The final is is Wednesday, Dec. 10, 8-9:45 AM in the morning. **You are allowed 20 sheets of notes and a calculator.** The final covers the first 3 exam reviews, the HW, the quizzes and the material on this review sheet. Numbers refer to types of problems on exam.

See old final except 7),11),12); see the quizzes and exams, especially Exam 1: 1), 3); Exam 2: 1), 2), 5), 7); Exam 3: 1), 2), 5), 6); and quiz 11.

131) An approximation is $\ddot{a}_x^{(m)} = a_x^{(m)} + \dfrac{1}{m} \approx \ddot{a}_x - \dfrac{m-1}{2m}$ where $\ddot{a}_x$ is given by the illustrative life table. Also $\overline{a}_x \approx \ddot{a}_x - 0.5$ since a continuous annuity is the limiting case of an $m$thly annuity as $m \to \infty$. The approximation is good for $m \geq 12$.

132) Know how to get $\ddot{a}_x$, $1000A_x$, and $1000[\ {}^2A_x\ ]$ from the illustrative life table.

133) See 70)-73) for the linear UDD and exponential constant force approximations to quantities like ${}_tp_x$ and ${}_tq_x$ when x is an integer and $0 \leq t < 1$. Sometimes want approximations when the subscript x is replaced by $x+v$ where $0 \leq v < 1$ and $0 \leq v+t < 1$. The exact, UDD and exponential constant force approximations are usually close. Note that the exponential constant force approximation does not depend on v.

linear or UDD approx	exponential or constant force approx
${}_tq_{x+v} \approx \dfrac{(t)q_x}{1-v(q_x)}$	${}_tq_{x+v} \approx 1-(p_x)^t \approx\ {}_tq_x$
${}_tp_{x+v} \approx 1 - \dfrac{(t)q_x}{1-v(q_x)}$	${}_tp_{x+v} \approx (p_x)^t \approx\ {}_tp_x$

136) Referring to 112), increasing whole life insurance pays t units at time t and has $v_t = e^{-\delta t}$ and $b_t = t$ for $t \geq 0$. Hence the APV $= (\overline{IA})_x = \int_0^\infty t e^{-\delta t} f_T(t)dt$. Suppose $T \sim EXP(\mu)$, then $(\overline{IA})_x = \dfrac{\mu}{(\mu+\delta)^2}$.

137) See 38)-41). The probability that (x) will die between $x+n$ and $x+n+m$ $= P(x+n < X < x+n+m | X > x) = {}_{n|m}q_x = {}_np_x - {}_{n+m}p_x = {}_{n+m}q_x - {}_nq_x = {}_np_x\ {}_mq_{x+n}$ $= \dfrac{F(x+n+m)-F(x+n)}{S(x)} = \dfrac{S(x+n)-S(x+n+m)}{S(x)} = \dfrac{l_{x+n}-l_{x+n+m}}{l_x} =$ $\dfrac{S(x+n)}{S(x)}\ \dfrac{S(x+n)-S(x+n+m)}{S(x+n)} = P(n < T_x \leq n+m) = S_{T_x}(n) - S_{T_x}(n+m) =$ $F_{T_x}(n+m) - F_{T_x}(n) = \dfrac{{}_md_{x+n}}{l_x}$.

138) A select life table is for people selected to receive life insurance, and $[x]$ denotes someone selected to receive life insurance at age x. $S_{[x]+s}(t) = {}_tp_{[x]+s} =$ P(a life currently aged $x+s$ who was select at age x survives to age $x+s+t$). Also, $S_{[x]}(t) = {}_tp_{[x]} = S_T(t;x)$ for $t \geq 0$. The rules for select quantities like $l_{[x]}$, ${}_tp_{[x]}$, and ${}_tq_{[x]}$ are similar to the rules for l_x, ${}_tp_x$, and ${}_tq_x$. One difference is the recursion formula. Let $l_{[x]}$ be the earliest select age (acting like the radix l_0), then $l_{[x]+t} = l_{[x]}S_t(t;x)$.

139) ${}_np_{[x]} = \dfrac{l_{[x]+n}}{l_{[x]}}$, $p_{[x]} = \dfrac{l_{[x]+1}}{l_{[x]}}$, ${}_np_{[x]+k} = \dfrac{l_{[x]+k+n}}{l_{[x]+k}}$ (treat $[x]+k$ like x^*).

140) ${}_{n|m}q_{[x]} = {}_{n}p_{[x]} - {}_{n+m}p_{[x]} = {}_{n+m}q_{[x]} - {}_{n}q_{[x]} = {}_{n}p_{[x]}\ {}_{m}q_{[x]+n} = \dfrac{l_{[x]+n} - l_{[x]+n+m}}{l_{[x]}}.$

141) ${}_{n|m}q_{[x]+k} = \dfrac{l_{[x]+k+n} - l_{[x]+k+n+m}}{l_{[x]+k}}.$ (Again, treat $[x]+k$ like x^*.) This symbol gives the conditional probability of failure between ages $x+k+n$ and $x+k+n+m$ of a person known to be alive at $x+k$ who was select at age x.

142) The select period k is the time over which selection is assumed to have an efect on failure. Write $l_{[x]+t}$ for $t \leq k$ but $l_{[x]+t} = l_{x+t}$ for $t > k$.

143) An n year endowment insurance is a term insurance plus a pure endowment insurance. See 100). Hence

$$\overline{A}_{x:\overline{n}|} = \overline{A}^{1}_{x:\overline{n}|} + A_{\underset{x:\overline{n}|}{\ \ 1}} = \overline{A}^{1}_{x:\overline{n}|} + {}_{n}E_x = \overline{A}_x + {}_{n}E_x(1 - \overline{A}_{x+n}) = \overline{A}_x + A_{\underset{x:\overline{n}|}{\ \ 1}}(1 - \overline{A}_{x+n})$$

$$\stackrel{E}{=} \quad \frac{\mu + \delta e^{-n(\mu+\delta)}}{\mu+\delta}.$$

144) A whole life insurance is a term insurance plus a deferred insurance. Hence

$$\overline{A}_x = \overline{A}^{1}_{x:\overline{n}|} + {}_{n|}\overline{A}_x.$$

Also

$${}_{n|}\overline{A}_x = {}_{n}E_x\ \overline{A}_{x+n} = A_{\underset{x:\overline{n}|}{\ \ 1}}\ \overline{A}_{x+n}.$$

145) Deferred term insurance pays 1 unit at time t only if $n < t \leq n+m$ with $b_t = 0$ for $t \leq n$ and $t > n+m$ and $b_t = 1$ for $n < t < n+m$. Then $z_t = b_t v_t$ and $Z_T = v_T = e^{-\delta t}$ for $n < T \leq n+m$ and $Z_T = 0$ for $T < n$ or $T > n+m$. Then

$$E(Z_T) = {}_{n|m}\overline{A}_x = {}_{n|}\overline{A}^{1}_{x:\overline{m}|} = \overline{A}_x({}_{n}E_x - {}_{n+m}E_x) = \int_n^{n+m} e^{-\delta t} f_T(t)dt$$

$$\stackrel{E}{=} \frac{\mu}{\mu+\delta}[e^{-n(\mu+\delta)} - e^{-(n+m)(\mu+\delta)}].$$