but in general WSIR's rejection percentages are higher than those for SSIR.

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
1. t	60	.065	.052	.055	.049	.051	.052	.054	.051	.047	.052	.052
χ^2		.328	.109	.077	.072	.069	.060	.059	.058	.062	.063	.063
SSIR		.001	.005	.006	.018	.015	.033	.048	.072	.087	.287	.150
WSIR		.171	.015	.015	.025	.027	.050	.054	.079	.101	.231	.147

Table 4.24. Test For $H_0 : \beta_i = 0, H_1 : \beta_i \neq 0$ With Type 6 $\boldsymbol{x}$

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
t	150	.036	.039	.049	.050	.050	.042	.037	.057	.050	.048	.048
χ^2		.082	.056	.062	.058	.053	.045	.041	.060	.051	.051	.051
SSIR		.000	.001	.001	.004	.012	.024	.025	.052	.069	.359	.220
WSIR		.001	.002	.007	.008	.017	.026	.035	.047	.074	.282	.218
t	500	.039	.052	.036	.057	.057	.050	.052	.059	.059	.058	.058
χ^2		.047	.054	.040	.060	.059	.051	.055	.060	.060	.058	.058
SSIR		.000	.000	.001	.001	.011	.007	.021	.045	.074	.456	.339
WSIR		.000	.000	.000	.002	.007	.011	.025	.045	.064	.423	.375
2. t	60	.050	.051	.042	.037	.041	.034	.038	.056	.078	.282	.038
χ^2		.334	.097	.067	.056	.058	.042	.046	.063	.091	.300	.044
SSIR		.000	.004	.002	.004	.011	.010	.029	.068	.170	.528	.051
WSIR		.166	.014	.008	.012	.014	.028	.033	.058	.094	.221	.015
t	150	.041	.038	.036	.029	.036	.038	.045	.043	.072	.405	.076
χ^2		.078	.049	.044	.037	.041	.038	.048	.045	.074	.412	.079
SSIR		.000	.001	.002	.006	.004	.010	.031	.061	.133	.623	.049
WSIR		.001	.003	.006	.003	.013	.018	.026	.045	.100	.240	.040
t	500	.039	.039	.028	.022	.026	.025	.021	.050	.063	.580	.104
χ^2		.046	.041	.029	.024	.027	.026	.022	.050	.064	.580	.105
SSIR		.000	.000	.000	.002	.002	.005	.017	.040	.139	.739	.108
WSIR		.000	.000	.001	.002	.004	.011	.025	.047	.092	.311	.139
3. t	60	.048	.040	.030	.031	.035	.038	.035	.054	.089	.276	.021
χ^2		.321	.081	.064	.042	.044	.045	.043	.066	.094	.285	.025
SSIR		.000	.003	.004	.005	.009	.015	.036	.082	.161	.464	.037
WSIR		.183	.019	.020	.017	.023	.036	.054	.067	.095	.251	.152
t	150	.041	.030	.021	.024	.019	.025	.025	.034	.080	.374	.036
χ^2		.076	.038	.025	.028	.022	.028	.030	.040	.083	.382	.036
SSIR		.001	.001	.000	.003	.003	.008	.018	.046	.149	.556	.023
WSIR		.004	.005	.003	.008	.007	.021	.019	.041	.084	.329	.264
t	500	.034	.027	.021	.012	.013	.018	.015	.019	.055	.528	.050
χ^2		.040	.033	.021	.015	.013	.019	.015	.019	.056	.529	.050
SSIR		.000	.000	.000	.000	.003	.005	.006	.026	.101	.696	.036
WSIR		.000	.000	.000	.006	.009	.008	.015	.044	.057	.458	.401
4. t	60	.045	.030	.033	.035	.043	.034	.038	.054	.090	.280	.101
χ^2		.318	.069	.070	.047	.048	.043	.046	.065	.098	.297	.108
SSIR		.000	.003	.003	.008	.015	.011	.035	.077	.171	.495	.080
WSIR		.180	.017	.016	.016	.020	.036	.050	.073	.096	.241	.145
t	150	.035	.025	.014	.028	.022	.034	.033	.026	.072	.411	.170
χ^2		.070	.032	.016	.034	.025	.036	.039	.031	.076	.418	.177

Table 4.24. (Continued)

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
SSIR		.000	.001	.001	.004	.002	.010	.026	.060	.118	.623	.098
WSIR		.003	.004	.005	.008	.006	.026	.025	.040	.080	.320	.244
t	500	.027	.030	.021	.018	.020	.022	.023	.026	.064	.562	.275
χ^2		.033	.034	.021	.018	.022	.023	.024	.026	.064	.565	.277
SSIR		.000	.000	.000	.000	.002	.005	.015	.038	.126	.711	.164
WSIR		.000	.000	.002	.005	.010	.010	.025	.035	.055	.461	.406
5. t	60	.056	.031	.032	.030	.035	.037	.043	.046	.087	.109	.021
χ^2		.333	.074	.056	.046	.043	.052	.048	.062	.096	.116	.031
SSIR		.000	.001	.003	.007	.007	.025	.029	.067	.132	.264	.026
WSIR		.191	.018	.024	.014	.015	.021	.030	.048	.102	.214	.021
t	150	.034	.026	.021	.025	.028	.034	.035	.054	.061	.133	.033
χ^2		.066	.039	.028	.033	.033	.037	.039	.058	.063	.140	.035
SSIR		.000	.004	.004	.005	.010	.015	.020	.055	.094	.318	.050
WSIR		.006	.002	.003	.003	.009	.016	.022	.044	.083	.259	.025
t	500	.022	.027	.021	.027	.019	.019	.036	.049	.062	.159	.066
χ^2		.029	.031	.021	.027	.019	.020	.036	.051	.064	.162	.069
SSIR		.000	.000	.001	.002	.002	.007	.025	.044	.097	.356	.163
WSIR		.000	.000	.001	.003	.006	.010	.023	.041	.085	.302	.114
6. t	60	.053	.036	.044	.042	.039	.038	.036	.051	.072	.062	.062
χ^2		.322	.073	.066	.057	.051	.048	.040	.057	.080	.066	.066
SSIR		.001	.002	.008	.012	.020	.028	.044	.060	.099	.316	.167
WSIR		.201	.018	.022	.017	.028	.028	.041	.064	.095	.254	.144
t	150	.030	.028	.019	.032	.039	.041	.043	.048	.066	.054	.054
χ^2		.068	.036	.025	.045	.043	.042	.046	.050	.069	.059	.059
SSIR		.001	.002	.002	.005	.008	.021	.026	.040	.087	.379	.242
WSIR		.002	.003	.005	.006	.012	.026	.031	.053	.077	.327	.247
t	500	.023	.035	.027	.039	.032	.036	.045	.056	.065	.071	.071
χ^2		.026	.037	.028	.039	.032	.036	.045	.057	.065	.073	.073
SSIR		.001	.000	.000	.002	.006	.013	.029	.048	.068	.486	.363
WSIR		.000	.000	.001	.007	.005	.014	.020	.043	.055	.470	.412
7. t	60	.049	.054	.036	.046	.043	.047	.042	.052	.082	.155	.031
χ^2		.328	.098	.067	.062	.051	.059	.053	.062	.091	.163	.040
SSIR		.002	.004	.005	.009	.012	.026	.026	.056	.116	.330	.035
WSIR		.182	.014	.013	.010	.009	.025	.025	.057	.102	.217	.017
t	150	.043	.024	.035	.025	.028	.044	.043	.049	.051	.246	.040
χ^2		.073	.036	.040	.030	.033	.048	.046	.053	.055	.249	.043
SSIR		.000	.000	.005	.007	.012	.017	.016	.052	.088	.427	.045
WSIR		.002	.002	.004	.003	.010	.014	.020	.042	.099	.237	.033

Table 4.24. (Continued)

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
t	500	.023	.025	.031	.033	.028	.025	.029	.045	.060	.293	.093
χ^2		.027	.030	.033	.036	.028	.025	.029	.047	.062	.296	.095
SSIR		.000	.000	.001	.006	.006	.005	.018	.047	.105	.538	.130
WSIR		.000	.000	.001	.003	.004	.011	.024	.043	.094	.329	.152

Table 4.24. (Continued)

The results in Table 4.24 are based on $\boldsymbol{x}_i \sim MVT_3$. As mentioned before, $\boldsymbol{x}_i$ has no second moments. So it is not a surprise that almost all the results for the original data set are not good. Most of the best results are obtained at 10% or 20% trimming. The adaptive trimming has better results than the original data set. For some models, the adaptive trimming works very well. As with the previous predictor distribution, the two SIR methods have similar results.

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
1. t	60	.045	.036	.045	.044	.050	.040	.040	.038	.046	.045	.045
χ^2		.298	.087	.070	.069	.068	.054	.048	.046	.057	.046	.046
SSIR		.001	.003	.007	.010	.016	.025	.042	.050	.071	.131	.055
WSIR		.167	.015	.012	.016	.024	.021	.039	.040	.064	.121	.091
t	150	.055	.052	.068	.060	.043	.053	.069	.061	.063	.067	.067
χ^2		.086	.070	.073	.064	.049	.061	.075	.066	.065	.069	.069
SSIR		.002	.003	.005	.007	.009	.010	.025	.031	.064	.143	.113
WSIR		.007	.001	.006	.010	.012	.014	.032	.047	.062	.147	.142
t	500	.038	.055	.044	.047	.049	.052	.055	.044	.049	.048	.048
χ^2		.047	.059	.049	.049	.050	.053	.055	.044	.050	.049	.049
SSIR		.000	.000	.002	.007	.005	.011	.017	.028	.055	.173	.152
WSIR		.000	.001	.000	.001	.007	.012	.017	.039	.049	.159	.159
2. t	60	.049	.037	.036	.030	.030	.021	.034	.030	.072	.167	.040
χ^2		.337	.098	.065	.042	.042	.033	.038	.043	.082	.184	.050
SSIR		.001	.005	.003	.003	.009	.009	.016	.041	.092	.293	.022
WSIR		.175	.016	.013	.010	.014	.022	.032	.036	.072	.142	.018
t	150	.040	.042	.041	.026	.021	.031	.029	.037	.039	.211	.037
χ^2		.076	.057	.049	.035	.022	.032	.032	.040	.043	.222	.042
SSIR		.000	.000	.001	.005	.008	.009	.020	.045	.084	.379	.034
WSIR		.004	.003	.002	.003	.009	.018	.022	.026	.050	.152	.024
t	500	.034	.030	.035	.032	.029	.026	.020	.027	.040	.335	.131
χ^2		.042	.031	.037	.033	.030	.026	.022	.027	.041	.337	.132

Table 4.25. Test For $H_0 : \beta_i = 0, H_1 : \beta_i \neq 0$ With Type 7 $\boldsymbol{x}$

