Chapter 9

More on Experimental Designs

The one and two way Anova designs, completely randomized block design and split plot designs are the building blocks for more complicated designs. Some split plot designs can be written as a linear model, $Y = \boldsymbol{x}^T\boldsymbol{\beta} + e$, but the errors are dependent with a complicated correlation structure.

9.1 Split Plot Designs

Definition 9.1. Split plot designs have two units. The large units are called **whole plots** and contain blocks of small units called **subplots**. The whole plots get assigned to Factor A while the subplots get assigned to factor B (randomly if the units are experimental but not randomly if the units are observational). A and B are crossed so the AB interaction can be studied.

The split plot design depends on how whole plots are assigned to A. Three common methods are described below, and methods a) and b) are described in more detail in the following subsections. The randomization and split plot Anova table depend on the design used for assigning the whole plots to factor A.

a) The whole plots are assigned to A completely at random, as in a one way Anova.

b) The whole plots are assigned to A and to a blocking variable as in a completely randomized block design (if the whole plots are experimental, but a complete block design is used if the whole plots are observational).

c) The whole plots are assigned to A, to row blocks and to column blocks as in a Latin Square.

The key feature of a split plot design is that there are two units of different sizes: one size for each of the 2 factors of interest. The larger units are assigned to A. The large units contain blocks of small units assigned to factor B. Also factors A and B are crossed.

9.1.1 Whole Plots Randomly Assigned to A

Shown below is the split plot Anova table when the whole plots are assigned to factor A as in a one way Anova design. The whole plot error is error(W) and can be obtained as an A*replication interaction. The subplot error is error(S). $F_A = MSA/MSEW$, $F_B = MSB/MSES$ and $F_{AB} = MSAB/MSES$. R computes the three test statistics and pvalues correctly, but for SAS F_A and the pvalue p_A need to be computed using MSA, MSEW, df_A and df_{eW} obtained from the Anova table. Sometimes "error(W)" is also denoted as "residuals." There are ma whole plots, and each whole plot contains b subplots. Thus there are mab subplots.

Source	df	SS	MS	F	p-value
A	$a-1$	SSA	MSA	F_A	p_A
error(W) or A*repl	$a(m-1)$	SSEW	MSEW		
B	$b-1$	SSB	MSB	F_B	p_B
AB	$(a-1)(b-1)$	SSAB	MSAB	F_{AB}	p_{AB}
residuals or error(S)	$a(m-1)(b-1)$	SSES	MSES		

The tests of interest for this split plot design are nearly identical to those of a two way Anova model. Y_{ijk} has $i = 1, ..., a$, $j = 1, ..., b$ and $k = 1, ..., m$. Keep A and B in the model if there is an AB interaction.

a) **The 4 step test for AB interaction** is
i) Ho there is no interaction H_A there is an interaction
ii) F_{AB} is obtained from output.
iii) The pvalue is obtained from output.
iv) If pvalue $< \delta$ reject Ho and conclude that there is an interaction between A and B, otherwise fail to reject Ho and conclude that there is no interaction between A and B.

b) **The 4 step test for A main effects** is
i) Ho $\mu_{100} = \cdots = \mu_{a00}$ H_A not Ho
ii) F_A is obtained from output.
iii) The pvalue is obtained from output.

iv) If pvalue $< \delta$ reject Ho and conclude that the mean response depends on the level of A, otherwise fail to reject Ho and conclude that the mean response does not depend on the level of A.

c) **The 4 step test for B main effects** is

i) Ho $\mu_{010} = \cdots = \mu_{0b0}$ $\quad H_A$ not Ho

ii) F_B is obtained from output.

iii) The pvalue is obtained from output.

iv) If pvalue $< \delta$ reject Ho and conclude that the mean response depends on the level of B, otherwise fail to reject Ho and conclude that the mean response does not depend on the level of B.

