Exam 3 is Wed. Nov. 20. **You are allowed 11 sheets of notes and a calculator.** Emphasis on Exam 3 is HW7-10 and Q7-9. Numbers refer to types of problems on exam.

From Exam 1 review, know pages 2-4. **Know all of Exam 2 review.**

62) If the data $Y_1, ..., Y_n$ is arranged in ascending order from smallest to largest and written as $Y_{(1)} \leq \cdots \leq Y_{(n)}$, then the $Y_{(i)}$'s are called the *order statistics.*

63) Let $\lceil x \rceil$ denote the smallest integer greater than or equal to x (eg, $\lceil 7.7 \rceil = 8$).

64) Consider intervals that contain c cases $(Y_{(1)}, Y_{(c)}), (Y_{(2)}, Y_{(c+1)}), ..., (Y_{(n-c+1)}, Y_{(n)})$. Compute $Y_{(c)} - Y_{(1)}, Y_{(c+1)} - Y_{(2)}, ..., Y_{(n)} - Y_{(n-c+1)}$. Then the estimator shorth$(c) = (Y_{(d)}, Y_{(d+c-1)})$ is the interval with the shortest length. A large sample $100(1-\alpha)\%$ prediction interval (PI) (L_n, U_n) is such that $P(Y_f \in (L_n, U_n)) \to 1 - \alpha$ as $n \to \infty$. The shorth(c) interval is a large sample $100(1-\alpha)\%$ PI if $c/n \to 1 - \alpha$ as $n \to \infty$ that often has the asymptotically shortest length. Be able to compute the shorth(c) interval given c and a small data set. See HW7 A).

65) If the data are iid and $c = \lceil n(1-\alpha) \rceil$, then for large n the coverage of the shorth(c) PI is about $1 - \alpha - 1.12\sqrt{\alpha/n}$.

66) Suppose there is data $Y_1, ..., Y_t$. Want to predict Y_{t+l} for $l = 1, ..., L$. Consider ARIMA(p,d,q) models where Y_t, W_t or Z_t follows a stationary, invertible ARMA(p,q) model. The l step ahead forecast for Y_{t+l} is $\hat{Y}_t(l)$. To find $\hat{Y}_t(l)$, suppose the model for Y_t is $Y_t = m_t + e_t$. Then $\hat{Y}_{t+l} = \hat{m}_{t+l}$. Write $\hat{Y}_t(l) = \hat{m}_{t+1}$ but add asterisks to the terms in $\hat{m}_{t+1}$. Using R notation, for $d = 0$,

$\hat{Y}_t(l) = \hat{\tau} + \hat{\phi}_1 Y^*_{t+l-1} + \cdots + \hat{\phi}_p Y^*_{t+l-p} + \hat{\theta}_1 \hat{e}^*_{t+l-1} + \cdots + \hat{\theta}_1 \hat{e}^*_{t+l-q}$. For $d = 1$,

$\hat{Y}_t(l) = \hat{\tau} + (1 + \phi_1) Y^*_{t+l-1} + (\phi_2 - \phi_1) Y^*_{t+l-2} + \cdots + (\phi_p - \phi_{p-1}) Y^*_{t+l-p} - \phi_p Y^*_{t+l-p-1} + \hat{\theta}_1 \hat{e}^*_{t+l-1} + \cdots + \hat{\theta}_1 \hat{e}^*_{t+l-q}$. For $d = 2$,

$\hat{Y}_t(l) = \hat{\tau} + (2 + \hat{\phi}_1) Y^*_{t+l-1} + (\hat{\phi}_2 - 2\hat{\phi}_1 - 1) Y^*_{t+l-2} + (\hat{\phi}_1 - 2\hat{\phi}_2 + \hat{\phi}_3) Y^*_{t+l-3} + \cdots + (\hat{\phi}_{p-2} - 2\hat{\phi}_{p-1} + \hat{\phi}_p) Y^*_{t+l-p} + (\hat{\phi}_{p-1} - 2\hat{\phi}_p) Y^*_{t+l-p-1} + \hat{\phi}_p Y^*_{t+l-p-2} + \hat{\theta}_1 \hat{e}^*_{t+l-1} + \cdots + \hat{\theta}_1 \hat{e}^*_{t+l-q}$.