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
SSIR		.000	.000	.000	.004	.008	.008	.011	.032	.054	.471	.053
WSIR		.000	.001	.002	.003	.002	.005	.018	.036	.068	.177	.121
3. t	60	.055	.030	.033	.022	.023	.021	.030	.034	.067	.181	.023
χ^2		.341	.096	.057	.036	.026	.025	.034	.040	.087	.187	.030
SSIR		.001	.004	.002	.003	.004	.007	.018	.037	.099	.291	.015
WSIR		.194	.020	.017	.014	.020	.025	.031	.040	.068	.144	.119
t	150	.039	.034	.023	.015	.018	.020	.016	.032	.041	.240	.031
χ^2		.070	.043	.028	.021	.019	.020	.020	.033	.045	.243	.032
SSIR		.000	.002	.000	.000	.001	.004	.005	.028	.066	.357	.006
WSIR		.004	.004	.006	.005	.014	.017	.030	.035	.057	.157	.148
t	500	.032	.022	.015	.019	.014	.018	.008	.014	.026	.348	.047
χ^2		.036	.023	.017	.022	.015	.019	.008	.016	.028	.350	.049
SSIR		.000	.000	.001	.001	.002	.003	.006	.009	.036	.463	.018
WSIR		.000	.002	.002	.004	.005	.011	.015	.035	.045	.185	.184
4. t	60	.040	.036	.032	.025	.025	.020	.024	.029	.058	.167	.079
χ^2		.329	.092	.048	.038	.032	.029	.029	.039	.066	.175	.089
SSIR		.002	.002	.002	.005	.005	.010	.028	.043	.091	.312	.050
WSIR		.180	.023	.021	.017	.025	.025	.030	.048	.063	.137	.104
t	150	.033	.029	.018	.019	.020	.022	.025	.035	.037	.243	.133
χ^2		.057	.039	.024	.021	.021	.023	.028	.036	.040	.248	.139
SSIR		.000	.000	.001	.001	.002	.009	.020	.035	.061	.343	.061
WSIR		.003	.004	.004	.006	.015	.015	.028	.037	.054	.161	.155
t	500	.028	.019	.014	.021	.022	.022	.016	.025	.038	.352	.265
χ^2		.036	.022	.016	.023	.023	.022	.016	.025	.039	.352	.265
SSIR		.000	.000	.001	.000	.002	.007	.009	.022	.058	.424	.129
WSIR		.000	.002	.001	.005	.010	.017	.017	.035	.050	.191	.190
5. t	60	.041	.036	.022	.023	.029	.032	.031	.039	.055	.108	.026
χ^2		.294	.083	.051	.037	.035	.042	.042	.043	.068	.117	.031
SSIR		.000	.004	.005	.001	.006	.020	.034	.048	.080	.175	.033
WSIR		.196	.019	.013	.007	.016	.014	.033	.033	.072	.141	.020
t	150	.037	.029	.030	.021	.019	.032	.033	.034	.041	.092	.030
χ^2		.083	.039	.035	.026	.023	.038	.036	.041	.043	.097	.032
SSIR		.001	.000	.005	.004	.007	.012	.017	.039	.070	.190	.051
WSIR		.004	.002	.004	.004	.005	.010	.019	.034	.056	.148	.027
t	500	.031	.026	.017	.024	.032	.027	.027	.034	.051	.139	.105
χ^2		.035	.027	.022	.025	.033	.027	.027	.034	.052	.139	.105
SSIR		.000	.000	.002	.004	.004	.006	.013	.020	.056	.202	.150
WSIR		.000	.002	.001	.001	.003	.008	.016	.029	.067	.179	.128

Table 4.25. (Continued)

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
6. t	60	.034	.030	.030	.023	.025	.030	.022	.033	.051	.054	.054
χ^2		.293	.072	.051	.033	.033	.040	.032	.035	.062	.060	.060
SSIR		.000	.003	.007	.011	.016	.019	.025	.038	.054	.156	.088
WSIR		.191	.018	.020	.019	.027	.026	.038	.039	.066	.155	.121
t	150	.041	.032	.027	.032	.023	.041	.047	.053	.052	.055	.055
χ^2		.080	.046	.035	.036	.028	.044	.051	.055	.055	.056	.056
SSIR		.000	.003	.002	.010	.009	.017	.025	.038	.054	.163	.138
WSIR		.009	.003	.007	.006	.013	.022	.019	.025	.054	.176	.169
t	500	.034	.020	.026	.028	.035	.035	.027	.037	.042	.061	.061
χ^2		.042	.025	.028	.029	.035	.035	.027	.038	.043	.061	.061
SSIR		.000	.001	.001	.004	.010	.016	.022	.030	.041	.203	.182
WSIR		.000	.001	.000	.001	.003	.007	.013	.033	.047	.195	.194
7. t	60	.042	.033	.035	.038	.032	.037	.043	.031	.049	.098	.018
χ^2		.290	.088	.052	.056	.038	.046	.056	.038	.057	.106	.022
SSIR		.000	.002	.006	.004	.010	.012	.031	.039	.072	.172	.021
WSIR		.200	.020	.017	.009	.013	.020	.026	.033	.063	.142	.019
t	150	.050	.034	.029	.029	.030	.034	.037	.038	.037	.100	.024
χ^2		.087	.043	.038	.033	.033	.037	.041	.041	.041	.102	.027
SSIR		.003	.004	.002	.001	.006	.009	.023	.032	.054	.171	.034
WSIR		.007	.002	.002	.002	.006	.010	.016	.033	.050	.148	.022
t	500	.027	.028	.033	.034	.039	.032	.035	.048	.052	.168	.100
χ^2		.037	.030	.033	.035	.040	.032	.035	.050	.052	.168	.100
SSIR		.000	.000	.000	.003	.008	.008	.012	.033	.071	.262	.138
WSIR		.000	.000	.000	.004	.004	.010	.009	.037	.071	.170	.119

Table 4.25. (Continued)

The results in Table 4.25 are based on $\boldsymbol{x}_i \sim MVT_5$. Now the predictor has both first and second moments. Compared with Table 4.24, the rejection percentages of 0% trimming here are lower. Generally we can get the best results after trimming 10% of the data. The other results are similar to the previous table.

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
1. t	60	.043	.063	.066	.054	.056	.051	.051	.051	.051	.056	.056
χ^2		.341	.115	.090	.074	.073	.064	.062	.057	.055	.062	.062
SSIR		.001	.006	.005	.013	.016	.018	.030	.034	.052	.075	.050
WSIR		.167	.022	.018	.020	.018	.017	.027	.038	.054	.075	.066

Table 4.26. Test For $H_0 : \beta_i = 0, H_1 : \beta_i \neq 0$ With Type 8 $\boldsymbol{x}$

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
t	150	.058	.047	.047	.044	.045	.047	.049	.060	.048	.049	.049
χ^2		.095	.065	.055	.054	.049	.050	.051	.065	.050	.052	.052
SSIR		.001	.002	.006	.012	.010	.012	.018	.032	.041	.068	.068
WSIR		.003	.004	.000	.009	.013	.022	.019	.035	.035	.070	.070
t	500	.051	.034	.032	.043	.048	.046	.047	.039	.044	.046	.046
χ^2		.059	.039	.034	.043	.048	.047	.048	.042	.045	.047	.047
SSIR		.000	.000	.000	.001	.004	.007	.015	.025	.027	.063	.063
WSIR		.000	.000	.000	.000	.004	.009	.016	.022	.027	.057	.057
2. t	60	.040	.050	.048	.030	.036	.037	.042	.038	.045	.070	.033
χ^2		.327	.095	.078	.050	.052	.050	.047	.044	.047	.080	.038
SSIR		.001	.002	.005	.003	.009	.010	.017	.023	.042	.097	.018
WSIR		.178	.024	.017	.014	.019	.014	.016	.027	.043	.067	.011
t	150	.045	.042	.028	.030	.024	.024	.028	.023	.034	.072	.046
χ^2		.083	.056	.032	.036	.027	.028	.029	.027	.035	.073	.046
SSIR		.001	.001	.005	.002	.007	.006	.012	.015	.029	.089	.017
WSIR		.005	.006	.003	.001	.013	.010	.021	.022	.039	.069	.023
t	500	.046	.028	.028	.022	.024	.023	.015	.027	.039	.071	.070
χ^2		.053	.028	.029	.022	.024	.023	.016	.027	.039	.072	.071
SSIR		.000	.000	.000	.001	.002	.002	.009	.015	.027	.090	.038
WSIR		.000	.000	.000	.003	.004	.005	.010	.017	.032	.055	.048
3. t	60	.041	.044	.028	.022	.026	.026	.029	.025	.029	.050	.026
χ^2		.307	.091	.062	.036	.033	.037	.038	.036	.033	.058	.032
SSIR		.000	.003	.003	.004	.005	.006	.010	.019	.032	.093	.004
WSIR		.192	.023	.029	.018	.020	.033	.031	.029	.060	.077	.070
t	150	.040	.026	.018	.025	.012	.019	.015	.018	.036	.081	.036
χ^2		.071	.035	.024	.030	.012	.022	.018	.018	.036	.081	.036
SSIR		.000	.000	.002	.005	.001	.002	.008	.010	.028	.105	.002
WSIR		.007	.005	.003	.008	.014	.014	.019	.036	.037	.070	.069
t	500	.037	.023	.021	.017	.019	.018	.005	.014	.026	.100	.058
χ^2		.041	.023	.023	.017	.020	.020	.005	.015	.027	.101	.059
SSIR		.000	.000	.000	.001	.000	.000	.003	.004	.015	.129	.004
WSIR		.000	.001	.001	.003	.005	.007	.019	.022	.030	.060	.060
4. t	60	.035	.044	.032	.019	.030	.026	.027	.027	.035	.050	.038
χ^2		.292	.086	.057	.037	.041	.033	.039	.035	.043	.057	.046
SSIR		.000	.001	.001	.003	.004	.007	.017	.028	.044	.087	.016
WSIR		.186	.024	.031	.020	.021	.029	.030	.037	.057	.074	.062
t	150	.038	.028	.022	.022	.010	.021	.028	.018	.035	.076	.073
χ^2		.072	.038	.025	.028	.011	.025	.030	.019	.036	.077	.073

Table 4.26. (Continued)

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
SSIR		.000	.000	.001	.004	.002	.005	.010	.013	.042	.094	.037
WSIR		.008	.004	.005	.006	.006	.012	.021	.033	.043	.066	.065
t	500	.031	.025	.024	.019	.022	.021	.015	.022	.025	.086	.086
χ^2		.037	.025	.026	.019	.023	.022	.015	.022	.026	.088	.088
SSIR		.000	.000	.000	.000	.000	.003	.005	.012	.033	.098	.055
WSIR		.000	.000	.000	.002	.005	.006	.012	.021	.033	.064	.064
5. t	60	.045	.035	.031	.031	.026	.032	.039	.038	.035	.063	.028
χ^2		.319	.081	.057	.047	.039	.040	.050	.045	.043	.071	.033
SSIR		.001	.004	.009	.005	.009	.017	.024	.033	.037	.069	.021
WSIR		.178	.023	.019	.007	.017	.018	.022	.027	.042	.080	.012
t	150	.034	.023	.022	.015	.017	.022	.034	.029	.036	.066	.047
χ^2		.059	.036	.026	.019	.020	.026	.035	.030	.042	.070	.052
SSIR		.000	.003	.001	.003	.003	.015	.016	.021	.039	.078	.038
WSIR		.009	.002	.001	.003	.007	.010	.015	.024	.039	.066	.018
t	500	.023	.023	.025	.012	.021	.027	.019	.026	.037	.059	.059
χ^2		.025	.027	.026	.013	.024	.029	.020	.026	.040	.060	.060
SSIR		.000	.000	.002	.001	.007	.007	.008	.017	.027	.062	.062
WSIR		.000	.002	.000	.001	.002	.003	.009	.017	.035	.075	.068
6. t	60	.042	.040	.019	.028	.034	.030	.031	.030	.041	.049	.049
χ^2		.288	.077	.048	.041	.045	.043	.046	.040	.052	.055	.055
SSIR		.000	.000	.007	.018	.011	.016	.018	.033	.042	.063	.048
WSIR		.184	.024	.028	.014	.021	.026	.031	.045	.053	.073	.068
t	150	.031	.030	.032	.024	.027	.038	.030	.036	.049	.055	.055
χ^2		.057	.043	.035	.030	.031	.042	.030	.036	.051	.058	.058
SSIR		.001	.002	.001	.005	.004	.006	.023	.025	.031	.068	.067
WSIR		.009	.000	.004	.006	.007	.014	.020	.024	.045	.068	.068
t	500	.023	.018	.025	.020	.030	.030	.025	.035	.040	.059	.059
χ^2		.029	.019	.028	.022	.030	.030	.026	.035	.041	.060	.060
SSIR		.000	.000	.001	.002	.009	.014	.015	.018	.037	.070	.070
WSIR		.000	.001	.000	.001	.008	.010	.019	.023	.036	.068	.068
7. t	60	.038	.037	.046	.037	.031	.035	.034	.036	.035	.058	.015
χ^2		.332	.095	.069	.054	.044	.046	.047	.045	.041	.063	.022
SSIR		.001	.005	.008	.008	.007	.008	.022	.024	.039	.073	.015
WSIR		.187	.015	.018	.008	.017	.014	.017	.031	.044	.074	.010
t	150	.041	.034	.033	.028	.029	.023	.038	.032	.037	.060	.032
χ^2		.065	.047	.037	.033	.033	.026	.045	.034	.038	.063	.034
SSIR		.001	.001	.001	.000	.005	.012	.019	.021	.037	.058	.022
WSIR		.005	.002	.002	.002	.004	.010	.018	.020	.041	.065	.024