Source	df	SS	MS	F	p-value
variety	7	763.16	109.02	1.232	0.3421
MSEW	16	1415.83	88.49		
treatment	3	30774.3	10258.1	423.44	0.00
variety*treatment	21	2620.1	124.8	5.150	0.00
error(S)	48	1162.8	24.2		

Example 9.1. This split plot data is from Chambers and Hastie (1993, p. 158). There are 8 varieties of guayule (rubber plant) and 4 treatments were applied to seeds. The response was the rate of germination. The whole plots were greenhouse flats and the subplots were 4 subplots of the flats. Each flat received seeds of one variety (A). Each subplot contained 100 seeds and was treated with one of the treatments (B). There were $m = 3$ replications so each variety was planted in 3 flats for a total of 24 flats and $4(24) = 96$ observations.

Factorial crossing: Variety and treatments (A and B) are crossed since all combinations of variety and treatment occur. Hence the AB interaction can be measured.

Blocking: The whole plots are the 24 greenhouse flats. Each flat is a block of 4 subplots. Each of the 4 subplots gets one of the 4 treatments.

Randomization: The 24 flats are assigned to the 8 varieties completely at random. Use the `sample(24)` command to generate a random permutation. The first 3 numbers of the permutation get variety one, the next 3 get variety 2, ..., the last 3 get variety 8. The use the `sample(4)` command 24 times, once for each flat. If 2, 4, 1, 3 was the permutation for the ith flat, then the 1st subplot gets treatment 3, the 2nd gets treatment 1, the 3rd gets treatment 4, and the 4th subplot gets treatment 2.

a) Perform the test corresponding to A.

b) Perform the test corresponding to B.

c) Perform the test corresponding to AB.

Solution: a) Ho $\mu_{100} = \cdots = \mu_{800}$ Ha not Ho
$F_A = 1.232$
pval = 0.3421
Fail to reject Ho, the mean rate of germination does not depend on variety. (This test would make more sense if there was no variety * treatment interaction.)
b) Ho $\mu_{010} = \cdots = \mu_{040}$ Ha not Ho
$F_B = 423.44$
pval = 0.00
Reject Ho, the mean rate of germination depends on treatment.
c) Ho no interaction Ha there is an interaction
$F_{AB} = 5.15$
pval = 0.00
Reject Ho, there is a variety * treatment interaction.

9.1.2 Whole Plots Assigned to A as in a CRBD

Shown below is the split plot Anova table when the whole plots are assigned to factor A and a blocking variable as in a completely randomized block design. The whole plot error is error(W) and can be obtained as an block*A interaction. The subplot error is error(S). $F_A = MSA/MSEW$, $F_B = MSB/MSES$ and $F_{AB} = MSAB/MSES$. Factor A has a levels and factor B has b levels. There are r blocks of a whole plots. Each whole plot contains b subplots, and each block contains a whole plots and thus ab subplots. Hence there are ra whole plots and rab subplots.

SAS computes the last two test statistics and pvalues correctly, and the last line of SAS output gives F_A and the pvalue p_A. The initial line of output for A is not correct. The output for blocks is probably not correct.

Source	df	SS	MS	F	p-value
blocks	$r-1$				
A	$a-1$	SSA	MSA	F_A	p_A
error(W) or block*A	$(r-1)(a-1)$	SSEW	MSEW		
B	$b-1$	SSB	MSB	F_B	p_B
AB	$(a-1)(b-1)$	SSAB	MSAB	F_{AB}	p_{AB}
error(S)	$a(r-1)(b-1)$	SSES	MSES		

The tests of interest for this split plot design are nearly identical to those of a two way Anova model. Y_{ijk} has $i = 1, ..., a$, $j = 1, ..., b$ and $k = 1, ..., r$. Keep A and B in the model if there is an AB interaction.

a) **The 4 step test for AB interaction** is
i) Ho there is no interaction H_A there is an interaction
ii) F_{AB} is obtained from output.
iii) The pvalue is obtained from output.
iv) If pvalue $< \delta$ reject Ho and conclude that there is an interaction between A and B, otherwise fail to reject Ho and conclude that there is no interaction between A and B.

