Exam 3 review, Thursday, April 21. TI30 calculator but no notes. Emphasis is sections 1.3, 1.4, 1.5, 1.6, 2.1, 2.2, 2.3, 3.1, 3.2, 3.3, 3.5, 3.6, 4.1, 4.2, 4.3, 4.4, 4.5, 4.7, 5.1, 5.2, 5.3, 5.4; HW2-19; Q1-21; with more emphasis on derivatives and integrals. No differentials (2.8), no Newton's Method (4.6). Problems F1) to F21) from Exam 1 and 2 reviews are still fair game.

1) You will need to find a few limits: a) Horizontal (and vertical) asymptotes, b) the derivative in disguise, and c) the integral in disguise.

2) You may need to find the derivative from the definition.

3) Be able to find the derivatives from the rules, especially the power rule. See p. 98.

4) Know the derivatives of the trig functions and of $f(x) = e^x$. See p. 111 and 165. Note that the cofunctions cos, cot, and csc all have minus signs with their derivatives.

5) Find the derivative via the product or quotient rule. See p. 107 and 109. Remember that the quotient rule is $\dfrac{dn' - nd'}{d^2}$ where d is the denominator and n is the numerator.

6) Know how to find 2nd, 3rd, etc derivatives, p. 90-91. In particular, the 2nd derivative is the derivative of the first (usual) derivative.

7) Know that velocity v(t) is the derivative of the position function s(t), and acceleration a(t) is the derivative of v(t). See p. 91.

8) The chain rule is **very important** for exam 3 and the final. [f(g(x))]' = [f'(g(x))][g'(x)] See section 2.5.

9) Know the derivative of $|x|$, $\ln x$, e^x, $\log_a x$, and a^x. Know the derivatives of the trig functions, the inverse trig functions (especially $\sin^{-1}(x), \cos^{-1}(x)$ and $\tan^{-1}(x)$) and the hyperbolic functions. See p. 164-169, 183, 186. Know the chain rule with these functions.

10) Know that the derivative of $\ln|g(x)| = g'(x)/g(x)$.

11) Memorize the table of integrals on p. 284 and 1-11, 16, 17 on the basic forms: reference p. 6 in the back of the book.

12) The power rule for integration is $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ for $n \neq -1$.
A special case (n = 0) is $\int dx = \int x^0 dx = x + C$.
The power rule for $n = -1$ is $\int \frac{1}{x} dx = \int x^{-1} dx = \ln(|x|) + C$.

13) For trig integrals, note that the $-$ cofunction is always in the antiderivative.

14) The Fundamental Theorem of Calculus has two parts: Suppose $f(x)$ is continuous on $[a, b]$.
I) If $F(x) = \int_a^x f(t)dt$, then $F'(x) = f(x)$.
II) $\int_a^b f(x)dx = F(b) - F(a)$ where F is any antiderivative of f: $F'(x) = f(x)$.

15) The Fundamental Theorem of Calculus combined with the chain rule says that the derivative of $F(g(x)) = \int_a^{g(x)} f(t)dt$ is $\dfrac{d}{dx}\displaystyle\int_a^{g(x)} f(t)dt = f(g(x))g'(x) = F'(g(x))g'(x)$.

16) $\frac{d}{dx}\int_{g(x)}^{b} f(t)dt = \frac{d}{dx}\int_{b}^{g(x)} [\,-f(t)\,]dt = -f(g(x))g'(x)$ using $\int_b^a f(t)dt = -\int_a^b f(t)dt$.

Types of problems:

1) Be able to state Rolle's theorem and the mean value theorem (for the average rate of change). Know how to find the tangent line parallel to the secant line through points (a,f(a)) and (b, f(b)). See p. 210-212.

2) Know how to use critical numbers to make test intervals for whether f is increasing or decreasing: $f'(x) > 0$ on test interval means f is increasing on the interval and $f'(x) < 0$ on test interval means f is decreasing on the interval. KNOW THE 1ST DERIVATIVE TEST on p. 218. (If c is a critical number of continuous f and if f' changes from positive to negative at c, then f has a local maximum at c. If f' changes from negative to positive at c, then f has a local minimum at c. If f' does not change sign at c, then f has no local max or min at c.)

3) Know how to find the critical points of f'(x) to determine test intervals for whether f is concave up or down. See p. 220: $f''(x) > 0$ on interval means f is concave up on the test interval and $f''(x) < 0$ on interval means f is concave down on the test interval.