Table 4.26. (Continued)

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
t	500	.038	.022	.022	.033	.033	.033	.019	.026	.043	.053	.053
χ^2		.043	.026	.024	.036	.033	.033	.020	.026	.045	.053	.053
SSIR		.000	.000	.000	.001	.002	.007	.010	.009	.030	.066	.064
WSIR		.001	.000	.000	.001	.003	.005	.009	.010	.021	.063	.061

Table 4.26. (Continued)

The results in Table 4.26 are based on $\boldsymbol{x}_i \sim MVT_{19}$. As mentioned in Section 4.2, this kind of distribution behaves like the multivariate normal distribution. Our results also reflect this tendency. Unlike the previous two tables, most of the best results here are obtained at the original data set, which is similar to the results based on the predictors with the multivariate normal distribution.

In general, for type 2 to 8 $\boldsymbol{x}$ distributions, adaptive trimming usually helped for OLS except for the type 4 model; whereas for SIR, adaptive trimming helped about as often as it failed.

3. Test For $H_0 : \boldsymbol{\beta}_O = \mathbf{0}, H_1 : \boldsymbol{\beta}_O \neq \mathbf{0}$

For this case, the coefficient in the true model is $\boldsymbol{\beta} = [1, 1, 0, 0]'$. Similar to the previous test, we put our results for the different distributions in different tables. Tables 4.27 to 4.34 have the results for the type 1 to 8 predictor distributions. The columns in these tables are the same as the tables for the previous tests.

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
1. F	60	.054	.063	.040	.050	.041	.056	.041	.039	.041	.044	.041
χ^2		.416	.134	.090	.070	.076	.076	.056	.060	.053	.049	.045
SSIR		.002	.007	.006	.007	.012	.023	.036	.037	.046	.063	.036
WSIR		.217	.012	.012	.016	.014	.023	.045	.056	.059	.080	.057
F	150	.052	.042	.048	.059	.056	.042	.048	.047	.048	.046	.046
χ^2		.115	.073	.064	.069	.063	.050	.057	.049	.057	.049	.049
SSIR		.000	.002	.003	.005	.009	.010	.020	.021	.040	.058	.050
WSIR		.002	.003	.001	.007	.006	.017	.017	.027	.039	.054	.051
F	500	.044	.038	.057	.054	.062	.063	.058	.049	.050	.059	.059
χ^2		.058	.043	.059	.056	.065	.064	.060	.051	.051	.062	.062

Table 4.27. Test For $H_0 : \boldsymbol{\beta}_o = \mathbf{0}, H_1 : \boldsymbol{\beta}_o \neq \mathbf{0}$ With Type 1 $\boldsymbol{x}$

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
SSIR		.000	.000	.002	.002	.005	.012	.013	.017	.035	.051	.051
WSIR		.000	.000	.000	.003	.006	.013	.014	.024	.041	.063	.063
2.F	60	.052	.050	.028	.031	.030	.026	.026	.039	.034	.057	.033
χ^2		.424	.111	.075	.046	.046	.036	.036	.049	.037	.064	.041
SSIR		.001	.003	.004	.004	.009	.010	.012	.017	.027	.065	.010
WSIR		.212	.023	.014	.007	.015	.016	.029	.034	.044	.077	.030
F	150	.045	.041	.044	.043	.035	.028	.024	.022	.028	.047	.046
χ^2		.101	.062	.056	.052	.044	.030	.030	.028	.031	.050	.049
SSIR		.002	.000	.005	.003	.006	.004	.006	.010	.023	.062	.010
WSIR		.006	.002	.006	.003	.007	.007	.012	.016	.035	.062	.039
F	500	.041	.031	.033	.035	.031	.018	.026	.022	.024	.062	.062
χ^2		.059	.037	.038	.040	.031	.021	.026	.024	.025	.063	.063
SSIR		.000	.000	.000	.000	.001	.001	.006	.008	.018	.055	.042
WSIR		.000	.000	.001	.002	.007	.007	.011	.012	.028	.054	.053
3. F	60	.044	.039	.024	.022	.023	.020	.016	.034	.032	.058	.030
χ^2		.428	.095	.055	.037	.034	.026	.024	.037	.041	.065	.038
SSIR		.001	.005	.003	.004	.004	.004	.004	.013	.028	.080	.004
WSIR		.229	.025	.020	.014	.024	.022	.030	.047	.050	.071	.066
F	150	.048	.039	.030	.026	.026	.020	.005	.012	.009	.036	.029
χ^2		.095	.057	.044	.032	.030	.023	.007	.013	.011	.041	.034
SSIR		.000	.000	.003	.001	.001	.000	.001	.006	.016	.061	.001
WSIR		.007	.003	.004	.001	.005	.012	.025	.027	.038	.062	.062
F	500	.042	.031	.025	.015	.015	.009	.011	.012	.013	.061	.061
χ^2		.055	.034	.026	.017	.016	.010	.011	.012	.013	.061	.061
SSIR		.000	.000	.000	.000	.000	.000	.001	.006	.003	.045	.003
WSIR		.000	.000	.001	.001	.006	.009	.014	.011	.026	.044	.044
4. F	60	.053	.029	.019	.018	.019	.020	.012	.029	.028	.053	.039
χ^2		.403	.092	.046	.035	.027	.021	.027	.042	.040	.071	.054
SSIR		.001	.002	.001	.003	.005	.005	.008	.014	.028	.076	.009
WSIR		.228	.025	.013	.017	.022	.024	.042	.049	.059	.082	.070
F	150	.041	.039	.027	.013	.028	.021	.013	.016	.018	.046	.045
χ^2		.086	.053	.033	.018	.032	.023	.015	.019	.019	.048	.048
SSIR		.000	.000	.002	.001	.002	.000	.001	.007	.015	.058	.012
WSIR		.007	.003	.004	.003	.005	.013	.06	.025	.043	.059	.059
F	500	.027	.026	.021	.012	.012	.011	.016	.014	.020	.057	.057
χ^2		.035	.029	.021	.012	.013	.012	.017	.014	.021	.058	.058
SSIR		.000	.000	.000	.001	.000	.000	.001	.002	.009	.053	.024
WSIR		.000	.000	.000	.002	.004	.013	.013	.010	.026	.042	.042

Table 4.27. (Continued)

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
5. F	60	.047	.031	.032	.017	.025	.023	.023	.027	.029	.057	.026
χ^2		.408	.086	.056	.038	.037	.033	.036	.038	.037	.074	.038
SSIR		.001	.005	.008	.007	.009	.013	.019	.028	.039	.074	.023
WSIR		.260	.032	.014	.019	.022	.009	.012	.033	.040	.066	.021
F	150	.033	.030	.009	.013	.017	.018	.014	.022	.037	.048	.047
χ^2		.078	.038	.017	.015	.021	.022	.016	.023	.039	.051	.050
SSIR		.000	.000	.000	.002	.004	.007	.014	.018	.029	.057	.040
WSIR		.010	.005	.002	.001	.003	.005	.010	.012	.024	.052	.026
F	500	.018	.019	.022	.015	.009	.011	.018	.018	.018	.042	.042
χ^2		.024	.023	.023	.016	.010	.012	.019	.019	.018	.044	.044
SSIR		.000	.000	.001	.002	.002	.004	.010	.014	.021	.051	.051
WSIR		.000	.000	.000	.001	.002	.005	.007	.008	.019	.049	.049
6. F	60	.044	.044	.038	.020	.026	.020	.021	.027	.030	.055	.055
χ^2		.405	.107	.069	.041	.045	.034	.034	.034	.040	.058	.058
SSIR		.000	.006	.008	.007	.011	.018	.019	.035	.032	.061	.041
WSIR		.262	.032	.019	.019	.024	.021	.027	.045	.050	.076	.068
F	150	.041	.026	.018	.019	.019	.027	.025	.023	.027	.036	.036
χ^2		.091	.042	.024	.025	.026	.034	.026	.024	.032	.045	.045
SSIR		.000	.000	.007	.004	.009	.008	.014	.024	.045	.070	.069
WSIR		.006	.003	.002	.002	.002	.006	.013	.022	.033	.052	.052
F	500	.024	.025	.027	.019	.015	.011	.015	.020	.028	.056	.056
χ^2		.035	.026	.029	.021	.016	.011	.017	.020	.028	.056	.056
SSIR		.000	.000	.001	.001	.005	.006	.010	.015	.025	.044	.044
WSIR		.000	.000	.001	.001	.004	.008	.017	.020	.027	.054	.054
7. F	60	.048	.039	.034	.026	.022	.023	.031	.034	.039	.059	.025
χ^2		.407	.101	.079	.053	.047	.042	.042	.041	.047	.072	.030
SSIR		.002	.004	.007	.006	.008	.008	.013	.023	.035	.060	.011
WSIR		.245	.031	.020	.011	.013	.016	.017	.031	.048	.074	.028
F	150	.053	.036	.023	.024	.024	.018	.019	.029	.042	.056	.051
χ^2		.101	.059	.036	.038	.032	.024	.022	.035	.044	.061	.055
SSIR		.000	.002	.004	.004	.002	.004	.007	.012	.028	.063	.037
WSIR		.007	.004	.002	.001	.003	.010	.010	.015	.032	.062	.039
F	500	.032	.024	.028	.022	.027	.031	.023	.017	.029	.053	.053
χ^2		.045	.032	.028	.025	.027	.033	.023	.017	.032	.054	.054
SSIR		.000	.000	.001	.001	.003	.005	.004	.014	.018	.063	.062
WSIR		.001	.000	.001	.000	.002	.005	.004	.012	.017	.055	.055

Table 4.27. (Continued)

The predictors used in Table 4.27 have the multivariate normal distribution $N_4(\mathbf{0}, \boldsymbol{I})$. We compared this table with Table 4.19. We found that most of the results here are similar to Table 4.19 except that the adaptive trimming does not have the same results as the 0% trimming as frequently in this table.