b) **The 4 step test for A main effects** is
i) Ho $\mu_{100} = \cdots = \mu_{a00}$ H_A not Ho
ii) F_A is obtained from output.
iii) The pvalue is obtained from output.
iv) If pvalue $< \delta$ reject Ho and conclude that the mean response depends on the level of A, otherwise fail to reject Ho and conclude that the mean response does not depend on the level of A.

c) **The 4 step test for B main effects** is
i) Ho $\mu_{010} = \cdots = \mu_{0b0}$ H_A not Ho
ii) F_B is obtained from output.
iii) The pvalue is obtained from output.
iv) If pvalue $< \delta$ reject Ho and conclude that the mean response depends on the level of B, otherwise fail to reject Ho and conclude that the mean response does not depend on the level of B.

Source	df	SS	MS	F	p-value
Block	5	4.150	0.830		
Variety	2	0.178	0.089	0.65	0.5412
Block*Variety	10	1.363	0.136		
Date	3	1.962	0.654	23.39	0.00
Variety*Date	6	0.211	0.035	1.25	0.2973
error(S)	45	1.259	0.028		

Example 9.2. The Anova table above is for the Snedecor and Cochran (1967, p. 369-372) split plot data where the whole plots are assigned to factor A and to blocks in a completely randomized block design. Factor A = variety of alfalfa (ladak, cossack, ranger). Each field had two cuttings, with the second cutting on July 7, 1943. Factor B = date of third cutting (none, Sept. 1, Sept. 20, Oct. 7) in 1943. The response variable was yield (tons per acre) in 1944. The 6 blocks were fields of land divided into 3 plots of land, one for each variety. Each of these 3 plots was divided into 4 subplots for date of third cutting. So each block had 3 whole plots and 12 subplots.

a) Perform the test corresponding to A.

b) Perform the test corresponding to B.

c) Perform the test corresponding to AB.

Solution: a) Ho $\mu_{100} = \cdots = \mu_{300}$ Ha not Ho
$F_A = 0.65$
pval = 0.5412
Fail to reject Ho, the mean yield does not depend on variety.
b) Ho $\mu_{010} = \cdots = \mu_{040}$ Ha not Ho
$F_B = 23.39$
pval = 0.0
Reject Ho, the mean yield depends on cutting date.
c) Ho no interaction Ha there is an interaction
$F_{AB} = 1.25$
pval = 0.2973
Fail to reject Ho, there is no interaction between variety and cutting date.

Warning: Although the split plot model can be written as a linear model, the errors are not iid and have a complicated correlation structure. It is also difficult to get fitted values and residuals from the software, so the model can't be easily checked with response and residual plots. These facts make the split plot model very hard to use for most researchers.

9.2 Review of the DOE Models

The three basic principles of DOE (design of experiments) are

i) use **randomization** to assign treatments to units.

ii) Use **factorial crossing** to compare the effects (main effects, pairwise interactions, ..., J-fold interaction) of $J \geq 2$ factors. If $A_1, ..., A_J$ are the factors with l_i levels for $i = 1, ..., J$; then there are $l_1 l_2 \cdots l_J$ treatments where each treatment uses exactly one level from each factor.

iii) **Blocking** is used to divide units into blocks of similar units where "similar" means the units are likely to have similar values of the response when given the same treatment. Within each block randomly assign units to treatments.

Next the 10 designs of Chapter 5 to Section 9.1 are summarized. If the randomization can not be done as described, then much stronger assumptions on the data are needed for inference to be approximately correct. There are three common ways of assigning units. For inference i) requires the least assumptions and iii) the most.

i) Experimental units are randomly assigned.

ii) Observational units are a random sample of units from a population of units. Each combination of levels determines a population. So a two way Anova design has ab populations.

iii) Units (such as time slots) can be assigned systematically due to constraints (eg physical, cost or time constraints).

I) One way Anova: Factor A has p levels.

a) For a fixed effects one way Anova model, the levels are fixed.

b) For a random effects one way Anova model, the levels are a random sample from a population of levels.