For $d = 1, 2$, usually $\hat{\tau} = 0$. Here $Y^*_{t+l-j} = Y_{t+l-j}$ if $l \leq j$ and $Y^*_{t+l-j} = \hat{Y}_t(l-j)$ if $l > j$. Need to find $\hat{Y}_t(1), \hat{Y}_t(2), ..., \hat{Y}_t(L)$ recursively. Also $\hat{e}^*_{t+l-j} = \hat{e}_{t+l-j}$ if $l \leq j$ and $\hat{e}^*_{t+l-j} = 0$ if $l > j$. If the index $t + l - j > t$, then Y_{t+l-j} and $\hat{e}_{t+l-j}$ are not available and are replaced by their "best" estimates.

67) The l step ahead forecast residual $\hat{e}_t(l) = Y_{t+l} - \hat{Y}_t(l)$.

68) The residuals $\hat{e}_t$ and the fitted or predicted values $\hat{Y}_t$ for the ARIMA(p,d,q) model are $\hat{e}_t = \hat{e}_{t-1}(1)$ and $\hat{Y}_t = \hat{Y}_{t-1}(1)$ computed using $\hat{\tau}, \hat{\phi}_1, ..., \hat{\theta}_q$ computed from $Y_1, ..., Y_n$ and $\hat{Y}_t = \hat{Y}_{t-1}(1)$ computed from $Y_1, .., Y_{t-1}, \hat{e}_j, ..., \hat{e}_{t-1}$ where often $j = 1$. So the residuals and fitted values are the 1 step ahead residuals and forecasts.

69) For an MA(q) model, $\hat{Y}_t(l) = \hat{\tau} = \hat{\mu}_Y$ and $\hat{e}_t(l) = Y_{t+l} - \hat{\mu}_Y$ for $l > q$.

70) For a stationary invertible ARMA(p,q) model, $\hat{Y}_t(l) \to \hat{\mu}_Y$ and $V(\hat{e}_t(l)) \to V(Y_t) = \gamma_0$ as $l \to \infty$ and $n \to \infty$.

71) The shorth(c) PI is good for a stationary invertible ARMA(p,q) model when l is large or for $l > q$ for MA(q) models.

72) When each future value becomes available, update your ARIMA(p,d,q) model and then update your remaining prediction intervals.

73) Typically there is more than 1 reasonable ARIMA (p,d,q) model. For each model, keep track of its prediction interval performance. Discard models with poor performance

and keep models that perform well (short PIs with coverage near the nominal value, eg 95%).

74) Normal PIs are made assuming the $\{e_t\}$ are a normal white noise. Check this assumption with QQ plots and histograms of the residuals. There are also tests with Ho: the white noise $\{e_t\}$ is a normal vs Ha: the white noise $\{e_t\}$ is not a normal.

75) For stationary invertible ARMA(p,q) models and for large lead times l or for an MA(q) model with $l > q$, if $t = n$ is large then the normal $100(1-\alpha)\%$ PI for Y_{t+l} is approximately $\overline{Y} \pm z_{1-\alpha/2} S$ where $S = \hat{\sigma}_Y = \sqrt{\frac{n}{n-1}\hat{\gamma}_0}$ is the sample standard deviation of $Y_1, ..., Y_t$.

76) For ARIMA(p,d,q) models, a normal $100(1-\alpha)\%$ PI for Y_{t+l} is
$\hat{Y}_t(l) \pm t_{1-\alpha/2,n-p-q}\sqrt{\hat{V}(\hat{Y}_t(l))} = \hat{Y}_t(l) \pm t_{1-\alpha/2,n-p-q} SE(\hat{Y}_t(l)) = (L_n, U_n)$. Suppose that as $n \to \infty$, $\hat{Y}_t(l) \to E(Y_{t+l}) = \mu_{t+l}$ and $SE(\hat{Y}_t(l)) \to SD(Y_{t+l}) = \sigma_{t+l}$. Then
$P[Y_{t+l} \in (L_n, U_n)] \approx P[Y_{t+l} \in (\mu_{t+l} - z_{1-\alpha/2}\sigma_{t+l}, \mu_{t+l} + z_{1-\alpha/2}\sigma_{t+l})] =$
$P[|Y_{t+l} - \mu_{t+l}| < z_{1-\alpha/2}\sigma_{t+l}]$ " $\geq$ " $1 - \frac{1}{z^2_{1-\alpha/2}}$ assuming Chebyshev's inequality holds to a good approximation. Hence a 95% PI could have coverage as low as 75% and a 99.7% PI could have coverage as low as 89%.