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
1. F	60	.033	.043	.060	.042	.049	.046	.051	.041	.058	.057	.057
χ^2		.389	.120	.107	.074	.077	.064	.062	.055	.072	.065	.065
SSIR		.002	.019	.030	.043	.060	.088	.245	.435	.604	.743	.607
WSIR		.286	.039	.029	.047	.069	.088	.227	.363	.523	.661	.586
F	150	.051	.054	.047	.055	.059	.058	.050	.054	.040	.045	.045
χ^2		.113	.079	.062	.063	.068	.064	.051	.059	.045	.049	.049
SSIR		.003	.004	.010	.019	.040	.067	.221	.402	.555	.723	.701
WSIR		.003	.010	.011	.028	.041	.082	.189	.315	.468	.662	.659
F	500	.059	.047	.050	.049	.048	.048	.050	.043	.059	.038	.038
χ^2		.067	.053	.055	.052	.051	.050	.052	.043	.062	.038	.038
SSIR		.001	.001	.003	.012	.019	.053	.179	.356	.560	.735	.735
WSIR		.000	.001	.003	.009	.021	.039	.147	.324	.466	.675	.675
2. F	60	.046	.039	.041	.033	.038	.077	.244	.271	.247	.257	.045
χ^2		.374	.091	.088	.059	.058	.103	.270	.296	.276	.276	.060
SSIR		.002	.011	.013	.022	.042	.126	.491	.723	.803	.881	.230
WSIR		.286	.039	.028	.035	.045	.095	.256	.459	.626	.746	.068
F	150	.043	.034	.024	.014	.015	.080	.239	.247	.212	.273	.056
χ^2		.103	.052	.034	.023	.020	.085	.252	.255	.224	.277	.058
SSIR		.004	.000	.005	.006	.015	.117	.520	.690	.775	.896	.255
WSIR		.012	.005	.013	.018	.033	.069	.260	.489	.640	.738	.080
F	500	.043	.028	.020	.023	.018	.043	.218	.207	.193	.214	.057
χ^2		.056	.036	.023	.027	.019	.045	.221	.210	.195	.215	.058
SSIR		.000	.001	.001	.006	.010	.054	.499	.658	.755	.889	.438
WSIR		.000	.000	.005	.015	.020	.069	.258	.504	.659	.740	.252
3. F	60	.054	.034	.027	.022	.030	.073	.248	.294	.297	.286	.028
χ^2		.388	.094	.060	.046	.045	.100	.286	.334	.321	.316	.042
SSIR		.003	.010	.007	.011	.034	.149	.500	.740	.818	.887	.163
WSIR		.311	.054	.047	.054	.064	.097	.243	.370	.557	.672	.592
F	150	.044	.028	.015	.014	.015	.086	.267	.223	.192	.270	.017
χ^2		.105	.043	.028	.015	.017	.086	.280	.240	.200	.280	.021
SSIR		.004	.000	.001	.005	.008	.116	.537	.706	.801	.879	.221
WSIR		.018	.008	.017	.024	.045	.074	.203	.351	.475	.678	.677
F	500	.035	.017	.020	.012	.016	.045	.191	.135	.107	.280	.019
χ^2		.044	.019	.022	.012	.017	.047	.194	.137	.110	.281	.019
SSIR		.000	.000	.000	.001	.001	.069	.458	.598	.683	.897	.318
WSIR		.000	.002	.006	.012	.031	.054	.162	.324	.490	.687	.687

Table 4.28. Test For $H_0 : \boldsymbol{\beta}_o = \mathbf{0}, H_1 : \boldsymbol{\beta}_o \neq \mathbf{0}$ With Type 2 $\boldsymbol{x}$

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
4. F	60	.044	.028	.024	.026	.025	.064	.202	.194	.238	.265	.187
χ^2		.382	.101	.059	.043	.042	.087	.231	.233	.261	.286	.214
SSIR		.002	.008	.008	.013	.037	.142	.504	.716	.798	.891	.475
WSIR		.305	.059	.049	.047	.063	.090	.224	.366	.553	.675	.595
F	150	.042	.024	.022	.014	.019	.073	.212	.174	.172	.247	.237
χ^2		.095	.037	.027	.023	.022	.081	.220	.182	.181	.256	.247
SSIR		.002	.000	.001	.005	.010	.105	.503	.686	.782	.874	.664
WSIR		.012	.008	.016	.027	.044	.065	.209	.343	.477	.666	.661
F	500	.022	.013	.020	.018	.022	.040	.201	.163	.139	.274	.274
χ^2		.030	.017	.022	.019	.023	.044	.206	.166	.143	.277	.277
SSIR		.000	.000	.000	.001	.007	.052	.510	.703	.778	.889	.855
WSIR		.000	.000	.003	.012	.028	.061	.153	.333	.477	.661	.661
5. F	60	.045	.021	.026	.028	.039	.057	.154	.192	.135	.098	.032
χ^2		.382	.089	.050	.050	.060	.075	.182	.221	.150	.114	.050
SSIR		.001	.008	.016	.021	.036	.103	.321	.552	.697	.764	.085
WSIR		.298	.049	.035	.040	.046	.095	.274	.460	.637	.736	.073
F	150	.029	.021	.026	.019	.020	.063	.140	.168	.116	.094	.036
χ^2		.085	.040	.030	.024	.024	.066	.150	.180	.125	.105	.039
SSIR		.002	.003	.007	.018	.025	.079	.336	.565	.671	.787	.104
WSIR		.013	.008	.008	.015	.026	.075	.263	.476	.634	.768	.081
F	500	.014	.022	.013	.025	.023	.046	.175	.155	.122	.068	.038
χ^2		.027	.025	.015	.025	.023	.048	.179	.0157	.125	.071	.041
SSIR		.000	.002	.004	.013	.012	.052	.331	.581	.704	.780	.325
WSIR		.000	.002	.003	.004	.014	.057	.266	.471	.662	.759	.284
6. F	60	.056	.040	.033	.034	.029	.050	.086	.092	.085	.073	.073
χ^2		.390	.095	.068	.053	.051	.070	.110	.112	.103	.089	.089
SSIR		.000	.006	.023	.032	.039	.117	.276	.463	.629	.777	.623
WSIR		.314	.062	.055	.060	.075	.115	.229	.370	.541	.657	.578
F	150	.034	.025	.026	.020	.024	.052	.109	.092	.081	.074	.074
χ^2		.091	.040	.039	.029	.030	.059	.121	.100	.091	.078	.078
SSIR		.003	.005	.014	.014	.026	.070	.246	.414	.579	.737	.721
WSIR		.018	.009	.015	.020	.025	.077	.214	.358	.488	.675	.675
F	500	.025	.023	.019	.035	.035	.048	.099	.096	.076	.082	.082
χ^2		.041	.027	.021	.036	.036	.051	.100	.098	.077	.084	.084
SSIR		.000	.000	.002	.011	.021	.063	.214	.384	.552	.771	.771
WSIR		.000	.000	.005	.009	.023	.037	.190	.342	.478	.689	.689
7. F	60	.047	.035	.032	.033	.041	.071	.164	.201	.188	.174	.028
χ^2		.393	.098	.069	.060	.057	.085	.189	.223	.208	.198	.043

Table 4.28. (Continued)

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
SSIR		.002	.009	.017	.024	.049	.103	.317	.604	.753	.860	.118
WSIR		.306	.044	.041	.048	.038	.087	.267	.467	.638	.744	.073
F	150	.041	.023	.025	.026	.026	.064	.150	.191	.157	.187	.042
χ^2		.091	.033	.037	.034	.033	.071	.159	.202	.168	.199	.047
SSIR		.002	.006	.007	.013	.020	.075	.338	.627	.748	.877	.102
WSIR		.019	.005	.011	.022	.033	.071	.261	.494	.640	.744	.074
F	500	.030	.029	.024	.024	.023	.042	.148	.177	.150	.157	.043
χ^2		.045	.035	.027	.025	.024	.043	.149	.179	.154	.161	.045
SSIR		.000	.001	.001	.008	.015	.064	.343	.637	.800	.859	.257
WSIR		.000	.002	.004	.010	.015	.069	.245	.491	.663	.740	.266

Table 4.28. (Continued)

The results in Table 4.28 are based on $\boldsymbol{x}_i \sim 0.6N_4(\mathbf{0}, \boldsymbol{I}) + 0.4N_4(\mathbf{0}, 25\boldsymbol{I})$. We compared this table with Table 4.20 and found that they are very similar. For example, the best results are obtained at 40% or 50% trimming except for the MLR model, the adaptive trimming has better results than the original data and so on.

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
1. F	60	.042	.060	.068	.049	.043	.045	.048	.042	.046	.040	.040
χ^2		.403	.144	.119	.080	.063	.066	.056	.057	.063	.049	.049
SSIR		.013	.047	.077	.136	.231	.402	.563	.665	.752	.818	.759
WSIR		.318	.101	.086	.127	.231	.368	.513	.612	.720	.787	.760
F	150	.051	.045	.057	.059	.055	.053	.065	.061	.051	.056	.056
χ^2		.108	.079	.077	.071	.074	.062	.071	.065	.053	.062	.062
SSIR		.005	.019	.037	.079	.235	.372	.505	.640	.692	.799	.794
WSIR		.018	.013	.045	.085	.221	.354	.491	.562	.665	.760	.760
F	500	.053	.046	.056	.052	.062	.051	.058	.048	.052	.052	.052
χ^2		.064	.051	.058	.054	.062	.052	.060	.049	.053	.052	.052
SSIR		.001	.007	.021	.055	.225	.355	.484	.624	.674	.797	.797
WSIR		.001	.005	.024	.054	.186	.336	.479	.584	.654	.774	.774
2. F	60	.055	.045	.055	.067	.131	.164	.135	.139	.111	.135	.045
χ^2		.407	.139	.099	.101	.173	.200	.177	.165	.134	.156	.062
SSIR		.007	.033	.057	.155	.377	.599	.707	.764	.824	.887	.345
WSIR		.345	.079	.081	.107	.251	.388	.562	.677	.773	.825	.138
F	150	.036	.033	.033	.053	.144	.155	.106	.074	.077	.134	.012
χ^2		.089	.052	.044	.061	.159	.169	.113	.083	.084	.142	.012
SSIR		.005	.015	.017	.092	.403	.592	.693	.734	.798	.879	.383

Table 4.29. Test For $H_0 : \boldsymbol{\beta}_o = 0, H_1 : \boldsymbol{\beta}_o \neq 0$ With Type 3 $\boldsymbol{x}$

