Exam 1 review. Thursday Feb. 11. Bring a TI-30 calculator but NO NOTES. Emphasis on sections 1.3, 1.4, 1.5, 1.6, 2.1, 2.2, 2.3; HW2-5; Q1-4. Know for trig functions that $0.707 \approx \sqrt{2}/2$ and $0.866 \approx \sqrt{3}/2$.

Let $\lim_{x\to a^*} f(x)$ denote $\lim_{x\to a} f(x)$, $\lim_{x\to a^-} f(x)$, or $\lim_{x\to a^+} f(x)$.

Types of problems:

1) Know the meaning of limit (p. 31), of one sided limits (p. 29-30), of continuity (p. 46), of infinite limits (p. 64-66), or of the derivative of a function (p. 77).

2) State why the limit or one sided limit does not exist: unbounded, jump, infinite oscillations (eg $\sin(1/x)$). See p. 28–31.

The following 3 are considered "easy."

3) Limit from graph. See practice exam one: II, HW2 1.3–3,4,5.

4) $\lim_{x\to a^*} f(x) = f(a)$ if f is continuous at a. See p. 46.

5) If there is a hole in the graph at $(x = a, y = L)$ then $\lim_{x\to a^*} f(x) = L$. See p. 26, 47.

6) Squeeze theorem on p. 41. Eg $\lim_{x\to 0^*} x^2 \sin(1/x) = 0$.

7) From a graph or function formula, is the discontinuity removable (hole) or non-removable (jump, unbounded, vertical asymptote). See p. 47, 57. Make sure that you examine **all roots** of the denominator. Q3 5

8) $\lim_{x\to a^*} f(g(x)) = f(g(a))$ if g is continuous at a and f is continuous at $g(a)$. See p. 51 and Q4–5.

9) Determine a constant so that f is continuous on some interval. Eg. $f(x) = \begin{cases} g(x), & x \le a \\ h(x) + c, & x > a. \end{cases}$ Then evaluate f at **a**. Need $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$. Typically get $g(a) = h(a)+c$ and solve for c. Alternatively get something like $f(x) = \begin{cases} 10 + \frac{\sin x}{x} & x \ne 0 \\ c & x = 0. \end{cases}$ If f is continuous, then $\lim_{x\to 0} f(0) = f(0)$, so $c = \lim_{x\to 0} f(x)$.

10) Find the vertical asymptotes. Again, examine all roots of the denominator. See p. 57.

11) $\lim_{x\to a^*} f(x) = \pm\infty$. Be careful of the sign. See p. 57, Q3 2.

12) Recognize operations, eg addition, subtraction, scalar multiplication, multiplication, division and composition that preserve continuity or that make finding limits easy. See p. 48-51.

13) Intermediate value theorem, p. 52, HW4 1.5 41.

14) $\lim_{x\to a^+} f(x) \ne \lim_{x\to a^-} f(x)$. Be careful of the sign of the one sided limit.

15) Be able to find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$. These are horizontal asymptotes. See p. 60.

16) From graph, find points where the derivative does not exist. This could happen if there is a corner, eg $f(x) = |x|$ where the left and right derivatives are not equal, if $f(x)$ is not continuous at $x = a$, or if f has a vertical tangent at $x = a$ (eg $f(x) = x^{1/3}$.) See p. 89–90.

17) Section 2.3: find the derivatives from the rules, especially the **power rule**. The sum and difference rules, sin rule, cos rule and exponential rule are also very important. Q4 6, HW5 2.3 3,5,6,8,A

The following problems are **very important for exam 1 and the final**. The notation F*** means it was on 3 out of 3 of the last 3 finals.

For F1)– F7), suppose $f(x) = \dfrac{g(x)}{h(x)}$ and $\dfrac{g(c)}{h(c)} = 0/0$ or ∞/∞ is indeterminant.

F1**) **Cancellation**: p. 38, S15–1b, F13–1a, Q1–6, Q2–1,2,5, Q4–1, S11 1c.

F2***) If $\lim\limits_{x\to\infty} \dfrac{7x}{\sqrt{x^2+x}} = L$ then $L = \lim\limits_{x\to\infty} \dfrac{7x}{\sqrt{x^2+x}} \dfrac{\frac{1}{\sqrt{x^2}}}{\frac{1}{\sqrt{x^2}}}$. Use the fact that for $x > 0$, $\sqrt{x^2} = |x| = x$ and that $\sqrt{a}(\frac{1}{\sqrt{b}}) = \sqrt{a/b}$ then simplify. Similarly if $\lim\limits_{x\to\infty} \dfrac{x^2}{\sqrt{1000+x^4}} = L$ then
$L = \lim\limits_{x\to\infty} \dfrac{x^2}{\sqrt{1000+x^4}} \dfrac{\frac{1}{\sqrt{x^4}}}{\frac{1}{\sqrt{x^4}}}$ and simplify. Similarly for a rational function, if
$\lim\limits_{x\to\infty} \dfrac{ax^n+...}{bx^m+...} = L$ then $L = \lim\limits_{x\to\infty} \left(\dfrac{ax^n+...}{bx^m+...}\right)\left(\dfrac{\frac{1}{x^m}}{\frac{1}{x^m}}\right)$. Notice that the highest power in the denominator is always used. Also note that this trick is used for $x \to \infty$. See S15–1c, F14–1c, S13–1d, Q3–3, Q4–3, S11–1a.

F3**) Rationalize the numerator (or denominator): p. 39, S15–1a, F14–1a, Q3–4.