R may not compute $SE(\hat{Y}_t(l))$ correctly if $d > 0$ and an intercept $\hat{\tau}$ is in the model.

77) The large sample shorth(c_1) $100(1-\alpha)\%$ PI (L_n, U_n) takes $\overline{e}_t = Y_t - \overline{Y}$. Then let shorth($\lceil n(1-\alpha)\rceil$) $= (\tilde{L}_n, \tilde{U}_n)$ be computed from the $\overline{e}_t$. Then $L_n = \overline{Y} + d_n \tilde{L}_n$ and $U_n = \overline{Y} + d_n \tilde{U}_n$ where $d_n = (1 + \dfrac{15}{n})\sqrt{\dfrac{n-1}{n+1}}$. For ARMA(p,q) models, this PI is too long for l near 1, but should be good for large l and if $l > q$ for an MA(q) model.

78) For ARIMA(p,d,q) models, let $c_2 = \lceil n(1-\alpha_n)\rceil$ and compute shorth(c_2) $= (\tilde{L}_n, \tilde{U}_n)$ of the l-step ahead forecast residuals $\hat{e}_t(l)$. Then a large sample $100(1-\alpha)\%$ PI for Y_{t+l} is $(L_n, U_n) = (\hat{Y}_n(l) + \tilde{L}_n, \hat{Y}_n(l) + \tilde{U}_n)$ where $1-\alpha_n = \min(1-\alpha+0.05, 1-\alpha+(p+q)/n)$ for $\alpha > 0.1$ and $1-\alpha_n = \min(1-\alpha/2, 1-\alpha+10(p+q)\alpha/n)$ for $\alpha \leq 0.1$.

79) PIs 77) and 78) attempt to compensate for the undercoverage of the shorth(c) interval in 65). The distribution of the white noise is assumed to be unknown, rather than a normal white noise. The PIs 77), 78) for $l = 1$ and 76) are compared for MA(2) data for four white noise distributions in HW7b. Expect PI 76) to be short with good coverage for a normal white noise. The coverage of PI 76) may not be near the nominal values when the white noise is not normal. Sometimes the length may be short and sometimes long. For $l > 2$ and large n, PI 77) should have good coverage and short length. Expect the 1-step ahead PI 78) to be shorter than the 1-step ahead PI 77). The endpoints of PI 77) do not depend on l so the length is the same for $l = 1, ..., L$. The coverage does vary some with l. Expect the 1-step ahead PI 78) to have coverage near the nominal value and to often be shorter than PI 76) for a nonnormal white noise, at least if n is large.

80) Suppose the coverage of a nominal $100(1-\alpha)\%$ PI is $1-\delta_n$. If $(1-\delta_n) < (1-\alpha)$, then the PI is liberal, while if $(1-\delta_n) > (1-\alpha)$, then the PI is conservative. In a simulation with m runs, the standard error of the coverage of a $100(1-\alpha)\%$ PI is $SE = \sqrt{\alpha(1-\alpha)/m}$. So a CI for the coverage is $[(1-\alpha) - cSE, (1-\alpha) + cSE)]$ with $c \in (2, 3)$. An observed coverage below the lower CI limit suggests that the PI is liberal while an observed coverage above the upper CI limit suggests that the PI is conservative.

81) Seasonal patterns often occur in time series. Let s be the seasonal period. Then $s = 12$ for monthly data and $s = 4$ for quarterly data are common. Plotting a letter for the month or A, B, C, D for the quarter in the plot of the time series can help show the seasonal pattern.

82) $Y_t \sim$ ARIMA(p,d,q)$\times(P, D, Q)_s$ is the multiplicative seasonal ARIMA model. Output in symbols and for a data set are shown below.