Randomization: Let $n = \sum_{i=1}^{p} m_i$ and do the `sample(n)` command. Assign the first m_1 units to treatment (level) 1, the next m_2 units to treatment 2, ..., the last m_p units to treatment p.

II) Two way Anova: Factor A has a levels and factor B has b levels. The two factors are crossed, forming ab cells.

Randomization: Let $n = mab$ and do the `sample(n)` command. Randomly assign m units to each of the ab cells. Assign the first m units to the $(A, B) = (1, 1)$ cell, the next m units to the (1,2) cell, ..., the last m units to the (a, b) cell.

III) k way Anova: There are k factors $A_1, ..., A_k$ with $a_1, ..., a_k$ levels, respectively. The k factors are crossed, forming $\prod_{i=1}^k a_i$ cells.

Randomization: Let $n = m \prod_{i=1}^k a_i$ and do the `sample(n)` command. Randomly assign m units to each cell. Each cell is a combination of levels, so the $(1, 1, ..., 1, 1)$ cell gets the 1st m units.

IV) Completely randomized block design: Factor A has k levels (treatments), and there are b blocks (a blocking variable has b levels) of k units.

Randomization: Let $n = kb$ and do the `sample(k)` command b times. Within each block of k units, randomly assign 1 unit to each treatment.

V) Latin squares: Factor A has a levels (treatments), the row blocking variable has a blocks of a units and the column blocking variable has a blocks of a units. There are a^2 units since the row and column blocking variables are crossed. The treatment factor, row blocking variable and column blocking variable are also crossed. A Latin square is such that each of the a treatments occurs once in each row and once in each column.

Randomization: Pick an $a \times a$ Latin square. Use the `sample(a)` command to assign row levels to numbers 1 to a. Use the `sample(a)` command to assign column levels to numbers 1 to a. Use the `sample(a)` command to assign treatment levels to the first a capital letters. If possible, use the `sample(`a^2`)` command to assign units, 1 unit to each cell of the Latin square.

VI) 2^k factorial design: There are k factors, each with 2 levels.

Randomization: Let $n = m2^k$ and do the `sample(n)` command. Randomly assign m units to each cell. Each cell corresponds to a run which is determined by a string of k $+$'s and $-$'s corresponding to the k main effects.

VII) 2_R^{k-f} fractional factorial design: There are k factors, each with 2 levels.

Randomization: Let $n = 2^{k-f}$ and do the `sample(n)` command. Randomly assign 1 unit to each run which is determined by a string of k $+$'s and $-$'s corresponding to the k main effects.

VIII) Plackett Burman $PB(n)$ design: There are k factors, each with 2 levels.

Randomization: Let $n = 4J$ for some J. Do the `sample(n)` command.

Randomly assign 1 unit to each run which is a string of $n-1$ +'s and −'s. (Each run corresponds to a row in the design matrix, so we are ignoring the column of 1's corresponding to I in the design matrix.)

IX) Split plot design where the whole plots are assigned to A as in a one way Anova design: The whole plot factor A has a levels and each whole plot is a block of b subplots used to study factor B which has b levels. Split plot designs have two types of units: the whole plots are the larger units and the subplots are the smaller units.

Randomization: a) Suppose there are $n = ma$ whole plots. Randomly assign m whole plots to each level of A with the `sample(n)` command. Assign the first m units (whole plots) to treatment (level) 1, the next m units to treatment 2, ..., the last m units to treatment a.

b) Do the `sample(b)` command ma times, once for each whole plot. Within each whole plot, randomly assign 1 subplot (unit) to each of the b levels of B.

X) Split plot design where the whole plots are assigned to A and a blocking variable as in a completely randomized block design: The whole plot factor A has a levels and each whole plot is a block of b subplots used to study factor B which has b levels. Split plot designs have two types of units: the whole plots are the larger units and the subplots are the smaller units. There are also r blocks of a whole plots. Each whole plot has b subplots. Thus there are ra whole plots and rab subplots.