4) Know how to apply the 2nd derivative test and when it fails (eg if f"(x) = 0 at a critical number of f). See p. 221: If f'' is continuous near c and $f'(c) = 0$, then $f''(c) > 0$ means f has a local minimum at $(c, f(c))$ while $f''(c) < 0$ means f has a local maximum at $(c, f(c))$. If $f''(c) = 0$ the 2nd derivative test fails, so use the 1st derivative test.

5) Given a formula for $F(x)$ and given a formula for $F'(x) = f(x)$, know that $\int f(x)dx = \int F'(x)dx = F(x) + C$ and that $\int_a^b f(x)dx = \int_a^b F'(x)dx = F(b) - F(a)$.

6) Be able to solve a differential equation given $f''(x), f'(e) = a$, and $f(d) = b$. Note that $\int f''(x)dx = f'(x) + C$. Use $f'(e) = a$ to solve for C. Then $\int f'(x)dx = f(x) + C$. Use $f(d) = b$ to solve for C.

7) Know how to find the mean value of f on an interval [a,b] by using the mean value theorem for integrals: ave val $= \frac{1}{b-a}\int_a^b f(x)dx$.

8) Use the right endpoint rule to find the definite integral $\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^n f(a + i\frac{(b-a)}{n})\frac{b-a}{n}$. **Know** that $\sum_{i=1}^n c = nc$, $\sum_{i=1}^n i = \frac{n(n+1)}{2}$, $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$, and $\sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4}$. Know $\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^n c = c$, $\lim_{n\to\infty}\frac{1}{n^2}\sum_{i=1}^n i = 1/2$, $\lim_{n\to\infty}\frac{1}{n^3}\sum_{i=1}^n i^2 = 1/3$, and $\lim_{n\to\infty}\frac{1}{n^4}\sum_{i=1}^n i^3 = 1/4$.

9) Recognize that $\lim_{n\to\infty}\sum_{i=1}^{n} f(a + i\frac{(b-a)}{n})\frac{b-a}{n} = \int_a^b f(x)dx$.

From reviews 1 and 2), know how to do all of the problems that are important for the final F1) – F21). The problems with three stars (F1, F2, F12, F13, F14, F15, F16, F17, F18, F20 and F21) are especially important. F3, F4 and F10 are also likely to appear on exam 3.

The following **new problems** are **very important for exam 3** and the final.

F22***) Know how to find the absolute maximum and minimum of a function f (usually) on an interval [a,b]. Solve $f'(x) \equiv 0$ and the places where f'(x) does not exist. These points are the critical numbers PROVIDED THAT THEY ARE in the interval (a, b). Often $f'(x) = \dfrac{g(x)}{h(x)}$. Then $f'(x)$ does not exist at the roots of $h(x)$. Evaluate f at a, b, and the critical numbers. One of these points will be the min and one the max. Give both coordinates x = c and y = f(c). S15 9, F14 8, F13 6, S11 8.

F23) Given the graph of the **derivative** f' of f, find open intervals where f is **increasing, decreasing, concave up and down, inflection points**, and the x coordinates of local mins and maxs. A local min occurs where f changes from decreasing to increasing. A local max occurs where f changes from increasing to decreasing. An inflection point occurs where concavity changes (form up to down or from down to up). Know the definitions of the terms in bold face.
Know that f is increasing on intervals where $f' > 0$.
Know that f is decreasing on intervals where $f' < 0$.
Know that f is concave upward on intervals where f' is increasing.
Know that f is concave downward on intervals where f' is decreasing.
See S11 10.

F24***) Given the formula of the **derivative** f' of f, find open intervals where f is **increasing, decreasing, concave up and down, inflection points**, and the x coordinates of local maxs and local mins. If **f is not continuous at critical number c** (eg VA), then f can not have an inflection point or local max or min at c. Variant: you are given f, f' and f". Variant: you are given f and need to find f' and f". S15 8, S14 7, F13 8

F25**) Given the information from F22) or F23) be able to graph a reasonable f. Alternatively, you may be given information about horizontal and vertical asymptotes, intervals where $f' > 0, f' < 0, f" > 0$, and $f" < 0$. Use this information to sketch a reasonable f. The y intercept is (0,f(0)) if it exists. There is at most one y intercept. For the x intercepts (x,0), solve $f(x) = 0$ for x. S15 8e, F13 8c,

F26***) MIN-MAX optimization story problems. You are given an equation Q of two or more variables to minimize or maximize and an equation C of constraints. The **key step** is to use equation C to eliminate all but one of the variables, say x, and write Q as a function of x. After this step, the problem is like F21). You need to show that your solution for x does indeed minimize or maximize Q. This is simple if Q is a parabola. If Q is defined on a closed interval then evaluate Q at the endpoints and all critical numbers as in F22. The FIRST DERIVATIVE TEST FOR ABSOLUTE EXTREME VALUES on p. 234 is also useful: if $f'(x) > 0$ for $x < c$ and $f'(x) < 0$ for $x > c$ then f(c) is the abs max of f, while if $f'(x) < 0$ for $x < c$ and $f'(x) > 0$ for $x > c$ then f(c) is the abs min of f. See F15 10, F14 9, F13 9, S11 12.