Label	coef	SE	LCI	UCI	Z	p-value
ar1	$\hat{\phi}_1$	$se(\hat{\phi}_1)$	95% CI	for ϕ_1	$Z_{ar,1} = \hat{\phi}_1/se(\hat{\phi}_1)$	for Ho: $\phi_1 = 0$
$\vdots$						
arp	$\hat{\phi}_p$	$se(\hat{\phi}_p)$	95% CI	for ϕ_p	$Z_{ar,p} = \hat{\phi}_p/se(\hat{\phi}_p)$	for Ho: $\phi_p = 0$
ma1	$\hat{\theta}_1$	$se(\hat{\theta}_1)$	95% CI	for θ_1	$Z_{ma,1} = \hat{\theta}_1/se(\hat{\theta}_1)$	for Ho: $\theta_1 = 0$
$\vdots$						
maq	$\hat{\theta}_q$	$se(\hat{\theta}_q)$	95% CI	for θ_q	$Z_{ma,q} = \hat{\theta}_q/se(\hat{\theta}_q)$	for Ho: $\theta_q = 0$
sar1	$\hat{\Phi}_1$	$se(\hat{\Phi}_1)$	95% CI	for Φ_1	$Z_{sar,1} = \hat{\Phi}_1/se(\hat{\Phi}_1)$	for Ho: $\Phi_1 = 0$
$\vdots$						
sarP	$\hat{\Phi}_P$	$se(\hat{\Phi}_P)$	95% CI	for Φ_P	$Z_{sar,P} = \hat{\Phi}_P/se(\hat{\Phi}_P)$	for Ho: $\Phi_P = 0$
sma1	$\hat{\Theta}_1$	$se(\hat{\Theta}_1)$	95% CI	for Θ_1	$Z_{sma,1} = \hat{\Theta}_1/se(\hat{\Theta}_1)$	for Ho: $\Theta_1 = 0$
$\vdots$						
smaQ	$\hat{\Theta}_Q$	$se(\hat{\Theta}_Q)$	95% CI	for Θ_Q	$Z_{sma,Q} = \hat{\Theta}_Q/se(\hat{\Theta}_Q)$	for Ho: $\Theta_Q = 0$
intercept	$\hat{\mu}_Y$	$se(\hat{\mu}_Y)$	95% CI	for μ_Y	$Z_o = \hat{\mu}_Y/se(\hat{\mu}_Y)$	for Ho: $\mu_Y = 0$

The intercept is actually the mean μ_Y and the $\hat{\theta}$ and $\hat{\Theta}$ are negatives of those in the book.

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Y <- log(AirPassengers) #ARIMA(p=2,d=1,q=3)x(P=1,D=0,Q=1)_{s=12} model
out<-arima(Y,c(2,1,3),seasonal=list(order=c(1,0,1), period=12),method="ML")
resplots(Y,out)
           coef          se        LCI        UCI          Z         pval
ar1   0.2160718 0.049836860  0.1163981  0.3157455   4.335582 1.453752e-05
ar2  -0.8823111 0.046145658 -0.9746024 -0.7900198 -19.120133 0.000000e+00
ma1  -0.5815285 0.088658705 -0.7588459 -0.4042110  -6.559181 5.410428e-11
ma2   1.0484197 0.044655669  0.9591083  1.1377310  23.477863 0.000000e+00
ma3  -0.4803567 0.090599145 -0.6615550 -0.2991584  -5.302000 1.145406e-07
sar1  0.9914837 0.004656271  0.9821712  1.0007962 212.935126 0.000000e+00
sma1 -0.5276964 0.080199086 -0.6880946 -0.3672983  -6.579831 4.709833e-11
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83) The model in 82) has parameters $\phi_1, ..., \phi_p$, $\theta_1, ..., \theta_q$, $\Phi_1, ..., \Phi_P$, $\Theta_1, ..., \Theta_Q$ with intercept τ and mean $E(Y_t) = \mu_Y$. The default is $\tau = 0$ and $\mu_Y = 0$ if $d > 0$ or $D > 0$. The 95% confidence intervals for a parameter τ_k are given by (LCI,UCI).

84) The 4 step test of hypotheses is i) Ho: $\tau_k = 0$ Ha: $\tau_k \neq 0$. ii) Get the test statistic Z from output. iii) Get the pval from output. iv) State whether you reject Ho or fail to reject Ho and give a conclusion. Reject Ho if pval $\leq \alpha$ and fail to reject Ho if pval $> \alpha$. **Use $\alpha = 0.05$ if α is not given.** For $\tau_k = \phi_k$ (θ_k), conclude Y_{t-k} (e_{t-k}) is needed in

the model if Ho is rejected. Conclude Y_{t-k} (e_{t-k}) is not needed in the model given the other terms are in the model if you fail to reject Ho. (Could use W_{t-k} if $d = 1$ or Z_{t-k} if $d = 2$.)