Randomization: a) Do the `sample(a)` command r times, once for each block. For each block of a whole plots, randomly assign 1 whole plot to each of the a levels of A.

b) Do the `sample(b)` command ra times, once for each whole plot. Within each whole plot, randomly assign 1 subplot to each of the b levels of B.

Try to become familiar with the designs and their randomization so that you can recognize a design given a story problem.

Example 9.3. Cobb (1998, p. 200-212) describes an experiment on weight gain for baby pigs. The response Y was the average daily weight gain in pounds for each piglet (over a period of time). Factor A consisted of 0 mg of an antibiotic or 40 mg of an antibiotic while factor B consisted of 0 mg of vitamin B12 or 5 mg of B12. Hence there were 4 diets $(A, B) =$ (0,0), (40,0), (0,5) or (40,5). If there were 12 piglets and 3 were randomly assigned to each diet, what type of experimental design was used?

Solution: A and B are crossed with each combination of (A, B) levels forming a diet. So the two way Anova (or 2^2 factorial) design was used.

Example 9.4. In 2008, a PhD student was designing software to analyze a complex image. 100 portions of the image need to be analyzed correctly, and the response variable is the proportion of errors. Sixteen test images are available and thought to be representative. The goal is to achieve an average error rate of less than 0.3 if many images were examined. The student has identified 3 factors to reduce the error rate. Each factor has 2 levels. Thus there are 8 versions of the software that analyze images.

The student selects a single test image and runs a 2^3 design with 8 time slots as units. Factor A is active but factors B and C are inert. When A was at the (+) level the error rate was about 0.27. Briefly explain why this experiment does not give much information about how the software will behave on many images.

Solution: More images are needed, 1 image is not enough.

(A better design is a completely randomized block design that uses each of the 16 images as a block and factor A = "software version" with 8 levels. The units for the block are 8 time slots so each of the 8 versions of the software is tested on each test image.)

9.3 Summary

1) The analysis of the response, not that of the residuals, is of primary importance. The response plot can be used to analyze the response in the background of the fitted model. For linear models such as experimental designs, the estimated mean function is the identity line and should be added as a visual aid.

2) Assume that the residual degrees of freedom are large enough for testing. Then the response and residual plots contain much information. Linearity and constant variance may be reasonable if the plotted points scatter about the identity line in a (roughly) evenly populated band. Then the residuals should scatter about the $r = 0$ line in an evenly populated band. It is easier to check linearity with the response plot and constant variance with the residual plot. Curvature is often easier to see in a residual plot, but the response plot can be used to check whether the curvature is monotone or not. The response plot is more effective for determining whether the signal

to noise ratio is strong or weak, and for detecting outliers, influential cases or a critical mix.

3) The three basic principles of DOE (design of experiments) are

i) use **randomization** to assign units to treatments.

ii) Use **factorial crossing** to compare the effects (main effects, pairwise interactions, ..., J-fold interaction) for $J \geq 2$ factors. If $A_1, ..., A_J$ are the factors with l_i levels for $i = 1, ..., J$; then there are $l_1 l_2 \cdots l_J$ treatments where each treatment uses exactly one level from each factor.

iii) **Blocking** is used to divide units into blocks of similar units where "similar" means the units are likely to have similar values of the response when given the same treatment. Within each block randomly assign units to treatments.

4) Split plot designs have two units. The large units are called whole plots and contain blocks of small units called subplots. The whole plots get assigned to Factor A while the subplots get assigned to factor B (randomly if the units are experimental but not randomly if the units are observational). A and B are crossed so the AB interaction can be studied.

5) The split plot design depends on how whole plots are assigned to A. Three common methods are a) the whole plots are assigned to A completely at random, as in a one way Anova, b) the whole plots are assigned to A and to a blocking variable as in a completely randomized block design (if the whole plots are experimental, a complete block design is used if the whole plots are observational), c) the whole plots are assigned to A, to row blocks and to column blocks as in a Latin Square.