$f(x) = \dfrac{g(x)}{h(x)}$ and $\dfrac{g(c)}{h(c)} = 0/0$, indeterminant. With trig functions, you need one of the following tricks:

F4**) Use $\lim\limits_{x\to 0^*} \dfrac{\sin ax}{ax} = 1$ or $\lim\limits_{x\to 0^*} \dfrac{ax}{\sin ax} = 1$. Variant: $\tan(ax) = \dfrac{\sin(ax)}{\cos(ax)}$. F14–1b, F13–1b, Q3–1, S11–1b.

F5) Use $\lim\limits_{x\to 0^*} \dfrac{1-\cos ax}{ax} = 0$.

F6*) If you have a trig function in both the numerator and the denominator of the form 0/0, multiply by $\frac{\frac{1}{x}}{\frac{1}{x}}$ and simplify. So $\lim\limits_{x\to 0} \dfrac{\sin 3x}{\tan 5x} = \lim\limits_{x\to 0} \dfrac{\sin 3x}{\tan 5x} \dfrac{\frac{1}{x}}{\frac{1}{x}} = \cdots = 3/5$. S15–1d

F7) You may need to find $\lim\limits_{x\to -\infty} \dfrac{7x}{\sqrt{x^2+x}} = L$, then $L = \lim\limits_{x\to -\infty} \dfrac{7x}{\sqrt{x^2+x}} \dfrac{\frac{1}{\sqrt{x^2}}}{\frac{1}{\sqrt{x^2}}}$. Use the fact that for $x < 0$, $\sqrt{x^2} = |x| = -x$ and that $\sqrt{a}(\frac{1}{\sqrt{b}}) = \sqrt{a/b}$, then simplify.

For F8 and F9, the indeterminant form is $\infty - \infty$ or $-\infty + \infty$.

F8) If $L = \lim\limits_{x\to\infty} x - \sqrt{x^2+bx+c}$ then $L = \lim\limits_{x\to\infty} (x - \sqrt{x^2+bx+c})\dfrac{x+\sqrt{x^2+bx+c}}{x+\sqrt{x^2+bx+c}}$
$= \lim\limits_{x\to\infty} \dfrac{-bx-c}{x+\sqrt{x^2+bx+c}} \dfrac{\frac{1}{\sqrt{x^2}}}{\frac{1}{\sqrt{x^2}}} = -b/2$.

F9) If $L = \lim_{x\to-\infty} x + \sqrt{x^2+bx+c}$ then $L = \lim_{x\to-\infty} (x + \sqrt{x^2+bx+c})\dfrac{x-\sqrt{x^2+bx+c}}{x-\sqrt{x^2+bx+c}}$
$= \lim_{x\to-\infty} \dfrac{-bx-c}{x-\sqrt{x^2+bx+c}} \dfrac{\frac{1}{\sqrt{x^2}}}{\frac{1}{\sqrt{x^2}}} = -b/2$. But now $\sqrt{x^2} = |x| = -x$.

F10) Finding the limit of a difference quotient $L = \lim_{h\to 0^*} \dfrac{f(a+h)-f(a)}{h}$. This is a "**derivative in disguise**" problem, $L = \frac{d}{dx} f(x)\Big|_a = f'(a)$. (Often use Δx instead of h.) See S10–2c.

Also $L = \lim_{h\to 0^*} \dfrac{f(x+h)-f(x)}{h} = \dfrac{d}{dx} f(x) = f'(x)$.

A variant is to find the limit of slope of the secant line which is the slope of the tangent line, so $L = \lim_{x\to a^*} \dfrac{f(x)-f(a)}{x-a} = \dfrac{d}{dx} f(x)\Big|_a = f'(a)$.

F11**) The function $f(x)$ has a vertical asymptote at $x = a$, so $lim_{x\to a} f(x)$ does not exist but the left or right limit is $\pm\infty$. A variant is to find the left or right limit. If $K > 0$, $\lim_{x\to a^-} \dfrac{K}{x-a} = \dfrac{K}{-0} = -\infty$, and $\lim_{x\to a^+} \dfrac{K}{x-a} = \dfrac{K}{0} = \infty$. See Q3–2, Q4–2, S15–1b, F13–1c.

For $\sin(x)$ and $\cos(x)$ you should know the zeroes and places where the functions $= \pm 1$. For $\ln(x)$ and $\ln(x-a)$. you should know that the zero is at $a+1$ and the vertical asymptote at $x = a$. For e^x and e^{-x} know where the horizontal asymptote is and know that $e^0 = 1$. Know that the vertical asymptote of $1/(x-a)^n$ is at $x = a$. Know how to sketch these functions. See graphs on p. R2, R3, R4, A7, A17, A18, 31, 61, 145, 157, 160

F12***) **USING THE DEFINITION OF THE DERIVATIVE** find the derivative of $f(x)$. You need to find $\lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$. Tricks:

a) If $f(x) = \sqrt{x+b}$, rationalize the numerator.

b) If f is a fraction, eg $f(x) = 1/(a+x)$, then get the difference of fractions over a common denominator, eg $f(x+h) - f(x) = \dfrac{-h}{(a+x+h)(a+x)}$.

c) Use cancellation.

d) Use the addition formulas for sin and cos .

See S15–2, F14–2, F13–2, HW5 2.2–19,20, S11–4, Q5.

F13***) Find the **equation of the tangent line** to the graph of $y = f(x)$ at the point (a, f(a)) (or at $x = a$, where you need to find $f(a)$, or at $y = d$ where you need to solve for $x = a$, or at the place where f crosses the x axis: so y = 0, solve for x=a, or at the y axis, so x = 0).

Step 1: find the derivative $f'(x)$.

Step 2: the slope m of the tangent line is $m = f'(a)$.

Step 3: since $y = mx + b$, $y = f(a)$, and $x = a$, sub in $f(a) = ma + b$ to find b.

Step 4: in the equation $y = mx + b$, sub in your values for m and b.

See S15–3, F14–5, S13–5, HW5 2.1–A,5, Q5