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
WSIR		.020	.019	.029	.096	.251	.432	.586	.684	.764	.847	.105
F	500	.033	.039	.029	.047	.165	.130	.087	.066	.066	.138	.017
χ^2		.044	.042	.031	.048	.173	.134	.088	.068	.066	.139	.019
SSIR		.000	.008	.011	.053	.428	.545	.679	.737	.794	.877	.537
WSIR		.001	.006	.025	.068	.288	.470	.616	.692	.766	.823	.227
3. F	60	.050	.043	.044	.068	.131	.170	.168	.147	.136	.150	.011
χ^2		.406	.123	.076	.108	.170	.212	.202	.174	.157	.174	.017
SSIR		.005	.028	.057	.163	.410	.637	.753	.789	.842	.895	.299
WSIR		.370	.086	.087	.135	.235	.379	.507	.623	.688	.800	.766
F	150	.036	.027	.026	.048	.134	.119	.081	.080	.079	.133	.006
χ^2		.097	.042	.032	.061	.158	.130	.085	.084	.081	.142	.006
SSIR		.006	.012	.015	.103	.412	.619	.689	.743	.813	.885	.332
WSIR		.029	.022	.048	.101	.246	.371	.479	.557	.681	.744	.742
F	500	.023	.033	.018	.035	.117	.057	.028	.022	.039	.160	.006
χ^2		.037	.034	.019	.038	.118	.058	.029	.023	.040	.160	.007
SSIR		.000	.001	.005	.047	.327	.474	.568	.672	.728	.894	.446
WSIR		.002	.008	.024	.064	.220	.366	.473	.580	.689	.775	.775
4. F	60	.051	.041	.042	.079	.121	.138	.118	.133	.137	.153	.105
χ^2		.405	.118	.074	.101	.154	.170	.145	.153	.166	.168	.118
SSIR		.005	.029	.051	.172	.409	.642	.721	.773	.818	.892	.643
WSIR		.362	.087	.084	.130	.226	.384	.496	.633	.695	.802	.770
F	150	.034	.028	.025	.050	.133	.089	.071	.065	.081	.125	.125
χ^2		.082	.045	.034	.065	.150	.102	.082	.071	.087	.132	.132
SSIR		.004	.008	.017	.112	.380	.609	.692	.755	.810	.858	.767
WSIR		.026	.024	.046	.099	.255	.384	.482	.567	.668	.752	.751
F	500	.018	.031	.022	.047	.146	.070	.063	.064	.064	.157	.157
χ^2		.027	.039	.024	.048	.151	.073	.068	.067	.065	.159	.159
SSIR		.000	.001	.006	.054	.440	.603	.702	.768	.821	.884	.867
WSIR		.003	.004	.027	.062	.223	.361	.480	.590	.675	.765	.765
5. F	60	.049	.043	.047	.061	.099	.117	.092	.079	.058	.045	.031
χ^2		.377	.112	.087	.092	.143	.148	.119	.099	.073	.054	.062
SSIR		.002	.030	.052	.116	.259	.437	.593	.681	.731	.807	.120
WSIR		.361	.075	.071	.115	.257	.399	.577	.677	.769	.819	.110
F	150	.029	.024	.038	.043	.139	.118	.098	.073	.047	.047	.025
χ^2		.081	.047	.043	.058	.153	.135	.106	.080	.054	.050	.030
SSIR		.008	.015	.030	.084	.289	.505	.626	.700	.762	.819	.121
WSIR		.017	.031	.042	.091	.265	.459	.599	.673	.774	.845	.095
F	500	.024	.024	.025	.037	.168	.117	.098	.067	.047	.033	.027

Table 4.29. (Continued)

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
χ^2		.029	.029	.025	.039	.174	.120	.102	.071	.049	.033	.028
SSIR		.000	.005	.006	.055	.337	.521	.640	.700	.770	.819	.274
WSIR		.004	.006	.017	.054	.323	.483	.614	.689	.773	.826	.222
6. F	60	.041	.039	.050	.060	.091	.096	.091	.075	.063	.068	.068
χ^2		.384	.125	.091	.096	.125	.117	.114	.094	.081	.084	.084
SSIR		.003	.036	.050	.144	.288	.462	.558	.679	.736	.813	.767
WSIR		.397	.087	.088	.157	.245	.388	.509	.625	.696	.792	.772
F	150	.026	.040	.033	.047	.101	.106	.072	.058	.050	.041	.041
χ^2		.086	.063	.047	.056	.114	.114	.077	.063	.057	.047	.047
SSIR		.006	.022	.034	.083	.242	.417	.564	.639	.718	.809	.809
WSIR		.025	.025	.050	.105	.239	.385	.493	.590	.663	.766	.765
F	500	.019	.025	.035	.040	.111	.107	.099	.073	.064	.060	.060
χ^2		.028	.033	.039	.043	.111	.109	.106	.075	.065	.062	.062
SSIR		.000	.004	.027	.046	.243	.399	.512	.596	.689	.813	.813
WSIR		.005	.007	.020	.048	.230	.356	.461	.558	.651	.761	.761
7. F	60	.042	.051	.046	.057	.095	.110	.105	.099	.083	.083	.029
χ^2		.401	.142	.085	.101	.139	.145	.141	.119	.096	.097	.056
SSIR		.007	.030	.049	.095	.276	.466	.665	.742	.827	.853	.161
WSIR		.366	.075	.065	.102	.256	.384	.568	.680	.768	.821	.127
F	150	.029	.023	.032	.061	.106	.122	.103	.071	.058	.084	.014
χ^2		.068	.042	.047	.075	.117	.140	.111	.076	.064	.089	.017
SSIR		.003	.014	.030	.100	.298	.532	.661	.756	.814	.850	.129
WSIR		.024	.017	.033	.098	.249	.437	.581	.687	.769	.844	.094
F	500	.034	.031	.029	.042	.141	.095	.087	.065	.052	.076	.015
χ^2		.042	.038	.030	.044	.145	.099	.088	.068	.056	.078	.016
SSIR		.002	.004	.012	.055	.335	.556	.671	.746	.842	.875	.284
WSIR		.003	.005	.017	.056	.306	.479	.613	.685	.762	.825	.222

Table 4.29. (Continued)

The results in Table 4.29 are based on $\boldsymbol{x}_i \sim 0.4N_4(\mathbf{0}, \boldsymbol{I}) + 0.6N_4(\mathbf{0}, 25\boldsymbol{I})$ and they are similar to the results in Table 4.21.

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
1. F	60	.042	.037	.036	.041	.047	.049	.051	.052	.048	.063	.063
χ^2		.380	.115	.086	.083	.069	.071	.068	.069	.061	.076	.076
SSIR		.002	.010	.007	.009	.015	.026	.040	.063	.098	.525	.194
WSIR		.233	.020	.020	.024	.027	.042	.045	.067	.098	.404	.146
F	150	.051	.049	.065	.056	.060	.054	.057	.057	.043	.045	.045
χ^2		.114	.074	.078	.070	.070	.063	.066	.065	.049	.047	.047

Table 4.30. Test For $H_0 : \boldsymbol{\beta}_o = \mathbf{0}, H_1 : \boldsymbol{\beta}_o \neq \mathbf{0}$ With Type 4 $\boldsymbol{x}$

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
SSIR		.003	.003	.003	.005	.013	.022	.035	.037	.079	.572	.359
WSIR		.002	.002	.007	.006	.012	.023	.031	.034	.070	.442	.360
F	500	.050	.046	.044	.046	.037	.047	.041	.043	.045	.052	.052
χ^2		.062	.050	.046	.054	.039	.050	.043	.044	.047	.052	.052
SSIR		.000	.000	.001	.002	.005	.005	.022	.025	.048	.568	.541
WSIR		.000	.000	.000	.003	.003	.005	.011	.024	.039	.481	.480
2. F	60	.045	.044	.038	.036	.029	.022	.034	.038	.153	.641	.094
χ^2		.389	.115	.078	.064	.045	.044	.042	.047	.167	.663	.109
SSIR		.000	.005	.008	.008	.008	.017	.011	.034	.205	.864	.109
WSIR		.258	.019	.019	.021	.014	.029	.034	.037	.099	.416	.033
F	150	.052	.044	.038	.030	.039	.016	.029	.031	.143	.641	.121
χ^2		.106	.065	.052	.035	.047	.021	.035	.033	.147	.645	.124
SSIR		.000	.002	.001	.000	.002	.007	.008	.023	.156	.885	.110
WSIR		.005	.001	.005	.005	.007	.014	.011	.028	.072	.503	.052
F	500	.041	.034	.029	.026	.015	.018	.014	.026	.065	.680	.112
χ^2		.051	.040	.035	.028	.018	.018	.016	.028	.066	.683	.113
SSIR		.000	.000	.001	.001	.001	.003	.008	.021	.089	.889	.128
WSIR		.000	.000	.001	.003	.004	.007	.012	.026	.067	.534	.250
3. F	60	.046	.044	.033	.030	.020	.016	.022	.024	.153	.641	.031
χ^2		.380	.115	.072	.060	.039	.024	.028	.039	.166	.656	.040
SSIR		.000	.003	.011	.004	.001	.008	.008	.020	.181	.808	.047
WSIR		.267	.026	.020	.024	.028	.031	.047	.054	.115	.496	.187
F	150	.049	.036	.025	.019	.016	.015	.020	.024	.127	.666	.045
χ^2		.096	.047	.030	.025	.021	.016	.023	.026	.130	.678	.046
SSIR		.000	.002	.000	.000	.003	.004	.003	.014	.175	.885	.046
WSIR		.007	.003	.010	.007	.012	.013	.030	.027	.081	.506	.421
F	500	.037	.027	.019	.012	.008	.009	.010	.016	.096	.666	.081
χ^2		.046	.032	.020	.013	.008	.010	.010	.016	.098	.668	.083
SSIR		.000	.000	.000	.000	.001	.000	.001	.004	.105	.875	.071
WSIR		.000	.000	.001	.005	.004	.011	.019	.032	.048	.552	.552
4. F	60	.043	.035	.040	.031	.023	.020	.021	.032	.141	.607	.176
χ^2		.386	.109	.066	.051	.038	.029	.031	.038	.159	.626	.191
SSIR		.000	.004	.007	.004	.001	.006	.013	.028	.200	.860	.118
WSIR		.265	.027	.019	.024	.026	.033	.049	.059	.112	.502	.192
F	150	.037	.029	.018	.016	.021	.015	.019	.022	.119	.598	.352
χ^2		.093	.042	.030	.019	.027	.017	.026	.026	.127	.600	.356
SIR		.000	.000	.001	.000	.003	.005	.005	.018	.168	.878	.186
WSIR		.006	.003	.008	.014	.016	.021	.028	.027	.080	.501	.416

Table 4.30. (Continued)

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
F	500	.028	.027	.020	.012	.015	.015	.011	.013	.091	.591	.554
χ^2		.034	.030	.022	.013	.015	.016	.012	.013	.091	.591	.554
SSIR		.000	.000	.000	.000	.000	.001	.002	.007	.085	.875	.400
WSIR		.000	.000	.000	.002	.003	.008	.016	.034	.051	.534	.534
5. F	60	.044	.028	.030	.023	.019	.025	.026	.025	.100	.316	.026
χ^2		.376	.097	.063	.044	.028	.031	.038	.035	.112	.335	.034
SSIR		.000	.003	.009	.006	.012	.013	.029	.041	.123	.575	.050
WSIR		.294	.038	.019	.019	.013	.018	.035	.041	.090	.438	.035
F	150	.033	.025	.015	.019	.014	.021	.023	.034	.085	.345	.064
χ^2		.076	.041	.024	.021	.015	.024	.026	.036	.088	.355	.071
SSIR		.000	.003	.002	.001	.008	.009	.016	.026	.088	.628	.095
WSIR		.011	.003	.007	.005	.007	.015	.009	.024	.080	.507	.061
F	500	.028	.018	.017	.018	.013	.013	.016	.019	.063	.344	.095
χ^2		.034	.020	.018	.018	.013	.016	.016	.021	.064	.346	.096
SSIR		.000	.000	.001	.000	.002	.006	.014	.026	.065	.661	.255
WSIR		.001	.000	.001	.000	.002	.005	.011	.016	.063	.539	.267
6. F	60	.051	.033	.030	.031	.028	.028	.027	.037	.064	.137	.137
χ^2		.370	.096	.057	.050	.042	.042	.036	.049	.077	.164	.164
SSIR		.000	.000	.004	.016	.012	.014	.029	.037	.116	.649	.240
WSIR		.293	.042	.026	.025	.029	.037	.047	.055	.112	.515	.197
F	150	.041	.026	.025	.021	.017	.018	.024	.036	.071	.158	.158
χ^2		.081	.042	.036	.025	.021	.021	.027	.045	.073	.164	.164
SSIR		.002	.000	.003	.005	.003	.011	.018	.035	.087	.659	.434
WSIR		.005	.003	.011	.009	.011	.021	.023	.034	.086	.529	.445
F	500	.027	.025	.025	.016	.024	.024	.021	.034	.050	.149	.149
χ^2		.040	.030	.027	.019	.026	.026	.024	.034	.054	.151	.151
SSIR		.000	.001	.000	.002	.001	.013	.023	.017	.067	.639	.604
WSIR		.000	.002	.003	.006	.007	.016	.024	.030	.046	.574	.574
7. F	60	.051	.043	.033	.036	.024	.035	.041	.045	.094	.424	.040
χ^2		.400	.120	.064	.052	.040	.050	.053	.054	.109	.448	.049
SSIR		.000	.003	.010	.008	.014	.018	.031	.032	.105	.683	.072
WSIR		.270	.027	.021	.021	.014	.023	.031	.036	.083	.429	.031
F	150	.053	.026	.038	.032	.037	.023	.022	.025	.072	.475	.059
χ^2		.100	.045	.048	.038	.040	.023	.027	.032	.078	.484	.064
SSIR		.004	.002	.001	.008	.004	.009	.008	.025	.089	.785	.053
WSIR		.006	.003	.004	.007	.005	.006	.009	.027	.074	.480	.048
F	500	.028	.026	.029	.025	.024	.018	.017	.026	.047	.509	.102
χ^2		.037	.027	.031	.026	.025	.018	.017	.030	.047	.513	.103

Table 4.30. (Continued)

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
SSIR		.000	.001	.002	.002	.003	.007	.010	.024	.060	.864	.087
WSIR		.000	.000	.001	.004	.003	.007	.013	.021	.051	.533	.269

Table 4.30. (Continued)

We will compare Table 4.30 with Table 4.22. The predictors for both tables have the distribution $0.9N_4(\mathbf{0}, \boldsymbol{I}) + 0.1N_4(\mathbf{0}, 25\boldsymbol{I})$. Like Table 4.22, the best results here are obtained at 10% or 20%.