6) The split plot Anova table when whole plots are assigned to levels of A as in a one way Anova is shown on the following page. The whole plot error is error(W) and can be obtained as an A*replication interaction. The subplot error is error(S). $F_A = MSA/MSEW$, $F_B = MSB/MSES$ and $F_{AB} = MSAB/MSES$. R computes the three test statistics and pvalues correctly, but for SAS F_A and the pvalue p_A need to be computed using MSA, MSEW, df_A and df_{e_W} obtained from the Anova table.

Source	df	SS	MS	F	p-value
A	$a-1$	SSA	MSA	F_A	p_A
error(W) or A*repl	$a(m-1)$	SSEW	MSEW		
B	$b-1$	SSB	MSB	F_B	p_B
AB	$(a-1)(b-1)$	SSAB	MSAB	F_{AB}	p_{AB}
residuals or error(S)	$a(m-1)(b-1)$	SSES	MSES		

7) The tests of interest corresponding to 6) are nearly identical to those of a two way Anova model. Y_{ijk} has $i = 1, ..., a$, $j = 1, ..., b$ and $k = 1, ..., m$. Keep A and B in the model if there is an AB interaction.

a) **The 4 step test for AB interaction** is
i) Ho there is no interaction H_A there is an interaction
ii) F_{AB} is obtained from output.
iii) The pvalue is obtained from output.
iv) If pvalue $< \delta$ reject Ho and conclude that there is an interaction between A and B, otherwise fail to reject Ho and conclude that there is no interaction between A and B.

b) **The 4 step test for A main effects** is
i) Ho $\mu_{100} = \cdots = \mu_{a00}$ H_A not Ho
ii) F_A is obtained from output.
iii) The pvalue is obtained from output.
iv) If pvalue $< \delta$ reject Ho and conclude that the mean response depends on the level of A, otherwise fail to reject Ho and conclude that the mean response does not depend on the level of A.

c) **The 4 step test for B main effects** is
i) Ho $\mu_{010} = \cdots = \mu_{0b0}$ H_A not Ho
ii) F_B is obtained from output.
iii) The pvalue is obtained from output.
iv) If pvalue $< \delta$ reject Ho and conclude that the mean response depends on the level of B, otherwise fail to reject Ho and conclude that the mean response does not depend on the level of B.

8) The split plot Anova table when whole plots are assigned to levels of A as in a completely randomized block design is shown on the following page. The whole plot error is error(W) and can be obtained as an block*A interaction. The subplot error is error(S). $F_A = MSA/MSEW$, $F_B = MSB/MSES$ and $F_{AB} = MSAB/MSES$. SAS computes the last two test statistics and pvalues correctly, and the last line of SAS output

gives F_A and the pvalue p_A. The initial line of output for A is not correct. The output for blocks is probably not correct.

Source	df	SS	MS	F	p-value
blocks	$r-1$				
A	$a-1$	SSA	MSA	F_A	p_A
error(W) or block*A	$(r-1)(a-1)$	SSEW	MSEW		
B	$b-1$	SSB	MSB	F_B	p_B
AB	$(a-1)(b-1)$	SSAB	MSAB	F_{AB}	p_{AB}
error(S)	$a(r-1)(b-1)$	SSES	MSES		

9) The tests of interest corresponding to 8) are nearly identical to those of a two way Anova model and point 7). Y_{ijk} has $i = 1, ..., a$, $j = 1, ..., b$ and $k = 1, ..., r$. Keep A and B in the model if there is an AB interaction.

a) **The 4 step test for AB interaction** is
i) Ho there is no interaction H_A there is an interaction
ii) F_{AB} is obtained from output.
iii) The pvalue is obtained from output.
iv) If pvalue $< \delta$ reject Ho and conclude that there is an interaction between A and B, otherwise fail to reject Ho and conclude that there is no interaction between A and B.

b) **The 4 step test for A main effects** is
i) Ho $\mu_{100} = \cdots = \mu_{a00}$ H_A not Ho
ii) F_A is obtained from output.
iii) The pvalue is obtained from output.
iv) If pvalue $< \delta$ reject Ho and conclude that the mean response depends on the level of A, otherwise fail to reject Ho and conclude that the mean response does not depend on the level of A.

c) **The 4 step test for B main effects** is
i) Ho $\mu_{010} = \cdots = \mu_{0b0}$ H_A not Ho
ii) F_B is obtained from output.
iii) The pvalue is obtained from output.
iv) If pvalue $< \delta$ reject Ho and conclude that the mean response depends on the level of B, otherwise fail to reject Ho and conclude that the mean response does not depend on the level of B.