The conclusion is modified for seasonal parameters because often the model will have additional terms near Y_{t-ks} or near e_{t-ks}. For $\tau_k = \Phi_k$ conclude Φ_k and Y_{t-ks} are needed in the model if Ho is rejected. Conclude Φ_k and Y_{t-ks} are not needed in the model given the other terms are in the model if you fail to reject Ho.

For $\tau_k = \Theta_k$ conclude Θ_k and e_{t-ks} are needed in the model if Ho is rejected. Conclude Θ_k and e_{t-ks} are not needed in the model given the other terms are in the model if you fail to reject Ho. See HW8 Ckl).

85) Purely seasonal models: $MA(Q)_s$: $Y_t = \tau - \Theta_1 e_{t-s} - \Theta_2 e_{t-2s} - \cdots - \Theta_Q e_{t-Qs} + e_t = \tau + \Theta(B)e_t$ where $\Theta(B) = 1 - \Theta_1 B^s - \Theta_2 B^{2s} - \cdots - \Theta_Q B^{Qs}$. Recall that $B^k D_t = D_{t-k}$.

$AR(P)_s$: $Y_t = \tau + \Phi_1 Y_{t-s} + \Phi_2 Y_{t-2s} + \cdots + \Phi_P Y_{t-Ps} + e_t$, or $\Phi(B)Y_t = \tau + e_t$.

$ARMA(P,Q)_s : Y_t = \tau + \Phi_1 Y_{t-s} + \Phi_2 Y_{t-2s} + \cdots + \Phi_P Y_{t-Ps} - \Theta_1 e_{t-s} - \Theta_2 e_{t-2s} - \cdots - \Theta_Q e_{t-Qs} + e_t$, or $\Phi(B)Y_t = \tau + \Theta(B)e_t$ where $\Phi(B) = 1 - \Phi_1 B^s - \Phi_2 B^{2s} - \cdots - \Phi_P B^{Ps}$. The $ARMA(P,Q)_s$ model is an ARMA(p=Ps,q=Qs) model where the AR parameters are 0 except at lags s, 2s, ..., Ps, and the MA parameters are 0 except at lags s, 2s, ..., Qs. Purely seasonal models are rare.

86) Let $\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p$ and $\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q$. The multiplicative ARMA(p,q)$\times(P,Q)_s$ model satisfies $\phi(B)\Phi(B)Y_t = \tau + \theta(B)\Theta(B)e_t$. For small p,q,P,Q, be able to solve this equation for Y_t to write down the model for Y_t. See HW8 B). This model is an ARMA(p + Ps, q + Qs) model where the nonzero coefficients are determined only by $p + P + q + Q$ coefficients, the AR characteristic polynomial is $\phi(B)\Phi(B)$ and the MA characteristic polynomial is $\theta(B)\Theta(B)$.
Note: Some texts use the notation $\Phi(B^s)$ for $\Phi(B)$ and $\Theta(B^s)$ for $\Theta(B)$.

87) Let $\bigtriangledown Y_t = (1-B)Y_t = Y_t - Y_{t-1}$, $\bigtriangledown^d Y_t = (1-B)^d Y_t$, $\bigtriangledown_s Y_t = (1-B^s)Y_t = Y_t - Y_{t-s}$, $\bigtriangledown_s^D Y_t = (1-B^s)^D Y_t$ where usually $d \le 1$ and $D \le 1$, $d = 2$ is rare and $D = 2$ is very rare. The differenced time series $X_t = \bigtriangledown^d \bigtriangledown_s^D Y_t$. Then $Y_t \sim$ ARIMA(p,d,q)$\times(P,D,Q)_s$ if $X_t \sim$ ARMA(p,q)$\times(P,Q)_s$. Also, $\phi(B)\Phi(B) \bigtriangledown^d \bigtriangledown_s^D Y_t = \tau + \theta(B)\Theta(B)e_t$ where the default is $\tau = 0$ if $d > 0$ or $D > 0$.