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
1. F	60	.047	.057	.058	.054	.046	.045	.041	.051	.057	.045	.045
χ^2		.391	.152	.112	.091	.077	.064	.051	.071	.071	.058	.058
SSIR		.000	.001	.002	.007	.014	.018	.054	.079	.181	.471	.327
WSIR		.189	.009	.003	.005	.011	.034	.070	.112	.197	.426	.232
F	150	.055	.039	.050	.046	.055	.063	.054	.045	.052	.047	.047
χ^2		.100	.061	.060	.059	.064	.069	.060	.052	.059	.050	.050
SSIR		.000	.000	.000	.002	.003	.012	.032	.099	.168	.451	.360
WSIR		.000	.001	.001	.003	.005	.024	.035	.074	.180	.455	.361
F	500	.042	.051	.056	.054	.052	.058	.051	.046	.051	.047	.047
χ^2		.049	.057	.060	.058	.053	.060	.052	.046	.053	.050	.050
SSIR		.000	.000	.000	.001	.001	.006	.014	.059	.166	.471	.393
WSIR		.000	.000	.000	.000	.001	.007	.043	.068	.165	.486	.458
2. F	60	.046	.056	.054	.043	.033	.031	.033	.031	.039	.068	.067
χ^2		.394	.149	.101	.071	.052	.051	.042	.046	.051	.076	.075
SSIR		.000	.001	.003	.004	.010	.020	.028	.095	.180	.493	.237
WSIR		.235	.018	.004	.005	.020	.024	.054	.105	.182	.427	.252
F	150	.055	.040	.036	.044	.035	.027	.025	.042	.023	.058	.058
χ^2		.102	.059	.050	.053	.041	.032	.034	.044	.024	.058	.058
SSIR		.000	.000	.000	.003	.001	.004	.023	.082	.113	.457	.194
WSIR		.004	.001	.000	.000	.005	.014	.028	.085	.180	.464	.364
F	500	.039	.048	.043	.045	.054	.052	.053	.053	.051	.056	.054
χ^2		.052	.055	.050	.047	.059	.055	.056	.057	.051	.057	.055
SSIR		.000	.000	.000	.000	.002	.010	.019	.057	.112	.415	.201
WSIR		.000	.000	.000	.001	.002	.007	.028	.079	.160	.474	.437
3. F	60	.058	.041	.038	.023	.018	.010	.016	.019	.034	.078	.006
χ^2		.366	.121	.073	.045	.029	.015	.022	.027	.046	.084	.009
SSIR		.000	.000	.001	.003	.003	.003	.016	.053	.155	.499	.067
WSIR		.229	.014	.004	.003	.017	.023	.048	.092	.184	.450	.254

Table 4.31. Test For $H_0 : \boldsymbol{\beta}_o = \mathbf{0}, H_1 : \boldsymbol{\beta}_o \neq \mathbf{0}$ With Type 5 $\boldsymbol{x}$

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
F	150	.028	.026	.026	.014	.012	.010	.017	.011	.011	.068	.006
χ^2		.075	.038	.033	.020	.017	.011	.017	.013	.012	.068	.007
SSIR		.000	.000	.000	.000	.000	.000	.006	.015	.056	.414	.028
WSIR		.004	.001	.000	.001	.004	.012	.032	.073	.170	.440	.354
F	500	.035	.028	.014	.017	.027	.018	.017	.012	.004	.060	.007
χ^2		.045	.032	.015	.019	.027	.018	.017	.013	.004	.060	.007
SSIR		.000	.000	.000	.000	.000	.000	.001	.001	.016	.347	.026
WSIR		.000	.000	.000	.000	.002	.005	.028	.077	.183	.469	.434
4. F	60	.057	.039	.037	.030	.024	.021	.018	.029	.039	.070	.050
χ^2		.376	.117	.075	.053	.038	.029	.028	.040	.048	.081	.058
SSIR		.000	.000	.002	.003	.006	.008	.027	.075	.174	.487	.134
WSIR		.232	.014	.004	.002	.019	.025	.049	.094	.183	.449	.255
F	150	.032	.029	.028	.023	.021	.016	.023	.034	.012	.053	.036
χ^2		.078	.044	.037	.028	.033	.016	.027	.036	.012	.058	.058
SSIR		.000	.000	.001	.000	.003	.005	.005	.018	.168	.878	.186
WSIR		.004	.001	.000	.000	.004	.012	.032	.071	.172	.439	.352
F	500	.032	.032	.021	.029	.043	.037	.041	.043	.027	.062	.044
χ^2		.039	.035	.022	.035	.045	.040	.043	.043	.031	.062	.045
SSIR		.000	.000	.000	.001	.000	.001	.012	.023	.077	.405	.158
WSIR		.000	.000	.000	.000	.004	.006	.025	.079	.190	.466	.431
5. F	60	.055	.040	.040	.033	.026	.023	.026	.035	.040	.054	.008
χ^2		.389	.126	.068	.060	.044	.030	.035	.047	.054	.061	.016
SSIR		.000	.001	.002	.004	.008	.022	.037	.072	.164	.400	.130
WSIR		.248	.009	.007	.001	.014	.021	.028	.072	.168	.432	.041
F	150	.030	.029	.027	.021	.019	.018	.024	.046	.037	.046	.020
χ^2		.077	.042	.038	.028	.025	.019	.032	.049	.045	.051	.021
SSIR		.000	.001	.000	.000	.003	.007	.025	.079	.157	.484	.284
WSIR		.000	.002	.001	.000	.006	.010	.019	.053	.147	.451	.069
F	500	.033	.033	.019	.026	.040	.041	.049	.073	.063	.051	.048
χ^2		.043	.037	.023	.026	.042	.041	.050	.076	.064	.054	.050
SSIR		.000	.000	.000	.000	.002	.008	.029	.071	.189	.487	.433
WSIR		.000	.000	.000	.000	.002	.008	.016	.060	.170	.504	.311
6. F	60	.058	.059	.040	.037	.026	.018	.022	.034	.042	.057	.057
χ^2		.413	.147	.088	.063	.053	.037	.030	.040	.055	.069	.069
SSIR		.001	.001	.001	.002	.004	.011	.025	.104	.179	.501	.339
WSIR		.000	.000	.000	.000	.002	.008	.016	.060	.170	.504	.311
F	150	.034	.044	.033	.036	.032	.017	.022	.043	.037	.039	.039
χ^2		.088	.062	.045	.048	.041	.022	.027	.047	.040	.045	.045

Table 4.31. (Continued)

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
SSIR		.000	.001	.000	.000	.002	.002	.012	.068	.150	.463	.333
WSIR		.195	.012	.011	.011	.013	.019	.060	.109	.213	.427	.236
F	500	.045	.054	.045	.050	.033	.022	.046	.057	.060	.056	.056
χ^2		.056	.065	.046	.051	.035	.023	.048	.059	.063	.057	.057
SSIR		.000	.000	.000	.000	.000	.002	.013	.061	.168	.473	.382
WSIR		.001	.000	.001	.002	.004	.017	.029	.077	.163	.418	.338
7. F	60	.045	.056	.055	.052	.052	.044	.050	.056	.048	.060	.060
χ^2		.403	.151	.107	.098	.077	.057	.067	.066	.060	.069	.069
SSIR		.000	.001	.003	.008	.008	.032	.049	.082	.176	.429	.337
WSIR		.000	.000	.000	.000	.003	.007	.029	.071	.146	.469	.430
F	150	.045	.039	.053	.048	.050	.054	.047	.047	.033	.038	.038
χ^2		.097	.062	.059	.057	.063	.062	.057	.053	.040	.042	.042
SSIR		.001	.000	.001	.000	.003	.009	.031	.077	.175	.476	.434
WSIR		.223	.012	.005	.006	.019	.022	.052	.108	.189	.451	.240
F	500	.043	.053	.053	.055	.050	.071	.057	.052	.039	.060	.060
χ^2		.054	.056	.058	.058	.053	.072	.058	.053	.039	.064	.064
SSIR		.000	.000	.000	.000	.002	.009	.024	.053	.148	.484	.464
WSIR		.003	.001	.000	.000	.005	.015	.030	.073	.150	.473	.375

Table 4.31. (Continued)

The results in Table 4.31 are based on $\boldsymbol{x}_i \sim LN(\mathbf{0}, \boldsymbol{I})$ and they are similar to the results in Table 4.23.