9.4 Complements

See Robinson, Brenneman and Myers (2009) for a comparison of completely randomized designs, completely randomized block designs and split plot designs. Some history of experimental designs is given by Box (1980, 1984). Also see David (1995, 2006-7) and Hahn (1982).

The importance of DOE is discussed in Gelman (2005), and a review is given by Steinberg and Hunter (1984). For experiments done as class projects, see Hunter (1977).

9.5 Problems

Problems with an asterisk * are especially important.

Source	df	SS	MS	F	p-value
Block	2	77.55	38.78		
Method	2	128.39	64.20	7.08	0.0485
Block*Method	4	36.28	9.07		
Temp	3	434.08	144.69	41.94	0.00
Method*Temp	6	75.17	12.53	2.96	0.0518
error(S)	12	50.83	4.24		

9.1. The Anova table above is for the Montgomery (1984, p. 386-389) split plot data where the whole plots are assigned to factor A and to blocks in a completely randomized block design. The response variable is tensile strength of paper. Factor A is (preparation) method with 3 levels (1, 2, 3). Factor B is temperature with 4 levels (200, 225, 250, 275). The pilot plant can make 12 runs a day and the experiment is repeated each day, with days as blocks. A batch of pulp is made by one of the 3 preparation methods. Then the batch of pulp is divided into 4 samples, and each sample is cooked at one of the four temperatures.

a) Perform the test corresponding to A.

b) Perform the test corresponding to B.

c) Perform the test corresponding to AB.

Source	df	SS	MS	F	p-value
Block	1	0.051	0.051		
Nitrogen	3	37.32	12.44	29.62	0.010
Block*Nitrogen	3	1.26	0.42		
Thatch	2	3.82	1.91	9.10	0.009
Nitrogen*Thatch	6	4.15	0.69	3.29	0.065
error(S)	12	1.72	0.21		

9.2. The Anova table above is for the Kuehl (1994, p. 473-481) split plot data where the whole plots are assigned to factor A and to blocks in a completely randomized block design. The response variable is the average chlorophyll content (mg/gm of turf grass clippings). Factor A is nitrogen fertilizer with 4 levels (1, 2, 3, 4). Factor B is length of time that thatch was allowed to accumulate with 3 levels (2, 5, or 8 years).

There were 2 blocks of 4 whole plots to which the levels of Factor A were assigned. The 2 blocks formed a golf green which was seeded with turf grass. The 8 whole plots were plots of golf green. Each whole plot had 3 subplots to which the levels of Factor B were randomly assigned.

a) Perform the test corresponding to A.

b) Perform the test corresponding to B.

c) Perform the test corresponding to AB.

Source	df	SS	MS	F	p-value
Block	5	4.150	0.830		
Variety	2	0.178	0.089	0.65	0.5412
Block*Variety	10	1.363	0.136		
Date	3	1.962	0.654	23.39	0.00
Variety*Date	6	0.211	0.035	1.25	0.2973
error(S)	45	1.259	0.028		

9.3. The Anova table above is for the Snedecor and Cochran (1967, p. 369-372) split plot data where the whole plots are assigned to factor A and to blocks in a completely randomized block design. Factor A = variety of alfalfa (ladak, cossack, ranger). Each field had two cuttings, with the second cutting on July 7, 1943. Factor B = date of third cutting (none, Sept. 1, Sept. 20, Oct. 7) in 1943. The response variable was yield (tons per acre) in 1944. The 6 blocks were fields of land divided into 3 plots of land, one for each variety. Each of these 3 plots was divided into 4 subplots for date of third cutting. So each block had 3 whole plots and 12 subplots.

a) Perform the test corresponding to A.

b) Perform the test corresponding to B.

c) Perform the test corresponding to AB.