F27) Know how to use the **Fundamental theorem of Calculus part I**: $\frac{d}{dx}\int_a^x f(t)dt = f(x)$. A variant is $\frac{d}{dx}\int_x^a f(t)dt = -f(x)$.

F28***) Know how to use FTCI and the chain rule. F27) Is the special case with $g(x) = x$. a) $\frac{d}{dx}\int_a^{g(x)} f(t)dt = f(g(x))g'(x)$.
b) $\frac{d}{dx}\int_{g(x)}^a f(t)dt = -f(g(x))g'(x)$.
c) $\frac{d}{dx}\int_{h(x)}^{g(x)} f(t)dt = \frac{d}{dx}(F(g(x)) - F(h(x)) = F'(g(x))g'(x) - F'(h(x))h'(x) = f(g(x))g'(x) - f(h(x))h'(x)$.
See S15 4e, F14 3d, F13 3g, S11 3f.

F29*) Know the **Fundamental theorem of calculus part II** for definite integrals. $\int_a^b f(x)dx = F(b) - F(a)$ where the antiderivative F satisfies $F'(x) = f(x)$. Alternatively, $\int_a^b f'(x)dx = f(b) - f(a)$. The antiderivative rules on p. 284 are important. F13 11a

F30**) Know how to get an integral $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$. Sometimes f is defined on two intervals and sometimes $f(x) = |h(x)|$. See F08 7a, S09 6b, S10 11a

F31***) The indefinite integral $\int f(x)dx = F(x) + C$ where the antiderivative $F(x)$ satisfies $F'(x) = f(x)$. Alternatively, $\int f'(x)dx = f(x) + C$. Use this fact and the rules on p. 284 and near the back cover of the text to evaluate indefinite integrals. The power rule (the integrand $f(x) = x^n, n \neq -1$ and for $f(x) = 1/x$ if n = 1) is very important. So are the trig rules, the exponential rules and rules where $\sin^{-1}(x)$ and $\tan^{-1}(x)$ are the antiderivative. Occasionally there is a tricky ratio or product of powers, trig functions or exponentials. See S15 11a, F14 10a, F13 10a, S11 6be.

F32) The "integral in disguise". Find

$$\lim_{n\to\infty} \frac{b-a}{n}[f(a+\frac{b-a}{n}) + f(a+\frac{2(b-a)}{n}) + \cdots + f(a+\frac{i(b-a)}{n}) + \cdots + f(a+\frac{n(b-a)}{n})].$$

This limit is equal to $\int_a^b f(x)dx$.

In particular, $a = 0, b = 1$ are especially common: find

$$\lim_{n\to\infty} \frac{1}{n}[f(\frac{1}{n}) + f(\frac{2}{n}) + \cdots + f(\frac{i}{n}) + \cdots + f(\frac{n}{n})].$$

This limit is equal to $\int_0^1 f(x)dx$. Figure out what $f(1/n)$ is, then substitute x for $1/n$ to get $f(x)$.

F33*) Given $v(t)$ or a graph of $v(t)$ on $[t_1, t_2]$, find
a) the distance traveled $= \int_{t_1}^{t_2} |v(t)|dt$.
b) the displacement $\int_{t_1}^{t_2} v(t)dt = s(t_2) - s(t_1)$.
c) Sketch the graph of $a(t) = v'(t)$. (Usually slopes of line segments of $v(t)$.)
Often the graph consists of triangular regions. Find the area of each region. Then the distance traveled is the sum of the areas while the displacement is the sum of the areas that are above the x-axis minus the sum of the areas below the x-axis. A variant is that you are given a formula for $s(t)$, need to find $v(t) = s'(t)$ and $a(t) = v'(t) = s''(t)$. You may need to find $a(t)$ at a specific value of t (eg $a(t) = v'(t) =$ slope of tangent line of v at t). See p. 287 and S15 14.