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
1. F	60	.052	.040	.039	.041	.044	.058	.063	.047	.054	.049	.049
χ^2		.411	.120	.081	.060	.071	.080	.081	.067	.068	.059	.059
SSIR		.001	.002	.007	.008	.019	.024	.043	.064	.135	.358	.168
WSIR		.234	.019	.016	.013	.025	.029	.046	.086	.117	.339	.188
F	150	.046	.042	.051	.039	.062	.046	.046	.042	.045	.050	.050
χ^2		.110	.063	.066	.059	.069	.052	.055	.046	.049	.058	.058
SSIR		.001	.001	.003	.007	.011	.018	.027	.052	.113	.443	.297
WSIR		.004	.002	.005	.002	.014	.021	.036	.053	.100	.396	.292
F	500	.048	.052	.040	.041	.049	.052	.052	.049	.045	.047	.047
χ^2		.060	.055	.049	.042	.050	.055	.053	.050	.046	.047	.047
SSIR		.000	.000	.001	.004	.006	.015	.027	.050	.088	.569	.436
WSIR		.000	.000	.003	.004	.003	.012	.029	.048	.081	.504	.442
2. F	60	.042	.040	.038	.028	.025	.041	.045	.065	.084	.433	.063
χ^2		.408	.123	.072	.044	.036	.059	.054	.073	.110	.466	.085

Table 4.32. Test For $H_0 : \boldsymbol{\beta}_o = \mathbf{0}, H_1 : \boldsymbol{\beta}_o \neq \mathbf{0}$ With Type 6 $\boldsymbol{x}$

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
SSIR		.001	.004	.007	.005	.006	.019	.035	.094	.211	.722	.087
WSIR		.258	.019	.013	.010	.014	.031	.045	.069	.133	.345	.032
F	150	.042	.030	.037	.037	.024	.035	.031	.042	.068	.582	.098
χ^2		.105	.047	.048	.044	.031	.037	.037	.046	.074	.598	.105
SSIR		.000	.000	.001	.001	.003	.006	.023	.064	.151	.812	.091
WSIR		.001	.000	.001	.006	.013	.017	.032	.054	.124	.384	.047
F	500	.047	.041	.031	.020	.016	.025	.028	.030	.066	.744	.143
χ^2		.060	.042	.032	.022	.018	.030	.028	.030	.066	.746	.145
SSIR		.000	.000	.000	.000	.003	.005	.010	.056	.164	.904	.142
WSIR		.000	.000	.000	.003	.004	.008	.013	.039	.102	.480	.227
3. F	60	.044	.043	.031	.023	.020	.028	.039	.055	.111	.388	.025
χ^2		.410	.107	.060	.036	.038	.043	.053	.065	.131	.405	.034
SSIR		.000	.002	.001	.003	.007	.010	.027	.092	.256	.639	.049
WSIR		.271	.026	.016	.014	.022	.026	.060	.071	.107	.380	.202
F	150	.053	.030	.035	.026	.017	.021	.026	.035	.087	.530	.039
χ^2		.102	.040	.046	.030	.019	.026	.030	.041	.094	.540	.045
SSIR		.001	.000	.001	.002	.003	.005	.021	.055	.192	.735	.033
WSIR		.001	.001	.003	.007	.012	.017	.025	.051	.086	.479	.377
F	500	.047	.029	.020	.016	.010	.021	.017	.020	.066	.711	.077
χ^2		.059	.038	.022	.018	.010	.023	.017	.020	.066	.712	.077
SSIR		.000	.000	.000	.000	.000	.003	.006	.026	.126	.864	.058
WSIR		.000	.000	.000	.004	.007	.007	.014	.044	.080	.599	.532
4. F	60	.051	.036	.036	.024	.027	.030	.038	.056	.102	.417	.124
χ^2		.395	.099	.058	.047	.047	.041	.054	.064	.120	.427	.132
SSIR		.000	.003	.001	.003	.007	.014	.035	.095	.224	.702	.120
WSIR		.270	.025	.012	.018	.019	.027	.056	.068	.113	.374	.203
F	150	.046	.028	.033	.021	.018	.028	.032	.042	.071	.597	.242
χ^2		.102	.043	.045	.028	.024	.034	.037	.050	.078	.600	.248
SSIR		.000	.000	.000	.001	.003	.007	.024	.058	.150	.795	.143
WSIR		.001	.000	.003	.007	.009	.019	.030	.052	.082	.489	.378
F	500	.039	.020	.021	.019	.011	.026	.018	.028	.070	.757	.381
χ^2		.050	.028	.027	.020	.012	.027	.019	.028	.070	.758	.382
SSIR		.000	.000	.000	.000	.000	.005	.015	.039	.165	.875	.223
WSIR		.000	.001	.001	.001	.002	.012	.017	.040	.079	.572	.506
5. F	60	.049	.030	.025	.024	.037	.039	.046	.050	.076	.228	.033
χ^2		.416	.090	.072	.041	.053	.055	.057	.062	.097	.257	.043
SSIR		.000	.006	.006	.008	.015	.026	.036	.062	.153	.466	.054
WSIR		.286	.017	.009	.011	.015	.021	.040	.064	.117	.359	.023

Table 4.32. (Continued)

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
F	150	.038	.025	.021	.021	.024	.028	.035	.049	.062	.220	.037
χ^2		.074	.038	.032	.029	.028	.030	.038	.052	.064	.229	.038
SSIR		.000	.000	.000	.004	.005	.015	.029	.057	.141	.494	.090
WSIR		.005	.001	.000	.002	.005	.012	.017	.052	.118	.387	.040
F	500	.016	.012	.021	.019	.016	.020	.030	.028	.064	.324	.119
χ^2		.025	.018	.025	.019	.016	.021	.030	.029	.065	.326	.122
SSIR		.000	.000	.000	.003	.003	.008	.015	.038	.132	.605	.309
WSIR		.000	.000	.000	.001	.003	.010	.019	.045	.097	.519	.264
6. F	60	.054	.028	.032	.035	.028	.038	.043	.045	.063	.095	.095
χ^2		.405	.093	.074	.051	.046	.054	.058	.059	.075	.109	.109
SSIR		.000	.002	.004	.012	.013	.021	.045	.068	.123	.409	.197
WSIR		.293	.023	.015	.020	.022	.031	.053	.078	.124	.398	.223
F	150	.036	.040	.027	.033	.028	.032	.039	.057	.066	.102	.102
χ^2		.091	.061	.037	.044	.031	.032	.042	.064	.074	.107	.107
SSIR		.000	.002	.001	.002	.008	.018	.026	.054	.108	.521	.369
WSIR		.000	.001	.002	.007	.008	.021	.026	.047	.096	.502	.394
F	500	.026	.017	.027	.026	.023	.035	.032	.048	.064	.103	.103
χ^2		.036	.019	.029	.028	.023	.036	.035	.048	.066	.105	.105
SSIR		.000	.000	.002	.004	.004	.012	.024	.042	.099	.635	.485
WSIR		.000	.001	.000	.001	.004	.011	.028	.039	.100	.599	.535
7. F	60	.041	.035	.037	.028	.037	.033	.049	.049	.069	.253	.027
χ^2		.377	.112	.073	.044	.054	.047	.065	.065	.087	.266	.041
SSIR		.000	.005	.004	.003	.011	.013	.038	.063	.145	.499	.055
WSIR		.284	.034	.010	.007	.016	.016	.053	.071	.134	.356	.025
F	150	.041	.042	.032	.028	.027	.027	.042	.051	.062	.320	.047
χ^2		.105	.060	.044	.036	.031	.030	.047	.054	.071	.324	.050
SSIR		.000	.001	.002	.001	.007	.010	.023	.064	.125	.588	.082
WSIR		.003	.000	.002	.003	.007	.012	.026	.056	.123	.392	.044
F	500	.043	.033	.027	.030	.030	.029	.028	.039	.046	.444	.145
χ^2		.055	.038	.029	.031	.033	.031	.028	.043	.049	.447	.147
SSIR		.000	.000	.000	.000	.003	.007	.017	.036	.107	.732	.205
WSIR		.000	.000	.000	.002	.002	.011	.015	.037	.106	.491	.240

Table 4.32. (Continued)

The results in Table 4.32 are based on $\boldsymbol{x}_i \sim MVT_3$ and they are similar to the results in Table 4.24.

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
1. F	60	.045	.050	.043	.047	.055	.058	.048	.048	.067	.077	.070
χ^2		.384	.116	.084	.084	.084	.082	.068	.063	.077	.089	.082
SSIR		.000	.007	.005	.014	.015	.032	.037	.063	.094	.192	.098
WSIR		.207	.024	.007	.016	.025	.025	.051	.063	.095	.167	.125
F	150	.050	.042	.060	.053	.051	.047	.053	.046	.045	.046	.046
χ^2		.103	.072	.076	.062	.063	.055	.059	.050	.049	.049	.049
SSIR		.000	.000	.001	.005	.005	.015	.026	.037	.068	.178	.136
WSIR		.003	.000	.002	.009	.014	.019	.030	.038	.066	.166	.153
F	500	.059	.061	.050	.046	.058	.066	.060	.058	.050	.044	.044
χ^2		.075	.066	.058	.051	.059	.066	.063	.061	.051	.045	.045
SSIR		.000	.000	.000	.002	.005	.011	.015	.035	.060	.225	.206
WSIR		.000	.000	.000	.004	.007	.015	.020	.040	.052	.196	.196
2. F	60	.045	.048	.032	.027	.034	.039	.041	.047	.065	.252	.049
χ^2		.390	.125	.070	.056	.056	.045	.056	.055	.082	.281	.062
SSIR		.000	.007	.005	.006	.015	.022	.036	.054	.115	.433	.041
WSIR		.223	.029	.012	.016	.026	.030	.023	.045	.081	.182	.027
F	150	.047	.045	.041	.035	.032	.026	.032	.036	.050	.306	.086
χ^2		.101	.061	.052	.041	.037	.031	.035	.038	.054	.316	.089
SSIR		.000	.000	.002	.003	.002	.005	.012	.034	.106	.510	.046
WSIR		.006	.002	.003	.002	.010	.011	.021	.031	.067	.219	.050
F	500	.057	.040	.032	.028	.033	.025	.026	.032	.037	.453	.187
χ^2		.068	.043	.036	.033	.036	.027	.026	.032	.038	.453	.188
SSIR		.000	.000	.000	.002	.004	.014	.015	.022	.055	.660	.087
WSIR		.000	.000	.001	.001	.003	.006	.015	.027	.051	.221	.179
3. F	60	.047	.046	.029	.019	.033	.030	.034	.036	.068	.241	.028
χ^2		.391	.115	.054	.041	.047	.043	.045	.051	.080	.262	.036
SSIR		.001	.003	.007	.007	.008	.014	.023	.041	.121	.423	.015
WSIR		.234	.034	.013	.019	.022	.041	.042	.052	.086	.199	.142
F	150	.045	.035	.034	.029	.015	.021	.022	.029	.055	.305	.044
χ^2		.103	.051	.042	.035	.023	.023	.026	.031	.064	.317	.050
SSIR		.001	.000	.003	.001	.002	.002	.006	.028	.072	.507	.019
WSIR		.007	.004	.005	.006	.009	.017	.028	.041	.064	.208	.204
F	500	.049	.030	.019	.012	.019	.009	.013	.018	.024	.485	.064
χ^2		.060	.037	.021	.014	.019	.009	.014	.021	.024	.488	.065
SSIR		.000	.000	.000	.000	.001	.000	.002	.009	.035	.617	.017
WSIR		.000	.000	.000	.000	.002	.005	.011	.021	.034	.237	.237
4. F	60	.049	.042	.022	.020	.032	.029	.031	.039	.062	.247	.095
χ^2		.402	.099	.053	.039	.043	.048	.045	.046	.079	.267	.112

Table 4.33. Test For $H_0 : \boldsymbol{\beta}_o = \mathbf{0}, H_1 : \boldsymbol{\beta}_o \neq \mathbf{0}$ With Type 7 $\boldsymbol{x}$