88) If there is no differencing so $d = 0$ and $D = 0$, then $X_t = Y_t$. Always plot Y_t and X_t. The plot of Y_t is useful for determining if there is a seasonal pattern and if the time series is stationary or nonstationary. Then the ACF and PACF of Y_t helps determine d and D while the ACF and PACF of X_t helps determine p, q, P, and Q.

89) The ACF and PACF of purely seasonal $AR(P)_s$ and $MA(Q)_s$ models are exactly like those of AR(p) and MA(q) models, except the nonzero spikes are at lags $0, s, 2s, 3s, 4s, ...$ Hence the theoretical ACF of an $MA(Q)_s$ model cuts off after lag Qs while the PACF has exponential or damped exponential sinusoidal decay at lags $0, s, 2s, 3s, 4s, ...$ The theoretical PACF of an $AR(P)_s$ model cuts off after lag Ps while the ACF has exponential or damped exponential sinusoidal decay at lags $0, s, 2s, 3s, 4s, ...$

90) For a purely seasonal ARMA(P,Q)$_s$ model, the theoretical ACF can have spikes at lags 1, ..., Q, and the theoretical PACF can have spikes at lags 1, ..., P. Both plots have exponential or damped exponential sinusoidal decay at lags $0, s, 2s, 3s, 4s, ...$

91) For the purely seasonal ARIMA(P,D=1,Q) model, the ACF or PACF may show linear decay or the sinusoidal peaks show linear decay at lags $0, s, 2s, 3s, 4s, ...$

92) The sample ACF and PACF will show trends and ripples that are not in the theoretical ACF and PACF.

93) The ARIMA(p,d,q)$\times(P, D, Q)_s$ model is common. Suppose $d = 0$ and $D = 0$ or the ACF and PACF of the differenced time series X_t is used. The ACF and PACF will often have spikes near s, 2s, 3s, ...

Sometimes there are spikes near $s/2, s, 3s/2, 2s, 5s/2, ...$ Fit the model with s and see if the problems go away in the residual ACF and PACF before fitting a model with seasonal period $s/2$.

94) Suppose $P = 1$ and the CI for Φ_1 is short and contains 1, eg (0.98,1.01). Then $Y_t = \hat{\Phi}_1 Y_{t-s}+$ stuff $\approx Y_t = Y_{t-s}+$ stuff, or $Y_t - Y_{t-s} =$ stuff, but $Y_t - Y_{t-s}$ corresponds to $D = 1$. Often the $D = 1$ model will have fewer parameters than the $D = 0$ model. See HW 8C).

95) Suppose $p = 1$ and the CI for ϕ_1 is short and contains 1, eg (0.98,1.01). Then $Y_t = \hat{\phi}_1 Y_{t-1}+$ stuff $\approx Y_t = Y_{t-1}+$ stuff, or $Y_t - Y_{t-1} =$ stuff, but $Y_t - Y_{t-1}$ corresponds to $d = 1$. Often the $d = 1$ model will have fewer parameters than the $D = 0$ model.

96) Much of the fitting and checking an ARIMA(p,d,q)$\times(P, D, Q)_s$ model is similar to fitting and checking an ARIMA(p,d,q) model. You should be able to recognize good and bad plots for both models, and be able to find I_{min}, I_I, and good models to look at for both models, given a table of $\Delta(I) = I_I - I_{min}$. The table is made for P, D and Q fixed. See HW8 Cnop).

97) A good ARIMA(p,d,q)$\times(P, D, Q)_s$ model should have i) good response and residual plots, ii) few nonsignificant parameters, iii) ϕ_p, θ_q, Φ_P and Θ_Q should be significant and not near 1 in absolute value. iv) The ACF and PACF of the residuals should resemble those of a white noise. v) $X_t = \bigtriangledown^d \bigtriangledown_s^D Y_t$ should be stationary and invertible. vi) Want roots of $\phi(B) = 0, \Phi(B) = 0, \theta(B) = 0$ and $\Theta(B) = 0$ to be outside the unit circle. vii) Want AIC(I) and $\Delta(I)$ to be within 7 and preferably 4 of $AIC(I_{min})$ and $\Delta(I_{min})$. viii) The Ljung Box pvalues should all be above the horizontal 0.05 line. ix) Usually $d, D \leq 2$. Want $n > 10(p + q + P + Q)$ and maybe $n > 10(p + Ps + q + Qs)$.