9.4. Following Montgomery (1984, p. 386-389), suppose the response variable is tensile strength of paper. One factor is (preparation) method with 3 levels (1, 2, 3). Another factor is temperature with 4 levels (200, 225, 250, 275).

a) Suppose the pilot plant can make 12 runs a day and the experiment is repeated each day, with days as blocks. A batch of pulp is made by one of the 3 preparation methods. Then the batch of pulp is divided into 4 samples, and each sample is cooked at one of the four temperatures. Which factor, method or temperature is assigned to subplots?

b) Suppose the pilot plant could make 36 runs in one day. Suppose that 9 batches of pulp are made, that each batch of pulp is divided into 4 samples, and each sample is cooked at one of the four temperatures. How should the 9 batches be allocated to the three preparation methods and how should the 4 samples be allocated to the four temperatures?

c) Suppose the pilot plant can make 36 runs in one day and that the units are 36 batches of material to be made into pulp. Each of the 12 method temperature combinations is to be replicated 3 times. What type of experimental design should be used? (Hint: not a split plot.)

9.5. a) Download (www.math.siu.edu/olive/regdata.txt) into R, and type the following commands. Then copy and paste the output into *Notepad* and print the output.

```
attach(guay)
out <- aov(plants~variety*treatment + Error(flats),guay)
summary(out)
detach(guay)
```

This split plot data is from Chambers and Hastie (1993, p. 158). There are 8 varieties of guayule (rubber plant) and 4 treatments were applied to seeds. The response was the rate of germination. The whole plots were greenhouse flats and the subplots were subplots of the flats. Each flat received seeds of one variety (A). Each subplot contained 100 seeds and was treated with one of the treatments (B). There were $m = 3$ replications so each variety was planted in 3 flats for a total of 24 flats and $4(24) = 96$ observations.

b) Use the output to test whether the response depends on variety.

9.6. Download (www.math.siu.edu/olive/regdata.txt) into *R*, and type the following commands. Then copy and paste the output into *Notepad* and print the output.

```
attach(steel)
out <- aov(resistance~heat*coating + Error(wplots),steel)
summary(out)
detach(steel)
```

This split plot steel data is from Box, Hunter and Hunter (2005, p. 336). The whole plots are time slots to use a furnace, which can hold 4 steel bars at one time. Factor A = heat has 3 levels (360, 370, 380^o F). Factor B = coating has 4 levels (4 types of coating: c1, c2, c3 and c4). The response was corrosion resistance.

a) Perform the test corresponding to A.

b) Perform the test corresponding to B.

c) Perform the test corresponding to AB.

9.7. This is the same data as in Problem 9.6, using *SAS*. Copy and paste the SAS program from (www.math.siu.edu/olive/reghw.txt) into *SAS*, run the program, then print the output. Only include the second page of output.

To get the correct F statistic for heat, you need to divide MS heat by MS wplots.

9.8. a) Copy and paste the SAS program from (www.math.siu.edu/olive/reghw.txt) into *SAS*, run the program, then print the output. Only include the second page of output.

This data is from the SAS Institute (1985, p. 131-132). The B and AB Anova table entries are correct, but the correct entry for A is the last line of output where Block*A is used as the error.

b) Perform the test corresponding to A.

c) Perform the test corresponding to B.

d) Perform the test corresponding to AB.

9.9. Suppose the response variable is tensile strength of paper. One factor is preparation method with 3 levels (1, 2, 3). Another factor is temperature with 4 levels (200, 225, 250, 275). Suppose the pilot plant can make 36 runs in one day and that the units are 36 batches of material to be made into pulp. Each of the 12 method temperature combinations is to be replicated 3 times. What type of experimental design should be used?