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
SSIR		.001	.002	.005	.005	.008	.021	.028	.053	.132	.468	.057
WSIR		.245	.031	.013	.017	.023	.027	.039	.050	.080	.190	.143
F	150	.038	.029	.018	.027	.017	.020	.022	.034	.058	.334	.199
χ^2		.092	.042	.030	.031	.024	.025	.025	.035	.062	.346	.209
SSIR		.000	.000	.001	.000	.001	.003	.011	.027	.096	.488	.072
WSIR		.006	.004	.004	.007	.010	.012	.021	.036	.063	.214	.208
F	500	.027	.017	.017	.011	.017	.011	.018	.028	.037	.481	.373
χ^2		.041	.023	.018	.012	.020	.013	.018	.029	.040	.484	.377
SSIR		.000	.000	.000	.000	.000	.000	.002	.015	.068	.612	.146
WSIR		.000	.000	.000	.002	.002	.006	.010	.031	.052	.237	.237
5. F	60	.044	.033	.021	.031	.026	.032	.033	.037	.057	.145	.032
χ^2		.397	.112	.055	.045	.042	.042	.048	.049	.069	.163	.042
SSIR		.000	.004	.005	.008	.011	.020	.028	.053	.098	.263	.038
WSIR		.250	.022	.011	.010	.016	.018	.028	.047	.083	.202	.020
F	150	.031	.019	.019	.019	.019	.016	.017	.030	.049	.163	.042
χ^2		.073	.032	.026	.021	.022	.016	.023	.034	.051	.173	.050
SSIR		.000	.000	.003	.007	.008	.010	.013	.022	.075	.276	.077
WSIR		.004	.003	.001	.004	.002	.007	.019	.030	.060	.216	.045
F	500	.018	.018	.018	.020	.024	.020	.016	.036	.040	.170	.142
χ^2		.026	.020	.019	.022	.027	.021	.017	.036	.041	.170	.142
SSIR		.000	.000	.000	.001	.001	.005	.013	.037	.056	.296	.259
WSIR		.000	.000	.000	.000	.002	.000	.011	.016	.045	.233	.191
6. F	60	.044	.034	.034	.031	.031	.037	.028	.039	.050	.091	.091
χ^2		.397	.107	.068	.054	.046	.052	.040	.048	.061	.106	.106
SSIR		.000	.000	.004	.005	.016	.021	.035	.049	.083	.222	.131
WSIR		.267	.033	.020	.018	.022	.026	.032	.045	.071	.192	.135
F	150	.039	.019	.024	.026	.023	.026	.022	.037	.059	.097	.097
χ^2		.095	.038	.036	.030	.029	.028	.025	.041	.063	.102	.102
SSIR		.000	.001	.001	.005	.009	.011	.017	.025	.068	.216	.173
WSIR		.009	.004	.002	.004	.007	.012	.020	.038	.066	.236	.229
F	500	.030	.018	.021	.023	.030	.013	.034	.036	.049	.093	.093
χ^2		.049	.020	.025	.025	.030	.014	.036	.040	.050	.093	.093
SSIR		.000	.000	.000	.002	.003	.006	.016	.028	.048	.277	.234
WSIR		.000	.000	.000	.003	.003	.003	.012	.022	.038	.283	.283
7. F	60	.044	.034	.040	.032	.030	.040	.034	.039	.054	.134	.028
χ^2		.383	.108	.072	.055	.048	.052	.047	.048	.070	.158	.037
SSIR		.000	.003	.005	.006	.009	.016	.022	.035	.075	.248	.026
WSIR		.256	.027	.014	.018	.013	.020	.026	.046	.078	.192	.024

Table 4.33. (Continued)

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
F	150	.031	.030	.029	.026	.028	.023	.028	.034	.040	.151	.036
χ^2		.085	.054	.041	.036	.033	.027	.034	.040	.043	.159	.042
SSIR		.001	.000	.000	.001	.001	.002	.016	.031	.065	.256	.060
WSIR		.009	.002	.002	.004	.004	.009	.016	.037	.074	.219	.049
F	500	.045	.026	.027	.019	.024	.020	.021	.029	.032	.197	.151
χ^2		.056	.029	.028	.021	.025	.022	.024	.030	.033	.199	.153
SSIR		.000	.000	.000	.001	.000	.003	.006	.017	.051	.324	.229
WSIR		.000	.000	.000	.000	.000	.002	.007	.021	.051	.235	.197

Table 4.33. (Continued)

The results in Table 4.33 are based on $\boldsymbol{x}_i \sim MVT_5$ and they are similar to the results in Table 4.25.

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
1. F	60	.061	.053	.051	.047	.045	.049	.063	.054	.055	.043	.041
χ^2		.399	.132	.105	.078	.077	.072	.073	.066	.061	.050	.048
SSIR		.001	.003	.003	.015	.014	.020	.030	.040	.056	.073	.039
WSIR		.236	.018	.016	.019	.023	.031	.035	.056	.063	.093	.064
F	150	.048	.053	.053	.051	.052	.044	.047	.050	.040	.057	.057
χ^2		.109	.080	.068	.061	.061	.054	.053	.053	.046	.058	.058
SSIR		.000	.004	.001	.010	.004	.014	.021	.029	.052	.066	.059
WSIR		.002	.001	.000	.007	.006	.015	.025	.040	.054	.087	.080
F	500	.051	.039	.053	.059	.059	.061	.058	.052	.048	.054	.054
χ^2		.063	.051	.054	.060	.065	.063	.060	.055	.050	.055	.055
SSIR		.000	.000	.000	.000	.005	.004	.012	.017	.032	.075	.074
WSIR		.000	.000	.000	.001	.006	.004	.018	.021	.039	.069	.069
2. F	60	.048	.042	.038	.028	.038	.034	.034	.040	.034	.077	.035
χ^2		.388	.109	.068	.060	.053	.049	.042	.050	.050	.082	.041
SSIR		.000	.001	.005	.008	.007	.008	.018	.019	.042	.107	.013
WSIR		.232	.018	.018	.017	.010	.023	.030	.043	.062	.091	.020
F	150	.043	.041	.040	.035	.029	.024	.023	.023	.024	.085	.061
χ^2		.100	.067	.052	.045	.041	.028	.027	.028	.029	.093	.066
SSIR		.000	.000	.002	.004	.004	.007	.009	.016	.031	.122	.016
WSIR		.001	.000	.006	.007	.008	.008	.013	.023	.040	.085	.029
F	500	.052	.040	.039	.032	.037	.026	.031	.023	.032	.096	.096
χ^2		.058	.048	.043	.034	.038	.026	.033	.026	.032	.100	.100
SSIR		.000	.001	.001	.000	.000	.003	.007	.003	.018	.151	.056
WSIR		.000	.002	.000	.005	.005	.008	.012	.016	.034	.086	.081

Table 4.34. Test For $H_0 : \boldsymbol{\beta}_o = \mathbf{0}, H_1 : \boldsymbol{\beta}_o \neq \mathbf{0}$ With Type 8 $\boldsymbol{x}$

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
3. F	60	.051	.035	.033	.021	.032	.027	.027	.027	.034	.070	.028
χ^2		.392	.107	.058	.040	.046	.042	.038	.037	.042	.087	.038
SSIR		.000	.001	.002	.005	.005	.004	.014	.016	.045	.125	.005
WSIR		.254	.020	.017	.033	.023	.030	.037	.043	.061	.091	.074
F	150	.046	.037	.033	.026	.023	.015	.015	.015	.018	.087	.043
χ^2		.085	.061	.042	.028	.027	.017	.017	.019	.022	.097	.048
SSIR		.000	.001	.001	.001	.002	.002	.003	.004	.019	.123	.004
WSIR		.002	.000	.004	.010	.009	.020	.013	.024	.046	.077	.077
F	500	.041	.033	.026	.020	.013	.009	.011	.007	.018	.123	.096
χ^2		.055	.037	.030	.021	.015	.011	.013	.007	.018	.124	.096
SSIR		.000	.000	.000	.000	.000	.001	.001	.003	.012	.170	.001
WSIR		.000	.000	.002	.002	.003	.009	.019	.023	.034	.077	.077
4. F	60	.047	.039	.025	.020	.027	.031	.027	.029	.037	.073	.048
χ^2		.389	.105	.048	.036	.045	.036	.041	.043	.049	.079	.053
SSIR		.000	.000	.002	.003	.005	.006	.010	.022	.048	.120	.019
WSIR		.240	.019	.015	.026	.022	.032	.033	.038	.065	.095	.076
F	150	.037	.031	.022	.021	.021	.015	.015	.018	.027	.088	.085
χ^2		.077	.054	.033	.027	.029	.016	.019	.020	.029	.091	.088
SSIR		.000	.000	.001	.001	.000	.002	.004	.009	.031	.126	.016
WSIR		.000	.000	.005	.009	.010	.021	.029	.027	.036	.079	.079
F	500	.029	.029	.023	.018	.013	.013	.017	.008	.023	.093	.093
χ^2		.036	.035	.026	.021	.013	.013	.017	.009	.023	.094	.094
SSIR		.000	.000	.000	.000	.000	.002	.002	.007	.013	.132	.040
WSIR		.000	.000	.002	.002	.002	.006	.015	.019	.032	.084	.084
5. F	60	.051	.037	.030	.019	.025	.024	.026	.036	.037	.072	.031
χ^2		.403	.101	.060	.032	.038	.037	.042	.046	.048	.086	.043
SSIR		.000	.004	.010	.011	.011	.019	.020	.037	.054	.093	.030
WSIR		.274	.027	.019	.017	.015	.015	.015	.043	.050	.088	.011
F	150	.030	.021	.020	.024	.017	.017	.020	.025	.029	.076	.065
χ^2		.083	.038	.022	.030	.023	.021	.023	.030	.033	.080	.068
SSIR		.000	.002	.002	.004	.006	.008	.014	.019	.043	.094	.059
WSIR		.008	.001	.001	.005	.003	.011	.010	.017	.029	.080	.030
F	500	.023	.019	.021	.021	.019	.018	.025	.021	.029	.077	.077
χ^2		.027	.021	.023	.021	.020	.018	.026	.023	.031	.080	.080
SSIR		.000	.001	.001	.002	.004	.006	.013	.014	.030	.100	.100
WSIR		.000	.001	.000	.000	.002	.004	.007	.013	.026	.082	.081
6. F	60	.050	.050	.031	.024	.029	.022	.032	.034	.038	.058	.058
χ^2		.406	.113	.068	.045	.043	.038	.045	.043	.047	.073	.073

Table 4.34. (Continued)

Model	n	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%	ADAP
SSIR		.000	.003	.003	.004	.005	.016	.016	.033	.053	.095	.067
WSIR		.278	.024	.018	.023	.026	.028	.030	.050	.052	.089	.076
F	150	.036	.028	.019	.028	.020	.018	.022	.029	.045	.061	.061
χ^2		.084	.059	.028	.031	.028	.020	.027	.033	.049	.064	.064
SSIR		.001	.001	.000	.001	.007	.013	.014	.024	.041	.068	.064
WSIR		.007	.000	.001	.007	.016	.011	.012	.027	.041	.091	.091
F	500	.030	.022	.033	.021	.022	.021	.025	.021	.035	.062	.062
χ^2		.036	.025	.036	.022	.022	.021	.026	.022	.037	.066	.066
SSIR		.001	.000	.001	.001	.003	.005	.008	.012	.030	.058	.058
WSIR		.000	.001	.002	.001	.001	.004	.007	.013	.028	.080	.080
7. F	60	.043	.048	.044	.034	.030	.021	.032	.044	.037	.061	.031
χ^2		.398	.107	.083	.060	.042	.034	.041	.059	.052	.074	.038
SSIR		.000	.004	.004	.010	.008	.010	.020	.038	.046	.086	.017
WSIR		.267	.021	.016	.015	.015	.022	.029	.043	.051	.079	.020
F	150	.047	.030	.033	.032	.028	.030	.030	.026	.037	.059	.041
χ^2		.097	.049	.041	.038	.035	.033	.032	.031	.041	.062	.046
SSIR		.001	.002	.001	.006	.006	.008	.008	.014	.034	.093	.043
WSIR		.005	.000	.001	.005	.006	.009	.014	.015	.042	.098	.045
F	500	.039	.030	.030	.023	.029	.026	.027	.029	.034	.082	.082
χ^2		.047	.037	.032	.026	.031	.026	.028	.030	.034	.082	.082
SSIR		.000	.000	.000	.002	.002	.002	.004	.013	.024	.085	.084
WSIR		.000	.000	.001	.001	.002	.001	.007	.016	.020	.083	.083

Table 4.34. (Continued)

The results in Table 4.34 are based on $\boldsymbol{x}_i \sim MVT_{19}$ and they are similar to the results in Table 4.26.

In general, SIR does not have good results except for the type 1 distribution although the adaptive trimming sometimes helped SSIR. For OLS, the adaptive trimming usually helped if the results for the 0% were bad.

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